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Gravitational Field of the Quantized Electromagnetic Plane Wave

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The quantum and classical descriptions of an electro-magnetic field are connected by the correspondence principle. We consider the electromagntic field as a source for gravity and compare the metrics due to a classical and quantized electromagnetic field. The quantization of the source demands the quantization of gravity. We also show that (i) the conformal degree of the freedom is *not* an unphysical mode, and (ii) quantum gravitational effects can exist even when the light cones do not fluctuate.

1. CLASSICAL AND QUANTUM SOURCES

The source for gravity is the energy momentum tensor of matter fields. Classical gravity is described by Einstein's equations

$$R_{ik} - \tfrac{1}{2} g_{ik} R = 8\pi G T_{ik} \tag{1}$$

The equations of motion for the source is contained in (1) because of the divergence-free nature of the left-hand side, implying

$$T^{i}{}_{k;i} = 0 \tag{2}$$

Because of (2), we are not allowed to prescribe the equations of motion for the source in an independent manner. A consistent solution to (1) will automatically determine the evolution of the source via (2).

Unfortunately for the general relativist, matter fields obey quantum mechanical laws and not classical ones. Since three decades of work on

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"quantizing gravity" has led virtually nowhere, one has to be content with approximate answers to problems involving quantum fields and gravity. These approximations usually proceed in the following manner.

The source, in the right-hand side of (1), is separated into two parts: a classical part with $T^{\rm cl}_{ik}$ and quantum part with $T^{\rm qu}_{ik}$. In the lowest-order approximation, one replaces (1) and (2) by

$$R^i_{\ k} - \tfrac{1}{2}\delta^i_{\ k} R \approx 8\pi G (T^i_{\ k})^{\rm classical} \tag{3}$$

and

$$[T^{i({\rm qu})}_{k}]_{;i} = 0 \tag{4}$$

Equations (3) and (4) define the subject of "quantum fields in curved space-time." (For an overview, see [1]) A classical source $(T^i_{\ k})^{\rm cl}$ produces a classical background g_{ik} via (3). In this zeroth order of approximation, all effects of $(T^i_{\ k})^{\rm qu}$ on g_{ik} are neglected, making (3) a c number equation. On the other hand, (4) is to be treated as an operator equation. In terms of the basic field variables, (4) would become the usual field equation in the curved space-time. A suitable Hilbert space, Fock basis, etc., are constructed based on (4). Since the background g_{ik} is not static (in general), there is inequivalence between the in- and out-states of the theory. *Most of the work in this subject assumes the quantum field to be in a vacuum state asymptotically.*

Once in a while, the next stage in the approximation (the "back reaction") is attempted. This consists of replacing (3) by

$$R^i_{\ k} - \tfrac{1}{2}\delta^i_{\ k} R \approx 8\pi G\{(T^i_{\ k})^{\rm cl} + \langle 0, {\rm in}|\,(T^i_{\ k})^{\rm qu}\,|0, {\rm in}\rangle\} \tag{5}$$

Of course, one has to solve (5) and quantize the fields in (4) in a self-consistent manner. It is only natural that much less progress has been made with (4) and (5) than with (3) and (4).

It appears that one particular aspect of the above approximation has not received sufficient atention. Consider, for example, the following question: What is the difference (if any) between the gravitational field produced by a classical electromagnetic field and that produced by the "corresponding" quantized electromagnetic field? (As it stands, the question is meaningless because the term "corresponding" has not been made precise; but we take care of this ambiguity later.) An answer to this question is based on the equations

$$R^i_{\ k} - \tfrac{1}{2}\delta^i_{\ k} R = 8\pi G \langle\psi|\,T^{i({\rm qu})}_{k}\,|\psi\rangle_R \tag{6}$$

and

$$T^{i\,{\rm qu}}_{k;i} = 0 \tag{7}$$

Equation (6) differs from (5) in three aspects: First, we have set $T^i_{\ k}{}^{(\rm cl)}$ to zero; there is no source other than the particular quantum field. Second, we evaluate the expectation value of $T^i_{\ k}{}^{(\rm qu)}$ in some specified quantum state $|\psi\rangle$ which *need not* (and, in general, will not) be a vacuum state. One may say that if the quantum field is in the state $|\psi\rangle$, then the metric has a particular form g_{ik}. Last, we have added a subscript R to the expectation value in (6) to remind ourselves that these expectation values need to be regularized. Equation (7) has to be treated just like (4); needless to say, (6) and (7) have to be solved self-consistently.

