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Measuring Statistical Isotropy of CMB Anisotropy

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Abstract

The statistical expectation values of the temperature fluctuations and polarization of cosmic microwave background (CMB) are assumed to be preserved under rotations of the sky. We investigate the statistical isotropy (SI) of the CMB maps recently measured by the Wilkinson Microwave Anisotropy Probe (WMAP) using the bipolar spherical harmonic formalism proposed in Hajian & Souradeep 2003 for CMB temperature anisotropy and extended to CMB polarization in Basak, Hajian & Souradeep 2006. The *Bipolar Power Spectrum (BiPS)* had been measured for the full sky CMB anisotropy maps of the first year WMAP data and now for the recently released three years of WMAP data. We also introduce and measure directional sensitive *reduced Bipolar coefficients* on the three year WMAP ILC map. Consistent with our published results from first year WMAP data we have no evidence for violation of statistical isotropy on large angular scales. Preliminary analysis of the recently released first WMAP polarization maps, however, indicate significant violation of SI even when the foreground contaminated regions are masked out. Further work is required to confirm a possible cosmic origin and rule out the (more likely) origin in observational artifact such as foreground residuals at high galactic latitude.

Key words: cosmology, theory, cosmic microwave background

1 Introduction

In standard cosmology, CMB anisotropy signal is expected to be statistically isotropic, i.e., statistical expectation values of the temperature fluctuations

$\Delta T(\hat{q})$ are preserved under rotations of the sky. In particular, the angular correlation function $C(\hat{q}, \hat{q}') \equiv \langle \Delta T(\hat{q}) \Delta T(\hat{q}') \rangle$ is rotationally invariant for Gaussian fields. In spherical harmonic space, where $\Delta T(\hat{q}) = \sum_{lm} a_{lm} Y_{lm}(\hat{q})$ the condition of *statistical isotropy* (SI) translates to a diagonal $\langle a_{lm} a^*_{l'm'} \rangle = C_l \delta_{ll'} \delta_{mm'}$ where C_l, is the widely used angular power spectrum of the CMB anisotropy. Statistical isotropy of the CMB sky is essential for C_l to be a complete description of (Gaussian) CMB anisotropy and, hence, an adequate measure for carrying out cosmological parameter estimation of 'standard' (SI) model. Hence, it is crucial to be able to determine from the observed CMB sky whether it is a realization of a statistically isotropic process, or not. The detection of statistical isotropy (SI) violations in the CMB signal can have exciting and far-reaching implications for cosmology and the cosmological principle. For example, a generic consequence of cosmic topology is the breaking of statistical isotropy in characteristic patterns determined by the photon geodesic structure of the manifold as probed by the CMB photons traveling to us from the surface of last scattering over a distance comparable to the cosmic horizon, R_H (1; 2; 3). Mildly anisotropic cosmological models predict charateristic patterns hidden in the CMB sky. On the other hand, SI violation could also arise from foreground contamination, or, artifacts of observational and analysis techniques.

The CMB measurements of the *Wilkinson Microwave Anisotropy Probe* (*WMAP*) are consistent with predictions of the concordance ΛCDM model with (nearly) scale-invariant and adiabatic fluctuations expected to have been generated during an inflationary epoch in the early universe (4; 5; 6; 7; 8; 9; 10; 11). After the first year of *WMAP* data, the SI of the CMB anisotropy (*i.e.* rotational invariance of n-point correlations) has attracted considerable attention. Tantalizing evidence of SI breakdown (albeit, in very different guises) has mounted in the *WMAP* first year sky maps, using a variety of different statistics (12; 13; 14; 15; 16; 17; 18; 19; 20; 21; 22; 23; 24; 25; 26; 27; 28; 29; 30; 31; 32; 33; 34). The three-year WMAP maps are consistent with the first-year maps up to a small quadrupole difference. The two additional years of data and the improvements in analysis has not significantly altered the low multipole structures in the maps (10). Hence, 'anomalies' are expected to persist at the same modest level of significance but are now less likely to be artifacts of noise, uncorrected systematics, or the analysis of the first year data. The cosmic significance of these 'anomalies', however, remains debatable because of the aposteriori statistics employed to ferret them out of the data. *More importantly, what is missing is a common, well defined, mathematical language to quantify SI (as distinct from non Gaussianity) and the ability to ascribe statistical significance to the anomalies unambiguously.* We employ a well defined, mathematical language of Bipolar harmonic decomposition of the underlying correlation to quantify SI that can ascribe statistical significance to the anomalies unambiguously.

