General Relativity and Gravitation, Vol. 9, No. 12 (1978), pp. 1089–1100

Tachyons and the Second Law of Black Hole Physics

S. V. DHURANDHAR and J. V. NARLIKAR

Tata Institute of Fundamental Research, Bombay 400 005, India

Received October 25, 1977

Abstract

It is shown that the usual proof of the second law of black hole physics breaks down if there are tachyons present in the vicinity of a black hole. Explicit cases are discussed where a tachyon of positive energy falling into the Kerr singularity actually decreases the area of the Kerr black hole.

§(1): *Introduction*

In the important developments in black hole physics in the past decade the second law has played a crucial role. The result that the area of a black hole does not decrease in any normal physical process was used to establish an analogy between the surface area of a black hole and the entropy [1]. The actual identification between the entropy of a black hole and a multiple of its surface area emerged from the important work of Hawking [2]. This work showed that a black hole radiates and that a temperature proportional to the surface gravity can be associated with this process. The concept has no classical analogue; it is an essentially quantum mechanical result. In the Hawking process although the surface area (and hence the entropy) of the radiating black hole decreases in an apparent violation of the second law, the system consisting of the source and the sink still obeys the laws of thermodynamics.

In this paper we investigate the question of what happens when a black hole interacts with a field of tachyons (faster than light particles). Theoretical studies of tachyons indicate that their thermodynamics can be markedly different from that of ordinary matter. For example, the night sky background of tachyons is

0001-7701/78/1200-1089$05.00/0

not relativistically invariant [3]. Does the presence of such particles violate the laws of black hole physics? We will consider here more specifically the second law which primarily established the link between black hole physics and thermodynamics. It has already been shown [4, 5] that tachyon trajectories behave anomalously near black holes. For example, in the extended Kruskal-Szekeres manifold of a Schwarzschild black hole a tachyon radially free-falling into the event horizon from region I to II emerges through the horizon from region II to III without falling into the singularity of region II. Thus the horizon does not act as a one-way membrane. Does such anomalous behavior violate the second law even on a classical level?

To answer this question we will consider first how the proof of the second law is affected by the presence of tachyons. It will emerge that the so-called focussing theorem on which the proof is based is no longer valid, thus rendering the situation inconclusive. We will then show that under certain general conditions a tachyon falling into a black hole leads to a decrease of its surface area.

The discussion in the following two sections is based on the consideration of the energy tensor of a discrete set of tachyonic particles. Although this treatment is lengthier than the more familiar fluid-dust approximations, we find it in the long run more illuminating, especially in view of the explicit example of the discrete particle considered in Section 4.

§(2): *Tachyon Energy Tensors*

We will first construct and study the properties of the energy tensor of a system of tachyons in analogy with the energy tensor of a system of material particles.

Let $a, b, c, \ldots$ be material particles of masses $m_a, m_b, m_c, \ldots$ moving in a Riemannian space-time. Let da^i denote the coordinate differentials and ds_a the element of proper time of particle a at a typical world point A with coordinates a^i on its world line. We may then define the energy tensor of the system of particles by the formula (cf. [6])

$$T^{ik}(X) = \sum_a \int m_a \frac{\delta_4(X, A)}{\sqrt{-\bar{g}(X, A)}} \bar{g}^i{}_{i_A} \bar{g}^k{}_{k_A} \frac{da^{i_A}}{ds_a} \frac{da^{k_A}}{ds_a} ds_a \qquad (2.1)$$

Here $\bar{g}^i{}_{i_A}$ are the parallel propagators between A and X, and $\bar{g}(X, A)$ is the determinant of $\|\bar{g}_{ii_A}\|$. (We are using the signature $-, -, -, +$ here.) $\delta_4(X, A)$ is the four-dimensional delta function.

