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HOW TO AVOID THE AMBIGUITY IN APPLYING THE COPERNICAN PRINCIPLE FOR COSMIC TOPOLOGY: TAKE THE OBSERVATIONAL APPROACH

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ABSTRACT

It is often stated that homogeneity and isotropy of the Universe are assumptions of the almost Friedmann-Lemaître (FL) model (the hot big bang model), inspired from the Copernican Principle. However, only *local* homogeneity and isotropy are required by the model: multiply connected almost FL models are locally homogeneous and isotropic, but they can be *globally* anisotropic and/or *globally* inhomogeneous. Toy models are used here to show how global anisotropy and/or global inhomogeneity of an almost FL model could be shown directly in observations. This approach may avoid having to make any assumptions regarding global anisotropy and inhomogeneity.

INTRODUCTION

The Copernican principle (Copernicus 1543), defined for the present paper as the principle according to which the observer should not happen to be located in any special location in the Universe, is generally *applied* in one of three ways in twentieth century cosmology:

- (i) by assuming the Cosmological Principle, in which spatial sections of the Universe at constant cosmological time are close to homogeneous and isotropic when averaged on “large enough” length scales;
- (ii) by assuming the Perfect Cosmological Principle, in which all of space-time is close to homogeneous and isotropic, on average; or
- (iii) by assuming that the Universe is very chaotic, very far from homogeneity, but that by the Weak Anthropic Principle, we happen to live in a tiny bubble which is inflated to a locally close to flat Friedmann-Lemaître model;

or it is *rejected*

- (iv) by taking the “observational” point of view that the observer lives at the centre of a close to isotropic but radially very inhomogeneous three-dimensional space-time cone with a spherical (S^2) boundary defined by the age of the local patch of the Universe, so that no space outside of this sphere exists as far as science is concerned, and which happens to coincide with what one expects in case (i) above.

Case (i) underlies what is generally accepted as the standard hot big bang model, which can be more formally referred to as the almost Friedmann-Lemaître model (e.g. Schwarzschild 1900, 1998; de Sitter 1917; Friedmann 1923, 1997; Lemaître 1958; Weinberg 1972). This model may or may not have zero curvature, and it may or may not have trivial topology. Numerous observations now knit together to provide a very solid foundation for the almost Friedmann-Lemaître model, within the spatial, temporal and density limits of its validity.

Case (ii), that of a steady state or of a quasi-steady-state model (Hoyle, Burbidge & Narlikar 1993), is only defined in its general outline, and would require many more close links with recent observations in order to have predictive power. For example, the case (i) interpretation of quasars' redshifts as primarily due to expansion of the Universe, when applied to observations, implies that quasars and Lyman break galaxies at high redshift trace large scale structure at $L \sim 130 \pm 10h^{-1}$ Mpc (comoving) and that they function as a standard ruler which implies a low matter density $\Omega_m \sim 0.3$ (Roukema & Mamon 2000, 2001; Broadhurst & Jaffe 1999). This would be very difficult to explain if quasar (and Lyman break galaxy) redshifts were non-cosmological.

Case (iii) corresponds to the presently very popular chaotic inflation scenarios. Whether or not the Weak Anthropic Principle is consistent with the Copernican Principle is out of the scope of this paper.

Case (iii) is generally considered an extrapolation (towards short cosmological times and large spatial distances) of the almost Friedmann-Lemaître (FL) model. Case (ii) could also conceivably be defined as an extension of the FL model.

The anti-Copernican case (iv) is essentially identical to case (i) as far as observational analysis is concerned — as long as the topology of the Universe remains unmeasured. However, theoretical calculation is somewhat difficult if case (i) is not resorted to.

If the topology of space were measured by observation (Luminet & Roukema 1999, and see below), then the size of the Universe would be finite and less than the horizon diameter in at least one direction, i.e. finite. Models of case (ii) would then require serious revision and would no longer satisfy the Perfect Cosmological Principle, and case (iii) models would presumably require individual “universes” to be born as separate space-times and might have difficulty surviving Occam's Razor.