The observed CMB sky is a single realization of the underlying correlation, hence the detection of SI violation or correlation patterns pose a great observational challenge. For statistically isotropic CMB sky, the correlation function

$$C(\hat{n}_1, \hat{n}_2) \equiv C(\hat{n}_1 \cdot \hat{n}_2) = \frac{1}{8\pi^2} \int d\mathcal{R} C(\mathcal{R}\hat{n}_1, \mathcal{R}\hat{n}_2), \tag{1}$$

where $\mathcal{R}\hat{n}$ denotes the direction obtained under the action of a rotation $\mathcal{R}$ on $\hat{n}$, and $d\mathcal{R}$ is a volume element of the three-dimensional rotation group. The invariance of the underlying statistics under rotation allows the estimation of $C(\hat{n} \cdot \hat{n}')$ using the average of the temperature product $\widetilde{\Delta T}(\hat{n})\widetilde{\Delta T}(\hat{n}')$ between all pairs of pixels with the angular separation θ. In the absence of statistical isotropy, $C(\hat{n}, \hat{n}')$ is estimated by a single product $\widetilde{\Delta T}(\hat{n})\widetilde{\Delta T}(\hat{n}')$ and hence is poorly determined from a single realization. Although it is not possible to estimate each element of the full correlation function $C(\hat{n}, \hat{n}')$, some measures of statistical anisotropy of the CMB map can be estimated through suitably weighted angular averages of $\widetilde{\Delta T}(\hat{n})\widetilde{\Delta T}(\hat{n}')$. The angular averaging procedure should be such that the measure involves averaging over sufficient number of independent 'measurements', but should ensure that the averaging does not erase all the signature of statistical anisotropy. We proposed the Bipolar Power spectrum (BiPS) κ_ℓ ($\ell = 1, 2, 3, \ldots$) of the CMB map as a statistical tool of detecting and measuring departure from SI (35; 36) and reviewed in this article in sec. 2. The non-zero value of the BiPS spectrum imply the break down of statistical isotropy

$$\text{STATISTICAL ISOTROPY} \quad \Longrightarrow \quad \kappa_\ell = 0 \quad \forall \ell \neq 0. \tag{2}$$

The BiPS is sensitive to structures and patterns in the underlying total two-point correlation function (35; 36). The BiPS is particularly sensitive to real space correlation patterns (preferred directions, etc.) on characteristic angular scales. In harmonic space, the BiPS at multipole ℓ sums power in off-diagonal elements of the covariance matrix, $\langle a_{lm} a_{l'm'} \rangle$, in the same way that the 'angular momentum' addition of states lm, $l'm'$ have non-zero overlap with a state with angular momentum $|l - l'| < \ell < l + l'$. Signatures, like a_{lm} and a_{l+nm} being correlated over a significant range l are ideal targets for BiPS. These are typical of SI violation due to cosmic topology and the predicted BiPS in these models have a strong spectral signature in the bipolar multipole ℓ space (37). The orientation independence of BiPS is an advantage for constraining patterns (preferred directions) with unspecified orientation in the CMB sky such as that arising due to cosmic topology or, anisotropic cosmology (38).

The results of WMAP are a milestone in CMB anisotropy measurements since it combines high angular resolution, high sensitivity, with 'full' sky coverage allowed by a space mission. The frequency coverage allows for WMAP CMB sky maps to be foreground cleaned up to $l \sim 100$ (16; 39). The CMB anisotropy

map based on the WMAP data are ideal for testing statistical isotropy. Measurement of the BiPS on CMB anisotropy maps based on the first year WMAP data are consistent with statistical isotropy (40; 41; 42). BIPS analysis on the recent WMAP-3yr data are consistent with SI and further, indicate that BiPS of the three years maps show an improvement in SI – the deviations are smaller and fewer (43).

