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Quantum Fluctuations in the Conformally Flat and the Schwarzschild Space-Times

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Abstract

A general technique is described for dealing with the quantum fluctuations between conformally flat space-times. The second part of the paper deals with the Schwarzschild space-time. It is shown there that this space-time is stable against fluctuations of mass, but transitions between two space-times of different masses can be obtained via conformal fluctuations. Purely conformal fluctuations of the Schwarzschild metric are, however, damped at the event horizon. Similar conclusions are drawn about the Reissner–Nordstrom space-time.

§(1): *Introduction*

In the last two decades considerable work has been done on attempts to quantize gravity as described classically by Einstein's general relativity. Among the various approaches a promising one is that of path integrals. In a recent article Hawking [1] has reviewed the status of this particular approach, outlining its various advantages as well as difficulties. In particular, Hawking points out that some of the difficulties are lessened by going over from the Lorentzian to the Euclidean metric. Nevertheless, in view of the complexities of the problem, other techniques within the path integral method are also worth exploring. In particular, methods and concepts which have proved useful in the early development of quantum mechanics may turn out to be relevant in the problems of quantum gravity.

This paper discusses a specific elementary technique of path integration that has been successfully employed in the recent past to study quantum fluctuations of the space-time metric [2–4]. Although a fully quantized version of Einstein's theory of gravity still remains a long way off, these limited investigations convey some idea of the richness of quantum gravity when compared to its classical counterpart. In particular, these investigations have already demonstrated how the difference between the predictions of quantum gravity and classical gravity may become significant close to the so-called space-time singularities which arise in general relativity.

The investigations referred to above dealt with specific types of quantum fluctuations around certain well-known classical solutions. In [2] and [4], the spherically symmetric collapse of a dust ball and the Friedmann universe close to the big bang were considered. In [3] the anisotropic Bianchi type-I models were examined. The general feature that emerges from these investigations is as follows. The classical Einstein solution represents some sort of average around which the quantum fluctuations take place. However, close to the space-time singularity (which is found in all the above classical solutions) the quantum fluctuations diverge in such a way as to permit nonsingular, nonclassical states of space-time geometry to occur with finite probabilities. As discussed in [4] the expectation value of space-time curvature does not diverge as the classical singular epoch is approached, indicating that the overall effect of the quantum fluctuations in a spherically symmetric solution is to prevent the occurrence of a singularity. In another paper [5] one of us (T.P.) has examined the corresponding situation for anisotropic models.

In the present paper we are concerned with another class of spherically symmetric solutions which have played a key role in the development of general relativity. We will consider quantum fluctuations around the Schwarzschild solution and the Reissner–Nordstrom solution. In contrast to the solution discussed in [2–4], these are static solutions and as such we expect to see some difference in their behavior with respect to quantum fluctuations. This expectation is realized through the result that these solutions are relatively stable.

Before considering these problems, however, we analyze the formal problem of conformal fluctuations of the flat (empty) space-time. This problem was briefly discussed in [2], for conformal functions depending on time only. We now show how the general problem of arbitrary conformal fluctuations can be dealt with, against the flat Minkowski background. Although from the viewpoint of general relativity the Minkowski space-time, being free of matter and gravity, is hardly of any significant interest, conformal fluctuations around it do generate nontrivial space-time structures. For example, the Friedman/Robertson–Walker models are conformally flat. Our purpose in studying the elementary problem of conformal fluctuations around the Minkowski space-time is to see whether we come across any analogies with the vacuum fluctuations in quantum field theory. As the calculations of the following section show, some analogous structure does emerge in the problem we have posed here.

§(2): *Fluctuations in Conformally Flat Space-Times*

Consider fluctuations of the Minkowski space-time, which are represented by the line elements

$$ds^2 = \Omega^2 (x^\mu, t) [dt^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2] \tag{1}$$

Here x^μ ($\mu = 1, 2, 3$) are the rectangular Cartesian coordinates, t ($= x^4$) the time coordinate (c = speed of light = 1), and Ω an arbitrary $C^{(2)}$ function of x^μ and t. We will raise and lower tensor indices with respect to the Minkowski metric

$$\eta_{ik} = \text{diagonal } [-1, -1, -1, +1] \quad (i, k = 1, 2, 3, 4) \tag{2}$$

