LRS BIANCHI-TYPE I MODELS WITH A TIME DEPENDENT COSMOLOGICAL "CONSTANT"

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Abstract

Plane symmetric homogeneous models of the universe with a perfect fluid distribution of matter and a variable cosmological "constant" representing the energy density of vacuum have been presented. The models are highly anisotropic in the early stages of evolution and ultimately evolve to the standard model at the present epoch. Of the three models obtained in this paper, one has an increasing vacuum energy density with time in the early universe. Though highly negative in the very early universe, the vacuum density soon becomes positive, and meets the observational limits at the present epoch. The negative vacuum in the early universe in this model, which decays numerically, adds up to anisotropy and at the same time contributes to the isotropization process. The second model is found to admit matter creation from the decaying vacuum energy whereas the third one, in which the vacuum density is always one third of the corresponding radiation density, is devoid of any such creation.

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I. Introduction

One of the most intriguing problems of the current theoretical physics is, perhaps, that of understanding the smallness of the observed cosmological constant Λ. The theoretical expectations of Λ (which is also interpreted as the energy density of vacuum) exceed observational limits by some 120 orders of magnitude [1]. Although one can set the constant to zero by hand, but this amounts to a highly artificial fine tuning. However, this problem can easily be solved by assuming a dynamically decaying Λ due to its coupling with the matter fields of the universe. The idea is that Λ, which was very large in the early universe, relaxed to its present small value in the course of the expansion of the universe by creating massive or massless particles. Several authors have advocated a variable Λ to account for this fact [2-8]. Gasperini [5] argues that Λ can be interpreted as a measure of the temperature of cosmic vacuum which should decrease, like the radiation temperature, with cosmic expansion. Linde [9] proposed that Λ is function of temperature and is related with the process of broken symmetries. In this regard, Peebles and Ratra [10] have shown that if the potential energy of the scalar field ϕ during inflation has a power-law tail at large ϕ, the mass density associated with ϕ acts like a cosmological constant that decreases with time.

Besides the cosmological constant Λ, there is another quantity, the anisotropy of the universe which, like Λ, is supposed to play a fundamental role in the evolution and the dynamics of the early universe. Although the universe, on large scale, seems homogeneous and isotropic at present, there is no observational data that guaranties the isotropy in the era prior to the recombination [11]. In fact, there are theoretical arguments that sustain the existence of an anisotropic phase that approaches an isotropic one [12]. Many authors have considered anisotropic cosmological solutions as possible models for the beginning of the universe which isotropize very rapidly due to particle production in the course of their expansion [13]. It has been conjectured [14, 15] that any cosmology with a positive cosmological constant Λ would asymptotically approach a deSitter spacetime. The isotropization effect of Λ arises from its peculiar 'equation of state', i.e., from the negative stress of vacuum (see equation (6)). There have also been found limits on Bianchi type VII_h models that on the largest scales, the spacetime will depart from R-W metric by $\sigma/H < 10^{-9}$ [16].

In an attempt to find a more realistic model of the universe, it would be worthwhile to study the decay of Λ in an anisotropic background and to see how this decay effects the isotropization process. In this paper, we consider a Bianchi type I metric with plane symmetry which is one of the simplest models of an anisotropic universe that describes a homogeneous and spacially flat universe. By considering a time-dependent Λ in the general theory of relativity, we investigate three different models and discuss their implications.

II. Field Equations and the General Discussion

We consider a space-time admitting a Bianchi-type-I group of motions and having plane symmetry [17]:

$$ds^2 = -dt^2 + R^2\left[dx^2 + dy^2 + \left(1 + \alpha \int R^{-3}dt\right)^2 dz^2\right], \tag{1}$$

where α is a positive constant and R, some function of the cosmic time t. We assume that the cosmic matter is represented by the energy momentum tensor of a perfect fluid

$$T_{ij} = (\rho + p)v_i v_j + p g_{ij}, \tag{2}$$

where the energy density of the cosmic matter is the sum of rest mass and radiation energy densities and its pressure is due to the radiation only, i.e.,

$$\left\{\begin{array}{ll} \rho & = \rho_m + \rho_r \\ p & = p_r = \frac{1}{3}\rho_r \end{array}\right\} \tag{3}$$