Case (iv) would also be partly or totally avoidable: only the *apparent* space would be a sphere centred on the observer. At least some parts of the “edge of the Universe” of the case (iv) interpretation would no longer be edges.

On the other hand, whether or not case (i) would remain valid if the Universe is found to be multiply connected would depend on the resolution of the ambiguity in the meanings of the words “homogeneous” and “isotropic”. The solution of the Einstein-Hilbert equations is only found as a local solution, so only *local* homogeneity and isotropy are required. In general, although Friedmann-Lemaître models satisfy *local* homogeneity and isotropy, they need not satisfy *global* homogeneity and isotropy. Observational support for local homogeneity and isotropy does not imply global homogeneity and isotropy.

In this paper, the relevance of different interpretations of these two words in the context of the global geometry of space (curvature and topology) is explained and illustrated by simple toy models in two dimensions. By attempting to measure the more subtle elements of inhomogeneity and anisotropy which would reveal the global shape of the Universe, some of the ambiguity in deciding how to apply the Copernican principle to observational cosmology could be avoidable: empirical results sometimes help resolve theoretical dilemmas.

Proper distance (Weinberg 1972, Eq. 14.2.21) in comoving units is used throughout this paper unless otherwise stated. The Hubble constant is parametrised here as $h \equiv H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Values of the density parameter, Ω_m , the dimensionless cosmological constant, Ω_Λ , the dimensionless curvature, $\Omega_\kappa \equiv \Omega_m + \Omega_\Lambda - 1$, and the curvature radius $R_C \equiv (c/H_0)\Omega_\kappa^{-1/2}$, are indicated where used.

COSMIC TOPOLOGY, HOMOGENEITY AND ANISOTROPY

Local Homogeneity and Isotropy

When justifying the assumptions of (i) homogeneity and (ii) isotropy, these are generally interpreted to mean that

- (a) the comoving spatial density of astrophysical objects is approximately constant, and

- (b) the solid angular distribution of distant objects (or microwave background fluctuations) is approximately constant, after correction for the very anisotropic biases due to living in a dusty disk galaxy and due to our peculiar velocity with respect to the comoving reference frame,

respectively.

Both are reasonably consistent with numerous observations, and the latter has been further reinforced recently by the only cosmological class of objects known to be isotropic *in spite* of galactic absorption and our peculiar velocity: the gamma ray bursts. These may in fact reveal the births of black holes, i.e. of regions of extreme curvature (inhomogeneity) on very small scales, that are briefly visible before being engulfed by event horizons (Joshi et al. 2000).

These two properties (a), (b) are generally referred to as “local homogeneity and isotropy”. Solutions to the Einstein-Hilbert equations are normally only found as local solutions of differential equations, and so only local assumptions of homogeneity and isotropy have any effect on Friedmann-Lemaître models.

One might also refer to these as first order homogeneity and isotropy, since only one-point statistics of density are considered.

Cosmic Topology and Global Anisotropy

For the earliest known article on cosmic topology, see Schwarzschild (1900, 1998). A recent review is that of Luminet & Roukema (1999). The multiply connected models which have been compared with observations are all FL models, i.e. they are *locally homogeneous and isotropic*.

A simple way to illustrate multiply connected 3-manifolds which may correspond to the comoving 3-space we live in is to start with the 3-torus, T^3 . This can be thought of

- (i) *physically*, as a cube of which opposite faces are identified so that space is continuous, with no boundaries, but finite,
- (ii) *observationally*, as a tiling of Euclidean space R^3 by multiple copies of the single *physical* “cube”, or
- (iii) as the analogy one dimension higher of a 2-torus embedded in R^3 but endowed with an intrinsically flat metric, i.e. a metric different to the normal metric of R^3 .

