5D Radiating black holes in Einstein-Yang-Mills-Gauss-Bonnet gravity

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We derive nonstatic spherically symmetric solutions of a null fluid, in five dimension (5D), to Einstein-Yang-Mills (EYM) equations with the coupling of Gauss-Bonnet (GB) combination of quadratic curvature terms, namely, 5D-EYMGB radiating black hole solution. It is shown that, in the limit, we can recover known radiating black hole solutions. The spherically symmetric known 5D static black hole solutions are also retrieved. The effect of the GB term and Yang-Mills (YM) gauge charge on the structure and location of horizons, of the 5D radiating black hole, is also discussed.

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I. INTRODUCTION

Recent years have witnessed a renewed interest to study black hole solutions in string-generated gravity models which mainly is accomplished by studying solutions of the Einstein theory supplemented by Gauss-Bonnet (GB) term [1, 2]. String theory also predicts quantum corrections to classical gravity theory and the GB term is the only one leading to second order differential equations in the metric. On the other hand the black hole solutions in gravity coupled to fields of different types have always drew in much attention, in particular, a great interest in solutions to Einstein-Yang-Mills (EYM) systems [3–10]. Wu and Yang [11] obtained static symmetric solution of Yang-Mills equation for the *isospin* gauge group $SO(3)$. The remarkable feature of this Wu-Yang *ansatz* is that the field has no contribution from gradient and instead has pure YM non-Abelian component. A curved-space generalization of the Wu-Yang solutions [11] for the gauge group $SO(3)$ is shown to be a special case of Yasskin's [3] solutions. It is known that non-Abelian gauge theory coupled to gravitation, i.e., EYM results to precisely the geometry of Reissner-Nordström with the charge that determines the geometry is gauge charge [3–5]. Indeed, Yasskin [3] gave an explicit theorem so that from each solution of the Einstein-Maxwell equations one can get solutions of EYM equations. One would like to study how these features get modified in higher-dimensional (HD) spacetimes and whether this theorem holds in HD spacetimes. Recent developments in string theory indicate that gravity may be truly HD theory, becoming effectively four dimensional (4D) at lower energies. Since non-Abelian gauge fields also feature in the low energy effective action of string theory, it is interesting to study the properties of the corresponding EYM in presence of GB terms. Mazharimousavi and Halilsoy [6, 7] have found static spherically symmetric HD black holesolutions to coupled set of equations of the EYMGB, for $SO(N-1)$ gauge group, systems which are based on the Wu and Yang [11] *ansatz*. The corresponding static topological black holes have been found independently by others [8–10].

It would be interesting to further consider nonstatic generalization of Mazharimousavi and Halilsoy solutions [6, 7]. It is the purpose of this letter to obtain an exact nonstatic solution of the 5D EYMGB theory in the presence of a null fluid and by employing the Wu-Yang *ansatz*. We shall present a class of 5D nonstatic solutions describing the exterior of radiating black holes with null fluid endowed with gauge charge, i.e., an exact Vaidya like solution in 5D EYMGB theory. The Vaidya geometry permitting the incorporations of the effects of null fluid offers a more realistic background than static geometries, where all back reaction is ignored. It may be noted one of few nonstatic black hole solutions is Vaidya [12] which is a solution of Einstein's equations with spherical symmetry for a null fluid (radially propagating radiation) source. It is possible to model the radiating star by matching them to exterior Vaidya spacetime (see [13, 14] for reviews on Vaidya solution and [15] for it's higher dimensional version). This letter also examines the effect of the GB terms and YM gauge charge on the structure and location of the horizons for the radiating black holes. A black holehas three horizon like surface [16, 17]: timelike limit surface (TLS), apparent horizons (AH) and event horizons (EH). In general the three horizon does not coincide and they are sensitive to small perturbation. For a classical Schwarzschild black hole (which does not radiate), the three surfaces EH, AH, and TLS are all identical. Upon switching on the Hawking evaporation this degeneracy is partially lifted even if the spherical symmetry stays. We have then AH=TLS, but the EH is different from AH=TLS. In particular, the AH is located inside the EH, the portion of spacetime between the two surfaces forming the so-called quantum ergosphere. If we break spherically symmetry preserving stationarity (e.g., Kerr black hole), then AH=EH but EH $\neq$ TLS. Here the ergosphere is the space between the horizon EH=AH and the TLS, usually called the static limit [16]. In both cases particles and light signals can escape from within the ergosphere and reach infinity. The characteristics of EH and AH associated with black holes in 5D EYMGB

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are also discussed.

