THE RADIATION OF MICROWAVES AND INFRARED BY SLENDER GRAPHITE NEEDLES

(Letter to the Editor)

F. HOYLE
Dockray, Penrith, Cumbria

J. V. NARLIKAR
Tata Institute of Fundamental Research, Bombay, India

and

N. C. WICKRAMASINGHE
Department of Applied Mathematics and Astronomy, University College, Cardiff, Wales, U.K.

(Received 12 March, 1984)

Abstract. It has become clear from sources observed by IRAS that there are many examples which attain maximum emission per unit (frequency) bandwidth longward of 100 μm. Here, we show that a large emissivity at such long wavelengths can be obtained by needle-shaped particles of free carbon. Indeed for needles with a sufficiently high ratio of length to diameter the large emissivity extends from the infrared through the whole microwave region of the spectrum.

1. Graphite Whiskers

Carbon particles in the form of graphite would seem to qualify as a candidate for widespread distribution in the Universe. On the Earth, deposits of graphite occur in certain metamorphic rocks in the form of tabular sheet-like accumulations or needle-shaped filaments. Similar graphitic structures occur within carbonaceous meteorites.

Graphite is an extremely refractory material with sublimation temperatures under certain conditions in excess of 3800 K. The optical and electrical properties of graphite are markedly anisotropic due to its hexagonal crystal structure. Hexagonal units, each comprised of 6 carbon atoms, are arranged into tabular stacks to form flakes or wrapped helically into long cylindrical whiskers. The axis of symmetry in each case is the axis of maximum electrical conductivity.

A major component of interstellar grains is now thought to be in the form of spherical (polycrystalline) graphite particles with average radii ~ 0.02 μm. Such particles might arise in mass loss from carbon stars (Hoyle and Wickramasinghe, 1962), or less popularly, they could be thought to arise as degradation products of microorganisms in space (Hoyle and Wickramasinghe, 1983). In the latter event, whilst charring of spherical microorganisms could give rise to graphite spheres, degradation of filamentary

Astrophysics and Space Science **103** (1984) 371–377. 0004–640X/84/1032–0371$01.05.

organisms – some of which have considerable lengths – would produce long whiskers.

More conservatively perhaps, long whiskers could condense readily from carbon vapour under conditions which might be astrophysically quite common. The condensation of graphite whiskers is now well attested in the laboratory. The first discovery of whiskers was reported by Davis *et al.* (1953) in the following words:

> In the course of experimental work on the deposition of carbon in the brickwork of blast furnaces (deposition which may cause the disintegration of the bricks), it has been found by electron micrography that the carbon is deposited as minute vermicular growths which can penetrate considerable thicknesses of brickwork. The carbon is formed by the interaction of carbon monoxide and iron oxide in the so-called iron spots in the brick. It has been found possible to reproduce this reaction in the laboratory by exposing samples of brick containing iron spots to the action of carbon monoxide at an optimum temperature of about 450 °C. Moreover, a similar form of carbon growth is observed if iron ore, magnetite or any form of iron oxide is substituted for the brick samples.
>
> The carbon vermicules have a characteristic appearance which is easily recognized – they may vary in thickness from 100 A or so up to about 0.2 micron, and they are generally somewhat helical in form.

Subsequently, more extensive experiments by Bacon (1960) showed that graphite whiskers condense readily from a barely saturated vapour in a dc graphite arc at a temperature of 3900 K and at a pressure of 92 atmospheres of argon gas. According to Bacon (1960),

> Diameters range from a fraction of a micron to over five microns, with recoverable lengths up to 3 cm. They (whiskers) consist of one or more concentric tubes, each tube being in the form of a scroll, or rolled-up sheet of graphite layers, extending continuously along the length of the whisker, with the *c*-axis exactly perpendicular to the whisker axis. They exhibit a high degree of flexibility, tensile strengths up to 2000 kg mm^{-2}, Young's modulus in excess of 7×10^{12} dyne cm^{-2}, ...

It would seem that whiskers with cross-sectional radius $\sim 0.5\,\mu$ and with lengths of several mm (radius/length $\sim 1/10^4$) are extremely stable, very common, and such condensations would seem to occur naturally under conditions where a carbon vapour becomes marginally saturated with respect to graphite at temperatures of 2000 K or more. Since whiskers grow in length exponentially with time, we would expect such particles to arise with considerable frequency wherever conditions become favourable for whisker growth.