For a demonstration of basic properties of locally homogeneous and isotropic manifolds, it is useful to subtract one dimension, i.e. to consider 2-manifolds. This is the approach adopted here for pedagogical purposes. For the physical Universe, the results extrapolate to the three-dimensional case.

The cube (i) is referred to as a *fundamental domain*, and the space (ii) is referred to as the *universal covering space* or in a new coinage, *apparent space*. Both are equivalent ways of thinking about the same manifold. The former is useful for conventional physical reasoning, according to which any particle or object only exists at one point in a space, but difficult to use for interpreting observations. The latter is useful for observational analyses, but surprising and sometimes confusing for physical reasoning, since objects “exist” many times in it.

In two dimensions, the corresponding space is the flat 2-torus T^2 , of which the fundamental domain is a square (in general a parallelogram, but a square is used for the present paper), and the universal covering space is R^2 .

Figure 1 shows what such a universe would look like. For reference, the reader should remember that the horizon size for likely values of the curvature parameters ($\Omega_m \approx 0.3, \Omega_\Lambda \approx 0.7$) is $R_H \approx 10h^{-1}$ Gpc, and that objects like quasars or “Lyman break galaxies” are seen up to typical redshifts of $z \sim 2 - 4$, which correspond to roughly half this distance. Clusters of galaxies and galaxies are mostly only seen to a few $100h^{-1}$ Mpc.

Present constraints on the size of the Universe (defined here as the injectivity diameter $2r_{inj}$, which is the length of the shortest spatial geodesic linking an object to an image of itself) are $2r_{inj} \gtrsim 1h^{-1}$ Gpc, whether the Universe is hyperbolic, flat or spherical. So, in order to see multiple images of a single object, catalogues of objects which are seen to typically a few tenths of R_H would be required. This implies that effects of aging of the objects, or of viewing angles, or of movement relative to the comoving reference frame, would make multiple images of a single object appear quite different, so that proving the identity of two images would be difficult.

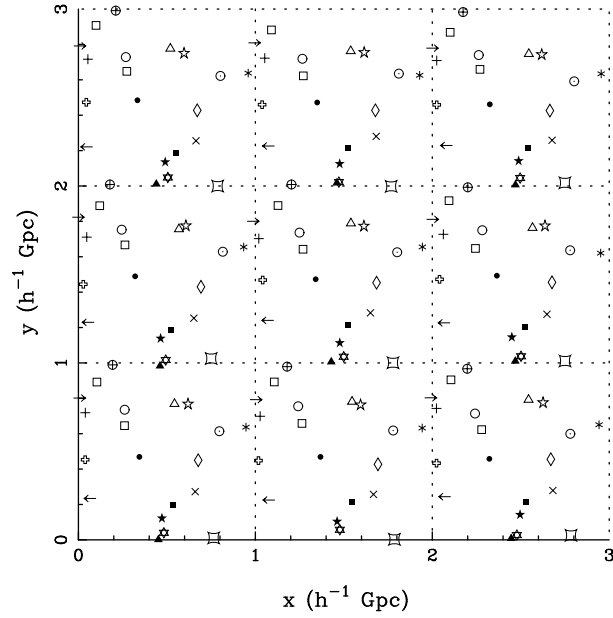


Fig. 1. Distribution of images of $N = 20$ objects at cosmological distances in a multiply connected two-dimensional T^2 model of side length $L = 1h^{-1}$ Gpc, in the apparent space (covering space). The N objects are placed randomly according to a uniform distribution in the fundamental domain. Each physically distinct object is shown by a separate symbol. Multiple topological *images* of a single *object* are each shown by the same symbol. Random uniform offsets of $\pm 25h^{-1}$ Mpc are added in order that correlations in pair separation plots (below) are easily visible to the eye. More realistic offsets due to peculiar velocities and/or measurement error would be of the order of $\sim 1 - 10h^{-1}$ Mpc over these time intervals, i.e. slightly smaller. Dotted lines are used to guide the eye, but have no physical meaning. This model is locally homogeneous and isotropic.