The above problem has been analyzed previously in the literature in the context of $|\psi\rangle$ being a vacuum state ([1], chap. 6). Classically, we expect the vacuum state to lead to a flat space-time. Quantum mechanically, the existence of *topological* contribution to $T^i_{\ k}$ allows nonflat solutions to (6).

In this paper, we are concerned with the opposite limit, that of the semiclassical transition between (1) and (6). Naive application of the correspondence principle would lead us to expect the solution of (1) to arise as a "suitable" limit of the solution to (6). We attempt to define and analyze this limit in a very specific context, the metric due to a plane electromagnetic wave traveling along the z axis. The classical solution is reviewed briefly in Section 2, and the quantum theoretical extension is discussed in Section 3. The conclusions are summarized in the last section.

2. CLASSICAL SOLUTION

Consider a line element of the form

$$ds^2 = L^2(\bar u)(d\bar x^2 + d\bar y^2) - d\bar u\, d\bar v \tag{8}$$

in which $\bar u$, $\bar v$ are the null coordinates

$$\bar u = \bar t - \bar z \qquad \bar v = \bar t + \bar z \tag{9}$$

The only nonvanishing component of R_{ik} corresponding to (8) is ([2] p. 962.)

$$R_{\bar u\bar u} = -\frac{2}{L}\frac{d^2 L}{d\bar u^2} \tag{10}$$

It is easy to verify that (8) can represent the metric due to a plane electromagnetic wave traveling along the $\bar z$ axis. Suppose the vector potential $A_i(x)$ describing such a wave is chosen to be

$$A_{\bar x} = A(\bar u) \qquad A_i = 0 \qquad \text{for} \quad i \neq \bar x \tag{11}$$

The field F_{ik} and the energy momentum tensor T_{ik} due to (11) have the following nonvanishing components

$$F_{\bar{u}\bar{x}} = F_{\bar{t}\bar{x}} = -F_{\bar{z}\bar{x}} = \left(\frac{dA}{d\bar{u}}\right) \tag{12}$$

$$T_{\bar{u}\bar{u}} = \frac{1}{4\pi L^2}\left(\frac{dA}{d\bar{u}}\right)^2 \tag{13}$$

Maxwell's equations

$$\frac{1}{(-g)^{1/2}}\,\partial_i((-g)^{1/2}F^{ik}) = 0 \tag{14}$$

are identically satisfied for arbitrary $A(\bar{u})$. Einstein's equations reduce to

$$L\frac{d^2L}{d\bar{u}^2} + G\left(\frac{dA}{d\bar{u}}\right)^2 = 0 \tag{15}$$

determining L in terms of $A(\bar{u})$.

One of the simplest choices that can be made for $A(\bar{u})$ is the monochromatic wave

$$A(\bar{u}) = A_0 \sin\omega(\bar{t} - \bar{z}) = A_0 \sin(\omega\bar{u}) \tag{16}$$

For this choice (15) has the following particular solution

$$L(\bar{u}) = G^{1/2}A_0\cos(\omega\bar{u}) \tag{17}$$

It should be noted that (17) is a very special solution and in no way represents the most general metric corresponding to (16). For example, consider the short wave length limit ($\omega \to \infty$) in which one can average the physical quantities over many wave lengths. Then, denoting average values by an overbar

$$\overline{(dA/d\bar{u})^2} = A_0^2\omega^2\,\overline{\cos^2\omega\bar{u}} = \tfrac{1}{2}A_0^2\omega^2 \tag{18}$$

If this averaged value is used in (15) we get

$$L\frac{d^2L}{d\bar{u}^2} + \frac{1}{2}GA_0^2\omega^2 = 0 \tag{19}$$

which is solved in the parametric form by

$$L^2(\eta) = L_0^2\exp\{-GA_0^2\omega^2\eta^2\} \tag{20}$$

$$\bar{u}(\eta) = L_0\int_0^\eta \exp\left\{-\frac{1}{2}GA_0^2\omega^2\tau^2\right\}d\tau \tag{21}$$

There is no way to reproduce L of (20) by averaging over (17). Since (15) is a nonlinear equation, its solutions cannot be decomposed into a simple set of basis solutions.