2. Radiation According to the Johnson Noise Formula

The *Electrical Engineers' Handbook* (Wiley Handbook Series, H. Pender and K. McIlwain (eds.)) contains the following definition of the noise formula for an object of resistence R:

> The open-circuit rms noise voltage across a resistor is $\sqrt{4kTR\Delta\nu}$, where k is the Boltzmann constant, T the temperature, and $\Delta\nu$ the effective noise bandwidth of the instrument used to measure the voltage.

Although electrical engineers would attest to the correctness of this statement, for the reason that it has been amply demonstrated under macroscopic laboratory conditions, two conditions must be satisfied for it to hold good. One is that the frequency ν must be such that $h\nu \ll kT$, h being the Planck constant. Otherwise, as was pointed out by

Dicke, the product kT in the formula must be replaced by

$$\frac{h\nu}{\exp(h\nu/kT)-1}\,. \tag{1}$$

The second condition is that surface effects on the resistor be negligible. Otherwise, surface charges tend to suppress the open-circuit noise voltage, an effect that is particularly important when the resistor is a small particle. However, it will be useful to begin by calculating with the formula given above, bringing in these conditions at a later stage.

Since a voltage V (measurable with a voltmeter) requires a current V/R to flow in the resistor, it follows that the rms current oscillating with frequency in the small range ν to $\nu + \Delta\nu$ must be $\sqrt{4kT\Delta\nu/R}$ when the above formula is used. In this connection it may be noted that the instantaneous voltage measured across the resistor has the form $A(t)\cos 2\pi\nu t$, with the amplitude $A(t)$ varying slowly with respect to the time t, slowly on the time-scale $1/\nu$. It is the rms value of $A(t)$ that is $\sqrt{4kTR\Delta\nu}$. The current at any moment is $A(t)\cos 2\pi\nu t/R$, so that the rms amplitude of the current is obtained simply by dividing the rms value of $A(t)$ by R.

The statements just made rest on the technical meaning of the term 'resistor', which implies an object with zero inductance, otherwise the current would not keep immediately in phase with the voltage. No real physical object has a strictly zero inductance of course. For example, a current $A(t)\cos 2\pi\nu t/R$ flowing in a small needle-shaped resistor of length l will inevitably radiate like an oscillating dipole, thereby generating an inductance effect. However, so long as the radiation resistance of such a dipole is small compared with R, the concept of the needle acting as a 'resistor' is not seriously impaired. This condition is satisfied generally for microwaves but should be checked in detail for explicit applications at shorter wavelengths.

The power radiated by a current $A(t)\cos 2\pi\nu t/R$ flowing in a small needle-shaped particle of length l is*

$$\frac{8\pi^2}{3c}\,\frac{l^2}{\lambda^2}\,\frac{A^2(t)}{R^2}\,, \tag{2}$$

where $\lambda = c/\nu$. Insertion of the mean-square value $4kTR\Delta\nu$ for $A^2(t)$ gives for the power radiated into the frequency range ν to $\nu + \Delta\nu$ the expression

$$\frac{32}{3}\,\frac{\pi^2 l^2}{c^3 R}\,kT\nu^2\Delta\nu\,. \tag{3}$$

For a needle of radius a composed of material of conductivity σ the resistance R is given by $R = l/(\pi a^2\sigma)$, so that (3) takes the form

$$\frac{32}{3}\,\frac{\pi^2\sigma}{c^3}\,(\pi a^2 l)\,(kT\nu^2\Delta\nu)\,. \tag{4}$$

* For example, M. Abraham and R. Becker, *The Classical Theory of Electricity and Magnetism*, Blackie, London, p. 227.

Thus the power radiated is proportional to the volume of the particle and to its conductivity, and is of the form of the Rayleigh-Jeans longwave part of the Planck distribution. And if the Johnson formula is modified by replacing kT by (1), the power radiated becomes

$$\frac{32}{3}\frac{\pi^2\sigma}{c^3}\pi a^2 l \frac{h\nu^3\Delta\nu}{\exp(h\nu/kT)-1}, \tag{5}$$

which has the Planck form at all frequencies, a remarkable result.