For a tachyon the mass is imaginary. If in the preceding case $a, b, c, \ldots$ are tachyons instead of ordinary particles, we can redefine T^{ik} by analytic continuation. Define in this case

$$m_a = iM_a, \qquad ds_a = i d\sigma_a \qquad (2.2)$$

where M_a (the meta mass) and $d\sigma_a$ are real quantities. Then the corresponding expression for tachyons is

$$\mathcal{T}^{ik}(X) = \sum_a \int M_a \frac{\delta_4(X, A)}{\sqrt{-\bar{g}(X, A)}} \bar{g}^i{}_{i_A} \bar{g}^k{}_{k_A} \frac{da^{i_A}}{d\sigma_a} \frac{da^{k_A}}{d\sigma_a} d\sigma_a \tag{2.3}$$

(We have used $\mathcal{T}$ to distinguish it from T.)

Consider now a timelike vector ϑ^i at X representing a material observer's four-velocity. The expression $T^{ik}\vartheta_i\vartheta_k$ is an invariant and it represents the energy density of the system of particles in the rest frame of this observer. Denoting it by $T_{\hat{4}\hat{4}}$ ($\equiv T^{\hat{4}\hat{4}}$), we have from (2.1)

$$T_{\hat{4}\hat{4}} = \sum_a \int m_a \frac{\delta_4(X, A)}{\sqrt{-\bar{g}(X, A)}} \vartheta_i \bar{g}^i{}_{i_A} \frac{da^{i_A}}{ds_a} \vartheta_k \bar{g}^k{}_{k_A} \frac{da^{k_A}}{ds_a} ds_a \tag{2.4}$$

The delta function ensures that only the immediate neighborhood of X need be considered. Consider the right-hand side of (2.4) in a locally inertial frame at X so that $\vartheta^i = (0, 0, 0, 1)$. Let the particle of the system at X have a four-velocity $\mathbf{u}^i$ and mass m. Using the Minkowski coordinates in the neighborhood of X as $(x, y, z, t) \equiv (\mathbf{r}, t)$ with origin at X, we get

$$\begin{aligned} T_{\hat{4}\hat{4}} &= m\mathbf{u}^4 \sum_a \int \delta_3(\mathbf{r}_a)\, \delta(t_a)\, dt_a \\ &= m\mathbf{u}^4 \sum_a \delta_3(\mathbf{r}_a) = nm\mathbf{u}^4 = nm\mathbf{u}^i\vartheta_i \end{aligned} \tag{2.5}$$

In (2.5) we have taken $(\mathbf{r}_a, t_a)$ as the coordinates of the particle in this reference frame. n is the particle number density. Thus $T^{ik}\vartheta_i\vartheta_k$ is proportional to $\mathbf{u}^i\vartheta_i$ with a positive constant of proportionality. Since $\mathbf{u}^i\vartheta_i > 0$ for any two future directed timelike vectors, $T^{ik}\vartheta_i\vartheta_k > 0$.

For tachyons, however, a similar analysis does not lead to the same conclusion. Let $M\mathbf{U}^i$ denote the four-momentum of a tachyon passing through X, in the corresponding situation. Then (2.5) is replaced by

$$\mathcal{T}^{ik}\vartheta_i\vartheta_k = \mathcal{T}_{\hat{4}\hat{4}} = nM\mathbf{U}^4 \equiv nM\mathbf{U}^i\vartheta_i \tag{2.6}$$

Although (2.6) is formally similar to (2.5), we will now show that $\mathbf{U}^i\vartheta_i$ can have any sign, depending on the orientation of the timelike vector ϑ_i and the spacelike vector $\mathbf{U}^i$.

Theorem. (i) Given a spacelike vector $\mathbf{U}$, there exists a nonempty open convex set $\mathcal{K}$ of future pointing timelike vectors such that the scalar product of $\mathbf{U}$ with any member of $\mathcal{K}$ is negative.

(ii) There exists a nonempty subset $\mathfrak{N}$ of $\partial\mathcal{K}$ open in $\partial\mathcal{K}$ of future pointing null vectors such that the scalar product of **U** with any member of $\mathfrak{N}$ is negative.

(iii) If $\mathfrak{A}$ is the set of future directed timelike vectors and + denotes a disjoint union, then

$$\partial K = \mathfrak{N} + \overline{\{(\mathbf{U})^{\perp} \cap \mathfrak{A}\}}$$

were $(\mathbf{U})^{\perp}$ is the set of vectors orthogonal to **U**.