The geometry of the space time is governed by the gravitational effects of matter and vacuum through the Einstein field equations

$$R_{ij} - \frac{1}{2}R^h_h g_{ij} = -8\pi G\left(T_{ij} - \frac{\Lambda(t)}{8\pi G}g_{ij}\right). \tag{4}$$

In view of the vanishing divergence of the Einstein tensor, the incorporation of a time dependent cosmological constant Λ(t) in the Einstein field equation amounts to assuming that along with its usual energy momentum tensor of the matter content T_{ij}, the energy momentum tensor of the universe possesses an additional piece $-\frac{\Lambda}{8\pi G}g_{ij}$ representing the energy momentum tensor of vacuum:

$$T^{\text{vac}}_{ij} = -\frac{\Lambda}{8\pi G}g_{ij} \tag{5}$$

which can be regarded as the energy momentum tensor of a perfect fluid with its energy density ρ_v and pressure p_v satisfying the equation of state

$$p_v = -\rho_v = -\frac{\Lambda}{8\pi G}. \tag{6}$$

Thus the total energy momentum tensor of the universe, due to the matter and the vacuum, can be written as

$$T_{ij}^{\text{tot}} \equiv T_{ij} + T_{ij}^{\text{vac}} = (\rho_t + p_t)v_i v_j + p_t g_{ij}, \tag{7}$$

where $\rho_t = \rho + \rho_v$ and $p_t = p + p_v$. Obviously it is the total energy momentum tensor, due to the matter and vacuum, which is conserved and not the two separately. In the context of equations (1), (2) and (6), the field equation (4) yields two independent equations

$$8\pi G \rho_t = \frac{\dot{R}^2}{R^2} + \frac{2\alpha \dot{R}}{R^4(1 + \alpha \int R^{-3} dt)}, \tag{8}$$

$$8\pi G p_t = -2\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2}. \tag{9}$$

The volume expansion rate θ and shear σ are obtained as

$$\theta = 3\frac{\dot{R}}{R} + \frac{\alpha}{R^3(1 + \alpha \int R^{-3} dt)}, \tag{10}$$

and

$$\sigma^2 = \frac{\alpha^2}{3R^6(1 + \alpha \int R^{-3} dt)^2}. \tag{11}$$

A representative length ℓ, representing the volume behaviour of the cosmic fluid, can be obtained by using

$$\dot{\ell}/\ell = \theta/3, \tag{12}$$

which amounts to

$$\ell = R\left(1 + \alpha \int R^{-3} dt\right)^{\frac{1}{3}}. \tag{13}$$

The density ρ and pressure p in terms of the Hubble parameter H, the deceleration parameter q and the shear σ may now be written as

$$8\pi G(\rho + \rho_v) = 3H^2 - \sigma^2, \tag{14}$$

and

$$8\pi G(p-\rho_v)=H^2(2q-1)-\sigma^2, \tag{15}$$

where $H=\dot{\ell}/\ell,\quad q=-(\ddot{\ell}/\ell)(1/H^2)$. The shear σ, in terms of ℓ comes out as

$$\sigma^2=\frac{\alpha^2}{3\ell^6}. \tag{16}$$

thus the energy density ρ_β, associated with the anisotropy by $\rho_\beta=\sigma^2/8\pi G$, decays very rapidly due to the expansion by converting into photons.

The isotropization effect of Λ is described by equation (14) which can alternatively be written as

$$\left(\frac{\sigma}{\theta}\right)^2=\frac{1}{3}\left(1-\Omega-\frac{\Lambda}{3H^2}\right), \tag{17}$$

where $\Omega\equiv\frac{\rho}{\rho_c}$, $\rho_c\equiv\frac{3H^2}{8\pi G}$. This constrains the value of Λ (>0) by $0\leq\frac{\Lambda}{3H^2}=\frac{\rho_v}{\rho_c}\leq 1-\Omega$. It is clear from equation (17) that if $\frac{-\Lambda}{3H^2}$ decreases with time, it will promote the isotropization process. Thus a positive Λ further contributes to bringing about isotropy (in a deflationary universe). However, a negative as well as a decaying Λ (>0) allows more room for the anisotropy than does a constant, positive Λ.