First 'full-sky' CMB polarization maps have been recently delivered by WMAP (11). The statistical isotropy of the CMB polarization maps are an independent probe of statistical isotropy. Since CMB polarization is generated on at the surface of last scattering, violations of statistical isotropy are pristine cosmic signatures and more difficult to attribute to the local universe. The bipolar power spectrum has been defined and implemented for CMB polarization and show great promise (44).

2 The Bipolar representation of SI violation

Although it is not possible to estimate each element of the full correlation function $C(\hat{n}_1, \hat{n}_2)$, some measures of statistical isotropy of the CMB map can be estimated through suitably weighted angular averages of $\Delta T(\hat{n}_1)\Delta T(\hat{n}_2)$. The angular averaging procedure should be such that the measure involves averaging over sufficient number of independent measurements to reduce the cosmic variance, but should ensure that the averaging does not erase all the signature of statistical anisotropy. The Bipolar power spectrum (BiPS) is a measure of statistical isotropy proposed by us in ref. (35)

$$\kappa^\ell \;=\; (2l+1)^2 \int d\Omega_{n_1} \int d\Omega_{n_2} \, [\frac{1}{8\pi^2} \int d\mathcal{R} \chi^\ell(\mathcal{R}) \, C(\mathcal{R}\hat{n}_1, \, \mathcal{R}\hat{n}_2)]^2 \qquad (3)$$

that also has another important desirable property of being independent of the overall orientation of the sky. In the above expression, $C(\mathcal{R}\hat{n}_1, \, \mathcal{R}\hat{n}_2)$ is the two point correlation at $\mathcal{R}\hat{n}_1$ and $\mathcal{R}\hat{n}_2$ which are the coordinates of the two pixels $\hat{n}_1$ and $\hat{n}_2$ after rotating the coordinate system by element $\mathcal{R}$ of the rotation group. χ^ℓ is the trace of the finite rotation matrix in the ℓM-representation $\chi^\ell(\mathcal{R}) \;=\; \sum_{M=-\ell}^{\ell} D^\ell_{MM}(\mathcal{R})$ which is called the *characteristic function*, or the character of the irreducible representation of rank ℓ. For a statistically isotropic model, $C(\hat{n}_1, \, \hat{n}_2)$ is invariant under rotation, and therefore $C(\mathcal{R}\hat{n}_1, \, \mathcal{R}\hat{n}_2) \;=\; C(\hat{n}_1, \, \hat{n}_2)$ and then the orthonormality of $\chi^\ell(\omega)$, implies the condition for SI, $\kappa^\ell \;=\; \kappa^0 \delta_{\ell 0}$ stated in eq.(2).

The harmonic space representation BiPS provides complementary understanding and allows for rapid numerical computation. Two point correlations of

CMB anisotropy, $C(\hat{n}_1, \hat{n}_2)$, are functions on $S^2 \times S^2$, and hence can be expanded as

$$C(\hat{n}_1, \hat{n}_2) = \sum_{l_1,l_2,\ell,M} A_{\ell M|l_1 l_2} Y^{l_1 l_2}_{\ell M}(\hat{n}_1, \hat{n}_2)\,. \tag{4}$$

Here $A_{\ell M|l_1 l_2}$ are the Bipolar Spherical harmonic (BipoSH) coefficients of the expansion and $Y^{l_1 l_2}_{\ell M}(\hat{n}_1, \hat{n}_2)$ are bipolar spherical harmonics. Bipolar spherical harmonics form an orthonormal basis on $S^2 \times S^2$ and transform in the same manner as the spherical harmonic function with ℓ, M with respect to rotations (45). Consequently, inverse-transform of $C(\hat{n}_1, \hat{n}_2)$ in eq. (4) to obtain the BipoSH coefficients of expansion is unambiguous.

Most importantly, the Bipolar Spherical Harmonic (BipoSH) coefficients, $A_{\ell M|l_1 l_2}$, are linear combinations of *off-diagonal elements* of the harmonic space covariance matrix,

$$A_{\ell M|l_1 l_2} = \sum_{m_1 m_2} \langle a_{l_1 m_1} a^*_{l_2 m_2}\rangle (-1)^{m_2} \mathcal{C}^{\ell M}_{l_1 m_1 l_2 -m_2} \tag{5}$$

where $\mathcal{C}^{\ell M}_{l_1 m_1 l_2 m_2}$ are Clebsch-Gordan coefficients. This clearly shows that $A_{\ell M|l_1 l_2}$ completely represent the information of the covariance matrix.

