SPHERICAL AND NON-SPHERICAL GRAVITATIONAL COLLAPSE IN MONOPOLE-ANTI-DE SITTER-VAIDYA SPACETIME

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In the present work, we investigate the influence of the monopole field on the occurrence of the spacetime singularities in the gravitational collapse of anti-de Sitter-Vaidya spacetime. It has been found that the spherically symmetric monopole anti-de Sitter-Vaidya spacetime contradicts the *CCH*, whereas the non-spherical collapse respects it.

Key words : cosmic censorship, naked singularity gravitational collapse.
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I. INTRODUCTION

Most important and challenging problem in classical general relativity is to prove(or disprove) Penrose's cosmic censorship hypothesis(CCH)[1], which states that singularities formed in generic gravitational collapse of physical matters cannot be observed. Despite several attempts by many researches neither general proof nor precise mathematical formulation of the hypothesis has been available so far, on the contrary several examples of naked singularities have been found. These include dust [2-6], radiation [7-12], perfect fluid [13-14], null strange quark fluids [15-16] etc.

What will be the final outcome of the collapse which is not spherically symmetric? Must non-spherical collapse produce black holes? Unlike the spherically symmetric case, very little is known about the cosmic censorship in non-spherical gravitational collapse. In this direction, Thorne has proposed the hoop conjecture [17], which states that black holes with horizons form when and only when a mass M gets compacted into a region whose circumference in every direction is $C \leq 4\pi M$. Many researchers have attempted to give precise reformulation to this conjecture [18-21], but no proof nor mathematical formulation for this conjecture is available so far. If the hoop conjecture is true, then the non-spherical gravitational collapse with one or two dimensions sufficiently larger than others must produce naked singularities.

The status of the cosmological constant, Λ as a universal constant in general relativity has been well established now [22]. The astronomical observations of high red-shift type Ia supernova suggest that the universe may have a nonzero cosmological constant $\Lambda > 0$ [23]. Recent cosmological observations suggest for the presence of a positive cosmological constant, while low energy limit of super symmetry theories indicate a negative sign. [cf.24]. Positive cosmological constant leads to slow down the gravitational collapse, while negative cosmological constant support the gravitational forces. Hence it would be quite interesting to analyse the gravitational collapse of a spacetime containing cosmological constant.

In the present work we discuss the gravitational collapse of null fluid around monopole field in an anti-de Sitter background. Much work on monopole field has been appeared so far [25-27]. In [28] gravitational collapse in constant potential bath has been discussed. In this paper we analyze the collapse in spherically symmetric as well as non-spherically symmetric spacetimes.

This brief report is organized as follows: In Sec. II we analyze the spherical gravitational collapse of monopole-anti-de Sitter-Vaidya spacetime. In Sec.III we discuss the nature of the singularities arising in non-spherical collapse (toroidal, cylindrical, or planar symmetry). In Sec. IV we conclude the brief report.

II. MONOPOLE-ANTI-DE SITTER-VAIDYA SPACETIME

The general spherically symmetric metric [29, 30] is given by

$$ds^2 = -\left[1 - \frac{2m(v,r)}{r}\right]dv^2 + 2dv\,dr + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right), \qquad (1)$$

where v is an advanced Eddington time coordinate, r is the radial coordinate with $0 < r < \infty$ and $m(v,r)$ is the mass function giving gravitational mass inside the sphere of radial coordinate r.

Non-vanishing components of the Einstein tensor for the above metric are given by

$$G_0^0 = G_1^1 = \frac{-2m'}{r^2}, \qquad G_0^1 = \frac{2\dot{m}}{r^2}, \qquad G_2^2 = G_3^3 = \frac{-m''}{r} \qquad (2)$$

where $\{x^\mu\} = \{v, r, \theta, \phi\}$, $\mu = 0,1,2,3$ and $(\cdot)$ and $(')$ represent partial derivatives with respect to v and r respectively. Combining Eq.(2) with the Einstein field equations $G_{\mu\nu} = \kappa T_{\mu\nu}$, we find that the corresponding energy momentum tensor can be written in the form [30,31]

$$T_{\mu\nu} = T_{\mu\nu}^{(n)} + T_{\mu\nu}^{(m)}, \qquad (3)$$

where

$$T_{\mu\nu}^{(n)} = \sigma l_\mu l_\nu \ , \tag{4}$$

$$T_{\mu\nu}^{(m)} = (\rho + P)(l_\mu \eta_\nu + l_\nu \eta_\mu) + P g_{\mu\nu} \ . \tag{5}$$

Using Eqs. (2)-(5) we can write the expression for σ, ρ and P as

$$\sigma = \frac{2\dot{m}}{\kappa r^2} \quad , \quad \rho = \frac{2m'}{\kappa r^2} \quad , \quad P = \frac{-m''}{\kappa r} \ , \tag{6}$$

where κ is the gravitational constant.

Here ρ, P are energy density and pressure, while σ is the energy density of the Vaidya null radiation.