3. A Uniform Spherical Cloud of Radiating Needle-Shaped Particles

Consider a cloud of radius r in which the number of similar-radiating needles is $(4\pi r^3/3)n$, the temperature of the needles being supposedly maintained at a value T. For a particle density that is sufficiently large the cloud radiates with an intensity equal to the black-body emission by a sphere of radius r at temperature T: namely,

$$4\pi r^2 \frac{c}{4}\frac{8\pi h}{c^3}\frac{\nu^3\Delta\nu}{\exp(h\nu/kT)-1}. \tag{6}$$

An approximation to this situation occurs for a number density n obtained by multiplying (5) by $5\pi r^3 n/3$ and then equating it to (6), we see that

$$2\pi a^2 lnr = \frac{9c}{8\pi\sigma}. \tag{7}$$

Since the left-hand side of (7) is the volume of the grain material across a diameter of the cloud, we have the right-hand side as the determinant of the column density at which a cloud of particles becomes 'optically thick' with respect to the black-body frequency distribution at the temperature of the particles themselves. Or if one imagines all the particles in the cloud to be projected normally onto a plane, they would fall in a circular area of the plane of radius r with a thickness of the order of the right-hand side of (7), which for the usually-quoted low-frequency conductivity of graphite, $\sigma \cong 10^{15}\ \text{s}^{-1}$, is $\sim 10^{-5}$ cm, a remarkably small thickness. Very little in the way of graphite needles is required to generate an approximation to the black-body distribution, all this being on the supposition that the Johnson–Dicke noise formula holds good. We now proceed to examine the second of the two conditions mentioned above.

4. A Condition on the Ratio of Length to Diameter for Needle-Shaped Particles

Writing $\pm Q(t)$ for the charges built up by the noise current over the frequency range ν to $\nu + \Delta\nu$, with the charges appearing towards the ends of needle-shaped particles of length l and diameter $2a$, we have

$$\dot{Q} = A(t)\cos 2\pi\nu t/R, \tag{8}$$

$$Q = A(t)\sin 2\pi\nu t/2\pi R\nu. \tag{9}$$

The field arising from the maximum value of Q, $A(t)/2\pi R\nu$ can be regarded as equivalent to a voltage $\sim 4A(t)/lR\nu$ tending to oppose the noise voltage $A(t)$. It follows that unless $l\nu R > 4$ the noise formula is grossly changed under open-circuit conditions. Putting $R = l/\pi a^2\sigma$, the requirement on the ratio of length to diameter is, therefore, seen to be

$$\frac{l}{2a} > \left(\frac{\pi\sigma}{\nu}\right)^{1/2} = \left(\frac{\pi\sigma}{c}\right)^{1/2} \lambda^{1/2} \cong 300\lambda^{1/2} \tag{10}$$

for graphite, with the wavelength λ in cm. Hence, to generate microwaves with the high efficiency calculated above graphite needles must be very slender.

As we mentioned earlier, there are good arguments for thinking that graphite particles in space would be needle-shaped with length to cross-sectional radius ratios $\sim 10^4$. Thus, graphite particles which condense as hot carbon vapour cools slowly are found to appear as long threads or 'whiskers', as discussed above in Section 1. Moreover, graphite of biological origin is very frequently found in fossil specimens to be filamentous, threadlike forms being exceedingly common throughout biology. Such knowledge as we have would, therefore, suggests that long needle-shaped particles might well be widespread also in astronomy. Indeed, if prejudices in this direction were themselves widespread in astronomy, the observed existence of cosmic microwaves would have been taken as strong evidence for the existence of such particles, whereas the passion with which most astronomers want to think of the observed microwaves as a relic of the origin of the Universe has led them *nem. con.* to overlook this alternative possibility for their generation.

5. The Emissivity of Needle-Shaped Graphite Grains

A small macroscopic conducting sphere immersed in a radiation field, which need not be thermodynamic, is said to attain the black-sphere temperature if it absorbs all the incident radiation and if it radiates like a black body. A small dust grain also attains the black-sphere temperature if it radiates the Planck frequency distribution multiplied by an emissivity factor ε that is independent of λ. Needle-shaped grains that are sufficiently long and of sufficiently small width emit according to (5), from which it can be seen that insofar as the conductivity σ is considered to be independent of λ such grains attain the black-sphere temperature when immersed in a radiation field.

