Electrodynamics and Cosmology

Lectures by

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Lecture 1

SUMMARY OF BASIC IDEAS IN CLASSICAL ELECTRODYNAMICS

My first lecture will deal with the early development of electromagnetism. In later lectures I shall discuss the role of electromagnetism in cosmology.

When you look at the history of physics you find that the relative importance of astronomy has undergone great variation from time to time. For example, before Newton, astronomy was the most important branch of physics. However, with Newton came the important developments in the laws of motion and laboratory physics. Still later, toward the end of the 19^{th} century, laboratory physics attained a dominant role despite important advances in astronomy. The relative importance of laboratory physics has lasted until recently. However, as you have heard in the other lectures here, recent developments in astrophysics have brought it once again to the forefront of physics.

After Newton's law of gravitation

$$F = \frac{Gmm'}{r^2} \qquad (1.1)$$

received general acceptance, physicists tried to formulate a similar mathematical law for electromagnetism

$$F = \pm \frac{ee'}{r^2} \qquad (1.2)$$

Coulomb's law also seemed to work very well for a while but not as successfully as Newton's law. The reason is easy to see. The law of gravitation, although it worked very well, could not be applied with the same degree of freedom as Coulomb's law. The smallness of the gravitational constant prevents us from using laboratory masses in a variety of experiments. Large masses are only available to us through astronomy where we may observe the masses but we cannot disturb them. The interaction of isolated charges is much more easily studied in the laboratory since the interaction is relatively large. For typical elementary particles(such as an electron and a proton) the ratio of the electrical to the gravitational force is $\sim 10^{40}$. When people began to investigate Coulomb's law in the laboratory they found that it does not work very well. In particular, the discrepancy from Coulomb's law arises when the interacting charged particles are in motion relative to each other. For rapidly moving or oscillating charges, Coulomb's law simply breaks down.

Originally, physicists tried to patch up Coulomb's law by adding terms depending on the particles' velocity and acceleration, but this does not lead to a neat and simple law. Gauss realized that the problem was in part due to the use of an instantaneous action between particles. His was the first suggestion that the velocity of propagation between charged particles might propagate with the velocity of light. Gauss did not pursue this work and progress was left to others. Maxwell finally solved the problem in an altogether different way by introducing the notion of the electromagnetic field. Thus particles influenced one another through disturbances in this field. Maxwell also derived the equation showing that disturbances in the field propagated with the velocity of light. Maxwell's theory not only led to predictions in agreement with experiment but also inspired other theoretical advances such as the Special Theory of Relativity. Einstein wondered whether the role of velocity of light could be only an "accident" in Maxwell's theory, or part of some more general scheme. In the Special Theory of Relativity it seems very natural that Maxwell equations are invariant under Lorentz transformations which preserve the basic nature of the velocity of light.

In the beginning of this century a number of people felt that Gauss idea of a direct interparticle interaction should be revived. This was done by Schwarzschild (1903), Tetrode (1922) and Fokker (1929 a, b; 1932). Basically they suggested that the interaction takes place via retarded, rather than instantaneous, interaction at a distance. Consider the world lines of two particles of charges e and e' (Figure 1.1). The old formulation of instantaneous action would occur along lines AC while the Special Theory demanded that the propagation proceeded along DA or AB. In order to retain the Newtonian concept of action and reaction we must allow advanced effects along the light cone as well as retarded effects. If the concept of the law of action and reaction is to be valid then one must allow advanced interactions to proceed from B to A and A to D. We might ask "Why not drop the concept of action and reaction?" It turns out that you cannot formulate a Lagrangian for action at a distance consistent with the ideas of classical physics without the notion of action and reaction. So, if you want to have retarded action at a distance you must also have advanced interactions. Now advanced interactions seem to violate our sense of causality since the future will influence the past. Consider the following typical paradoxical situation. In figure 1.2 A and B are separated by one light hour and operate according to the rules: 1) A sends a signal to B at 4 p.m. if and only if he does not hear from B at 4 p.m.; 2) B sends a signal to A at 5 p.m. if and only if he hears from A at 5 p.m.. What does A do at 4 p.m.? If A sends a signal at 4 p.m. it means he did not receive a signal from B. Then A's signal would reach B at 5 p.m.. B would in turn send a signal to A. However, the advanced effect of B's signal would reach A at 4 p.m. which contradicts the assumption that A