We shall now show that, like the anisotropy energy, the decaying vacuum energy is also converted into photons. The divergence of equation (4) yields

$$\dot{\rho}_t+3(\rho_t+p_t)\dot{\ell}/\ell=0, \tag{18}$$

which may alternatively be written as

$$\frac{d}{dt}(\rho\ell^3)+p\frac{d\ell^3}{dt}+\ell^3\frac{d\rho_v}{dt}=0. \tag{19}$$

The integration of equation (19) in the early pure radiation era ($\rho_m=0$), with the equation of state given by equation (3), gives

$$\rho_r=\rho_{ro}\left(\frac{\ell_o}{\ell}\right)^4-\frac{1}{\ell^4}\int_{\ell_o}^{\ell}\ell'^4\frac{d\rho_v}{d\ell'}d\ell', \tag{20}$$

where $\rho_r=\rho_{ro}$ at $\ell=\ell_o$. When both matter and radiation are present ($\rho=\rho_m+\rho_r$), equation (19) reads

$$\frac{d}{dt}(\rho_m\ell^3)+\frac{d}{dt}(\rho_r\ell^3)+p\frac{d\ell^3}{dt}+\ell^3\frac{d\rho_v}{dt}=0. \tag{21}$$

Vacuum coupling primarily to non-relativistic matter would imply the continuous creation of baryon-antibaryon pairs. Naively, one would expect this effect to be greatly suppressed relative to radiation production [4]. Moreover, the observations so restrict the size of such a matter creation term that one would get

$$\frac{d}{dt}(\rho_m \ell^3) \simeq 0, \tag{22}$$

so that equation (20) may be considered approximately valid for the radiation component in both- the radiation dominated as well as matter dominated eras. Here ℓ_o may be taken as any finite value of ℓ except $\ell = 0$ which is a singularity in the present models. One observes that for $\dot{\rho}_v < 0$ with $\dot{\ell} > 0$ (which is the case in the last two models as we shall see later on), the integral in equation (20) is negative so that the decrease of ρ_v, as ℓ increases, generates a positive contribution to ρ_r (provided $p \neq \rho_v$. See section V).

The change in the entropy of the matter content of the universe, which follows from equation (19) as

$$TdS \equiv dE + dW = -\ell^3 d\rho_v, \tag{23}$$

is always increasing for a decaying vacuum energy (provided $p \neq \rho_v$) keeping, however, the total entropy of matter and vacuum taken together always conserved as is clear from equation (18). (The entropy discussed here is not concerned with the decay of the anisotropy energy since we consider here the average volume behaviour of the cosmic fluid.)

In order to obtain specific models, we consider in the following sections three dynamical laws for the variations of ρ_v by assuming three phenomenological ansatzes: (i) $\rho + p = \rho_c$, (ii) $q =$ constant and (iii) total active gravitational mass = constant, and investigate the cosmological consequences of the resulting models.

III. Critical Density Considerations

Observations indicate that the ratio of the present cosmic energy density ρ to the present critical density ρ_c is somewhere between 0.1 and 2 (the lower limit could be less) [2, 15] whereas the present galactic mass density of the universe, i.e., the mass of the visible matter is of the order of about a tenth or less than the critical density [18]. Some FLRW models [3] have been investigated in the past few years by considering $\rho = \rho_c$ for all time. However, taking $\rho = \rho_c$ in the present model amounts to a negative vacuum energy density throughout the evolution as equation (14) indicates:

$$\rho_v = -\frac{\sigma^2}{8\pi G}. \tag{24}$$

In the present case we rather assume

$$\rho + p = \rho_c, \tag{25}$$

a relation which has been found earlier in some FLRW models admitting a certain symmetry [19]. Although equation (25), taken together with the field equations also gives rise to a negative ρ_v in the early universe, it evolves to the positive observed value in the present universe as we shall see in what follows. One can see that equation (25) together with equation (3) yields

$$\Omega \simeq \begin{cases} 3/4, & \text{in the early radiation dominated (RD) era,} \\ 1, & \text{in the present matter dominated (MD) era,} \end{cases} \tag{26}$$

showing that equation (25) does not conflict with observations.