Proof. Let $\mathfrak{M}^4$ be a four-dimensional space-time manifold and P be a point in it. Let $\mathfrak{T}_P(\mathfrak{M}^4)$, the tangent space to $\mathfrak{M}^4$ at P, contain **U**. Let $(\mathbf{U})^{\perp} = \{\mathbf{w} \in \mathfrak{T}_P(\mathfrak{M}^4) \mid g(\mathbf{U}, \mathbf{w}) = 0\}$ where g is the nondegenerate Lorentzian tensor field of type (0, 2) representing the metric on $\mathfrak{M}^4$.

Since **U** is spacelike $(\mathbf{U})^{\perp}$ is a timelike hyperplane and has a basis such that the metric tensor restricted to $(\mathbf{U})^{\perp}$ has a diagonal form (−1, −1, 1). Therefore there exists a nonzero future-directed timelike vector $\mathbf{p} \in (\mathbf{U})^{\perp}$. Consider the subspace $\mathcal{L} = (\mathbf{p}) \oplus (\mathbf{U})$ of $\mathfrak{T}_P(\mathfrak{M}^4)$ and the mapping into it of the closed subset [0, 1] of $\mathbf{R}^1$ defined by

$$\boldsymbol{\vartheta}(\lambda) = \lambda \mathbf{U} + (1 - \lambda)\,\mathbf{p}, \qquad \lambda \in [0, 1] \tag{2.7}$$

Since g is a continuous function of $\mathcal{L} \times \mathcal{L}$ and since $g(\mathbf{p}, \mathbf{p}) > 0$, $g(\mathbf{U}, \mathbf{U}) < 0$, there exists a $\lambda_0 \in (0, 1)$ such that

$$\mathbf{n} = \lambda_0 \mathbf{U} + (1 - \lambda_0)\,\mathbf{p}, \qquad g(\mathbf{n}, \mathbf{n}) = 0 \tag{2.8}$$

For $0 < \lambda < \lambda_0$ the vectors $\boldsymbol{\vartheta}$ are future-directed and timelike, and they satisfy

$$g(\boldsymbol{\vartheta}, \mathbf{U}) = \lambda g(\mathbf{U}, \mathbf{U}) < 0 \tag{2.9}$$

The set $\{\boldsymbol{\vartheta}(\lambda) \mid 0 < \lambda < \lambda_0\} \subset K$ whence K is nonempty. We can write K in the form

$$K = \{\boldsymbol{\vartheta} \mid g(\boldsymbol{\vartheta}, \mathbf{U}) < 0\} \cap \{\boldsymbol{\vartheta} \mid g(\boldsymbol{\vartheta}, \boldsymbol{\vartheta}) > 0\} \tag{2.10}$$

for all future-directed $\boldsymbol{\vartheta}$ in $\mathfrak{T}_P(\mathfrak{M}^4)$. Since both the sets are open and convex, K is also open and convex. This proves (i).

By considering all future directed timelike vectors of type **p** described before, we can look on ∂K, the boundary of K, as made up of (i) vectors of $\mathfrak{N}$ and (ii) the nonspacelike vectors that have a zero scalar product with **U**. This is expressed by the statement

$$\partial K = \mathfrak{N} + \overline{\{(\mathbf{U})^{\perp} \cap \mathfrak{A}\}} \tag{2.11}$$

Since $\mathfrak{N}$ is the complement of a closed set in ∂K, it is open; which completes the proof.

Note that $\mathfrak{N}$ is open because its boundary in ∂K consists of the null vectors whose scalar product with **U** vanishes. This boundary is also the boundary of the set $\{(\mathbf{U})^{\perp} \cap \mathfrak{A}\}$.

Remarks. (1) If **U** were timelike, then $\mathfrak{K}$ and $\mathfrak{N}$ would be empty.

(2) The existence of a nonempty set $\mathfrak{K}$ implies, physically, the existence of material observers in whose rest frames $\mathcal{T}_{\hat{4}\hat{4}} < 0$. Similarly, the existence of a nonempty $\mathfrak{N}$ implies that there exist future-directed null rays $\mathbf{n}_i$ for which $\mathcal{T}^{ik}\, \mathbf{n}_i \mathbf{n}_k < 0$.