To discuss the conformal fluctuations we wish to compute the quantum mechanical amplitude for the function Ω to make a transition from the state $\Omega = \Omega_1$ at $t = t_1$ to the state $\Omega = \Omega_2$ at $t = t_2$ ($> t_1$). We will write in general

$$\Omega_1 = \alpha_1 (x^\mu), \qquad \Omega_2 = \alpha_2 (x^\mu) \tag{3}$$

Formally, the solution is given by the propagator

$$K [\alpha_2, t_2; \alpha_1, t_1] = \int \exp (iS [\Omega]) \mathfrak{D} \Omega \tag{4}$$

where we have taken $\hbar = 1$, $G = 1$ and S, the action, to be

$$S = \frac{1}{16\pi} \int_\vartheta R (-g)^{1/2} \, d^4x \tag{5}$$

The appearance of the Einstein-Hilbert action (5) in (4) serves to remind us that we are dealing with the quantum effects of gravity. The propagator (4) tells us that the dynamics of transitions $\Omega_1 \to \Omega_2$ must be subject to the same rules which in their classical limit lead us to general relativity.

The scalar curvature R is to be calculated for the metric (1). In general R involves second derivatives of the metric tensor and as such poses problems for an action principle and for path integration which are really suited to handling first derivatives only. This difficulty has been extensively discussed by Gibbons and Hawking [5] (see also Ref. [1]), who first pointed out that the addition of a surface integral on the boundary $\partial\vartheta$ of the space-time 4-volume ϑ can take away the variations of second derivatives. In [2] this point was discussed within the context of the present approach, and we will not repeat the arguments here. We will assume that the second derivatives of Ω have been transformed away by the divergence theorem and cancelled by the surface term. Accordingly (4) becomes

$$K [\alpha_2, t_2; \alpha_1, t_1] = \int \exp \left(- \frac{3i}{8\pi} \int_\vartheta \Omega_{,i} \Omega^{,i} d^4x \right) \mathfrak{D} \Omega \tag{6}$$

Since the integral in the exponent is quadratic in Ω, the above functional integral can be evaluated at once to give

$$K[\alpha_2, t_2; \alpha_1, t_1] = F(t_2, t_1) \exp\left(-\frac{3i}{8\pi}\int_{\vartheta} \bar{\Omega}_{,i}\bar{\Omega}^{,i} d^4x\right) \tag{7}$$

where $\bar{\Omega}$ is the solution of the classical wave equation

$$\Box\bar{\Omega} = 0 \tag{8}$$

satisfying the boundary conditions

$$\bar{\Omega}(t_1, x^\mu) = \alpha_1(x^\mu), \qquad \bar{\Omega}(t_2, x^\mu) = \alpha_2(x^\mu) \tag{9}$$

It is worth pointing out here that the approach being followed here is that of Feynman [7] and not that of Hawking [1]. In the former, the exponential inside the path integral has imaginary exponent and it therefore oscillates. This poses difficulties of convergence, although using mathematical tricks meaningful results are usually obtained (see for example [8]). In Hawking's approach to achieve convergence the exponent is made real by a complexification of space-time. In this case the negative sign in the exponent of (6) may pose difficulties. In our approach in the explicit examples considered so far [2-4] as well as in the present paper, no difficulty arises from this "wrong" sign.

The problem of evaluating $\bar{\Omega}$ can be solved by first Fourier analyzing α_1 and α_2. Writing $\mathbf{x} = (x^\mu)$ and $\mathbf{k} = (k^\mu)$, we get

$$A_1(\mathbf{k}) = \int \alpha_1(x^\mu)\, e^{-i\mathbf{k}\cdot\mathbf{x}} d^3\mathbf{x}$$
$$A_2(\mathbf{k}) = \int \alpha_2(x^\mu)\, e^{-i\mathbf{k}\cdot\mathbf{x}} d^3\mathbf{x} \tag{10}$$