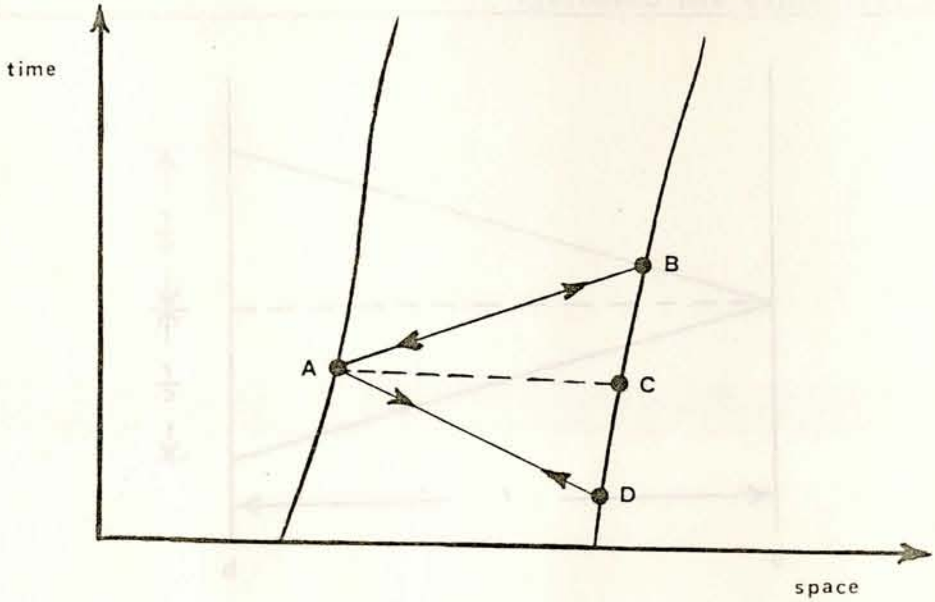

Figure 1.1

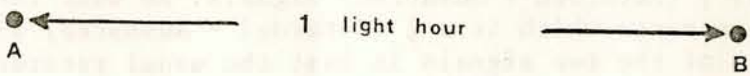

Figure 1.2

did not receive a signal at 4 p.m.. If conversely you assume that A did not send a signal at 4 p.m. you come to the conclusion that A in fact should have sent a signal at 4 p.m.. We find then that neither possibility is self consistent. In the 1940's Wheeler and Feynman addressed themselves to this problem. They suggested that you can make use of the Fokker, Tetrode and Schwartzschild formalism provided you take into account the rest of the universe. This is where cosmology first comes into picture. Suppose particle (a) and particle (b) described by world lines in figure 1.3 are separated by a distance r. Then if (a) sends a foreward signal to (b) it will reach (b) at a time r/c later. The advanced reaction from (b) will come back along the same path so that (a) will experience a reaction which is instantaneous with the emission of the signal regardless of the distance of separation, r. Therefore, we must include the interaction with all the matter in the universe

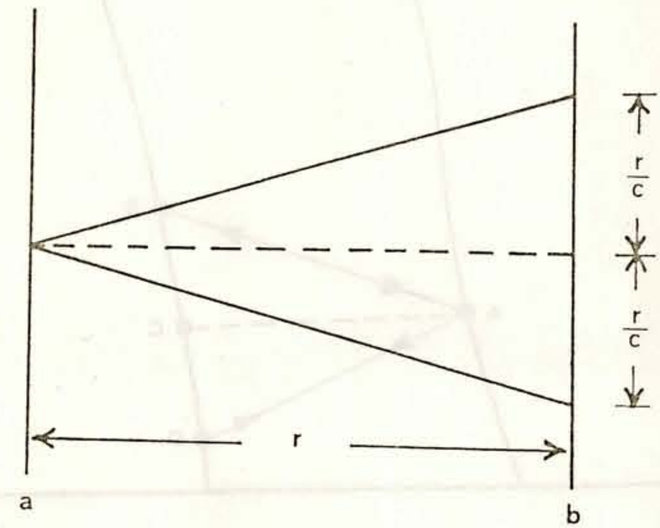

Figure 1.3

and calculate the response on particle (a).

First, I will describe what we expect from the response of the universe and in later lectures I will describe the detailed calculations. Normally, the particles radiate both forward and backward in time i.e. $\frac{1}{2}$ (Retarded + Advanced) signals. We want the universe to yield a response which is a $\frac{1}{2}$ (Retarded - Advanced) signal so that the sum of the two signals is just the usual retarded signal. We shall see later that the Wheeler-Feynman analysis gives just this result in certain cases.

We now present some mathematical relations leading up to the Wheeler-Feynman result. We start with a review of Hamilton's principle of least action.

Classical Action Principle

A free particle of mass m, moving along a path $\underset{\sim}{a}$ between two points fixed at times t_1 and t_2 possesses an action J, where

$$J = \frac{1}{2}\int_{t_1}^{t_2} m\dot{\underset{\sim}{a}}^2(t)dt. \qquad \text{(non-relativistic)} \qquad (1.3)$$

Now Hamilton's principle requires that we choose that path, $\underset{\sim}{a}(t)$, which makes the action stationary

$$\delta J = 0. \qquad (1.4)$$

Using the usual Euler-Lagrange variational techniques we obtain

$$m\ddot{\underset{\sim}{a}} = 0 \qquad (1.5)$$

Of course this is Newton's law restated for a free particle. We use the variational principle because it allows us to treat more complicated problems in a simpler manner than if treated by Newton's law. One cannot hope to be like Newton and guess the right answer!