II. VAIDYA LIKE SOLUTION IN 5D EYMGB THEORY

We consider $SO(4)$ gauge theory with structure constant $C^{(\alpha)}_{(\beta)(\gamma)}$, the YM fields $F^{(\alpha)}_{ab}$ and the YM potential $A^{(\alpha)}_a$. The gauge potentials $A^{(\alpha)}_a$ and the Yang-Mills fields $F^{(\alpha)}_{ab}$ are related through the equation

$$F^{(\alpha)}_{ab} = \partial_a A^{(\alpha)}_b - \partial_b A^{(\alpha)}_a + \frac{1}{2\sigma} C^{(\alpha)}_{(\beta)(\gamma)} A^{(\beta)}_a A^{(\gamma)}_b. \qquad (1)$$

We note that the internal indices $\{\alpha, \beta, \gamma, ...\}$ do not differ whether in covariant or contravariant form. The action which describes EMYGB theory in 5D reads [6, 7]:

$$\mathcal{I}_{\mathcal{G}} = \frac{1}{2}\int_M dx^5 \sqrt{-g}\left[(R + \omega' L_{GB}) - \sum_{\alpha=1}^{6} F^{(\alpha)}_{ab} F^{(\alpha)ab}\right]. \qquad (2)$$

Here, $g = \det(g_{ab})$ is the determinant of the metric tensor, R is the Ricci Scalar and $\omega' = \omega/2$ with ω the coupling constant of the GB terms. This type of action is derived in the low-energy limit of heterotic superstring theory [18]. In that case, ω is regarded as the inverse string tension and positive definite, and we consider only the case with $\omega \geq 0$ in this paper. Expressed in terms of Eddington advanced time coordinate (ingoing coordinate) v, with the metric ansatz of 5D spherically symmetric spacetime [15, 19, 20]:

$$ds^2 = -A(v,r)^2 f(v,r)\, dv^2 + 2A(v,r)\, dv\, dr + r^2 d\Omega_3^2, \qquad (3)$$

where $d\Omega_3^2 = d\theta^2 + \sin^2\theta d\phi^2 + \sin^2\theta \sin^2\phi^2 d\psi^2$. Here A is an arbitrary function of v and r and $\{x^a\} = \{v,\ r,\ \theta,\ \phi,\ \psi\}$. We wish to find the general solution of the Einstein equation for the matter field given by Eq. (13) for the metric (3), which contains two arbitrary functions. It is the field equation $G^0_1 = 0$ that leads to $A(v,r) = g(v)$ [15, 19]. This could be absorbed by writing $d\tilde{v} = g(v)dv$. Hence, without loss of generality, the metric (3) takes the form ,

$$ds^2 = -f(v,r)dv^2 + 2dvdr + r^2 d\Omega_3^2. \qquad (4)$$

We introduce the Wu-Yang *ansatz* in 5D [6, 7] as

$$\begin{aligned} A^{(\alpha)} &= \frac{Q}{r^2}\left(x_i dx_j - x_j dx_i\right), \qquad (5)\\ 2 &\leq i \leq 4,\\ 1 &\leq j \leq i-1,\\ 1 &\leq (\alpha) \leq 6, \end{aligned}$$

where the super indices α is chosen according to the values of i and j in order[6, 7]. It is easy to see that for the metric 4), the YM matter field equations admit solution $\sigma = Q$ [6, 7]. The Wu-Yang solution appears highly non-linear because of mixing between spacetime indices and gauge group indices. However, it is linear as expressed in the non-linear gauge fields because purely magnetic gauge charge is chosen along with position dependent gauge field transformation [3]. The YM field 2-form is defined by the expression

$$F^{(\alpha)} = dA^{(\alpha)} + \frac{1}{2Q} C^{(\alpha)}_{(\beta)(\gamma)} A^{(\beta)} \wedge A^{(\gamma)}. \qquad (6)$$