We can write the solution $\bar{\Omega}$ as

$$\bar{\Omega}(x^\mu, t) = \int \Big\{ A_1(k) \cos k(t_1 - t_1)\, e^{-i\mathbf{k}\cdot\mathbf{x}} + \frac{\sin k(t_1 - t)}{\sin k(t_1 - t_2)} [A_2(k) - A_1(k) \cdot \cos k(t_1 - t_2)\, e^{-i\mathbf{k}\cdot\mathbf{x}}] \Big\} \frac{d^3\mathbf{k}}{(2\pi)^3} \tag{11}$$

where $k = |\mathbf{k}|$. Therefore,

$$\int \bar{\Omega}_{,i}\bar{\Omega}^{,i} d^4x = \int \frac{2k}{\sin k(t_1 - t_2)} \Big\{ A_2(\mathbf{k}) A_1(-\mathbf{k}) - \frac{1}{2}\cos k(t_1 - t_2) \cdot [A_1(\mathbf{k}) A_1(-\mathbf{k}) + A_2(\mathbf{k}) A_2(-\mathbf{k})] \Big\} \frac{d^3\mathbf{k}}{(2\pi)^3} \tag{12}$$

The function $F(t_2, t_1)$ can be easily evaluated by specializing to the case Ω_1 = const, Ω_2 = const. Writing $\Omega_1 = \alpha_1$, $\Omega_2 = \alpha_2$, where α_1 and α_2 are constants, we get from (10)

$$A_1(\mathbf{k}) = \alpha_1 \int e^{-i\mathbf{k}\cdot\mathbf{x}} d^3\boldsymbol{x}$$

$$A_2(\mathbf{k}) = \alpha_2 \int e^{-i\mathbf{k}\cdot\mathbf{y}} d^3\boldsymbol{y}$$

Substitute these expressions into (12). Perform the y integration in the first term, getting $\delta(\mathbf{k})$. The x integration then gives V, the 3-volume of the space where quantum fluctuations are being considered. (A warning: V has the meaning of proper volume only in the Minkowski space-time). Then the k integration gives the first term of (12) as

$$-\frac{2V\alpha_1\alpha_2}{t_2 - t_1}$$

The second term containing $\cos k(t_1 - t_2)$ can be similarly evaluated and we get

$$\int \Omega_{,i}\bar{\Omega}^{,i} d^4x = \frac{V(\alpha_1 - \alpha_2)^2}{t_2 - t_1} \tag{13}$$

In fact this was the special case discussed in [2] where Ω was considered to depend on t only. Using the transitivity of the Kernel (7) it then follows that

$$F(t_2, t_1) = \left[\frac{3V}{8\pi^2 iG(t_2 - t_1)}\right]^{1/2} \tag{14}$$

Formally, there is analogy between our solution of the above problem and the path integral quantization of a scalar field. The analogy is only formal, however; for whereas the scalar field quantization shows the behavior of the field in question in a flat space-time, in our case the fluctuations of Ω lead us to non-Euclidean space-time geometries. If the initial state of geometry is specified by a wave functional $\Psi_1[\alpha_1(x^\mu)]$, our kernal (7) can in principle give us the final wave functional at $t = t_2$ to be

$$\Psi_2[\alpha_2(x^\mu)] = \int K[\alpha_2, t_2; \alpha_1, t_1]\, \Psi_1[\alpha_1(x^\mu)]\, \mathfrak{D}\alpha_1(x^i) \tag{15}$$

In this way arbitrary transitions of space-time geometry can be described under conformal fluctuations.

The above functional integral is a formal statement of our quantum gravity problem and we briefly clarify its operational meaning. A given function $\alpha_1(x^\mu)$ specifies the line element of space-time at $t = t_1$. In a classical situation we are able to say (in the form of an initial-value problem) what $\alpha_1(x^\mu)$ is. In the quantum situation such a definitive statement is not possible. Instead we have to attach a probability amplitude that $\alpha_1 (x^\mu)$ has a specific functional form. The wave functional Ψ is supposed to specify the probability amplitude. In this sense (15) is no different from the corresponding statement in quantum field theory where we are interested in transitions of fields from given states (at t_1) to given states (at t_2). The difference here is that in the present case the transitions are of space-time geometries conformal to Minkowski space.

In the early development of quantum mechanics wave packets played a useful role in forming the link between quantum and classical ideas. A wave packet is usually specified by a Gaussian whose mean tells us where the particle is essentially located and whose standard deviation indicates the extent of uncertainty of the particle's position. We can use this analogy in the present case also by arguing that a wave-packet form for Ψ tells us with reasonable accuracy what α_1 is while its spread reflects our inability to specify α_1 precisely.