We can extend the non-relativistic to a relativistic treatment. The simplest scalar quantity one can associate with a particle's motion is *mda* and the action is given by

$$J = -m\int da, \tag{1.6}$$

where da is an element of proper time along the path of the particle defined by $da^2 = \eta_{ik}da^i da^k$, where the a^i are the coordinates of the particle and η_{ik} is the Minkowski metric of signature -2. The speed of light c = 1.

If we again take the variation of the action equal to zero we obtain the relativistic equation of motion of a free particle,

$$m\frac{d^2a^i}{da^2} = 0. \tag{1.7}$$

We can extend this idea to other branches of physics such as Maxwell's electrodynamics. Here the simplest action which describes the equations of motion of both fields and particles is

$$J = -\sum_a m_a\int da - \frac{1}{16\pi}\int F_{ik}F^{ik}d^4x - \sum_a e_a\int A_i da^i \tag{1.8}$$

where again the speed of light set equal to one, A_i is the four potential $A_i = \left[-\vec{A},\phi\right]$, $F_{ik} = \partial_i A_k - \partial_k A_i$, and e_a is the charge of particle (a).

Variation of the action with respect to the world-line of particle (a) yields the equation of motion

$$m_a\frac{d^2a^i}{da^2} = e_a F^i_{\ k}\frac{da^k}{da} \tag{1.9}$$

Varying the action with respect to the fields leads to the equation

$$F^{ik}{}_{,i} = 4\pi j^k, \tag{1.10}$$

where

$$j^k(x) = \sum_a e_a\int\frac{da^k}{da}\,\delta_4(x,A(a))da \tag{1.11}$$

and $A(a)$ is a point on the world-line of particle (a). We can compare the action just treated with that given in an action at a distance formulation

$$J = -\sum_a m_a \int da - \sum_{a<b}\sum e_a e_b \iint \delta(s_{AB}^2)\eta_{ik}\, da^i da^k \tag{1.12}$$

Here s_{AB}^2 is the square of the four-distance between point A, on the world-line of particle (a), and B on the world line of particle (b). In Minkowski coordinates

$$s_{AB}^2 = (t_B - t_A)^2 - (\vec{x}_B - \vec{x}_A)^2 \tag{1.13}$$

The symbol δ denotes the usual Dirac delta function. Finally the double sum does not permit self interactions.

The delta function implies that in order that there be a non-zero contribution to the integral, $s_{AB}^2 = 0$. Therefore we get interactions between particles (a) and (b) whenever

$$t_B - t_A = \pm |\vec{x}_B - \vec{x}_A| \tag{1.14}$$

The two possibilities account for the retarded and advanced interactions. We define the four-potential produced by a particle (a) at a field point X

$$A_i^{(a)}(X) = e_a \int \delta(s_{AX}^2)\eta_{ik}\, da^k \tag{1.15}$$

$$= \frac{1}{2} A_i^{(a)}(\text{ret}) + A_i^{(a)}(\text{adv})$$

and the current by

$$j_i^{(a)}(X) = e_a \int \delta_4(X,A)\eta_{ik}\, da^k \tag{1.16}$$

From these definitions we immediately get

$$\Box A_i^{(a)} = 4\pi j_i^{(a)}$$
$$A^{(a)i}{}_{,i} = 0 \tag{1.17}$$

That is, Maxwell's equations and the gauge condition are identically satisfied. When we vary the world line of particle (a) in the action, we obtain

$$m_a \frac{d^2 a_i}{da^2} = \sum_{b \neq a} F^{(b)i}{}_k \frac{da^k}{da} \tag{1.18}$$

The particle then moves under the influence of all the other particles in the universe.

Lecture 2

THE WHEELER-FEYNMAN ABSORBER THEORY

I will now review the Wheeler-Feynman treatment of action at a distance electrodynamics. As mentioned in the previous lecture the particle produces a signal which is the sum of one-half the retarded and one-half the advanced *fields*[1]. We will show that the contribution of the universe is just one-half the retarded minus one-half the advanced *field* of the particle,leaving just the retarded field of the particle. Moreover the half retarded minus half advanced effect of the universe is just what is needed to account for radiative damping, an effect which one ordinarily does not expect from a theory devoid of self-interactions.

Because the expression describing the effect of the absorber is quite long I present it term by term together with explanation. Since we expect the universe to make a contribution equal to one-half the retarded minus one-half the advanced field, and further the source contributes one-half the sum of advanced and retarded fields, we expect to see the full retarded wave leaving the source. Therefore I shall use the full retarded wave for the calculation, and look for a self-consistent solution. For simplicity we can imagine the source to be located at the center of a cavity in the distribution of absorber particles. The absorber extends outward from a radius R from the source. The calculation proceeds as follows:

1) $\vec{U} = \vec{U}_\circ e^{-i\omega t}$ - It represents the acceleration of the source. We assume a general acceleration can be Fourier analyzed, in which case this term represents a particular component.

2) $-\frac{e}{r}\sin\Theta$ - Here e is the charge of the source particle. The angle Θ is the angle between the vector $\vec{U}_\circ$ and the line joining the source and a point in the absorber.