The integrability conditions

$$dF^{(\alpha)} + \frac{1}{Q} C^{(\alpha)}_{(\beta)(c)} A^{(\beta)} \wedge F^{(\gamma)} = 0, \qquad (7)$$

as well as the YM equations

$$d * F^{(\alpha)} + \frac{1}{Q} C^{(\alpha)}_{(\beta)(\gamma)} A^{(\beta)} \wedge *F^{(\gamma)} = 0, \qquad (8)$$

are all satisfied. Here d is exterior derivative, $\wedge$ stands for wedge product and $*$ represents Hodge duality. All these are in the usual exterior differential forms notation. The GB Lagrangian is of the form

$$L_{GB} = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}. \qquad (9)$$

The action (2) leads to following set of field equations:

$$\mathcal{G}_{ab} \equiv G_{ab} + \omega' H_{ab} = T_{ab}, \qquad (10)$$

where

$$G_{ab} = R_{ab} - \frac{1}{2} g_{ab} R, \qquad (11)$$

is the Einstein tensor and

$$\begin{aligned} H_{ab} &= 2[RR_{ab} - 2R_{a\alpha}R^{\alpha}_b - 2R^{\alpha\beta}R_{a\alpha b\beta} + R^{\alpha\beta\gamma}_a R_{b\alpha\beta\gamma}]\\ &\quad -\frac{1}{2} g_{ab} L_{GB}, \qquad (12) \end{aligned}$$

is the Lanczos tensor.

The stress-energy tensor is written as

$$T_{ab} = T^G_{ab} + T^N_{ab}, \qquad (13)$$

where the gauge stress-energy tensor T^G_{ab} is

$$T^G_{ab} = \sum_{\alpha=1}^{6}\left[2F^{(\alpha)\lambda}_a F^{(\alpha)}_{b\lambda} - \frac{1}{2} F^{(\alpha)}_{\lambda\sigma} F^{(\alpha)\lambda\sigma} g_{ab}\right], \qquad (14)$$

The energy-momentum tensor of a null fluid is

$$T_{ab} = \psi(v,r)\beta_a\beta_b, \qquad (15)$$

where $\psi(v,r)$ is the non-zero energy density and β_a is a null vector with

$$\beta_a = \delta^0_a, \beta_a\beta^a = 0. \qquad (16)$$

Introducing

$$\begin{aligned} x_1 &= r\cos\psi\sin\phi\sin\theta,\\ x_2 &= r\sin\psi\sin\phi\sin\theta,\\ x_3 &= r\cos\phi\sin\theta,\\ x_4 &= r\cos\theta, \end{aligned}$$

and using *ansatz* (5) one obtains

$$\begin{aligned}
A^{(1)} &= -Q \sin^2\phi \sin^2\theta \, d\psi, \\
A^{(2)} &= Q \sin^2\theta \left(\cos\psi \, d\phi - \cos\phi \sin\psi \sin\phi \, d\psi\right), \\
A^{(3)} &= Q \sin^2\theta \left(\sin\psi \, d\phi + \cos\phi \cos\psi \sin\phi \, d\psi\right), \\
A^{(4)} &= Q\big(\sin\theta \left(\cos\psi \cos\phi d\phi - \sin\psi \sin\phi \, d\psi\right)\cos\theta \\
&+ \cos\psi \sin\phi \, d\theta\big), \\
A^{(5)} &= Q \left(\cos\phi \, d\theta - \cos\theta \sin\phi \sin\theta \, d\phi\right), \\
A^{(6)} &= Q \left(\cos\phi d\theta - \cos\theta \sin\phi \sin\theta \, d\phi\right).
\end{aligned}$$

We then find that the nonzero components would read as: $T_v^r = \psi(v,r)$, $T_v^v = T_r^r = -3Q^2/(2r^4)$ and $T_\theta^\theta = T_\phi^\phi = T_\psi^\psi = Q^2/r^4$. It may be recalled that energy-momentum tensor (EMT) of a Type II fluid has a double null eigenvector, whereas an EMT of a Type I fluid has only one timelike eigenvector [13, 21]. It may be noted that, the gauge field has only the angular components, $F^\alpha_{\theta_i\theta_j}$ with $i \neq j$, nonzero and they go as r^{-2} which in turn make T^G_{ab} go as r^{-4}.