§(3): *The Second Law of Black Hole Physics*

The deduction of this law is essentially based on the following two results.

Penrose's Theorem. This is summarized in the following steps (cf. [8], [9] for details).

(i) The union of all future horizons $\dot{J}^-(\mathcal{I}^+)$ is generated by null geodesics that have no future end points.

(ii) When followed into the past, a generator may (but need not) leave $\dot{J}^-(\mathcal{I}^+)$ and go into $J^-(\mathcal{I}^+)$. The event at which the generator leaves $\dot{J}^-(\mathcal{I}^+)$ is called a caustic.

(iii) A generator, when followed into the future, can never leave $\dot{J}^-(\mathcal{I}^+)$, nor can it intersect any other generator.

(iv) Through each caustic event there passes one and only one generator.

The Focusing Theorem (cf. [9]). Let $\mathcal{O}$ be the cross-sectional area of a bundle of rays (null geodesics) all lying in the same surface of constant phase. Let $\mathbf{k}^l$ be the tangent vector to the central ray of the bundle and λ the affine parameter along it. Then $\mathcal{O}$ satisfies the differential equation

$$\frac{d^2 \mathcal{O}^{1/2}}{d\lambda^2} = -(|\sigma|^2 - \tfrac{1}{2} R_{lm}\, \mathbf{k}^l \mathbf{k}^m)\, \mathcal{O}^{1/2} \tag{3.1}$$

where

$$|\sigma|^2 = \tfrac{1}{2}\, \mathbf{k}_{l;m} \mathbf{k}^{l;m} - \tfrac{1}{4} (\mathbf{k}^l{}_{;l})^2 \tag{3.2}$$

is the squared magnitude of shear and is positive.

The Einstein equations (in units with $G = 1$, $c = 1$) are

$$R_{ik} - \tfrac{1}{2} g_{ik} R = -8\pi T_{ik} \tag{3.3}$$

so that (3.1) becomes

$$\mathcal{O}^{-1/2} \frac{d^2 \mathcal{O}^{1/2}}{d\lambda^2} = -(|\sigma|^2 + 4\pi T_{lm}\, \mathbf{k}^l \mathbf{k}^m) \tag{3.4}$$

For material particles we have seen in Section 2 that the magnitude of $T_{lm} \boldsymbol{\vartheta}^l \boldsymbol{\vartheta}^m$ is nonnegative for any timelike vector $\boldsymbol{\vartheta}^l$, and hence by a limiting argument $T_{lm}\mathbf{k}^l\mathbf{k}^m \geqslant 0$ for any null vector $\mathbf{k}^l$. However, for a system of tachyons $\mathcal{T}_{lm} \boldsymbol{\vartheta}^l \boldsymbol{\vartheta}^m$ can have any sign so that such a conclusion cannot be drawn for

$\mathcal{T}_{lm}\mathrm{k}^l\mathrm{k}^m$. Indeed, in Section 2 we have shown the existence of future-directed null vectors, with $\mathcal{T}_{lm}\mathrm{k}^l\mathrm{k}^m < 0$. Thus, while the Penrose theorem is essentially unaffected by the presence of tachyons, the focusing theorem is no longer valid. That is we cannot conclude that

$$\frac{d^2\,\Theta^{1/2}}{d\lambda^2} \leqslant 0 \tag{3.5}$$

The proof of the second law of black hole physics as given by Hawking [10] depends on the Penrose theorem, the result (3.5) of the focusing theorem, and the nonexistence of naked singularities in space-time. We have seen that the focusing theorem is inconclusive where tachyons are involved. Thus, although the Penrose theorem and the naked singularity assumption are valid, we cannot conclude that the surface area of a black hole necessarily increases in any of its interaction with tachyons.

In the remaining part of this paper we will consider specific examples in which the tachyon interaction with a black hole decreases the area of the black hole. In the absence of a general result or a definitive theorem as in the case of ordinary particles, such specific cases illustrate the anomalous behavior of tachyons. Here $\mathcal{T}_{lm}\mathrm{k}^l\mathrm{k}^m$ is sufficiently large and negative to overcome the focusing effect of the shear term (3.2).