The product of 1) and 2) describes the strength of the retarded field at sufficiently large distances from the source.

3) $e^{i\omega r}$ - This is the phase change of the wave at a radial distance r from the source.

4) $2(1 + n - ik)^{-1}$ - A factor by which the outgoing disturbance must be reduced by reflection at the boundary of the absorber cavity.

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1. In the discussion of the action at a distance theory the word *field* will continue to be used. It is used for convenience and does not imply any independent existance of the electromagnetic disturbance.

5) $e^{i\omega(n-ik-1)(r-R)}$ - A factor allowing for the change of phase and amplitudes as the disturbance propagates through the absorber medium.

The product of the five terms above yields the magnitude of the electric field

$$\vec{E} = -\vec{U}_o\left(\frac{e}{r}\sin\theta\right)\left(\frac{2}{1+n-ik}\right)e^{-i\omega(t-r)}e^{i\omega(n-ik-1)(r-R)} . \qquad (2.1)$$

This field acts upon the absorber particles setting them in motion. We turn now to evaluating the response of the absorber to this field

6) $\frac{e_k}{m_k}P(\omega)E$ - This term is the acceleration of the absorber particle to the *electric field* E. $P(\omega)$ is the frequency dependence of the response of the particle. The complex index of refraction n-ik is related to $P(\omega)$ by the relation

$$(n-ik)^2 = 1 - \frac{4\pi e_k^2 N}{m_k\omega^2}P(\omega),$$

where N is the number of the absorber particles per unit volume.

7) $-\frac{e_k}{2r}\sin\theta\, e^{-i\omega r}$ - These terms correspond to terms 2) and 3) above, but are evaluated for the advanced field of the absorber at the source. The factor $\sin\theta$ arises because we are considering the resolved part of the absorber field parallel to the acceleration of the source.

So far we have calculated the field of an absorber particle back on the source particle. You may ask why I do not put the respective index into this part of the calculation. The reason is that we needed the net field that acted to accelerate the absorber particles, but now we want to calculate the direct elementary action of the absorber back on the source and not the net field back on the source. The product of terms 6) and 7) give the field of a single absorber particle on the source. Integrating the contribution of all the absorbers acting at the source and parallel to the acceleration $\vec{U}_o$ yields

$$\vec{R} = \frac{e\vec{U}_o}{1+n-ik}\frac{e_k^2}{m_k}\int_R^\infty\int_0^\pi\int_0^{2\pi} dr\,d\theta\,d\Phi\; P(\omega)N\sin^3\theta\, e^{-i\omega t}e^{i\omega(n-ik-1)(r-R)}$$

$$= -\frac{2}{3}\,ei\omega\vec{U}_o e^{-i\omega t} \qquad (2.2)$$

We see that all of the terms describing the characteristics of the universe have cancelled out. We are left with a contribution of the electric field which is parallel to the acceleration. If we now note that the Fourier component $(-i\omega)$ corresponds to differentiation, we have for the force acting on the source particle

$$e\vec{R} = \frac{2e^2}{3}\frac{d\vec{U}}{dt} \qquad (2.3)$$

This is the usual form of the non-relativistic radiation reaction. There is no net response force in any other direction.

In the calculation just completed we calculated the effect of the universe at the position of the particle. One can also calculate the *field* of the absorber in the neighborhood of the source. Wheeler and Feynman have done this. When you calculate this field, you find that the effect of the absorber is a field which is half of the difference of the retarded and advanced *fields* of the particle.

GENERAL METHOD

As we have seen the particular characteristics of the absorber have dropped out of the final result. This leads us to seek a more general way to arrive at the same result without going through all the intermediate details. This method is also found in Wheeler and Feynman (1945). We consider a completely absorbing universe, i.e. there is some boundary beyond which the *field* of the particles vanish:

$$\frac{1}{2}\sum_b (F^{(b)}_{ret} + F^{(b)}_{adv}) = 0 \qquad \text{(outside the absorber)} \qquad (2.4)$$

Here $F^{(b)}_{ret}$ (F^{b}_{adv}) is the retarded (advanced) field of the b^{th} particle. However, if the sum vanishes then

$$\sum_b F^{(b)}_{ret} = 0 \quad \text{and} \quad \sum_b F^{(b)}_{adv} = 0,$$

since there cannot be complete destructive interference between the outgoing retarded and incoming advanced waves. This implies that

$$\frac{1}{2}\sum_b (F^{(b)}_{ret} - F^{(b)}_{adv}) = 0 \qquad \text{(outside the absorber)} \qquad (2.5)$$