The only non-trivial components of the EGB tensor ($\mathcal{G}^a_b$), in a unit system with $\omega' = \omega/2$, take the form:

$$\mathcal{G}^v_v = \mathcal{G}^r_r = f' - \frac{2}{r}(1-f) + \frac{4\omega}{r^2}(1-f)f', \tag{17}$$

$$\mathcal{G}^\theta_\theta = \mathcal{G}^\phi_\phi = \mathcal{G}^\psi_\psi = f'' + \frac{4}{r}f' + \frac{2}{r^2}(1-f) + \frac{4\omega}{r^2}\left[f''(1-f) + f'^2\right], \tag{18}$$

$$\mathcal{G}^r_v = \frac{3}{2}\frac{\dot{f}}{r} + \frac{6\omega}{r^3}\dot{f}(1-f). \tag{19}$$

Then, $f(v,r)$ is obtained by solving only the (10), The equation $\mathcal{G}^v_v = T^v_v$ is integrated to give the general solution as

$$f(v,r) = 1 + \frac{r^2}{2\omega}\left[1 \pm \sqrt{1 + \frac{4\omega M(v)}{r^4} - \frac{8\omega Q^2 \ln r}{r^4} + \frac{4\omega^2}{r^4}}\right], \tag{20}$$

where $M(v)$ is positive and an arbitrary function of v identified as mass of the matter. The gauge charge Q can be either positive or negative. The special case in which $\dot{M}(v) = 0$ and $Q^2 = 0$, Eq. (17) leads to GB-Schwarzschild solution, of which the global structure is presented in [22]. The solution (20) is a general spherically symmetric solution of the 5D EYMGB theory with the metric (4) for the null fluid defined by the energy momentum tensor (15). Since YM T^G_{ab} go as r^{-4} (the same as for Maxwell field in $D = 4$), for $5D$. That is why its contribution in $f(v,r)$ will be the same for $5D$ as in 4D Reissner-Nordstrom (RN) black hole [20]. The nonradiating limit of this would be 5D-Yaskin black hole and not 5D analogue of Reissner-Nordström.

There are two families of solutions which correspond to the sign in front of the square root in Eq. (20). We call the family which has the minus (plus) sign the minus-(plus+) branch solution. From $\mathcal{G}^r_v = T^r_v$, we obtain the energy density of the null fluid as

$$\psi(v,r) = \frac{3}{2}\frac{\dot{M}(v)}{r^3}. \tag{21}$$

for both branches, where the dot denotes the derivative with respect to v. We first turn out attention to the three limiting case when the solution is known. These are (i) $M(v) \neq 0$, $Q = 0$ and $\omega \neq 0$ then 5D-EGB black holes [19, 23, 24]. The solution of the Eq. (17) is

$$f(v,r) = 1 + \frac{r^2}{2\omega}\left[1 \pm \sqrt{1 + \frac{4\omega M(v)}{r^4}}\right], \tag{22}$$

(ii) $M(v) \neq 0$, $Q \neq 0$ and $\omega = 0$ then the 5D-EYM black holes [20]. Now one has solution of the Eq. (17) as

$$f(v,r) = 1 - \frac{M(v)}{r^2} - \frac{2Q^2 \ln r}{r^2}, \tag{23}$$

and (iii) in the general relativistic limit $\omega \to 0$ and $Q^2 \to 0$, the minus-branch solution reduces to

$$f(v,r) = 1 - \frac{M(v)}{r^2}, \tag{24}$$

which is the $5D$ Vaidya solution [15, 19] in Einstein theory. It may noted that, in $5D$ Einstein theory, the density is still given by Eq. (21). There is no such limit for the plus-branch solution. The family of solutions discussed here belongs to Type II fluid. However, In the static case $M =$ constant and the matter field degenerates to type I fluid, we can generate static black hole solutions obtained in [6, 7] by proper choice of these constants. In the static limit, this metric can be obtained from the metric in the usual spherically symmetric form,