For future convenience we transform the metric in (8) into a conformally flat form via the coordinate transformation

$$u \equiv \int\frac{d\bar{u}}{L^2(\bar{u})}, \qquad x = \bar{x},\quad y = \bar{y},\quad z = \bar{z} \tag{22}$$

In terms of (x, y, v, u) (8) becomes

$$ds^2 = L^2(u)[dx^2 + dy^2 - du\,dv] \tag{23}$$

and (10) transforms to

$$R_{uu} = \left(\frac{\partial\bar{u}}{\partial u}\right)^2 R_{\bar{u}\bar{u}} = -2L\frac{d}{du}\left(\frac{1}{L^2}\frac{dL}{du}\right) \tag{24}$$

Similarly (11)–(13) transform to

$$A_x = A_{\bar{x}}[\bar{u}(u)] = A_x(u) \equiv A(u) \tag{25}$$

$$F_{ux} = \left(\frac{\partial\bar{u}}{\partial u}\right)F_{\bar{u}\bar{x}} = \left(\frac{dA}{du}\right) \tag{26}$$

$$T_{uu} = \left(\frac{\partial\bar{u}}{\partial u}\right)^2 T_{\bar{u}\bar{u}} = L^4\frac{1}{4\pi L^2}\frac{1}{L^4}\left(\frac{dA}{du}\right)^2 = \frac{1}{4\pi L^2}\left(\frac{dA}{du}\right)^2 \tag{27}$$

Einstein's equations ($R_{uu} = 8\pi T_{uu}$) are, of course, the same as (15) transformed to the u coordinate

$$L\frac{d}{du}\left(\frac{1}{L^2}\frac{dL}{du}\right) + \frac{G}{L^2}\left(\frac{dA}{du}\right)^2 = 0 \tag{28}$$

The solutions given in (16) and (20) can also be transformed to the u coordinate. However, a monochromatic plane wave in the $\bar{u}$ coordinate will not have any simple physical interpretation in terms of u. An observer using u coordinate will assume a monofrequency wave to be

$$A(u) = A_0\sin\omega u \tag{29}$$

Note that the phase is ωu and not $\omega\bar{u}$. Thus (28) becomes

$$L^3\frac{d}{du}\left(\frac{1}{L^2}\frac{dL}{du}\right) + \frac{1}{2}GA_0^2\omega^2\cos^2\omega u = 0 \tag{30}$$

Unfortunately (30) cannot be solved in closed form, except in the high frequency limit. In this limit, using (18) we replace (30) by

$$L^3 \frac{d}{du}\left(\frac{1}{L^2}\frac{dL}{du}\right)+\frac{1}{2}GA_0^2\omega^2=0 \tag{31}$$

which has the particular solution

$$L(u)=1+\tfrac{1}{2}G^{1/2}A_0\omega u \tag{32}$$

(The solution has the correct limiting behavior for $\omega=0$.) We now proceed to study the quantized analogues of these solutions.

3. METRIC FOR THE QUANTIZED SOURCE

We assume that as long as the quantum state of the electromagnetic field $|\psi\rangle$ is chosen suitably, the metric can be chosen to have the same form as before

$$ds^2=L^2(u)[dx^2+dy^2-du\,dv] \tag{33}$$

To obtain the quantum analogues of the solutions in the previous section, we have to proceed in the following manner: (i) Quantize the electromagnetic field in the background metric (33). (ii) Choose a particular state $|\psi\rangle$ for the field. (iii) Prescribe a regularization scheme and use it to evaluate the regularized expectation value $\langle T_{ik}\rangle_R$ in this particular state $|\psi\rangle$. (iv) Determine $L(u)$ via (6).