The half retarded minus the half advanced field satisfies the homogeneous Maxwell equation and, if it is zero everywhere on a closed boundary, then it must also vanish everywhere inside the boundary. We can now calculate the force acting on the charge "a". According to Wheeler-Feynman theory the field acting on "a" is

$$\frac{1}{2}\sum_{b\neq a}(F^{(b)}_{ret} + F^{(b)}_{adv}). \tag{2.6}$$

This can be broken up as follows:

$$\frac{1}{2}\sum_{b\neq a}(F^{(b)}_{ret} + F^{(b)}_{adv}) = \sum_{b\neq a}F^{(b)}_{ret} + \frac{1}{2}(F^{(a)}_{ret} - F^{(a)}_{adv}) - \frac{1}{2}\sum_{b}(F^{(b)}_{ret} - F^{(b)}_{adv}),$$

the last term of which vanishes. We are left with

$$\sum_{b\neq a}F^{(b)}_{ret} + \frac{1}{2}(F^{(a)}_{ret} - F^{(a)}_{adv}). \tag{2.7}$$

This means that charge "a" is acted upon by the retarded field of all the other particles in the universe and its radiation reaction.

TIME SYMMETRY

We started out with a theory which was explicitly time symmetric but which now, through the radiation reaction force, is no longer symmetric. It seems that we should be able to reverse the sign of the time and not affect the universe but we know that advanced and retarded fields would switch roles. Wheeler and Feynman (1945) recognized that the arrow of time is selected by the boundary conditions placed on the absorbing universe. These boundary conditions are thermodynamic in character, connected with the asymmetry between initial conditions and final conditions. In obtaining the usual radiation damping we assumed that the particles of the absorber were at rest before the source is set in motion, and move later as a result of absorbtion of energy from the source. In the time reversed picture the absorber particles were moving before the source is accelerated in just such a way as to come to rest and give up their energy to the source particle. Thermodynamics argues against such an unlikely situation. Hogarth (1962) examined the effect of cosmology on the Wheeler-Feynman theory. He noted that if the universe is expanding the invariance under $t \to -t$ is no longer valid for the expanding universe becomes a contracting one. This means we must check to see if the retarded or advanced solutions work in an expanding or contracting universe. In this way cosmology can lead us to a particular choice of models.

SELF INTERACTION

I will now turn to a discussion of the solutions of the equation on the non-relativistic level. The equation of motion in one dimension reduces to

$$m\ddot{x} = F + \frac{2e^2}{3}\dddot{x}; \tag{2.8}$$

where F is an external force. This is the same equation as one obtains from classical field theory. In the case F = 0 it reduces to

$$m\ddot{x} = \frac{2e^2}{3}\dddot{x}. \tag{2.9}$$

This equation has, as one solution,

$$\dot{x} = e^{\lambda t}, \text{ where } \lambda = \frac{3m}{2e^2}, \tag{2.10}$$

the classic runaway solution.
Dirac noted an equivalent problem in the relativistic treatment. Dirac suggested an ingenious way out of this difficulty. He considered what happens if the particle is acted upon by a delta function force

$$m\ddot{x} - \frac{2e^2}{3}\dddot{x}=\delta(t). \tag{2.11}$$

By applying final boundary conditions, $\dot{x}$ = const. for t > 0, he found he must have a pre-acceleration before the pulse hit the particle as shown in figure 2.1. The time scale over which the particle suffers a pre-acceleration is very short, of the order 10^{-23} seconds. Dirac attempted to relate the premonitory effects to light propagation inside the electron. In the Wheeler-Feynman theory such effects arise from the advanced disturbance of the universe. One advantage which is present in the Wheeler-Feynman theory is the impossibility of the runaway solution. If you impose the solution (2.10) you find that the fields become infinite in extent, and you can no longer satisfy the condition of complete absorption.