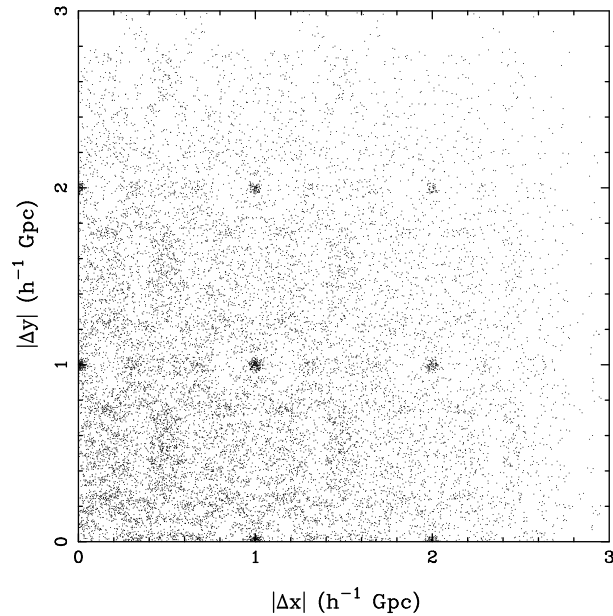


Fig. 2. Separation vectors $(|x_i - x_j|, |y_i - y_j|)$ for all pairs of images in the T^2 model shown in Fig. 1. The model is globally *anisotropic*, since the pattern of “spikes” at the points $\{(ih^{-1} \text{ Mpc}, jh^{-1} \text{ Mpc}), i, j \in \mathbb{N}\}$ changes under rotation of the $x - y$ axes. The model is globally *homogeneous*: the pattern is identical for different choices of the origin.

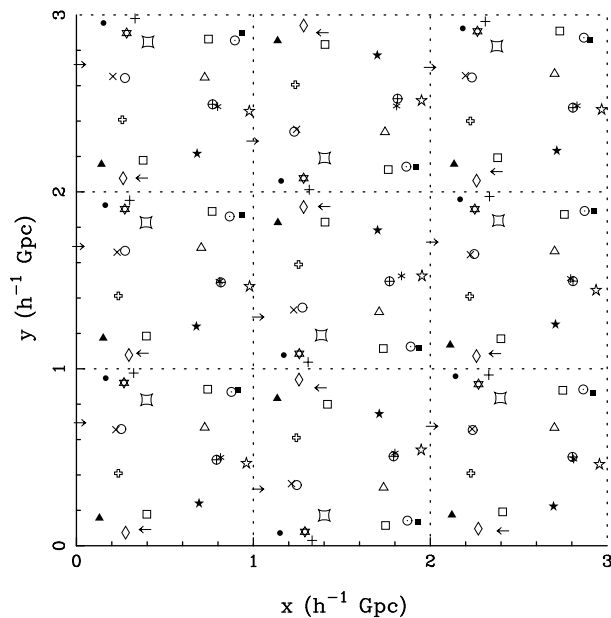


Fig. 3. As for Fig. 1, for a Klein bottle model, i.e. identical to T^2 except that before identifying the left and right hand sides of the fundamental domain, a twist or reversal is applied. This model is locally homogeneous and isotropic.

So, the identity between multiple copies of symbols in Fig. 1 would not be obvious in any category of presently known objects.

The model is *locally homogeneous and isotropic*: any study in a small region around a point is independent of the orientation of the axes of a chosen coordinate system and of the origin of that system.

The model of Fig. 1 is globally homogeneous, but globally anisotropic.

The anisotropy is shown by Fig. 2, which shows vectors separating all pairs of images in the simulated universe. Separations between multiple images of single objects correspond to multiples of the generators [the vectors $(0, 1)$ and $(1, 0)$] and clearly cause clustering around these points. As pointed out by Lehoucq, Luminet & Lachièze-Rey (1996), a one-dimensional pair separation histogram $\Delta N(\Delta r)$ should show sharp spikes corresponding to these cluster points. (If no offsets were added between multiple images, then the spikes would be Dirac δ functions.)