In order to carry out the first step, we note that Maxwell's equations (in four dimensions) are invariant under the following ("conformal") transformations

$$A_i\to A_i \qquad F_{ik}\to F_{ik} \qquad g_{ik}\to\Omega^2 g_{ik} \tag{34}$$

$$A^i\to\Omega^{-2}A^i \qquad F^{ik}\to\Omega^{-4}F^{ik} \qquad g^{ik}\to\Omega^{-2}g^{ik} \tag{35}$$

Thus if $(\bar{A}_i, \bar{g}_{ik})$ is a solution to Maxwell's equations, then so is $(\bar{A}_i, \Omega^2\bar{g}_{ik})$. By choosing $\bar{g}_{ik}$ to be the flat metric we immediately reach the conclusion that any flat space-time solutions of Maxwell's equations remain valid in any conformally flat space-time like (33) as well. Thus even in the background (33) one can decompose an electromagnetic field into plane wave modes, e.g., taking the gauge $A^i=(0,\mathbf{A})$

$$\mathbf{A}(\mathbf{x},t)=\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{L^2(u)}\left[\frac{\mathbf{a}_\mathbf{k}}{(2\omega_\mathbf{k})^{1/2}}e^{i(\mathbf{k}\cdot\mathbf{x}-\omega_\mathbf{k}t)}+\text{h.c.}\right] \tag{36}$$

where $\mathbf{a}_\mathbf{k}$ corresponds to the annihilation operator for the particular mode and "h.c." denotes the hermitian conjugate. [Note the $L^{-2}(u)$ factor in (36) which arises because the 3-vector $\mathbf{A}$ is defined via the "contravariant index" $A^i(x; L^2\eta)=L^{-2}A^i(x;\eta)$.[2]] The usual procedure for quantization of a field in curved space-time can be now carried out, rather trivially, because we are working with a conformally invariant theory in conformally flat space-time.

The states of the electromagnetic field can now be labeled by the set of occupation numbers $\{n_\mathbf{k}\}$. More general states can be constructed by superposition of these basis states.

Our next task is to choose a suitable state $|\psi\rangle$ such that the ansatz in (33) can be a solution to (6). Since, classically, (33) corresponds to a plane wave propagating along the z axis, it is reasonable to expect $|\psi\rangle$ to represent such a plane wave in the clssical limit. This can be achieved in two different ways:

(i) We can choose $|\psi\rangle$ to be a state with n photons in the mode $\mathbf{k}=(0,0,k_z)$ and zero photons at all other modes. In the limit of large n, this would represent a classical plane wave.

(ii) We can, instead, choose $|\psi\rangle$ to be a coherent state with the property

$$\begin{aligned}\langle\psi|A_i|\psi\rangle&=A(u) &&\text{for}\quad i=x\\ &=0 &&\text{for}\quad i\neq x\end{aligned} \tag{37}$$

The classical limit of such a state is clearly a plane wave.

In either case, honest evaluation (usually called "naive" evaluation) of the expectation value of T_{ik}, in the metric $L^2\eta_{ik}$

$$\tau_{ik}\equiv\langle\psi|(T_{ik})_{L^2\eta}|\psi\rangle \tag{38}$$

will diverge. There are many "prescriptions" which allow one to choose *a finite part* of (38) and throw away the rest. We use the simplest idea, based on the following requirement: when no topological contributions are present, T_{ik} must vanish in the vacuum state. We therefore replace τ_{ik} by

$$\begin{aligned}t_{ik}&\equiv\langle\psi|(T_{ik})_{L^2\eta}|\psi\rangle-\langle 0|(T_{ik})_{L^2\eta}|0\rangle\\ &\equiv\langle\psi|(T_{ik})_{L^2\eta}|\psi\rangle_R\end{aligned} \tag{39}$$

Here $|0\rangle$ represents the vacuum state in the background of (33). [This vacuum state is well-defined (via $\mathbf{a}_\mathbf{k}|0\rangle=0$) since (pair) creation of conformally invariant particles does not occur in conformally flat space-times.]

[2] Here and elsewhere $\eta=\eta_{ik}$ is the flat space metric.