Statistical isotropy implies that the covariance matrix is diagonal, $\langle a_{lm} a^*_{l'm'}\rangle = C_l\ \delta_{ll'}\delta_{mm'}$ and hence the angular power spectra carry all information of the field. When statistical isotropy holds BipoSH coefficients, $A_{\ell M|ll'}$, are zero except those with $\ell = 0, M = 0$ which are equal to the angular power spectra up to a $(-1)^l(2l+1)^{1/2}$ factor. Therefore to test a CMB map for statistical isotropy, one should compute the BipoSH coefficients for the maps and look for nonzero BipoSH coefficients. *Statistically significant deviations of BipoSH coefficient of map from zero would establish violation of statistical isotropy.*

It is impossible to measure all $A^{\ell M}_{l_1 l_2}$ individually from the single CMB sky map because of cosmic variance since they form an equivalent representation of a general two point correlation function. There are several ways of combining BipoSH coefficients that serve to highlight different aspects of SI violations. Further, one can exploit the freedom to smooth the map in real space by appropriate symmetric kernels $W(\hat{n}\cdot\hat{n}')$ to target specific regions of the multipole space by the corresponding window function W_l.

2.1 Bipolar Power spectrum (BiPS)

Among the several possible combinations of BipoSH coefficients, the Bipolar Power Spectrum (BiPS) has proved to be a useful tool with interesting features

(35). BiPS of CMB anisotropy is defined as a convenient contraction of the BipoSH coefficients

$$\kappa_\ell \;=\; \sum_{l,l',M} W_l W_{l'} \left| A_{\ell M|ll'} \right|^2 \geq 0 \tag{6}$$

where W_l is the window function that corresponds to smoothing the map in real space by symmetric kernel to allow targeting specific regions of the multipole. BiPS is interesting because it is orientation independent, *i.e.* invariant under rotations of the sky. SI condition implies a null BiPS, *i.e.* $\kappa_\ell = 0$ for every $\ell > 0$, ($\kappa_\ell = \kappa_0 \delta_{\ell 0}$). Non-zero components of BiPS imply break down of statistical isotropy, and this introduces BiPS as a measure of statistical isotropy. It is worth noting that although BiPS is quartic in a_{lm}, it is designed to detect SI violation and not non-Gaussianity (35; 36; 41; 42). An un-biased estimator of BiPS is given by

$$\tilde{\kappa}_\ell = \sum_{ll'M} \left| A_{\ell M|ll'} \right|^2 - \mathfrak{B}_\ell \,, \tag{7}$$

where $\mathfrak{B}_\ell$ is the bias that arises from the SI part of the map. The bias is given by the angular power spectrum, C_l that can be estimated from the map (done in the results quoted here), or, corresponding to the best fit theoretical model (done for WMAP-1 analysis) (35; 41; 43; 44).

2.2 Reduced Bipolar Coefficients

The BipoSH coefficients can be summed over l and l' to reduce the cosmic variance,

$$A_{\ell M} = \sum_{l=0}^{\infty} \sum_{l'=|\ell - l|}^{\ell + l} A_{\ell M|ll'}. \tag{8}$$

In any given CMB anisotropy map, $A_{\ell M}$ would fluctuate about zero. A severe breakdown of statistical isotropy will result in huge deviations from zero. Reduced bipolar coefficients are not rotationally invariant, hence they assign direction to the correlation patterns of a map. An interesting way of visualizing these coefficients is to make a *Bipolar map* from $A_{\ell M}$

$$\Theta(\hat{n}) = \sum_{\ell=0}^{\infty} \sum_{M=-\ell}^{\ell} A_{\ell M} Y_{\ell M}(\hat{n}). \tag{9}$$