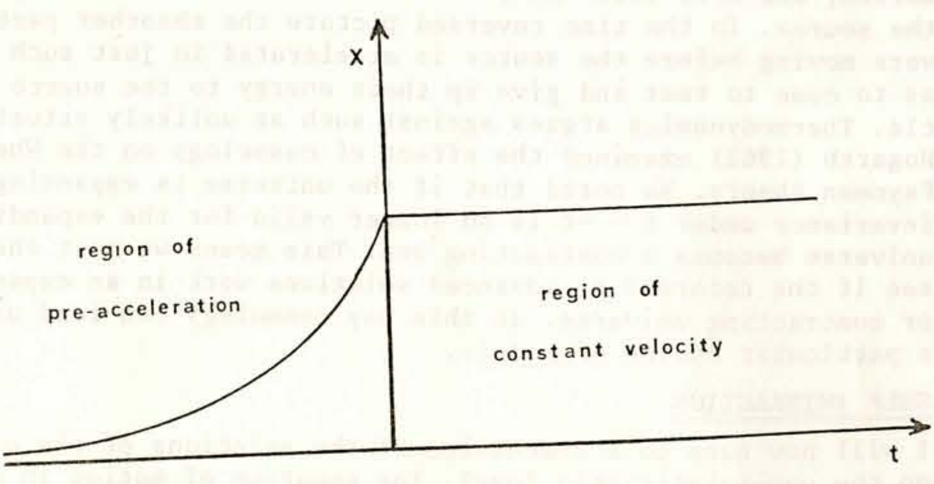

Figure 2.1

Lecture 3

THE RESPONSE OF THE UNIVERSE

This lecture will be concerned with the application of the Wheeler-Feynman theory to cosmological problems. However, before one can proceed, there are some technical problems which must be solved. The flat space action contained the expression $\delta(s^2)\eta_{ik}$ where s^2 and η_{ik} refer to flat space. Also, the δ - function is the propagator in flat space. It will be necessary to make this expression generally covariant. I will outline the way this is done while leaving out the details. The expression $\delta(s^2)\eta_{ik}$ is replaced by $\vec{G}_{i_A i_B}$. Let me explain what this means.We recall that in the picture of the worldlines of particles (a) and (b), $\delta(s^2)\eta_{ik}$ spanned the space between the points A and B. It has a foot on each end and must transform as a vector at each end. In the flat space case, an object which transforms as a vector at one point transforms as a vector everywhere. But, this is not true in curved space. Thus, the i_A subscript indicates that $\vec{G}_{i_A i_B}$ transforms as a vector at point A and the i_B that it transforms as a vector at point B. We call such an object a bivector.
If we consider the point A fixed, then $\vec{G}_{i_A i_B}$ is a function of B. It behaves like a δ - function on the light cone, but it also has support inside the light cone. The part of $\vec{G}$ with support inside the light cone is a result of curved space and is sometimes called gravitational scattering. It turns out to be not very important in our calculations as I will show. But it has the basic property that in curved space light travels not only on the light cone but also inside of it. Now, what one must do is take a cosmological model with curved spacetime and work out $\vec{G}$. At first sight this appears to be a complicated problem. However, one may utilize the conformal invariance of this theory to transform to a flat space where $\vec{G}_{i_A i_B} = \delta(s^2)\eta_{ik}$. I will describe how this is done.
Consider the Robertson-Walker line element

$$ds^2 = dt^2 - S^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]. \qquad (3.1)$$

$S(t)$ is the expansion factor, and k is a parameter which takes the values $0, \pm 1$. The line element can be rewritten as

$$ds^2 = e^{2\zeta}\left[d\tau^2 - d\rho^2 - \rho^2 d\Omega^2\right], \tag{3.2}$$

where $\rho = \rho(r,t), \tau = \tau(r,t)$ and $d\Omega^2 = d\Theta^2 + \sin^2\Theta d\phi^2$.

Expression (3.2) is just a factor $e^{2\zeta}$ multiplying a line element for flat space. Ignoring this factor you are in flat space where you can use $\vec{G} = \delta(s^2)\eta_{ik}$. Now, we can make a conformal transformation to flat space and use the simple flat space form of the Wheeler-Feynmann theory.

As an example, I will do the case $k = 0$. The cases $k = \pm 1$ can be done, but they are more complicated. For $k = 0$, the $(t,r) \rightarrow (\tau,\rho)$ transformation is

$$\tau = \int^t \frac{dt}{S(t)}, \ \rho = r, \text{ and } e^{2\zeta} = S^2(t).$$

In the Einstein-deSitter model

$$S(t) = (\frac{3Ht}{2})^{2/3} = \frac{1}{4} H^2\tau^2, \tag{3.3}$$

where H is the present value of the Hubble constant. In this case, both t and τ range between 0 and $+\infty$. The present time corresponds to $t = 2/3H$, or $\tau = 2/H$.

In the steady-state model we have

$$S(t) = e^{Ht} = -\frac{1}{H\tau}. \tag{3.4}$$

In this case t and τ range in the intervals $-\infty < t < \infty$ and $-\infty < \tau < 0$, while the present time corresponds to $t = 0$, or $\tau = -H^{-1}$.

Since physical processes are measured relative to the t coordinate, one does not need to be bothered by the finite range of the τ coordinate.

I now wish to consider the redshift phenomena. A retarded wave in the flat space which is conformal to the model universes we are considering will have the form $\exp\{-i\omega_\circ(\tau-r)\}$. Since the equations of electrodynamics are conformally invariant, it will have the same form in the geometries we are considering when measured relative to the τ coordinate. But $d\tau = dt/S(t)$, so that the wave will appear redshifted relative to the t coordinate. The observed frequency will be

$$\omega = \omega_o e^{-\zeta} \tag{3.5}$$

I must now repeat the calculation of the response of the universe which I did in the previous lecture. I will not go through it all but will point out where the essential differences arise. In the flat space case the wave contained a phase factor: $\exp\{i\omega_o(n-ik-1)r\}$. In the curved space case this must be replaced by:

$$\exp\left[i\omega_o\int_o^r (n - ik - 1)dr\right].$$

The integral appears because the index of refraction is measured relative to the t coordinate and the frequency changes in the t coordinate system as the wave propagates. We have taken the cavity radius R which appeared in the earlier calculation to be zero since we are only interested in the large r part of the integral. The argument of the exponential contains a real part $\omega_o\int kdr$. If we are going to get a damping over and above the $1/r$ dependence, it must come from here. Thus, the condition for a complete absorption of retarded waves is

$$\int_o^r kdr \to -\infty \qquad \text{as } r \text{ approaches } \infty,$$

or whatever is the maximum of the r variable in the geometry. A similar analysis for advanced wave absorption gives

$$\int_o^r kdr \to +\infty.$$

We now examine particular universes to see what type of refractive index we would actually have. First, I consider the case where the retarded wave is affected by radiative damping. If one is looking for self-consistent retarded solutions, then the radiative damping force and the equation of motion are given by (2.7) and (2.8). This leads to a refractive index

$$(n - ik)^2 = 1 - \frac{4\pi Ne^2}{m\omega^2}\left[1 - \frac{2ie^2}{3m}\omega + O(\omega^2)\right] \tag{3.6}$$