§(4): *Tachyons and the Kerr Black Hole*

In an earlier paper [5] we considered the tachyon trajectories in the vicinity of a Schwarzschild black hole. There are two types of trajectories. If the tachyon has a sufficient angular momentum relative to the black hole, it falls into the central singularity. We will call such trajectories with a bounded range of the geodesic affine parameter trajectories of class I. (In the case of ordinary massive particles or photons all trajectories that enter the event horizon are of class I.) For tachyons that follow radial geodesics or have a sufficiently small angular momentum, the trajectories start from region I of the Kruskal-Szekeres space-time, enter region II where they bounce at a finite Schwarzschild coordinate r and enter region III. Such trajectories will be called class II trajectories, and they have no analogues in the case of ordinary massive particles or photons. If we identify the (u, v) coordinates of region I with the $(-u, -v)$ coordinates of region III, such trajectories may lead to acausal effects [5]. Whatever the interpretation, it is possible to think of the class II tachyons as interacting with the black hole matter and reemerging from the event horizon with the result that the surface area of the black hole has decreased.

However, there may be controversial issues involved with regard to the type of topological identification made between the space-time region before the tachyon enters the event horizon and the region into which it emerges from the horizon (cf. [11], for example). Therefore we will consider here trajectories of class I. Contrary to expectation we will show that such trajectories do not necessarily lead to an increase in the surface area of the black hole. For our discussion we shall consider the explicit case of the Kerr black hole.

The Kerr Black Hole. We will consider the Kerr black hole of mass M and angular momentum S in the geometrical units $G = 1$, $c = 1$. In the Boyer-Lindquist coordinates (r, θ, ϕ, t) the line element is given by

$$ds^2 = \frac{\Delta}{\rho^2}(dt - a\sin^2\theta\, d\phi)^2 - \frac{\sin^2\theta}{\rho^2}[(r^2 + a^2)\,d\phi - a\,dt]^2 - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2 \tag{4.1}$$

where

$$\Delta = r^2 + a^2 - 2Mr, \qquad \rho^2 = r^2 + a^2\cos^2\theta, \qquad a = S/M \tag{4.2}$$

The event horizon is given by $r = r_+$ and the antievent horizon by $r = r_-$ where

$$r_\pm = M \pm \sqrt{M^2 - a^2} \tag{4.3}$$

The surface area of the black hole is given by

$$A = 4\pi(a^2 + r_+^2) \tag{4.4}$$

Suppose a tachyon with class I trajectory and with energy $M_0\Gamma$ and angular momentum $M_0 h$ (M_0 = meta mass of the tachyon) falls into the black hole. This results in a change of the black hole parameters from M to $M + M_0\Gamma$ and from S to $S + M_0 h$ (if we assume that the angular momentum added is the same direction as the original black hole angular momentum). Then for the area to decrease we require, with $h = M\eta$

$$\Gamma < \frac{aM}{a^2 + r_+^2}\eta \tag{4.5}$$

From this it is clear that type I tachyons can never decrease the area of a Schwarzschild black hole.[1] For a Kerr black hole the question is weather there exist any type I trajectories for which (4.5) holds.

Tachyon Trajectories. Writing $ds = i\,d\sigma$ we have the geodesic equations of tachyons in the form

$$\frac{d^2x^i}{d\sigma^2} + \Gamma^i_{kl}\frac{dx^k}{d\sigma}\frac{dx^l}{d\sigma} = 0 \tag{4.6}$$

[1] It is possible to decrease the area of a Schwarzschild black hole by a quantity of second order $\sim M_0^2/M^2$ if a tachyon having $h \simeq 2M$ and $\Gamma < M_0/2M$ is dropped into it. Indeed it would be a rare coincidence for a tachyon to have its energy and angular momentum lying in such a narrow range of values!