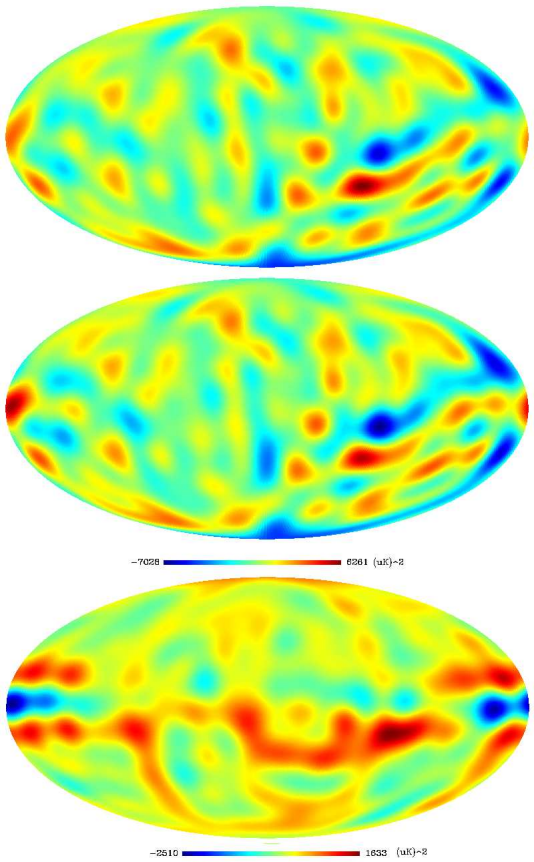


Fig. 1. **Top:** A *bipolar* map generated from bipolar coefficients, $A_{\ell M}$, of 3-year ILC map. **Middle:** bipolar map based on 1-year ILC map and **Bottom:** differences between the two maps (note the scales). The top map (ILC-3) has smaller fluctuations comparing to the middle one (ILC-1) except for the hot spot near the equator. Differences between these two maps mostly arise from a band around the equator *in bipolar space*. Both ILC maps are smoothed by a band pass filter, $W^S(l_t = 2, l_s = 10)$.

The symmetry $A_{\ell M} = (-1)^M A^*_{\ell -M}$ of reduced bipolar coefficients guarantees reality of $\Theta(\hat{n})$. The *bipolar map* based on bipolar coefficients of ILC-3 is shown on the top panel of Fig. 1. The map has small fluctuations except for a pair of hot and cold spots near the equator. To compare, we have also made a bipolar map of 1-year ILC map (ILC-1) from bipolar coefficients of ILC-1 (middle panel of Fig. 1). The difference map (Fig. 1 (bottom)) shows that differences between these two maps mostly arise from a band around the equator *in bipolar space*. As it is seen in Fig. 1, the bipolar map of ILC-3 has less fluctuations comparing to that of ILC-1. This is because almost all of $A_{\ell M}$'s of ILC-3 are smaller than those of ILC-1 (*i.e.* are closer to zero).

Figure 2 compares $A_{\ell M}$ and κ_ℓ of three year WMAP ILC map against an average of 1000 simulations of statistically isotropic maps with same angular power spectrum. As it can be seen many spikes presented in $A_{\ell M}$'s of ILC-1