The pairs of images contributing to these spikes can be referred to as *generator pairs* or as *Type II pairs* (Uzan, Lehoucq & Luminet 1999), which induce spikes in flat multiply connected spaces, but not in curved multiply connected spaces (Gomero et al. 1999; Uzan et al. 1999).

If the $x - y$ axes are chosen at a different angle, then the diagram would be rotated: the pattern of spikes is not invariant (i.e. varies) under rotation. This proves global anisotropy.

However, the diagram would be statistically identical under a shift (without rotation) of the origin: the space is homogeneous.

Cosmic Topology and Global Inhomogeneity

Use of “generator pairs”

Many 3-manifolds are globally inhomogeneous, but the effect is observationally quite subtle.

The effect is illustrated here for the Klein bottle. This is like T^2 , except that a twist is applied before identification of one pair of edges (here, the “vertical” edges). The distribution of images is shown in Fig. 3. Again, this model is *locally homogeneous and isotropic*.

Fig. 4 shows that, as for the T^2 model, the Klein bottle is *globally anisotropic*: rotation of the coordinate axes would modify the pattern of crystallographic spikes.

In order to show global inhomogeneity using “generator pairs”,

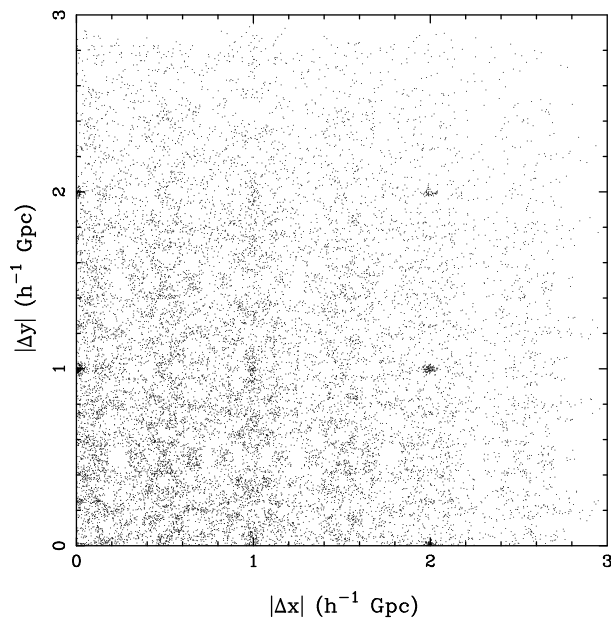


Fig. 4. Separation vectors $(|x_i - x_j|, |y_i - y_j|)$ for all pairs of images in the Klein bottle model shown in Fig. 3. As for Fig. 1, the model is globally *anisotropic*. Because of the orientation reversal, some of the separations corresponding to “cosmic crystallography spikes” (Lehoucq et al. 1996) present in Fig. 1 seem to have disappeared. This is a clue to establishing inhomogeneity.

- (i) knowledge of which pairs of images are due to multiple images of single objects is required, and
- (ii) this knowledge is required *despite the fact that the crystallographic spikes corresponding to “inhomogeneous generator pairs” are smeared out making them very difficult to detect.*

Both (i) and (ii) are, of course, easy for a simulated universe: in the observed Universe they are difficult.

Comparison of Figs 2 and 4 shows that some crystallographic spikes seem to disappear due to the twist in identifying the vertical edges of the fundamental domain.

Figures 5 and 6 show where these spikes have “disappeared” to, using property (i). Different choices of the y origin (see figure captions) yield different patterns. Together, the two figures show how the spikes have been smeared out.

Hence, the Klein bottle is *globally inhomogeneous*.

One way to apply this technique observationally would be

- (a) to use properties of galaxies or quasars which uniquely label and identify individual objects, despite the large fractions of the age of the Universe over which the objects age.

This seems difficult given present observational limitations and known techniques, but not impossible.