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2(d\Omega_3)^2. \tag{25}$$

with

$$f(r) = 1 + \frac{r^2}{2\omega}\left[1 \pm \sqrt{1 + \frac{4\omega M}{r^4} - \frac{8\omega Q^2 \ln r}{r^4} + \frac{4\omega^2}{r^4}}\right], \tag{26}$$

if $Q^2 \to 0$ this solution reduces to the solution which was independently discovered by Boulware and Deser [1] and Wheeler [2].

The Kretschmann scalar ($K = R_{abcd}R^{abcd}$, R_{abcd} is the 5D Riemann tensor) and Ricci scalar ($R = R_{ab}R^{ab}$, R_{ab} is the 5D Ricci tensor) for the metric (4) reduces to

$$\mathrm{K} = f''^2 + \frac{6}{r^4}f'^2 + \frac{12}{r^4}(1-f)^2, \tag{27}$$

and

$$R = f'' + \frac{6}{r}f' - \frac{6}{r^2}(1-f), \tag{28}$$

Radial (θ and ϕ = *const.*) null geodesics of the metric (4) must satisfy the null condition

$$2\frac{dr}{dv} = 1 + \frac{r^2}{4\omega}\left[1 \pm \sqrt{1 + \frac{4\omega M(v)}{r^4} - \frac{8\omega Q^2 \ln r}{r^4} + \frac{4\omega^2}{r^4}}\right], \tag{29}$$

The invariants are regular everywhere except at the origin $r = 0$, where they diverge. Hence, the spacetime has the scalar polynomial singularity [21] at $r = 0$. The nature (a naked singularity or a black hole) of the singularity can be characterized by the existence of radial null geodesics emerging from the singularity. The singularity is at least locally naked if there exist such geodesics, and if no such geodesics exist, it is a black hole. The study of causal structure of the spacetime is beyond the scope of this paper and will be discussed elsewhere [25].

Energy conditions: The family of solutions discussed here, in general, belongs to Type II fluid defined in [21]. When $m = m(r)$, we have ψ=0, and the matter field degenerates to type I fluid [13, 14]. In the rest frame associated with the observer, the energy-density of the matter will be given by

$$\mu = T^r_v, \quad \rho = -T^t_t = -T^r_r \tag{30}$$

and the principal pressures are $P_i = T^i_i$ (no sum convention) and due to isotropy $P = P_i$ for all i.

a) The weak energy conditions (WEC): The energy momentum tensor obeys inequality $T_{ab}w^a w^b \geq 0$ for any timelike vector [21], i.e., , i.e., $\psi \geq 0, \quad \rho \geq 0, \quad P \geq 0$,. We say that strong energy condition (SEC), holds for Type II fluid if, WEC is true., i.e., both WEC and SEC, for a Type II fluid, are identical [13].

b) The dominant energy conditions (DEC) : For any timelike vector w_a, $T^{ab}w_a w_b \geq 0$, and $T^{ab}w_a$ is non-spacelike vector , i.e., $\psi \geq 0, \quad \rho \geq P \geq 0$. Hence WEC and SEC are satisfied if $\dot{M}(v) \geq 0$. In addition DEC also holds.

III. HORIZONS OF 5D RADIATING BLACK HOLE

The line element of the radiating black hole in 5D EYMGB theory has the form (4) with $f(v,r)$ given by Eq. (20) and the energy momentum tensor (15). The luminosity due to loss of mass is given by $L = -dM/dv$, $L < 1$ where $L < 1$. Both are measured in the region where d/dv is time-like. In order to further discuss the physical nature of our solutions, we introduce their kinematical parameters. As first demonstrated by York [16] and later by others [17, 19], the horizons may be obtained to $O(L)$ by noting that a null-vector decomposition of the metric (4) is made of the form