(If we were looking for self-consistent advanced waves the $\dddot{x}$ term would have the opposite sign and things would be worked out in a similar way). In the steady-state model, N = constant, and if we recall that ω is getting red-shifted to small values, then we find that $k \propto 1/\omega$, the constant of proportionality being a negative quantity, and that

$$\int kdr \sim -\int\frac{dr}{\omega} \sim -\int\frac{d\tau}{\omega} \sim -\int\frac{d\omega}{\omega} \to -\infty.$$

This shows that the absorption is complete. For the Einstein-de Sitter model $N \propto S^{-3} \propto \omega^3$, and we get $k \sim \omega^2$. The result is that the integral $\int k dr$ converges and the absorption is not complete. When the same calculation is carried out for advanced waves, the result is that the absorption is complete for the Einstein-de Sitter model but not for the steady-state universe. If we recall that the advanced wave is indefinitely blue shifted in both cases, we can understand this result by noticing that the particle density rises in the Einstein-de Sitter model to absorb this energy, but it remains constant in the steady-state universe. These results, however, are not as reliable as for the retarded waves since we are ignoring quantum effects for high frequency waves, but are valid if quantum cross sections converge at high energies - as they must do. Figure 3.1 shows the models we have considered. The case $k = +1$ is also shown. In this case there are infinite density states in both the past and the future which could lead to complete absorption and the consistency of both advanced and retarded solutions. The table below shows the results for the various models.

Model	Future Absorber	Past Absorber	Nature of e.m. wave propagation
Minkowski	perfect	perfect	ambiguous
Einstein-de Sitter	imperfect	perfect	advanced
Closed Friedmann	perfect	perfect	ambiguous
Steady-state	perfect	imperfect	retarded

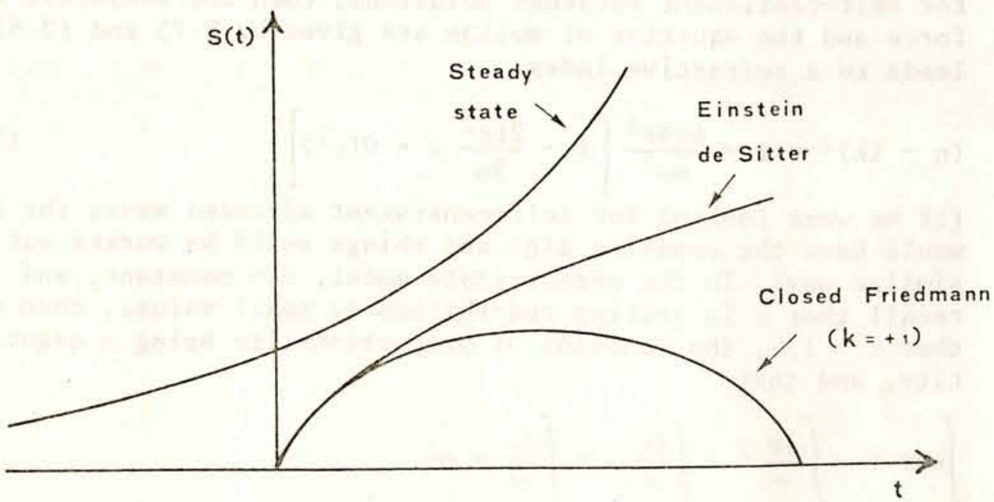

Figure 3.1

As another mechanism for absorption of the retarded wave we may consider collisional damping. In this case, for a particle in the absorber, we have

$$\ddot{x} + \nu\dot{x} = \frac{E}{m}$$

where ν is the collision frequency. The radiative damping $\dddot{x}$ term would also be here, but it is dominated by the $\nu\dot{x}$ term for the small red-shifted frequencies we are considering. If we wish to consider the absorption of advanced waves, it is not clear what to do with the sign of the $\nu\dot{x}$ term. Its origin is thermodynamical and it is difficult to say whether thermodynamics would go backward or not. When we work this out for retarded waves, we must be careful about the $\omega \to 0$ limit. If ω is very small, then the wave is damped by collisions before it can set up oscillations. But, if we take this into account correctly and work everything out, we get essentially the same result as for the radiative damping case.