We will consider geodesics in the $\theta = \pi/2$ "plane." The first integrals in terms of h and Γ are

$$r^2 \Delta \frac{dt}{d\sigma} = \Gamma\{(r^2 + a^2)^2 - a^2 \Delta\} - 2Marh \tag{4.7}$$

$$r^2 \Delta \frac{d\phi}{d\sigma} = h(\Delta - a^2) + 2Mar\Gamma \tag{4.8}$$

and the line element then gives

$$r^4 \left(\frac{dr}{d\sigma}\right)^2 = \alpha\Gamma^2 - 2\beta\Gamma + \gamma \tag{4.9}$$

where

$$\alpha = (r^2 + a^2)^2 - \Delta a^2, \qquad \beta = 2Marh, \qquad \gamma = h^2(a^2 - \Delta) + r^2 \Delta \tag{4.10}$$

For a tachyon heading inward from $r \gg M$, the expression on the right-hand side of (4.9) decides whether the trajectory is of class I or class II. If this expression has a zero for some r in $0 < r < r_+$, then the trajectory bounces inside the horizon and is of type II. If $dr/d\sigma$ does not vanish for the entire range $r \geqslant 0$ of the trajectory, then the tachyon falls into the singularity at $r = 0$. Thus for type I trajectory we require expression (4.9) to have no zero in $r \geqslant 0$. The problem therefore reduces to looking for the roots of

$$\alpha\Gamma^2 - 2\beta\Gamma + \gamma = 0 \tag{4.11}$$

It is convenient to replace r, M, and a by dimensionless quantities defined by

$$R = \frac{r}{M}, \qquad Q = \frac{a}{M}, \qquad x = \frac{h}{M} = \eta, \qquad y = \Gamma \tag{4.12}$$

Then (4.11) becomes

$$0 = g(x, y; Q, R) \equiv R^2(R^2 + Q^2 - 2R) + (2R - R^2)x^2 - 4RQxy + \{R^2(Q^2 + R^2) + 2RQ^2\}y^2 \tag{4.13}$$

For a given R, $g = 0$ is a conic. Any point (x, y) on this conic represents a pair of values for the energy and angular momentum per unit meta mass of a tachyon whose trajectory will bounce at R. Clearly therefore in order to look for class I trajectories we have to avoid all the points in the positive quadrant of (x, y) plane that lie on any of the system of conics given by $g = 0$. It is therefore necessary to study the geometrical properties of these conics for the full range of R. We summarize the relevant properties, for $0 < Q < 1$ (see Figure 1).

(i) For $R_- < R < R_+$ the conic $g = 0$ is an ellipse. For $R > R_+$ or for

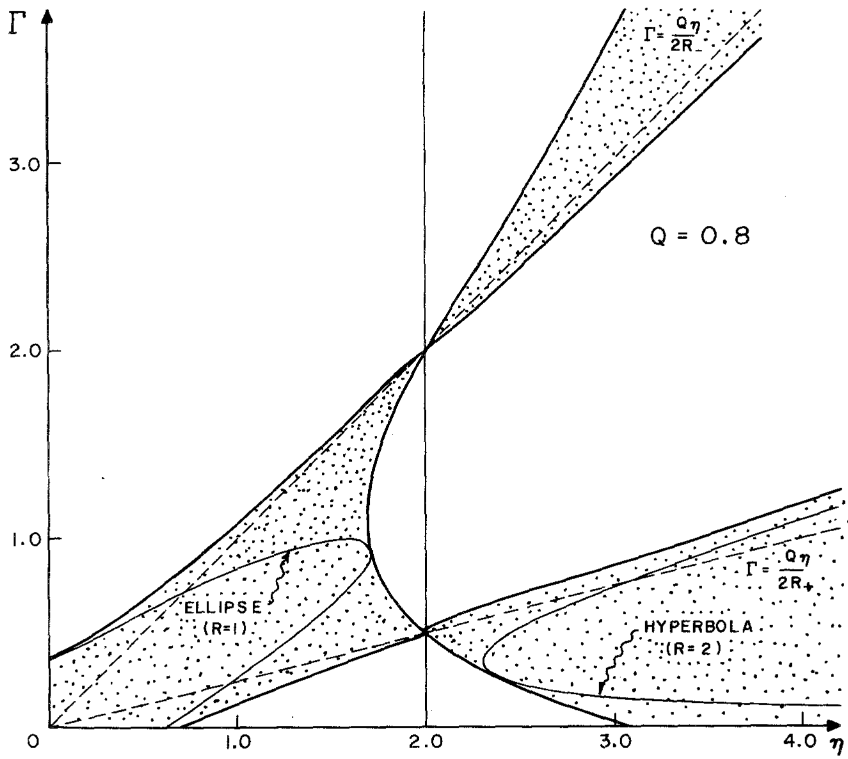

Fig. 1. Curves describing the dynamics of a tachyon in a Kerr black hole with $Q = 0.8$. The shaded region consists of the tachyons that bounce without reaching the central singularity. Tachyons avoiding this zone fall into the singularity, and those lying in the region below the line $\Gamma = Q\eta/2R_+$ *decrease* the surface area of the black hole.