The evaluation of (39) can again be accomplished by using conformal invariance. Note that

$$\langle\psi|(T_{ik})_{L^2\eta}|\psi\rangle=\langle\psi|(T_{ik})_\eta|\psi\rangle\frac{1}{L^2(u)}+C_{ik} \tag{40}$$

where the first term on the right-hand side corresponds to the classical transformation law for T_{ik}, while C_{ik} represents the quantum conformal anomaly. Usually one is interested in the effects of quantum fields in the vacuum state and, hence, C_{ik} makes an important contribution. In our case, however, C_{ik} cancels out when the subtraction on the right-hand side of (39) is performed. (Note that C_{ik} is independent of $|\psi\rangle$). Thus, we can take

$$(t_{ik})_{L^2\eta}=\frac{1}{L^2}(t_{ik})_\eta \tag{41}$$

as in the classical case.

The above prescriptions allow one to use the classical solutions of the previous section in the quantum domain as well! For example, it is well-known that the expectation value in the coherent state of quadratic polynomials of the field variables have the same form as the corresponding classical expressions. Thus, in a coherent state with

$$\langle \mathrm{coh}|A_x|\mathrm{coh}\rangle_\eta=A(u) \tag{42}$$

the energy momentum tensor will give the only nonzero expectation value

$$(t_{uu})_\eta=\frac{1}{4\pi}\left(\frac{dA}{du}\right)^2 \tag{43}$$

so that

$$(t_{uu})_{L^2\eta}=\frac{1}{4\pi L^2}\left(\frac{dA}{du}\right)^2 \tag{44}$$

Semiclassical equations

$$R_{ik}-\tfrac{1}{2}g_{ik}R=8\pi G\langle \mathrm{coh}|T_{ik}|\mathrm{coh}\rangle_R \tag{45}$$

are identical to the classical equation (28). Any solution to (28) can be interpreted as a solution to (45), which incorporates the "back reaction."

Let us next consider the n particle photon state in which all the photons are taken to be in the mode $\mathbf{k}=(0,0,k_z)$. In flat space-time the only nonvanishing expectation value is

$$(t_{uu})_\eta=\frac{1}{4\pi}\left(\frac{n}{2\omega_\mathbf{k}}\right)k_uk_u=\left(\frac{n\omega}{8\pi}\right) \tag{46}$$

so that

$$(t_{uu})_{L^2\eta}=\frac{1}{8\pi}\frac{n\omega}{L^2(u)} \tag{47}$$

The field equation (45) becomes

$$L^3\frac{d}{du}\left(\frac{1}{L^2}\frac{dL}{du}\right)+\frac{1}{2}Gn\omega=0 \tag{48}$$

We see that (48) has the same structure as (31) if we identify

$$A_0^2=n/\omega \tag{49}$$

which is the usual relation between the amplitude, frequency, and the mode number of a harmonic oscillator. The solution in (32) can be borrowed for the present case.

Thus we see that the line element due to an electromagnetic field in a monochromatic state $|n_\mathbf{k}\rangle$ can be represented by

$$ds^2=[1+\tfrac{1}{2}(Gb\omega)^{1/2}u]^2[dx^2+dy^2-du\,dv] \tag{50}$$

This is an exact result and is valid for all n and ω. [In contrast, (32) was valid only in the high frequency limit.] We now analyze some consequences of this result.

4. NEED TO QUANTIZE GRAVITY?

It is possible to have a framework in which gravity remains classical and the other fields are quantized? Such a framework would necessarily demand an equation like (6) to be valid exactly. In general, a formalism based on (6) would entail the following complications: (i) It is conceivable that the choice of states $|\psi\rangle$ is limited if both (6) and (7) have to be satisfied. In other words, one may have to restrict oneself to a subspace of the full Hilbert space in a rather unnatural fashion. (ii) In general, a time-dependent gravitational field will act as a classical source and create "particles," i.e., the states in the Fock basis, parametrized by the occupation numbers $\{n_\mathbf{k}\}$, will not be stationary states. (In the present paper, we bypassed this problem using conformal invariance.)