We have considered null vectors l_μ, η_μ such that

$$l_\mu = \delta_\mu^0 . \qquad \eta_\mu = \tfrac{1}{2}\left[1 - \frac{2m}{r}\right]\delta_\mu^0 - \delta_\mu^0 - \delta_\mu^1 \ ,$$

$$l_\lambda l^\lambda = \eta_\lambda \eta^\lambda = 0 \ , \ \ l_\lambda \eta^\lambda = -1 \ . \tag{7}$$

This fluid in general belongs to the type II fluids and the energy conditions for this type of fluids are given by

(i) The weak and strong energy condition:

$$\sigma > 0, \quad \rho \geq 0, \quad P \geq 0 \quad . \tag{8}$$

(ii) The dominant energy condition:

$$\sigma > 0, \quad \rho \geq P \geq 0 \quad . \tag{9}$$

The monopole solution defined by A. wang [30] is given by

$$m(v, r) = \frac{ar}{2} \ , \tag{10}$$

where a is an arbitrary positive constant.

For this solution we find that

$$\rho = \frac{a}{\kappa r^2} \quad , \quad \sigma = P = 0 \ . \tag{11}$$

Monopoles are formed due to a gauge-symmetry breaking and have many properties of elementary particles. Most of their energy is concentrated in a small region near the monopole core [32]. It has been stated in [26] that the spherically symmetric gravitational collapse of matter around the monopole is certainly leads to a black hole.

Since the energy-momentum tensor is linear in terms of the mass functions *m(v,r),* a linear superposition of particular solutions is also a solution of the Einstein field equations[30]. In particular, the combination

$$m(v,r) = \frac{ar}{2} + \frac{\Lambda r^3}{6} + \frac{\lambda v}{2} \quad , \tag{12}$$

would represent the monopole-anti-de Sitter-Vaidya solution. Here we have considered $\Lambda > 0$, for anti-de Sitter solution.

Substituting the above expression for *m(v,r)* into Eq. (1), we can write a line element for monopole-anti-de Sitter Vaidya spacetime as

$$ds^2 = -\left(1 - a - \frac{\Lambda r^2}{3} - \frac{\lambda v}{r}\right) dv^2 + 2dvdr + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) . \tag{13}$$

The solution being considered here is that of a radially injected flow in an initially region of the monopole-anti de Sitter universe. For $v < 0$, the spacetime is monopole-anti-de Sitter with $m(v)=0$. The radiation is focused into a central singularity at $r = 0$, $v = 0$, of growing mass $m(v)$. At v = T, say, the radiation is turned off. For v > T, the exterior spacetime settles to the Schwarzschild field embedded in a monopole-anti-de Sitter background. Thus for $v = 0$ to $v = T$, the metric is monopole-anti-de Sitter-Vaidya, whereas for $v > T$ it is Schwarzschild-monopole-anti-de Sitter space time. Note that presence of monopole breaks the asymptotic anti-de Sitter geometry of the space time.

It can be verified that with the choice of the mass function (12), all the energy conditions are preserved. To investigate the structure of the collapse we need to consider the radial null geodesics defined by $ds^2=0$. Thus equation for outgoing radial null geodesics for the metric (13) is given by

$$\frac{dr}{dv} = \frac{1}{2}\left(1 - a - \frac{\Lambda r^2}{3} - \frac{\lambda v}{r}\right) \quad . \tag{14}$$

It can be observed that the above differential equation has a singularity at *r=0*, *v=0*. To discuss the nature of this singularity, we analyse the outgoing radial null geodesics terminating at the singularity in the past.

We follow the technique described in [33].

Let

$$X_0 = \lim_{\substack{v\to 0\\ r\to 0}} X = \lim_{\substack{v\to 0\\ r\to 0}} \frac{v}{r} . \tag{15}$$

Hence Eq.(14) can be written as

$$X_0 = \lim_{\substack{v\to 0\\ r\to 0}} \frac{dv}{dr} = \frac{2}{1-a-\lambda X_0} \tag{16}$$

i.e.

$$\lambda X_0^2 + (a-1)X_0 + 2 = 0 \quad . \tag{17}$$

In order for the singularity at $v = 0$, $r = 0$ to be naked, radial null geodesics should be able to propagate outward, starting from the singularity. The variable X can be interpreted as the tangent to the outgoing geodesics, hence if Eq. (17) has at least one positive and real root, then the singularity could be naked. If Eq. (17) has no real and positive root, then the collapse ends into a black hole.

From Eq. (17) we obtain

$$X_0 = \frac{-(a-1) \pm \sqrt{(a-1)^2 - 8\lambda}}{2\lambda} \quad . \tag{18}$$

X_0 will be read and positive if

$$\lambda \le \frac{(a-1)^2}{8} \text{ and } a < 1 \; . \tag{19}$$

Note that in the absence of the monopole field (ie $a= 0$), the equation (17) reduces to same as that obtained in [9, 34] and the collapse ends into a naked singularity if

$$\lambda \le \frac{1}{8} \quad . \tag{20}$$

Note that for $0 < a < 1$, we have

$$\frac{(a-1)^2}{8} < \frac{1}{8} \; . \tag{21}$$

Hence from Eqs.(19)-(21), one can argue that due to inclusion of the monopole field to the Vaidya mass critical value of λ decreases.