$R < R_-$ it is a hyperbola. For $R = R_\pm$ the conic degenerates into a pair of coincident straight lines $y = Qx/2R_\pm$.

(ii) The conics are centred at the origin. The ellipses are confined to the region $-2 < x < 2$, while the two branches of the hyperbolas lie in the regions $|x| > 2$. Only in the degenerate cases of $R = R_\pm$ do the ellipses and the hyperbolas tend to reach $x = 2$ (at $y = Q/R_\pm$).

(iii) The asymptotes of the hyperbolas are given by straight lines $y = m_\pm x$, where

$$m_\pm = \frac{2Q \pm R\sqrt{R^2 + Q^2 - 2R}}{R(R^2 + Q^2) + 2Q^2} \tag{4.14}$$

(iv) For large values of R ($\gg R_+$) the branch of the hyperbola in the positive quadrant is very close to the x-axis and compressed between asymptotes making small angles with the x-axis ($m_\pm \sim (2Q \pm R^2)/R^3$). As R decreases, the major axis of the hyperbola makes a larger angle with the x-axis and its branch in the positive quadrant moves toward the line $x = 2$, touching it as $R \to R_+$. As R decreases further toward R_-, the corresponding ellipses shrink inward from the line until a critical value R_1 is reached. Afterward the ellipses open out again and touch the line $x = 2$ as $R \to R_-$. The major axis of the ellipse makes a larger and larger angle with the x-axis as R decreases. Finally as R decreases from R_- toward 0, we again have hyperbolas that move away from the line $x = 2$. As $R \to 0$ the hyperbola tends to be degenerate, with the asymptotes tending to the line $y = x/Q$.

(v) A comprehensive picture of the behavior of these conics is provided by their envelope with respect to the R_- parameter. This has the equation

$$\begin{aligned} F(x,y;Q) \equiv\ & 18(1+y^2)(Qy-x)^2(Q^2y^2-x^2+Q^2)-(Q^2y^2-x^2+Q^2)^2 \\ & +27(1+y^2)^2(Qy-x)^4-16(Qy-x)^2 \\ & +(1+y^2)(Q^2y^2-x^2+Q^2)^3 \end{aligned} \tag{4.15}$$

In Figure 1 the outline of the envelope is shown with a thick curve. In the shaded region within the envelope we can always find a conic for some R, implying that the tachyons corresponding to any point in this region cannot have a class I trajectory (i.e., it will bounce at that R).

Standard curve-tracing procedures and the help of an electronic computer are needed to trace the envelope. It has essentially three branches—corresponding to ellipses in $R_- < R < R_+$ and to hyperbolas in $0 < R < R_-$ and $R_+ < R < \infty$. The branches touch at $x = 2, y = Q/R_\pm$.

(vi) For the extreme Kerr black hole ($Q = 1$), the two values R_- and R_+ are equal. Thus the ellipses become degenerate and the hyperbolas form a continuous sequence from $R = 0$ to $R = \infty$ (see Figure 2).

Area Decrease of Kerr Black Hole. From the preceding discussion all points in the (x, y)-plane not lying in the shaded interior of the envelope represent tachyons falling into the Kerr singularity. Of these only those for which (4.5) holds, i.e., for which

$$y < \frac{Qx}{2R_+} \tag{4.16}$$

would lead to an area decrease. From Figure 1 and the preceding discussion we see that there is an unshaded region below the line $y = Qx/2R_+$ and that it has a nonzero measure.