Since techniques of functional analysis and their applications to quantum field theory are well known we will not go further with the exposition of the general case. We return to the simpler case of Ω being a function of time only. In this case α_1 and α_2 are independent of x^μ and our wave functionals are now functions $\Psi_1 (\alpha_1)$ and $\Psi_2 (\alpha_2)$. It is easy to verify that if we take Ψ_1 to be a Gaussian wave packet of width $\Delta (t_1) = \Delta_0$, then at t_2 the width of the wave packet Ψ_2 is increased to $\Delta (t_2)$, where

$$\Delta^2 (t_2) = \Delta_0^2 \left[1 + \left(\frac{t_2 - t_1}{T}\right)^2 \frac{1}{\Delta_0^4}\right] \tag{16}$$

where $T = 3V/4\pi$. If we express everything in cgs units we get

$$T \cong V_{\text{cm}^3} \times 10^{56} \text{ sec} \tag{17}$$

Now although the mean of the Gaussian is centered at $\Omega = 1$, the expectation value of $\Omega^2 (t_2)$ is given by

$$\langle \Omega^2 (t_2) \rangle = 1 + \Delta^2 (t_2) \tag{18}$$

Thus we can define an *effective metric* for the space-time by

$$\begin{aligned} ds_E^2 &= [1 + \Delta^2 (t)]\ \eta_{ik}\, dx^i\, dx^k \\ &= \left(1 + \Delta_0^2 + \frac{t^2}{\Delta_0^2 T^2}\right) \eta_{ik}\, dx^i\, dx^k, \qquad t = t_2 - t_1 \end{aligned} \tag{19}$$

Thus quantum fluctuations have a tendency to grow with time, and the effective metric (19) may be compared with the mini expanding structures suggested by Wheeler [9] and Misner [10]. The above effects are significant over very small volumes and over short time scales in view of (17).

Thus we see that the Minkowski space-time ceases to be a trivial structure in gravity once we depart from classical general relativity to its quantum counterpart. Although we have limited our discussion to conformally flat geometries, our explicit example illustrates the richness of quantum gravity in comparison with classical gravity. As surmised earlier this is analogous to the status of vacuum in quantum as opposed to classical field theory.

§(3): *The Schwarzschild Metric*

We next consider quantum fluctuations of the Schwarzschild space-time. Next to the trivial case of Minkowski space-time, this is the simplest solution of Einstein's equations. Following our work of the previous section we now investigate whether path integral techniques can help us identify specific quantum fluctuations of this classical situation. In the usual coordinates the Schwarzschild metric is given by

$$d\bar{s}^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \tag{20}$$

Compared to the Minkowski space-time the significant physical input in (20) is that of the mass parameter M and it is tempting to convert it into a dynamical quantum variable. However, an attempt to regard M as a function of t, and thus still preserving the spherically symmetric nature of (20) does not yield any dividend. For the scalar curvature then becomes

$$R = -\frac{d^2}{dt^2}\left[1 - \frac{2M(t)}{r}\right]^{-1} \tag{21}$$

Hence the gravitational action (5) becomes essentially path independent after the surface term mentioned in Section 2 is taken into account. The Schwarzschild metric is therefore stable against quantum fluctuations of the mass term.

If we were to talk about this process in the language of ordinary quantum mechanics, we could assert that a transition from the vacuum state characterized by $M = 0$ to a Schwarzschild state $M \neq 0$ is a forbidden one. This result can be easily generalized to the Reissner–Nordstrom metric, which is given by

$$d\bar{s}^2 = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 - \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \tag{22}$$

for which also R is expressible as a second-time derivative of the coefficient of dr^2 in (22), for M and Q both functions of t. Hence transitions from the vacuum or from the Schwarzschild state to the Reissner–Nordstrom state are forbidden.

This is of some interest from the point of view of black hole physics where the no-hair theorems also single out M and Q as the "undestroyable" measures of information about a black hole. Does the angular momentum also show stability vis-à-vis quantum fluctuations? We have not yet analyzed the Kerr metric from this point of view to be able to settle this question.