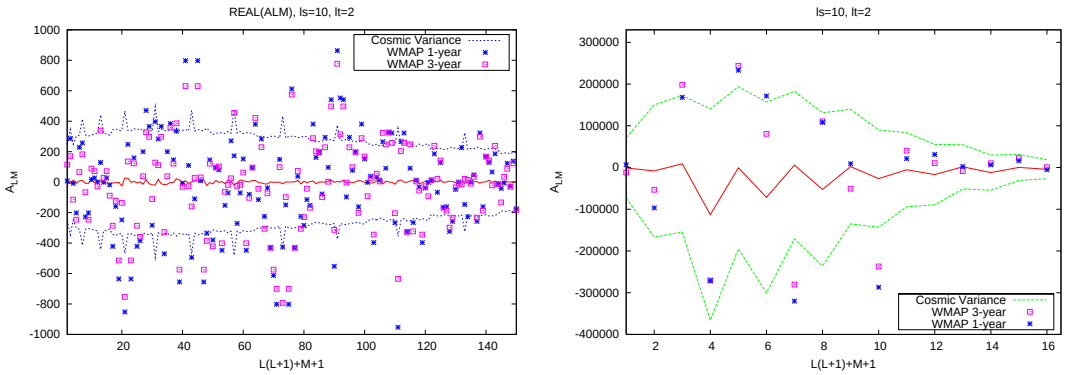


Fig. 2. *Left:* Real part of $A_{\ell M}$'s of ILC-3 (red square points) and ILC-1 (blue stars) for a $W_l^S(10,2)$ filter that roughly keeps multipoles between 2 and 15. ℓ and M indices are combined to a single index $n = \ell(\ell+1) + M + 1$. The blue dotted lines define 1-σ error bars derived from 1000 simulations of SI CMB anisotropy maps. Almost all of $A_{\ell M}$'s of ILC-3 are smaller than those of ILC-1 which means ILC-3 is more consistent with statistical isotropy. *Right:* The BiPS of ILC-3 (red square points) and ILC-1 (blue stars) for a $W_l^S(10,2)$ filter is consistent with statistical isotropy The green dotted lines define 1-σ error bars derived from 1000 simulations of SI CMB anisotropy maps.

have either disappeared or reduced in ILC-3 (*e.g.* those around $n = 20, 40$ and a big spike at $n = 111$). More detailed description of analysis and results are given in ref. (43).

The rotational properties of $A_{\ell M}$ suggests defining another rotationally invariant bipolar power spectrum in $D_\ell = \sum_{\ell M} A_{\ell M} A^*_{\ell M}$ – the same way a_{lm} are combined to construct the angular power spectrum, C_l. The measurement of *reduced Bipolar Power Spectrum (rBiPS)*, D_ℓ, on CMB maps is ongoing. We also defer a comparison of the two orientation insensitive measures of SI violation κ_ℓ (BiPS) and D_ℓ (rBiPS).

3 BiPS of CMB polarization maps

One of the firm predictions of this working 'standard' cosmological model is linear polarization pattern (Q and U Stokes parameters) imprinted on the CMB at the last scattering surface. A net pattern of linear polarization is retained due to local quadrupole intensity anisotropy of the CMB radiation Thomson scattering on the electrons at at the last scattering surface.

Recently WMAP has provided the first "full" sky CMB polarization maps. The wealth of information in the CMB polarization field will enable us to determine the cosmological parameters and test and characterize the initial perturbations and inflationary mechanisms with great precision. Cosmological polarized microwave radiation in a simply connected universe is expected to

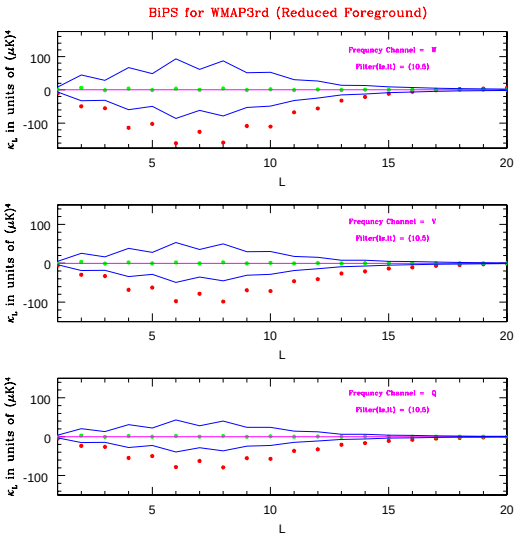


Fig. 3. BiPS for polarization maps(nside = 512) with polarization mask using band-pass filter $W_l^S(l_s = 10, l_t = 5)$. Red dots show the BiPS for WMAP3rd foreground cleaned polarization map with polarization mask after bias subtraction, green dots show the BiPS for 100 SI maps with polarization mask after bias subtraction and Blue lines show the $1-\sigma$ of the cosmic variance of BiPS for 100 SI maps with polarization mask.

be statistically isotropic. This is a very important feature which allows us to fully describe the field by its power spectrum that can have profound theoretical implications for cosmology. Statistical isotropy (SI) can now be tested with CMB polarization maps over large sky fraction. Importance of having statistical tests of departures from SI for CMB polarization maps lies not only in interesting theoretical motivations but also in testing the cleaned CMB polarization maps for residuals from polarized foreground emission.

The coordinate–free description of CMB polarization decomposes the two kinds of polarization pattern on the sky based on their different parities. In the spinor approach, the even parity pattern is called the E–mode and the odd parity pattern the B–mode. Hence the CMB sky maps are characterized by a triplet of random scalar fields: $X(\hat{n}) \equiv \{T(\hat{n}),\, E(\hat{n}),\, B(\hat{n})\}$. Statistical properties of each of these fields can be characterized by N-point correlation functions, $\langle X(\hat{n}_1)X(\hat{n}_2)\cdots X(\hat{n}_n)\rangle$ [1] .