Except in the ultraviolet, σ for graphite is usually considered to be only weakly dependent on λ, and so slender graphite needles take on the so-called black-sphere temperature when immersed in a radiation field involving mainly visible light or infrared. At the latter wavelengths the absorptivity α of bulk graphite is $\sim 10^5$ cm^{-1}, and the mass absorption coefficient $\kappa = \alpha/s$ (where s is the specific gravity of graphite) is about 40000 cm^2 g^{-1}. In the ultraviolet near 2000 Å, on the other hand, κ is about 600000 cm^2 g^{-1} for grains that are small enough, this being a consequence of the dramatic behaviour of the optical constants of graphite near 2000 Å, the famous 'graphite signature'. Thus, a sufficiently slender graphite needle heated in an ultraviolet radiation field, as for instance an interstellar grain in the vicinity of a B- or O-type star, absorbs considerably more strongly than it emits, by the ratio of the κ-values, 600 000 to 40 000, a factor ~ 15. In such a situation the temperature of the particle is raised above the black-sphere value by $\sim 15^{1/4}$, a factor of about 2.

All this is for graphite needles that are sufficiently long and sufficiently thin, these two conditions being not entirely the same, as we shall now see. Starting with a grain that is 'sufficiently long and sufficiently thin', what happens as the length l is considered to decrease with the width $2a$ remaining fixed? Defining λ_0 from (10) to be

$$\lambda_0^{1/2} = \frac{1}{300}\,\frac{l}{2a}\,, \tag{11}$$

the grain has the full radiating efficiency given by the Johnson–Dicke noise formula for $\lambda \leq \sim\lambda_0$. For $\lambda > \lambda_0$, however, surface effects arising from charge separation reduce the radiating efficiency by a factor $\sim(\lambda_0/\lambda)^2$, this being the well-known $1/\lambda^2$ behaviour of graphite. Expressed alternatively in terms of the mass absorption coefficient κ, we have

$$\begin{aligned} &\kappa \cong 40\,000 \text{ cm}^2\text{ g}^{-1}, \qquad \lambda \leq \lambda_0\,, \\ &\kappa \cong 40\,000(\lambda_0/\lambda)^2 \text{ cm}^2\text{ g}^{-1}, \qquad \lambda > \lambda_0\,, \end{aligned} \tag{12}$$

applicable for wavelengths going from microwaves to infrared to the visible.

To understand how grains immersed in a radiation field behave with respect to the value of λ_0, let $\lambda_{\max}$ be the wavelength at the maximum of the Planck distribution corresponding to the grain temperature. If λ_0 is greater than $\lambda_{\max}$, nothing is changed appreciably from the discussion given above. If, however, $\lambda_0 < \lambda_{\max}$, the grain is not able to radiate as efficiently as a longer needle would do, and its temperature is raised by a factor $\sim(\lambda_{\max}/\lambda_0)^{1/2}$.

Now, consider what happens if l is kept fixed and $2a$ is increased. Again defining λ_0 by (11), the same discussion holds good, the discussion so far being concerned with the radiating efficiency of the grain at long wavelengths in the infrared or microwave regions of the spectrum. But, an additional effect now appears at the much shorter wavelengths at which an interstellar grain is heated. The effect occurs with increasing $2a$ when the electromagnetic skin depth at the short wavelengths at which the particles are heated becomes smaller than a. The short-wave absorption cross-section is then given simply by the optical silhouette, so that the mass absorption coefficient becomes

$$\kappa = \frac{2al}{\pi a^2 ls} \simeq 40\,000\,\frac{1}{2a\pi s}\text{ cm}^2\text{ g}^{-1}\,, \tag{13}$$

where $2a$ is in microns and $s = 2.25$ is the specific gravity for graphite. As $2a\pi s$ exceeds unity with increasing diameter $2a$, the mass-absorption coefficient for heating in the ultraviolet by early-type stars, therefore, becomes less than the absorption coefficient at long waves. This is on the supposition that l is long enough for full radiating efficiency to be maintained at the long waves. The temperature is thus, rather remarkably, depressed *below* the black sphere value by the factor $\sim (2a\pi s)^{-1/4}$, the particle diameter $2a$ being in microns. Numerically, the depression of temperature occurs for $2a >$ about 0.14 μm.

References

Bacon, R.: 1960, *J. Appl. Phys.* **31**, 283.
Davis, W. R., Slawson, R. J., and Rigby, G. R.: 1953, *Nature* **171**, 756.
Hoyle, F. and Wickramasinghe, N. C.: 1962, *Monthly Notices Roy. Astron. Soc.* **124**, 417.
Hoyle, F. and Wickramasinghe, N. C.: 1983, *Astrophys. Space Sci.* **91**, 327.