Nevertheless, if we enlarge the scope of quantum fluctuations beyond M, the Schwarzschild metric does show possibilities of change. Below we consider the explicit case of conformal fluctuations coupled with mass fluctuations of the Schwarzschild space-time. Thus a typical quantum transition of (20) is described by the metric

$$ds^2 = \Omega^2(t)\left\{\left[1-\frac{2M(t)}{r}\right]dt^2 - \left[1-\frac{2M(t)}{r}\right]^{-1} dr^2 - r^2\,(d\theta^2 + \sin^2\theta\, d\phi^2)\right\} \tag{23}$$

To fix ideas we will consider a region of space-time contained between the spheres $r = R_1$ and $r = R_2$. It is convenient to define the quantum variable

$$Q(M) = \frac{3}{R_2^3 - R_1^3}\int_{R_1}^{R_2} \frac{r^2\,dr}{1-2M/r} \tag{24}$$

and denote by V the quantity $(R_2^3 - R_1^3)/3$. If we were dealing with a Euclidean space, V would correspond to the volume contained in a steradian of the spherical shell in question. Define further two new variables $X(t)$ and $Y(t)$ by

$$\Omega = (X - Y)^{1/5}, \qquad Q = (X + Y)(X - Y)^{3/5}. \tag{25}$$

With these definitions the gravitational action becomes

$$S = \frac{V}{10}\int_{t_1}^{t_2} (\dot{X}^2 - \dot{Y}^2)\,dt \tag{26}$$

where t_1 and t_2 represent the initial and final coordinate times of the quantum transition. It is easy to write down the quantum mechanical propagator corresponding to (26):

$$K[X_2, Y_2, t_2; X_1, Y_1, t_1] = F(t_2 - t_1)\exp\left\{\frac{iV}{10(t_2 - t_1)} \cdot [(X_2 - X_1)^2 - (Y_2 - Y_1)^2]\right\} \tag{27}$$

where

$$F(\tau) = (10\pi i\tau)^{-1}, \qquad \tau = t_2 - t_1 \tag{28}$$

Now a vacuum state is characterized by $M = 0$, $\Omega = 1$, i.e., by $Q = 1$, $\Omega = 1$. Hence if we identify the initial state by the vacuum state we get

$$X_1 = 1, \qquad Y_1 = 0 \tag{29}$$

For reasons explained in the last section we again concentrate on wave-packet solutions. If we consider a wave packet centered on (1, 0) in the (X, Y) plane, with a dispersion $\Delta(t_1) \equiv \Delta_0$ in both X_1 and Y_1, (27) tells us that this wave packet will diffuse so that at $t = t_2$, its dispersion $\Delta(t_2)$ will be higher. Restoring $c, \hbar$, and G, the result is given by

$$\Delta^2 = \Delta_0^2 + \left(\frac{5\hbar G}{4c^2 V\Delta_0}\right)^2 (t_2 - t_1)^2 \tag{30}$$

Since Δ^2 describes the dispersion in X_2 and Y_2, we can calculate the dispersions in Q and Ω by using (25). A simple calculation gives

$$Q^2 \cong 1 + \tfrac{16}{5}\Delta^2, \qquad \Omega^2 \cong 1 + \tfrac{2}{5}\Delta^2 \tag{31}$$

It is interesting to convert the dispersion in Q to a dispersion in M by using (24). In the weak-field approximation we get

$$Q(M) \cong 1 + \frac{3GM}{c^2}\cdot\frac{R_1 + R_2}{R_1^2 + R_1R_2 + R_2^2} \cong 1 + \frac{3GM}{c^2 R_1} \tag{32}$$

for $R_2 >> R_1$. From (31) and (32) we get

$$\langle M^2\rangle = M_0^2 + \left(\frac{4\hbar c^2 R_1^2}{9GVM_0}\right)^2 (t_2 - t_1)^2 \tag{33}$$

Notice that we do need an initial dispersion even in the vacuum state: $\langle M\rangle_{t_1} = 0$, $\langle M^2\rangle_{t_1} = M_0^2$. This corresponds to the zero point energy of vacuum. The minimum value of $\langle M^2\rangle$ is given by

$$\langle M^2\rangle_{\min} = \frac{8\hbar c^2 R_1^2}{9GV}(t_2 - t_1) = \mu^2 \quad \text{(say)} \tag{34}$$

It is convenient to express μ in terms of the Planck mass $\mu_p = (\hbar c/G)^{1/2}$:

$$\mu = \frac{2}{3}\mu_p\left[\frac{2c(t_2 - t_1)R_1^2}{V}\right]^{1/2} \tag{35}$$

We see therefore that although direct transitions from the vacuum state $M = 0$ to $M \neq 0$ are forbidden, if we introduce conformal degrees of freedom we can generate quantum fluctuations from an initial state of $M = 0$ to a final state of $M \neq 0$. In this way conformal fluctuations can bring about spontaneous mass production.