Gaussian CMB sky is completely described by two-point correlation functions of $X(\hat{n})$, or equivalently, the corresponding spherical harmonic coefficients

$$C^{XX'}(\hat{n},\hat{n'}) \;=\; \langle X(\hat{n})X'(\hat{n'})\rangle' \langle a^X_{lm} a^{X'*}_{l'm'}\rangle. \tag{10}$$

[1] For cut-sky, $E(\hat{n})$ and $B(\hat{n})$ mode decomposition is not unique (46; 47). But since mixing is linear there always exist two linearly independent modes. It is possible to formulate the SI of these linear independent modes.

It is possible to define generalized Bipolar Spherical Harmonic (BipoSH) coefficients, $A^{XX'}_{\ell M|l_1 l_2}$, that are linear combinations of off-diagonal elements of the $\langle a^X_{lm} a^{X'*}_{l'm'}\rangle$ covariance matrix,

$$A^{XX'}_{\ell M|l_1 l_2} \;=\; \sum_{m_1 m_2} \langle a^X_{l_1 m_1} a^{*X'}_{l_2 m_2}\rangle (-1)^{m_2} \mathcal{C}^{\ell M}_{l_1 m_1 l_2 - m_2} \tag{11}$$

where $\mathcal{C}^{\ell M}_{l_1 m_1 l_2 m_2}$ are Clebsch-Gordan coefficients. Incomplete sky coverage induces a contamination of E-mode of polarization by its B-mode and vice-versa. Then the modified temperature and polarization fields is related to their actual values of full sky coverage by a window matrix (46; 47) whose elements are basically window functions for temperature and polarization in harmonic space. It can be shown that the estimated BipoSH coefficients are in fact linear combinations of that for full-sky CMB maps,

$$\tilde{\mathbf{A}}_{\ell M|ll'} = \sum_{\ell' M' l_1 l_2} \mathbf{N}^{\ell M ll'}_{\ell' M' l_1 l_2}\; \mathbf{A}_{\ell M|l_1 l_2}\,. \tag{12}$$

Here bold-faced $\tilde{\mathbf{A}}_{\ell M|ll'}$ and $\mathbf{A}_{\ell M|l_1 l_2}$ are the column matrices corresponding to estimated and true BipoSH coefficients respectively, for the auto and cross-correlations $(TT, TE, TB, ET, EE, EB, BT, BE, BB)$ of temperature anisotropy and polarization. The elements of the matrix $\mathbf{N}^{\ell M ll'}_{\ell' M' l_1 l_2}$ depend on Clebsch-Gordan coefficients and window functions in harmonic space. Hence, the true BipoSH coefficients can be estimated from the pseudo-BipoSH coefficients by inverting the above equation.

Figure 3 shows BIPS for the foreground cleaned polarization maps recently released by WMAP filtered to retain power multipole between $l \sim 5$ to 15. The BiPS for CMB E-polarization maps at V and W from WMAP are compared to the mean and 1-σ deviations of BiPS measurements on 100 simulated maps The appropriate galactic mask for polarization maps suggested by WMAP was employed for all maps. These *preliminary* results indicate strong violation of statistical isotropy in the polarization. However, the dependence on frequency suggests a non cosmological origin in foreground residuals in parts of the sky beyond galactic mask. Interestingly enough,c onsistent with conclusions of the WMAP polarization analysis, the BiPS analysis also indicates that the V band is 'cleaner' than the W band.

4 Discussion

The Bipolar representation of the statitical correlations underlying the CMB anisotropy and polarization allows a unprejudiced evaluation of statistical

isotropy on largest observable cosmic scales. The null result of search for departure from statistical isotropy in the WMAP data provides strong evidence for cosmological principle and constrains non-trivial cosmic topology and ultra-large scale structure. It can potentially be used to constrain any other physics that violate global isotropy on cosmic scales. The null result has implications for the observation and data analysis techniques used to create the CMB anisotropy maps. Observational artifacts such as non-circular beam, inhomogeneous noise correlation, residual striping patterns, and residuals from foregrounds are potential sources of SI breakdown.

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