An alternative method would be

- (b) firstly to detect the crystallographic spikes of generator pairs which *do* appear, and which would imply a fundamental domain of size $2L = 2h^{-1}$ Gpc in the horizontal direction, and secondly to hypothesise that the real fundamental domain is just half this length.

This would make predictions of which pairs of objects should correspond to those seen separately in Figs 5 and 6. If the predicted multiply imaged pairs really were identical, then proving this by observation would be much easier than the “needle in a haystack” requirement of finding the corresponding pairs blindly as in the previous suggestion (a).

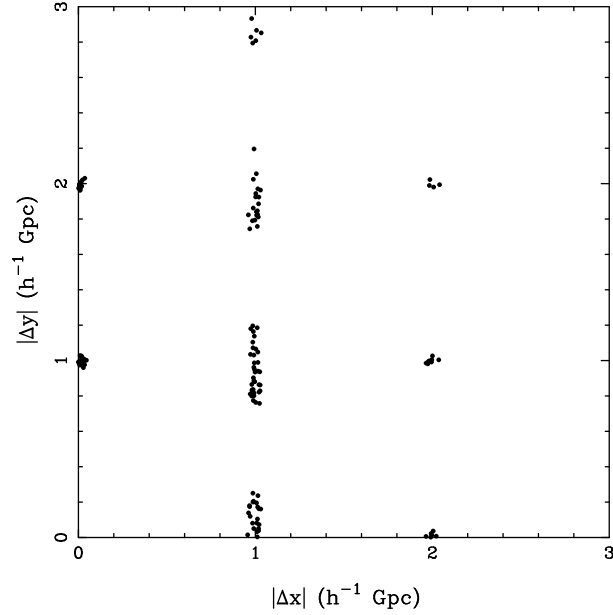


Fig. 5. Klein bottle image pair separations, as for Fig. 3, but only showing pair separations for pairs for which the two multiple images are those of a single physical object, where the y coordinate of an element of the pair satisfies $|(y - y_0) + nL/2| < L/8$, for some $n \in Z$, and where $L = 1h^{-1}$ Gpc and $y_0 = 0h^{-1}$ Gpc. One element of a pair of images of an object satisfies this relation if and only if the other satisfies it. In words, these are the objects which are close to either the horizontal “edges” or to the half-way dividing lines between the horizontal “edges”. Some of the “cosmic crystallography spikes” present in Fig. 2 but missing from Fig. 4 have now reappeared, but are spread out in the vertical direction.

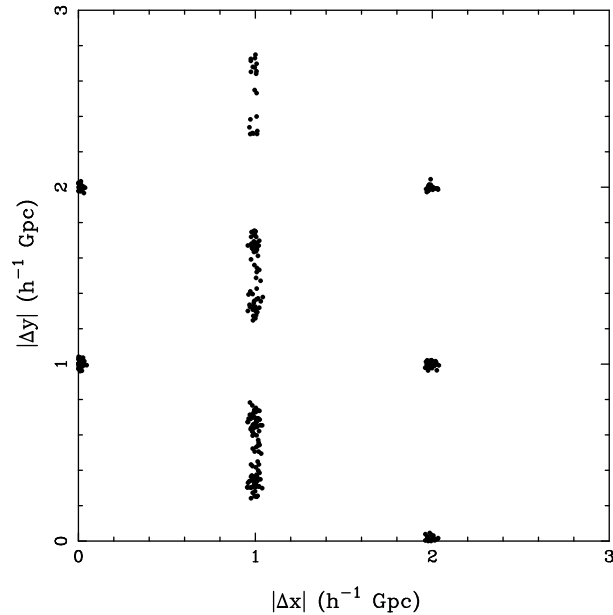


Fig. 6. Klein bottle image pair separations, as for Fig. 5, where the y coordinate of an element of the pair satisfies $|(y - y_0) + nL/2| < L/8$, for some $n \in Z$, and where $L = 1h^{-1}$ Gpc, but $y_0 = L/4 = 0.25h^{-1}$ Gpc. These pairs are the complement to the set in Fig. 5. The difference between Figs 5 and 6 shows that the Klein bottle is inhomogeneous: for different choices of the zero-point y_0 , the figure changes.