$$g_{ab} = -\beta_a l_b - l_a \beta_b + \gamma_{ab}, \tag{31}$$

where,

$$\beta_a = -\delta^v_a, \; l_a = -\frac{1}{2}f(v,r)\delta^v_a + \delta^r_a, \tag{32}$$

$$\gamma_{ab} = r^2\delta^\theta_a\delta^\theta_b + r^2\sin^2(\theta)\delta^\varphi_a\delta^\varphi_b + r^2\sin^2(\theta)\sin^2(\phi)\delta^\psi_a\delta^\psi_b, \tag{33}$$

$$l_a l^a = \beta_a\beta^a = 0, \;\; l_a\beta^a = -1, \;\; l^a\,\gamma_{ab} = 0, \;\; \gamma_{ab}\;\beta^b = 0, \tag{34}$$

with $f(v,r)$ given by Eq. (20). The Raychaudhuri equation of null geodesic congruence is

$$\frac{d\Theta}{dv} = \kappa\Theta - R_{ab}l^a l^b - \frac{1}{2}\Theta^2 - \sigma_{ab}\sigma^{ab} + \Omega_{ab}\Omega^{ab}, \tag{35}$$

with expansion Θ, twist Ω, shear σ, and surface gravity κ. The expansion of the null rays parameterized by v is given by

$$\Theta = \nabla_a l^a - \kappa, \tag{36}$$

where the ∇ is the covariant derivative and the surface gravity is

$$\kappa = -\beta^a l^b \nabla_b l_a. \tag{37}$$

In the case of spherically symmetric, the vorticity and shear of l_a are zero. Substituting Eqs. (20) and (32) into (37), we obtain surface gravity

$$\kappa = \frac{r}{2\omega}\left[1 - \sqrt{1 + \frac{4\omega M(v)}{r^4} - \frac{8\omega Q^2\ln r}{r^4} + \frac{4\omega^2}{r^4}}\right] + \frac{\frac{2M(v)}{r^3} + \frac{Q^2}{r^3} - \frac{4Q^2\ln r}{r^3} + \frac{2\omega}{r^3}}{\sqrt{1 + \frac{4\omega M(v)}{r^4} - \frac{8\omega Q^2\ln r}{r^4} + \frac{4\omega^2}{r^4}}}. \tag{38}$$

Then Eqs. (20), (32), (36), and (39) yields the expansion of null ray congruence:-

$$\Theta = \frac{3}{2r}\left[1 + \frac{r^2}{4\omega}\left[1 - \sqrt{1 + \frac{4\omega M(v)}{r^4} - \frac{8\omega Q^2\ln r}{r^4} + \frac{4\omega^2}{r^4}}\right]\right]. \tag{39}$$

The apparent horizon (AH) is the outermost marginally trapped surface for the outgoing photons. The AH can be either null or space-like, that is, it can 'move' causally or acausally [16]. The apparent horizons are defined as surface such that $\Theta \simeq 0$ which implies that $f = 0$. From the Eq. (39) it is clear that AH is the solution of

$$\left[1 + \frac{r^2}{2\omega}\left[1 - \sqrt{1 + \frac{4\omega M(v)}{r^4} - \frac{8\omega Q^2\ln r}{r^4} + \frac{4\omega^2}{r^4}}\right]\right] = 0. \tag{40}$$

i.e., zeros of

$$r^2 - M(v) + 2Q^2\ln(r) = 0. \tag{41}$$

For $Q \to 0$ and constant M, we have 5D Schwarzschild horizon $r = \sqrt{M}$. In general, Eq. (41), which admit solutions

$$r_{IAH} = \exp\left[-\frac{1}{2}\frac{Q^2\mathrm{LambertW}\,(0,x) + M(v)}{Q^2}\right]. \tag{42}$$

$$r_{OAH} = \exp\left[-\frac{1}{2}\frac{Q^2\text{LambertW}\left(-1,x\right)+M(v)}{Q^2}\right]. \quad (43)$$