Since $1/\lambda$ is the degree of inhomogeniety in the radiation collapse [34], Eq.(21) shows that collapse of the null fluid around monopole field requires higher inhomogeniety for the development of the naked singularity.

In the present analysis we should note that if $m(v,r) = \lambda v + f(v,r)$

and if

$$\lim_{\substack{v \to o \\ r \to 0}} \frac{f(v,r)}{r} = 0 \quad \text{or} \quad const.,$$

then inclusion of $f(v,r)$ to the Vaidya mass does not alter the result on the occurrence of naked singularity. This shows that the condition of the asymptotic flatness does not play any role in the formation of a naked singularity.

III. MONOPOLE-ANTI-DE SITTER VAIDYA SOLUTION IN TOROIDAL, CYLINDRICAL AND PLANAR SPACETIME.

In this section, we discuss the non-spherical gravitational collapse of monopole-anti-de Sitter-Vaidya space time.

Following the work in Refs.[30,34,35], it can be seen that the Einstein field equations also have the solution.

$$ds^2 = -\left[\alpha^2 r^2 - \frac{qm(v,r)}{r}\right] dv^2 + 2dvdr + r^2(d\theta^2 + d\phi^2), \tag{22}$$

where v is the advanced time coordinate and

$$\alpha = \sqrt{\frac{-\Lambda}{3}},$$

In this case σ is given by

$$\sigma = \frac{q\dot{m}}{\kappa r^2} \ . \tag{23}$$

Coordinates θ, ϕ describe the two dimensional zero-curvature spaces generated by two dimensional commutative Lie group G_2 of isometrics [34]. Referring to Refs.[34,35], we write the topology of two-dimensional space :

Topology of toroidal model is $S \times S$, cylindrically symmetric model has topology $R \times S$, while planar symmetrical model has $R \times S$. Ranges for θ and ϕ in these models are:

i) Toroidal : $0 \le \theta < 2\pi$ $\quad 0 \le \phi < 2\pi$

ii) Cylindrical : $-\infty < \theta < \infty$ $\quad 0 \le \phi < 2\pi$

iii) Planar : $-\infty < \theta < \infty$ $\quad -\infty < \phi < \infty$.

Depending upon the topology of the two dimensional space, parameter q has different values. For torus model, $m(v,r)$ is mass and $q = 2/\pi$. For the cylindrical case, $m(v,r)$ is mass per unit length and $q = 4/\alpha$, and for planar symmetrical model $m(v,r)$ is mass per unit area and $q = 2/\alpha^2$.

The values of parameter q are taken from Arnowitt-Deser-Misner (ADM) masses of the corresponding static black holes [35].

Let us take

$$qm(v,r) = \lambda v + ar \ . \tag{24}$$

Spacetime (22) then becomes

$$ds^2 = -\left[\alpha^2 r^2 - \frac{\lambda v}{r} - a\right] dv^2 + 2dvdr + r^2(d\theta^2 + d\phi^2) . \tag{25}$$

Equation for outgoing radial null geodesics for the above spacetime is given by

$$\frac{dv}{dr} = \frac{2}{\alpha^2 r^2 - \frac{\lambda v}{r} - a} . \tag{26}$$

It can be easily checked that the above differential equation has singularity at $v=0$, $r=0$. To analyse the nature of the singularity:

Let

$$X_0 = \lim_{\substack{v\to 0\\ r\to 0}} \frac{v}{r} = \lim_{\substack{v\to 0\\ r\to 0}} \frac{dv}{dr} = \frac{2}{-\lambda X_0 - a} \tag{27}$$

i.e.

$$\lambda X_0^2 + aX_0 + 2 = 0 \ . \tag{28}$$

It can be easily checked that the above algebric equation cannot have real and positive root. In other words, outgoing radial null geodesics having definite tangent at the singularity in the past do not exist in this case, hence the singularity is not naked and the collapse proceeds to form toroidal, cylindrical or planar black holes.

IV. CONCLUSION

Most important open problem in classical general relativity is to prove (or disprove) cosmic censorship hypothesis. However this hypothesis has not been proven yet, on the contrary number of spherically symmetric examples has been found, which suggest that the cosmic censorship might not be true. What will be the end state of collapse which is not spherically symmetric? Must non-spherical collapse produce black holes?

In the present brief report we have produced such a solution to the Einstein equations, which contradicts CCH in spherical collapse, whereas respect it in non-spherical collapse. From the present analysis we also observed that even a mass M does not become compacted into a region whose circumstance C in every direction satisfies $C > 4\pi M$, naked singularities are found in spherical collapse, while non-spherical collapse has produced black holes. Thus hoop conjecture is respected in spherical collapse whereas it is contradicted in non-spherical collapse.

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