To obtain the time-variation of the function ℓ, one may consider equations (14), (15) and (25) giving the differential equation

$$2\ddot{\ell} + \dot{\ell}^2\ell^{-1} + \frac{2}{3}\alpha^2\ell^{-5} = 0, \tag{27}$$

whose first integral yields

$$\dot{\ell}^2 = \frac{4a}{9}\ell^{-1} + \frac{2\alpha^2}{9}\ell^{-4}, \tag{28}$$

where the constant of integration has been specified as $\frac{4a}{9}, a > 0$ for mathematical ease. For the initial condition $\ell = 0$ at $t = 0$, equation (28) gives the solution

$$\ell = (at^2 + \sqrt{2}\alpha t)^{\frac{1}{3}}. \tag{29}$$

The time-variation of the function R then follows from equations (13) and (29) as

$$R \sim [at + \sqrt{2}\alpha]^{\frac{2+\sqrt{2}}{6}} \times t^{\frac{2-\sqrt{2}}{6}}. \tag{30}$$

Equations (27), (28) and (29) may be used to obtain

$$q = \frac{1}{2}\left[\frac{1 + 2\alpha^2 a^{-1}(at^2 + \sqrt{2}\alpha t)^{-1}}{1 + 2^{-1}\alpha^2 a^{-1}(at^2 + \sqrt{2}\alpha t)^{-1}}\right]. \tag{31}$$

From equations (12), (16) and (29), we find

$$\frac{\sigma}{\theta} = \frac{\alpha}{\sqrt{6}(\sqrt{2}at + \alpha)}, \tag{32}$$

which shows that $\frac{\sigma}{\theta} = \frac{1}{\sqrt{6}}$ initially. It decreases monotonically and approaches zero at $t \to \infty$ implying that the model approaches isotropy as $t \to \infty$, as mentioned in the preceding section. By the use of equation (32), equation (31) leads to

$$q = \frac{1}{2}\left[1 + 18\left(\frac{\sigma}{\theta}\right)^2\right] \tag{33}$$

indicating that $q = 2$ initially. It decreases continuously reducing to $\frac{1}{2}$ as $t \to \infty$. In order to obtain an estimate of the present value of q, we take $0 < \left(\frac{\sigma}{\theta}\right)_p \leq \frac{1}{4}$ [20] (hence $0 < \sigma_p \leq 1.2 \times 10^{-42} GeV$ for $H_p = 75\ kms^{-1}Mpc^{-1}$). Equation (33) then gives $0.5 < q_p \leq 1.06$. Here and henceforth the subscript p characterizes the present value of the quantity. Equations (3), (14) and (25) yield

$$\rho_m + \frac{4}{3}\rho_r = \frac{1}{12\pi G}\left[\frac{\sqrt{2}at + \alpha}{at^2 + \sqrt{2}\alpha t}\right]^2, \tag{34}$$

and

$$\frac{\rho_r}{3} - \rho_v = \frac{\alpha^2}{24\pi G}(at^2 + \sqrt{2}\alpha t)^{-2}. \tag{35}$$

Equation (35) indicates that the vacuum energy density ρ_v is always less than the radiation energy density ρ_r and it tends to $\frac{1}{3}\rho_r$ as $t \to \infty$. Thus ρ_v is, at most, of the order of ρ_r. This has also been found by Freese et al [4] by using a different parametrization. In the early pure radiation era ($\rho_m = 0$), equations (34) and (35) give

$$\rho_r = \frac{1}{16\pi G}\left[\frac{\sqrt{2}at + \alpha}{at^2 + \sqrt{2}\alpha t}\right]^2, \tag{36}$$

$$\rho_v = \frac{(\sqrt{2}at + \alpha + \sqrt{2}\alpha)(\sqrt{2}at + \alpha - \sqrt{2}\alpha)}{48\pi G(at^2 + \sqrt{2}\alpha t)^2} \tag{37}$$