Here

$$x = -\frac{\exp(-M(v)/Q^2)}{Q^2}.$$

Here r_{IAH} and r_{OAH} are respectively inner and outer horizons and the LambertW function satisfies $\text{LambertW}(x)\exp\left[\text{LambertW}(x)\right] = x$. The important feature of Eq. (43) is that it is ω independent. This lead to fact that it is similar to pure EYM case. Thus the GB term does not cause the AHs of the 5D-EYM black-holes to be distorted. The TLS for a black hole, with a small luminosity, is locus where $g_{vv} = 0$. Here one sees that $\Theta = 0$, implies $f = 0$ or $g_{vv}(r = r_{AH}) = 0$ implies that $r = r_{AH}$ is TLS and AH and TLS coincide in our non-rotational case. The pure charged case ($M(v) = 0$) is also important, then we have horizon without mass

$$r_{IAH} = \exp\left[-\frac{1}{2}\text{LambertW}\left(0,\frac{1}{Q^2}\right)\right], \quad (44)$$

$$r_{OAH} = \exp\left[-\frac{1}{2}\text{LambertW}\left(-1,\frac{1}{Q^2}\right)\right]. \quad (45)$$

For an outgoing null geodesic, $\dot{r}$ is given by Eq. (29). Differentiation of (29) w.r.t. v gives

$$\ddot{r} = \frac{r\dot{r}}{2\omega}\left[1-\sqrt{1+\frac{4\omega M(v)}{r^4}-\frac{8\omega Q^2\ln r}{r^4}+\frac{4\omega^2}{r^4}}\right] + \frac{\frac{L}{2r^2}+\frac{2M(v)\dot{r}}{r^3}+\frac{Q^2\dot{r}}{r^3}-\frac{4Q^2\ln r\dot{r}}{r^3}+\frac{2\omega\dot{r}}{r^3}}{\sqrt{1+\frac{4\omega M(v)}{r^4}-\frac{8\omega Q^2\ln r}{r^4}+\frac{4\omega^2}{r^4}}}. \quad (46)$$

At the time-like surface $r = r_{AH}$, $\dot{r} = 0$ and $\ddot{r} > 0$ for $L > 0$. Hence photons escape from $r = r_{AH}$ to the to reach arbitrarily large distances from the hole.

However, In the general GB term does change the location of AH, e.g., in the limit $Q \to 0$, in the 5D-EGB case the AHs reads [19]

$$r_{AH} = \sqrt{M(v) - 2\omega} \quad (47)$$

Further, In the relativistic limit $\omega \to 0, Q \to 0$ then $r_{AH} \to \sqrt{M(v)}$.

The future event horizon (EH) is the boundary of the causal past of future null infinity, and it represents the locus of outgoing future-directed null geodesic rays that never manage to reach arbitrarily large distances from the hole. This definition requiring knowledge of the entire future history of the hole. However, York [16], for a radiating black holes, argued that the question of the escape versus trapping of null rays is, physically, matter of qualitative degree and proposed a working definition of definition as follows: the EH are strictly null and are defined to order of $O(L)$ and Photons are in captivity by event horizon for times long compared to dynamical scale of the hole. It can be seen to be equivalent to the requirement that for acceleration of null-geodesic congruence at the EH,

$$\left[\frac{d^2r}{dv^2}\right]_{\text{EH}} \simeq 0. \quad (48)$$

This criterion enables us to distinguish the AH and the EH to necessary accuracy. An outgoing radial null geodesic which is parameterized by v satisfy

$$\frac{dr}{dv} = \frac{1}{2}\left[1+\frac{r^2}{4\omega}\left[1-\sqrt{1+\frac{4\omega M(v)}{r^4}-\frac{8\omega Q^2\ln r}{r^4}+\frac{4\omega^2}{r^4}}\right]\right]. \quad (49)$$

Then Eqs. (39) and (39) can be used to put Eq. (48) in the form

$$\kappa\Theta_{EH} \simeq \left[\frac{3}{2r}\frac{\partial f}{\partial v}\right]_{EH} \simeq \frac{1}{2r_{EH}^3}\frac{3L}{\sqrt{1+\frac{4\omega M(v)}{r_{EH}^4}-\frac{8\omega Q^2\ln r_{EH}}{r_{EH}^4}+\frac{4\omega^2}{r_{EH}^4}}} \quad (50)$$

where the expansion is

$$\Theta_{EH} \simeq \frac{3}{2r_{EH}}\left[1+\frac{r_{EH}^2}{4\omega}\left[1-\sqrt{1+\frac{4\omega M(v)}{r_{EH}^4}-\frac{8\omega Q^2\ln r_{EH}}{r_{EH}^4}+\frac{4\omega^2}{r_{EH}^4}}\right]\right]. \quad (51)$$