It is therefore possible to have tachyons with suitable energy and angular

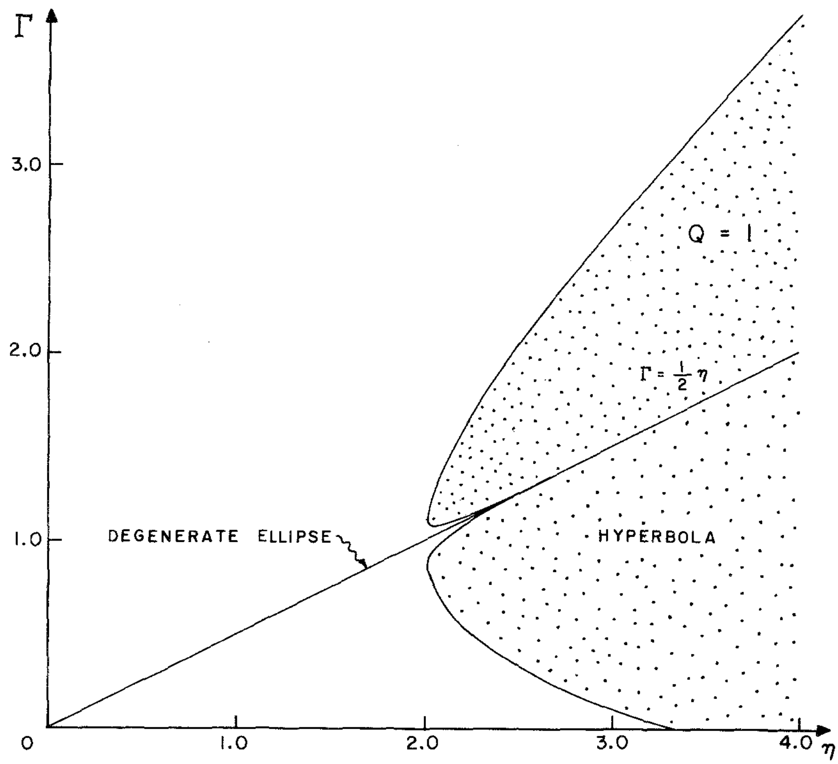

Fig. 2. Curves similar to those in Figure 1 are drawn for the extreme Kerr black hole. The main difference between this case and the $0 < Q < 1$ cases is the absence of the region containing the ellipses.

momentum which, when dropped into a Kerr black hole end at the space-time singularity and also *reduce* the surface area of the black hole.

§(5): *Conclusion*

We have shown that the proof of the second law of black hole physics breaks down in the presence of tachyons. We have also shown how the tachyons gobbled up by a black hole can lead to a decrease of its surface area. Since the second law is always obeyed by ordinary matter, its violation, on a classical level, would give an indication of the existence of tachyons.

For black holes associated with the end point of stellar evolution the masses are large enough to make the Hawking effect negligible. For such black holes, the area can decrease only through tachyon interactions. Hence we suggest this as a possible future device for the detection of tachyons.

References

1. Bekenstein, J. D. (1973). *Phys. Rev. D*, 7, 2333.
2. Hawking, S. W. (1975). *Comm. Math. Phys.*, **43**, 199
3. Bilaniuk, O.-M., and Sudarshan, E. C. G. (1969). *Physics Today*, **22**, 43.
4. Raychaudhuri, A. K. (1974). *J. Math. Phys.*, **15**, 856.
5. Narlikar, J. V., and Dhurandhar, S. V. (1976). *Pramana*, **6**, 388.
6. Hoyle, F. and Narlikar, J. V. (1974). *Action at a Distance in Physics and Cosmology* (W. H. Freeman and Co., San Francisco).
7. Synge, J. L. (1960). *Relativity: The General Theory* (North-Holland Publishing Company, Amsterdam).
8. Penrose, R. (1968). *Battelle Rencontres*, edited by C. M. De Witt and J. A. Wheeler (W. A. Benjamin, New York).
9. Misner, W., Thorne, K. S., and Wheeler, J. A. (1973). *Gravitation* (W. H. Freeman and Co., San Francisco).
10. Hawking, S. W. (1972). *Comm. Math. Phys.*, **25**, 152.
11. Israel, W. (1966). *Nature*, **211**, 466.