When both matter and radiation are present, we have $\rho_m \ell^3$ = constant = $\rho_{mp}\ell_p^3 \equiv E_{mp}$. Equations (34) and (35) then become

$$\rho_r = \frac{3}{4(at^2 + \sqrt{2}\alpha t)} \cdot \left[\frac{\sqrt{2}at + \alpha)^2)}{(12\pi G(at^2 + \sqrt{2}\alpha t)} - E_{mp} \right], \tag{38}$$

$$\rho_v = \frac{1}{4(at^2 + \sqrt{2}\alpha t)} \cdot \left[\frac{(\sqrt{2}at + \alpha + \sqrt{2}\alpha)(\sqrt{2}at + \alpha - \sqrt{2}\alpha)}{12\pi G(at^2 + \sqrt{2}\alpha t)} - E_{mp} \right]. \tag{39}$$

We note from equation (37) that $\rho_v \to -\infty$ as $t \to 0$. Further $\rho_v \substack{\leq \\ >} 0$ according as $(at^2 + \sqrt{2}\alpha t) \substack{\leq \\ >} \frac{\alpha^2}{2a}$ showing an increasing ρ_v with time in the early universe. Also equations (36) and (38) indicate that $\rho_r \ell^3$ is decreasing with time. Thus vacuum energy is being created from the decaying radiation. In order to have an estimate of ρ_{vp} (or equivalently of Λ_p), we eliminate σ^2 between equations (14) and (15) and use equation (25) therein, leading to

$$\Lambda = (3.5 - q)H^2 - 8\pi G\rho, \tag{40}$$

which for $H_p = 75\ kms^{-1}Mpc^{-1}$, $\rho_p = \frac{1}{10}\rho_c$ and q_p given by $0.5 < q_p < 1.06$, yields $\Lambda_p = 10^{-84}GeV^2$ which is well within the upper limit of the observed value of Λ_p obtained as $10^{-82}GeV^2$ [21].

IV. Solutions for a Constant q

Berman [6, 7] has obtained some flat FLRW models with decaying vacuum energy by assuming a constant deceleration parameter:

$$q = \text{constant} = m - 1, \tag{41}$$

which is sufficient to determine the metric. In the present anisotropic model, the assumption (41) amounts to

$$\ell = (nt)^{\frac{1}{m}}, \text{ with } \ell = 0 \text{ at } t = 0, \tag{42}$$

and hence

$$R \sim t^{\frac{1}{m}} \exp\left[\frac{\alpha m}{3n(3-m)}(nt)^{1-\frac{3}{m}} \right], m \neq 3, \tag{43}$$

$n > 0$ being some constant of integration. Equations (12), (16) and (42) yield

$$\frac{\sigma}{\theta} = \frac{\alpha m}{3\sqrt{3}n}(nt)^{1-\frac{3}{m}}, \tag{44}$$

showing that $\frac{\sigma}{\theta}$, being initially very large, tends to zero as $t \to \infty$ provided $m < 3$. With the help of equation (42), equations (14) and (15) respectively reduce to

$$\rho + \rho_v = \frac{3}{8\pi G}\left[(mt)^{-2} - \frac{\alpha^2}{9}(nt)^{\frac{-6}{m}}\right], \tag{45}$$

and

$$p - \rho_v = \frac{1}{8\pi G}\left[(2m-3)(mt)^{-2} - \frac{\alpha^2}{3}(nt)^{\frac{-6}{m}}\right], \tag{46}$$

yielding

$$\rho + p = \frac{2m}{3}\rho_c - \frac{\alpha^2}{12\pi G}(nt)^{\frac{-6}{m}}. \tag{47}$$

This shows that for $m = \frac{3}{2}$, the model would indicate $\rho+p = \rho_c$ at sufficiently large times. Moreover for $\alpha = 0$ and $m = \frac{3}{2}$, the resulting FLRW model would have $\rho = \rho_c$ for all time. We note from equations (45) and (46) that for $\alpha = 0$ and $m = 2$ and $\frac{3}{2}$ in the RD and MD phases, respectively, the model has $\rho_v = 0$; and hence is devoid of any creation. A similar case arises in one of Berman's models [6] as well, in which it was claimed that the creation of particles could be avoided for $m = \frac{3}{2}$ in the present universe. In fact $\rho_v = 0$ in his model too for $m = 2$ and $\frac{3}{2}$ in the RD and MD phases respectively as can be checked by using the correct form of equation (15) of that paper which should be

$$B = \left[\frac{3(1+\alpha)}{2m} - 1\right]A$$

for consistency. The problem of creation is then out of the question. (One may also see ref.[22])