For the null vectors l_a in Eq. (32) and the component of energy momentum tensor yields

$$R_{ab}l^al^b = \frac{3}{2r}\frac{\partial f}{\partial v}. \quad (52)$$

The Raychaudhuri equation, with $\sigma = \Omega = 0$ [16]:

$$\frac{d\Theta}{dv} = \kappa\Theta - R_{ab}l^al^b - \frac{1}{2}\Theta^2. \quad (53)$$

Since EH are defined to $O(L)$, we neglect Θ^2, as $\Theta^2 = O(L^2)$. Eqs. (50), (52) and (53), imply that

$$\left[\frac{d\Theta}{dv}\right]_{EH} \simeq 0. \quad (54)$$

Following [16, 19], for low luminosity, the surface gravity κ can be evaluated at AH and the expression for the EH can be obtained to $O(L)$. It can be shown that the expression for the event horizon is the same as that for the apparent horizon with M being replaced by M^* [19], where M^* is effective mass defined as follows: $M^*(v) = M(v) - L/\kappa$. From the eq. (47), it is clear that, in general, the presence of the coupling constant, of the GB terms, ω produces a change in the location of horizons. Such a change could have a significant effect in the dynamical evolution of these horizons. For nonzero ω the structure of the horizons is nontrivial. However, the eq. (43) is independent of the of the GB coupling constant ω, i.e., AH are exactly same as that in EYM without GB coupling constant ω. Thus the GB term does not alter the horizons of the 5D EYM black holes.

IV. DISCUSSION AND CONCLUSION

In this letter we have obtain an exact black hole solution that describe a null fluid in the framework of 5D EYMGB theory by employing 5D curved space generalization Wu-Yang *ansatz*. Thus we have an explicit nonstatic radiating black hole solution of 5D EYMGB theory. We have used the solution to discuss the consequence of GB term and YM charge on the structure and location of the horizons 5D radiating black hole. The AH's are obtained exactly and EH's are obtained to first order in luminosity by method developed by York [16]. We shown that a 5D radiating black hole in EYGB has three important horizon-like loci that full characterizes its structure, viz. AH, EH and TLS and we have relationship of the three surfaces $r_{EH} < r_{AH} = r_{TLS}$ and the region between the AH and EH is defined as *quantum ergosphere.* The presence of the coupling constant of the Gauss-Bonnet terms ω produces a change in the location of these horizons [19]. Such a change could have a significant effect in the dynamical evolution of these horizons. However, It turns out that the presence of the coupling constant of the GB terms $\omega > 0$ does not alter the location of the horizons from the analogous EYM case, i.e., horizons of the 5D EYM and 5D EYMGB are absolutely same when obtained by procedure suggested by York [16] to $O(L)$ by a null-vector decomposition of the metric.

In 4D, the Vaidya like solution of EYM yields same results as one would expect for the charged null fluid in EM theory, i.e., the geometry is precisely of the charged-Vaidya form and the charge that determines the geometry is magnetic charge. This is because T^G_{ab} go over r^{-4} which is exactly same as energy momentum of EM theory. However, this does not hold in 5D case because components of energy momentum tensor for EM and EYM theories are not same. Thus the Yaaskin's [3] theorem does not hold in 5D case. The 5D solution discussed here incorporates a logarithmic term unprecedent in 4D.

The family of solutions discussed here belongs to Type II fluid. However, if $M =$ constant and the matter field degenerates to type I fluid, we can generate static black hole solutions obtained in [6, 7] by proper choice of these constants. In particular, our results in the limit $\omega \to 0$ and $Q \to 0$ reduce *vis-à-vis* to 5D relativistic case.

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