These two difficulties suggests that a theory in which gravity is classical will appear to be quite unnatural and artificial. A more serious difficulty forcing us to abandon the concept of classical gravity arises when we consider the following situations.

Consider an atom with a ground state $|G\rangle$ and an excited state $|E\rangle$. Let the lifetime of the excited state be τ. It is assumed that the decay to the ground state occurs with a single photon emission. We also assume that the energy of the excited state is very high compared to the ground state. The atom is prepared in the excited state $|E\rangle$ and kept at the origin of a coordinate system. An observer at a large distance makes measurements on the space-time metric in order to determine the state of the atom. As long as the atom is in $|E\rangle$, the metric would closely approximate a Schwarzschild metric with the energy of the atom in state $|E\rangle$ acting as the source. Once the decay occurs, the metric would be (approximately) that of a single-photon state. (For the sake of argument, we may assume that the energy difference between the excited and ground states is far higher than the energy in the ground state. So, after the decay, the energy carried by the photon dominates.) Such an intuitive expectation will not be realized if we use $\langle\psi| T_{ik} |\psi\rangle$ to be the source. It can be shown that if gravity is treated as classical the metric continuously proceeds from that of the point particle to that of a single photon. (This is a slight adaptation of an argument originally due to Unruh [3]).

The above analysis has a counterpart in the case of the pure electromagnetic field itself. Suppose that the state of the electromagnetic field is given by

$$|\psi\rangle = \cos\theta |n_1\rangle + \sin\theta |n_2\rangle \tag{51}$$

where

$$\langle\psi|\psi\rangle = \langle n_1|n_1\rangle = \langle n_2|n_2\rangle = 1 \tag{52}$$

and

$$\langle n_1|n_2\rangle = 0 \tag{53}$$

(The state labeled $|n\rangle$ has n photons in a particular mode $\mathbf{k}$ and zero photons in other modes; we suppress the index $\mathbf{k}$.) The expectation value $\langle\psi| T_{ik} |\psi\rangle$ has the form

$$\langle\psi| T_{ik} |\psi\rangle = \cos^2\theta \langle n_1| T_{ik} |n_1\rangle + \sin^2\theta \langle n_2| T_{ik} |n_2\rangle \tag{54}$$

as long as $|n_1 - n_2| \neq 2$. We take $n_1 \gg n_2$. Following an analysis similar to that in (46)–(50) we get the conformal part of the metric to be

$$L^2(u) = [1 + \tfrac{1}{2}(Gn\omega)^{1/2} u]^2 \tag{55}$$

with

$$n = n_1 \cos^2\theta + n_2 \sin^2\theta \tag{56}$$

The *probability* for $|\psi\rangle$ to be n_1 photon state is $\cos^2\theta$ and the *probability* for $|\psi\rangle$ to be n_2 photon state is $\sin^2\theta$. Thus n in (56) does represent the *classical* mean value for n_1 and n_2.

Such a description, however, cannot be exact because it would involve discontinuous changes in the metric when the wave function "collapses." Suppose that a detector, capable of measuring the occupation number of the $\mathbf{k}$th mode interacts with the photon field. Such a detection (taking place at an event $(t, \mathbf{x})$, say) will change $|\psi\rangle$ to $|n_1\rangle$ (with a probability of $\cos^2\theta$) or to $|n_2\rangle$ (with a probability of $\sin^2\theta$). The metric, in the future domain of the event $(t, \mathbf{x})$, will have the form of (55) with n replaced by n_1 (with probability $\cos^2\theta$) or by n_2 (with probability $\sin^2\theta$). Whichever event occurs, the metric has to change discontinuously across the hypersurface of collapse. Our desire to keep gravity classical has led to such artificialities.