Equations (45) and (46), in the early RD era are

$$\rho_r = \frac{3}{16\pi G}\left[m(mt)^{-2} - \frac{\alpha^2}{3}(nt)^{\frac{-6}{m}}\right], \tag{48}$$

$$\rho_v = \frac{3}{16\pi G}\left[(2-m)(mt)^{-2} + \frac{\alpha^2}{9}(nt)^{\frac{-6}{m}}\right]. \tag{49}$$

When both matter and radiation are present, equations (45) and (46) become

$$\rho_r = \frac{3}{16\pi G}\left[m(mt)^{-2} - \frac{\alpha^2}{3}(nt)^{\frac{-6}{m}} - 4\pi G E_{mp}(nt)^{\frac{-3}{m}}\right], \tag{50}$$

$$\rho_v = \frac{3}{16\pi G}\left[(2-m)(mt)^{-2} + \frac{\alpha^2}{9}(nt)^{\frac{-6}{m}} - \frac{4\pi G}{3}E_{mp}(nt)^{\frac{-3}{m}}\right]. \tag{51}$$

We here note from equation (49) that $\rho_v > 0$ and $\dot{\rho}_v < 0$ for $m \leq 2$ (without restricting other constants) which shows creation of radiation from the decaying vacuum as has been mentioned in section II. This may also be the case for $m > 2$ with suitable choices of α and n. There is also creation of radiation from the vacuum energy in the MD era as is evidenced from equation (51) which shows that $\rho_v \ell^3$ can decrease with time. In order to obtain an estimate of the present value of the cosmological constant in this model, we use equation (14) giving

$$\Lambda = 3H^2 - \sigma^2 - 8\pi G\rho. \tag{52}$$

For the present values of the observables as taken in the previous section (thus imposing $0 < \frac{\alpha m}{3\sqrt{3}n}(nt_p)^{1-\frac{3}{m}} \leq \frac{1}{4}$) we get $\Lambda_p \simeq 10^{-84} GeV^2$ as in the previous section. (Equations (40) and (52) are the same via equation (33)).

V. Solutions for a Constant Active Gravitational Mass

A number of FLRW models have been investigated by assuming the conservation of active gravitational mass which is the consequence of a contracted Ricci-collineation along the fluid flow [19, 23, 24]. In the present anisotropic model, the active gravitational mass density $(\rho_t + 3p_t)$ is given from equations (14) and (15) as

$$\rho_t + 3p_t \equiv \rho + 3p - 2\rho_v = -\frac{1}{4\pi G}\left(3\frac{\ddot{\ell}}{\ell} + 2\sigma^2\right) \tag{53}$$

and the assumption of a constant active gravitational mass in a co-moving sphere, i.e.

$$(\rho + 3p - 2\rho_v)\ell^3 = \text{constant} = A \text{ (say)}, \tag{54}$$

then leads to

$$\ddot{\ell} + \frac{4\pi G A}{3}\ell^{-2} + \frac{2\alpha^2}{9}\ell^{-5} = 0. \tag{55}$$

Equation (55) indicates that $\ddot{\ell}$ may be positive for $A < 0$ (A may assume different constant values in the different phases of evolution [19]), implying that shear may not prevent inflation from occurring [25]. (Recently a number of pieces of evidences, especially studies of the Hubble diagram for type- Ia supernovae have lent support to the idea that the universe is dominated by a smooth component with an effective magnetic pressure, leading to an accelerating expansion [26]).