The Copernican principle in an inhomogeneous universe

Figs 5 and 6 illustrate one way in which the Copernican principle could be applied directly. One would not expect the observer to be at any special value of y_0 : the value $y_0 = 0.00 \pm 0.01h^{-1}$ Gpc for the toy model shown here would be surprising.

Use of “local pairs” or “local n -tuplets”

Inspection of Fig. 1 and a little reflection show that generator pairs are not the only pairs of images which occur repeatedly in a separation vector plot (or separation distance histogram).

Close pairs of images of *non*-identical objects within a single fundamental domain also define separation vectors which are repeated in the apparent space, though in a different way to generator pairs. These are *local pairs* or *Type I* pairs (Roukema 1996; Section 2 of Uzan et al. 1999). Since there are nine copies of the fundamental domain shown in Fig. 1, there are nine local pairs clustered around each of the $20 \times 19/2 = 190$ separation vectors of modulus smaller than $L = 1h^{-1}$ Gpc. This is visible as a “graininess” in Figs 2 and 4.

In a flat space, the same effect also occurs for pairs across many copies of the fundamental domain, but in general does not occur in curved spaces. Thus the term “local”: it is most useful just to consider relatively close pairs of images.

Fagundes & Gausmann (1999) noticed the effect of the local pairs in pair separation histograms, and Uzan et al. (1999) showed how to collect these together and detect multiple connectedness. If these led to detection of multiple connectedness for a model known (mathematically) to be inhomogeneous, then this would enable (a) above to be applied: the generator pairs could be used to observationally illustrate inhomogeneity. However, local pairs in themselves would not show inhomogeneity: they are local.

Similarly, local n -tuplets (Roukema 1996) would not directly show inhomogeneity. However, since non-orientable manifolds are inhomogeneous, the discovery of significant numbers of matching n -tuplets, where some of these required orientation reversals, would imply inhomogeneity.

For more details of global homogeneity and isotropy questions of 2-manifolds, and more particularly of 3-manifolds, see Thurston (1997).

CONCLUSION

The Copernican Principle, when used to motivate *local* homogeneity and isotropy of the Universe, does not necessarily imply *global* homogeneity and isotropy. If the topology of the Universe is observable, then this may reveal

- (i) that the Universe is globally anisotropic, though locally isotropic and both globally and locally homogeneous, as in the case of a T^2 model in two dimensions, or
- (ii) that the Universe is globally both anisotropic and inhomogeneous, even though locally isotropic and homogeneous, as in the case of a Klein bottle model in two dimensions.

Most techniques for measuring cosmic topology should easily lead to direct geometrical illustrations of global anisotropy if multiple connectedness is detected (unless the 3-manifold we live is globally isotropic: the projective space, S^3/Z_2 , which can be thought of by identifying all opposite points of the hypersphere S^3 , is globally both isotropic and homogeneous, even though multiply connected).

By contrast, global inhomogeneity in a flat multiply connected manifold would be difficult to detect directly. However, prior knowledge of multiple connectedness would help in obtaining direct illustrations of global inhomogeneity.

Present observations imply that the observable Universe is close to flat (Lange et al. 2000; Balbi et al. 2000; Roukema & Mamon 2000) and that multiply connected flat models of size $2r_{\text{inj}} \sim 2R_H/10$ are consistent with present large angle microwave background data (Roukema 2000). The observational approach to global geometry might just possibly provide an empirical solution to an otherwise philosophical question, by showing whether not both local *and* global homogeneity and/or isotropy are valid assumptions..

References

Balbi A. et al., Constraints on cosmological parameters from MAXIMA-1, *Astrophys. J.Lett.*, in press, arXiv:astro-ph/0005124, 2000.