§(4): *Conformal Fluctuations of the Schwarzschild Metric*

(i) Fluctuations near the Horizon: The analysis of the last section up to the equation (28) is exact and can be applied near the horizon where the gravitational effects are large. We will, however, suppress the degree of freedom contained in the mass and treat M = const. A simple calculation then gives

$$K[\Omega_2, t_2; \Omega_1, t_1] = \left[\frac{f(R_1, R_2)}{i\pi(t_2 - t_1)}\right]^{1/2} \exp\left[-\frac{if(R_1, R_2)(\Omega_2 - \Omega_1)^2}{t_2 - t_1}\right] \quad (36)$$

where

$$f(R_1, R_2) = \frac{1}{2}[(R_2 - 2M)^3 - (R_1 - 2M)^3] + \frac{3M}{2}[(R_2 - 2M)^2 - (R_1 - 2M)^2] + 6M^2(R_2 - R_1) + 12M^3 \ln\left(\frac{R_2 - 2M}{R_1 - 2M}\right) \quad (37)$$

The fluctuations are large when the exponent in (36) has the modulus comparable to unity, for this is when the exponential function oscillates significantly. However, as R_1, the inner radius of our space-time section under discussion approaches the horizon, the function $f(R_1, R_2)$ diverges logarithmically. As a result the modulus of the exponent in (36) diverges.

This is easily seen again by considering wave packets. The dispersion of a Gaussian wave packet depends on f as

$$\Delta(t_2) = \Delta(t_1)\left[1 + \frac{(t_2 - t_1)^2}{4\Delta^2(t_1)f^2}\right]^{1/2} \quad (38)$$

As $f \to \infty$, $\Delta(t_2) \to \Delta(t_1)$. That is, the dispersion stays almost unchanged.

This situation may be compared with the transition from quantum theory to classical theory as $\hbar \to 0$. An exactly similar divergence of the exponent takes place, with the result that only the classical paths ($\delta S = 0$) make a coherent contribution to the transition amplitude. We therefore see that as we approach the event horizon the effect of quantum fluctuations is damped out.

(ii) Matching with the Internal Solution: In [2], the quantum fluctuations of the dust ball were considered. The present work is complementary to that investigation in that a conformal transform of the Schwarzschild metric can be matched to the internal solution of the dust ball. The external and the internal solutions are matched at the boundary of the dust ball. Since the internal (comoving) time coordinate is a unique function of the external (Schwarzschild) time coordinate the conformal functions inside and outside can be easily related. The calculation is simple and need not be given here.

What type of fluctuations in the internal metric would correspond to the $M \neq$ const fluctuations of the Schwarzschild metric described in Section 3? Preserving spherical symmetry about the central point, the simplest line element which can be written as a generalization of the Robertson–Walker line element is

$$ds^2 = d\tau^2 - S^2(\tau)\left[\frac{d\rho^2}{1 - \alpha\rho^2} + B^2(\tau)\rho^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right] \quad (39)$$

where $B(\tau)$ is a function of τ. In the classical solution of the dust ball $B(\tau) = 1$, $\alpha =$ const. Any conformal transform of (39) with $B = 1$ can be matched to a conformal transform of the Schwarzschild solution with the two conformal functions depending upon their respective time coordinates as discussed earlier. Such a matching, however, keeps $M =$ const. A time-dependent $B(\tau)$ however, requires a time-dependent $M(t)$ as in (23). We have not, however, succeeded as yet in discussing the quantum fluctuations of $B(\tau)$ within the path integral framework.

§(5): *Conclusion*

We have described how arbitrary quantum transitions between conformally flat space-times can be represented by formal propagators. The formalism can be used to discuss the spontaneous eruptions of mini expanding universes in vacuum.

Our investigations of the Schwarzschild and Reissner–Nordstrom space-times show that these space-times are stable under fluctuations of mass and charge. By coupling these fluctuations with the conformal fluctuations, however, it is possible to achieve transitions between space-times of different mass and charge. Purely conformal fluctuations of these space-times are however damped near their respective horizons.

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