The integral of equation (55) yields

$$\dot{\ell}^2 = \frac{8\pi GA}{3}\ell^{-1} + \frac{\alpha^2}{9}\ell^{-4} + B, \tag{56}$$

where B is some constant of integration. Equations (14), (54) and (56) may be used to obtain

$$\rho + \rho_v = \frac{A}{\ell^3} + \frac{3B}{8\pi G\ell^2}, \tag{57}$$

and

$$p - \rho_v = -\frac{B}{8\pi G\ell^2}. \tag{58}$$

In order to obtain an exact solution of equation (56) we choose $B = 0$. Equation (58) then becomes

$$p = \rho_v, \tag{59}$$

showing that vacuum energy is always one-third of the corresponding radiation energy. Equation (57), by the use of equation (3) and (59) then yields

$$\rho_m + \frac{4}{3}\rho_r = \frac{A}{\ell^3}. \tag{60}$$

It is interesting to note that for $p = \rho_v$, the ansatz (54) follows directly from the conservation equation (19). Equation (56) now yields the solution

$$\ell = (6\pi GAt^2 + \alpha t)^{\frac{1}{3}} \tag{61}$$

by selecting the initial time at $\ell = 0$. The constant A may now be chosen as $A = (\rho_{mp}\ell_p^3 + \frac{4}{3}\rho_{rp}\ell_p^3) \equiv (E_{mp} + \frac{4}{3}E_{rp})$ as equations (54) or (60) indicate. Equations (13) and (61) yield

$$R \sim [6\pi GAt + \alpha]^{\frac{2}{3}}, \tag{62}$$

which is in agreement with equation (9) for $p = \rho_v$. The expression for the deceleration parameter q is now obtained as

$$q = \frac{1}{2}\left[\frac{1 + \alpha^2(6\pi GA)^{-1}(6\pi GAt^2 + \alpha t)^{-1}}{1 + \alpha^2(24\pi GA)^{-1}(6\pi GAt^2 + \alpha t)^{-1}}\right]. \tag{63}$$

Equations (12), (16) and (61) may be used to obtain

$$\frac{\sigma}{\theta} = \frac{\alpha}{\sqrt{3}(12\pi GAt + \alpha)}, \tag{64}$$

showing that $\frac{\sigma}{\theta} = \frac{1}{\sqrt{3}}$ initially. It decreases continuously and approaches 0 as $t \to \infty$ tending to isotropy. By the use of equation (64), equation (63) reduces to

$$q = \frac{1}{2}\left[1 + 9\left(\frac{\sigma}{\theta}\right)^2\right] \tag{65}$$

indicating that $q = 2$ at $t = 0$ and $q \to \frac{1}{2}$ as $t \to \infty$ as in section III. However, $0.5 < q_p \leq 0.8$, in this model, which is different from the corresponding one in section III. The value of Λ_p in the model is obviously the same as obtained in the previous sections.

It may be noted that ρ_r, ρ_v as well as ρ_m vary as ℓ^{-3}, indicating that there is no creation out of the vacuum, which decreases only due to the expansion of the universe. However, the anisotropy energy does decay into photons to bring about isotropy in the early universe. For $p = \rho_v$, the change in the entropy of the matter content due to the decrease of vacuum may be obtained from equation (19), which in this case reduces to

$$d(\rho\ell^3) + d(p\ell^3) = 0, \tag{66}$$

implying that $TdS \equiv dE + dW = 0$.

Finally a graphical representation of the evolutions of the different physical parameters in the models discussed in the previous sections (denoted by I, II, & III respectively in the graphs) has been provided which helps to have a better understanding of the models.

VI. Conclusion

In our attempt to solve the cosmological constant problem and find a more realistic model of the universe, we have obtained three plane symmetric Bianchi type I models with time-dependent Λ by assuming (i) $\rho+p=\rho_c$, (ii) q =constant and (iii) conservation of total active gravitational mass. Models are highly anisotropic in the early universe and start from a singularity. The critical density-model has highly negative Λ in the beginning which adds up to the anisotropy in the early universe. However the increasing (numerically decreasing) vacuum energy in the early universe in this model also contributes to the isotropization of the universe. When it becomes positive, it starts decaying, like the vacuum in the other two models, reducing asymptotically to one third of the corresponding radiation energy density. The positive, decaying vacuum energy in this model, as well as in the other two models, provides more room for anisotropy in the early universe than could be done by a constant (positive) vacuum. Though the former is still effective in the isotropization process via its peculiar equation of state. In all the three models, Λ as well as anisotropy are significant only in the early universe and the models reduce to the standard model in the present epoch.

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