- Broadhurst T., Jaffe A. H., Using the comoving maximum of the galaxy power spectrum to measure cosmological curvature. in the press, *Astrophys. J. Lett.*, *arXiv:astro-ph/9904348*, 2000.
- Copernicus, N., De revolvionibvs orbium coelestium, libri VI., *Norimbergae, apud Ioh. Petreium*, 1543.
- de Sitter W., Einstein's theory of gravitation and its astronomical consequences. Third paper, *Mon. Not. R. Astr. S.* **78**, 3, 1917.
- Fagundes H. & Gausmann E., Cosmic Crystallography in Compact Hyperbolic Universes, in Proceedings of the XIX Texas Symp. Rel. Astr. Cosmology, CD-ROM version, *arXiv:astro-ph/9811368*, 1999.
- Friedmann, A., Mir kak prostranstvo i vremya, (Leningrad: Akademiya), 1923.
- Friedmann, A., L'Univers comme espace et temps (translation of Friedmann 1923), in Essais de Cosmologie, (Paris: Le Seuil/Sources du Savoir) 1997.
- Gomero G. I., Teixeira A. F. F., Rebouças M. J., Bernui A., Spikes in Cosmic Crystallography, *arXiv:gr-qc/9811038*, 1999.
- Hoyle F., Burbidge G., Narlikar J.V., A quasi-steady state cosmological model with creation of matter, *Astrophys. J.* **410**, 437, 1993.
- Joshi P. S., Dadhich N. K., Maartens R., Gamma-ray bursts as the birth cries of black holes, *arXiv:gr-qc/0005080*, 2000.
- Lange A. E. et al., First estimations of cosmological parameters from BOOMERANG, *arXiv:astro-ph/0005004*, 2000.
- Lehoucq R., Luminet J.-P., Lachièze-Rey M., Cosmic crystallography, *Astron. & Astroph.* **313**, 339, 1996.
- Lemaître G., La Structure et l'Evolution de l'Univers, Onzième Conseil de Physique Solvay, ed. Stoops R., (Brussels: Stoops), 1958.
- Luminet J.-P., Roukema B. F., Topology of the Universe: Theory and Observations, in Cargèse summer school 'Theoretical and Observational Cosmology, ed. Lachièze-Rey M., Dordrecht: Kluwer, p117, *arXiv:astro-ph/9901364*, 1999.
- Roukema B. F., On Determining the Topology of the Observable Universe via 3-D Quasar Positions, *Mon. Not. R. Astr. S.* **283**, 1147, 1996.
- Roukema B. F., A Counterexample to Claimed COBE Constraints on Compact Toroidal Universe Models, *Class. & Quant. Grav.* **17**, 3951, *arXiv:astro-ph/0007140*, 2000.
- Roukema B. F., Mamon G. A., Tangential Large Scale Structure as a Standard Ruler: Curvature Parameters from Quasars, *Astron. & Astroph.* **358**, 395, *arXiv:astro-ph/9911413*, 2000.
- Roukema B. F., Mamon G. A., Lifting cosmic degeneracy within a single data set, *Astron. & Astroph.* **in press**, , *arXiv:astro-ph/0010511*, 2001.
- Schwarzschild, K., On the permissible curvature of space, *Vier.d.Astr.Gess.* **35**, 337, 1900.
- Schwarzschild, K., ‘ “On the permissible curvature of space” by K Schwarzschild’, *Class. & Quant. Grav.* **15**, 2539, 1998.
- Thurston, W. P., Three-Dimensional Geometry and Topology, ed. Levy, S., Princeton, U.S.A.: Princeton University Press, 1997.
- Uzan J.-Ph., Lehoucq R. & Luminet J.-P., A new crystallographic method for detecting space topology, *Astron. & Astroph.* **351**, 766, *arXiv:astro-ph/9903155*, 1999.
- Weinberg, S., Gravitation and Cosmology, New York, U.S.A.: Wiley, 1972.