It is more natural to assume that the gravitational field is also quantized. In such a case, a possible description will proceed somewhat like this: The quantum state $|\psi\rangle$ now describes the quantum state of the metric and the photon field, and is again parametrized by an integer n. One possibility would be to have basis states $|n\rangle$ with

$$\langle n| L(u) |n\rangle = [1 + (Gnw)^{1/2} u] \tag{57}$$

and

$$\langle n| T_{ik} |n\rangle_R = \tfrac{1}{2}(nk_i k_k/\omega) \tag{58}$$

Then if the state is $|\psi\rangle$, the conformal factor will have a value $L_1(u)$ with a *probability amplitude* $\cos\theta$ and $L_2(u)$ with a *probability amplitude* $\sin\theta$. *The measurement of either the photon occupation number or the conformal factor will collapse* $|\psi\rangle$ *to an eigenstate of the number operator.*

The description in the above paragraph attempts to *guess* at the possible quantized version of the gravity. Such guesswork, which ignores the dynamics of the system, is quite likely misleading. Neverthless, the attempt does illustrate the following points:

(i) A quantum description of gravity appears to be more natural than classical description. In the past, there have been attempts to justify the quantization of gravity based on measurement processes [4], [5], [6], and [3]. Of these, the works of DeWitt and Eppeley et al. concentrate on the quantum limitations of the *detecting system*. In contrast, our analysis is based on the quantum nature of the *source* and is independent of the detector physics. (This point of view is more along the lines of Page and Geilker and Unruh.) We have analyzed a fully general relativistic solution due to a quantized source, while the previously mentioned authors worked in a Newtonian approximation.

(ii) It is seen that the quantum nature of the source *demands* the quantization of the *conformal* degree of freedom of the metric. In our analysis, this is entirely due to the fact that the electromagnetic field is conformally invariant. *But it does show that the conformal degree of freedom is a physical degree of freedom.* This should have been obvious from the fact that there exist nontrivial space-time geometries in which conformal degree of freedom is the only degree of freedom (Robertson–Walker space-times, Plane wave metric of Section 2, etc. etc.). The above analysis confirms the suspicion that "transverse degrees of freedom" are not to be isolated in a simpleminded fashion in quantum gravity.

(iii) As a consequence of the above feature we see that the quantum dynamics of the source (in our special case) leaves the light cone structure unaltered. Clearly, nontrivial quantum gravitational physics can exist even without fluctuations in the light cone. (Of course, when more general sources are concerned, this situation will change; that, however, is irrelevant to the conclusion drawn here.)

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REFERENCES

1. Birrell, N. D., and Davies, P. C. W. (1982). *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge).
2. Misner, C. W., Thorne, K. S., and Wheeler, J. A. (1973). *Gravitation* (Freeman, San Francisco).
3. Unruh, W. G. (1984). In *Quantum Theory of Gravity*, Christensen, S. M., ed. (Adam Hilger, Bristol).
4. DeWitt, B. (1962). In *Gravitation: An Introduction to Current Research*, Witten, L., ed. (John Wiley & Sons, New York).
5. Eppeley, K., and Hannah, E. (1977). *Found. Phys.*, **7**, 51.
6. Page, D. N., and Geilker, C. D. (1981). *Phys. Rev. Lett.*, **47**, 979.

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Bianchi Type V Perfect Fluid Models with Source-Free Electromagnetic Fields

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The Einstein–Maxwell Field equations characterizing a nontilted Bianchi type V perfect fluid model with source-free electromagnetic field are solved exactly in the nonlocally rotationally symmetric case. It is found that these equations admit one and only one exact solution, expressible, however, in terms of two arbitrary functions.

1. INTRODUCTION

The relevance of the study of Bianchi type V cosmological models has already been discussed in one of our earlier papers [1] where we studied the perfect fluid models in some detail. As a natural sequel to that study, here we incorporate an electromagnetic field into the perfect fluid Bianchi type V models and solve the coupled field equations exactly. In literature, we find the study of such models to be scarce and limited. Ftaclas and Cohen [2] first made attempts to solve the coupled field equations for Bianchi type V nontilted models with the equation of state $\rho = p$, where ρ is the density and p is the pressure of the fluid. Lorenz [3] later solved these equations with the same equation of state but for a tilted model in the locally rotationally symmetric case.

In this paper, we have exactly solved the Einstein–Maxwell coupled field equations for nontilted diagonal Bianchi type V space-times in the nonlocally rotationally symmetric case. It is found that these equations admit one and only one exact solution expressible in terms of two arbitrary functions.

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