Other types of cosmological models can be considered to see how far one can go and still get perfect absorption. The flat space with constant density was a perfect future absorber, but the Einstein-de Sitter model was not. Thus, there should be some cases in between where the absorption is still perfect. Or, if you want to bring in creation of matter, you can work out what sorts of expansion factors will give perfect absorption. If you insist on solutions to Einstein's equations, the choices are limited. But, if you do not want to limit yourself, you can consider more general models where the expansion factor goes like $S(t) \sim t^n$, and the density is not proportional to $S(t)^{-3}$, but has some dependence of the type $N \propto t^m$. Hogarth and Davies have looked at some models like this one. It turns out that the conditions for perfect absorption are sensitive to the details of the models. Thus we see that the large scale nature of the universe bears on the results of laboratory experiments which measure the electromagnetic waves from accelerated particles.

We found that the steady-state model satisfies the condition of perfect absorption of the future waves. There are other models which do this, and one which has been considered is shown in figure 3.2. It is a steady state in the asymptotic past and future, and involves matter creation. A random observer in this model would most likely be at $|t|$ very large. If he is toward $t = +\infty$ he is essentially in the steady-state universe which gives consistent retarded waves. His universe would be expanding. An observer toward $t = -\infty$, where the model is contracting, would have consistent advanced waves. But, since he uses electromagnetic waves to view his universe, he would see it as expanding too! To make this more certain, one should examine the way thermodynamics goes relative

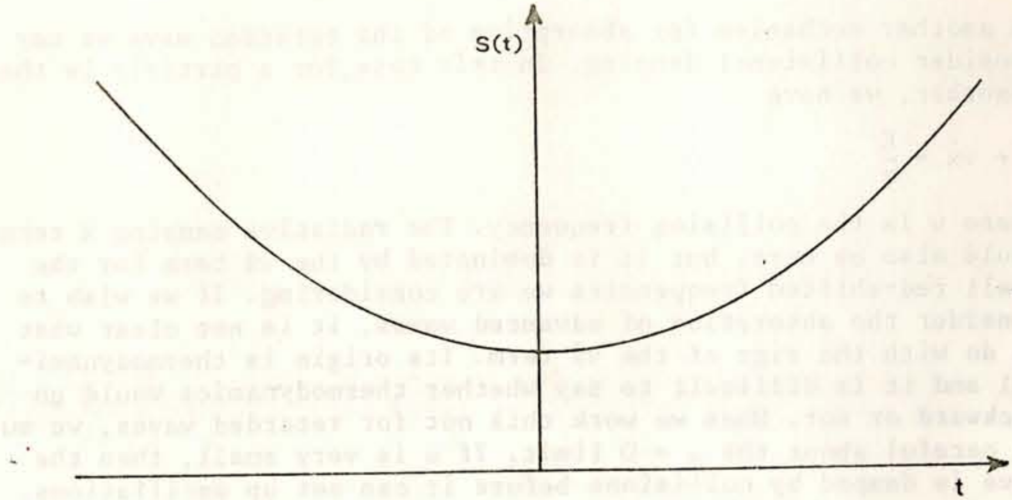

Figure 3.2

to the direction of electromagnetic wave propagation. Black body radiation shows a time asymmetry in the existence of spontaneous downward transitions. One should try to understand this asymmetry. To do so, we will need a quantum theory and this will be the topic of my next lecture. Let me briefly indicate which problems will arise. In the Maxwell theory, we have both the particle trajectories and the electromagnetic fields as dynamical variables. The quantization of the fields leads to the concept of photons and is responsible for the spontaneous downward transitions. However in the Wheeler-Feynman theory the fields are not separate dynamical variables to be quantized. Also, the non-local nature of the Wheeler-Feynman theory could be a source of difficulties. We will face these problems in the next lecture.

Lecture 4

QUANTUM FORMULATION

I would like to begin by discussing the ideas of path integration which were introduced by Feynman in 1948. We will use this method of quantization in order to avoid the problems which I discussed at the end of the last lecture.

This method is based on the following expression for the amplitude K (2, 1) that a particle at the space-time point 1 will be found at the space-time point 2:

$$K(2,1) = \sum_{\Gamma_{2,1}} \exp\left[\frac{iS}{\hbar}\right], \tag{4.1}$$

where S is the classical action for the system and the sum is extended over all paths $\Gamma_{2,1}$ that connect the point 1 to 2. This is a non-relativistic theory, so we will restrict ourselves to paths which do not go backward in time. That is, paths such as (a) and (b) in figure 4.1 are allowed, but not (c). What we are saying is that there is an amplitude exp $\left[iS(\Gamma_{2,1})/\hbar\right]$ for the particle to go from 1 to 2 by the path $\Gamma_{2,1}$. To get the total amplitude for the transition we must sum the contributions for all the paths. Now, we must recognize that there are not just a discrete number of paths but rather a continuum. The sum becomes an integral over the space of paths which I indicate by

$$K(2,1) = \int^{P} \exp\frac{iS(\Gamma)}{\hbar}\, \mathcal{D}^3\Gamma \tag{4.2}$$

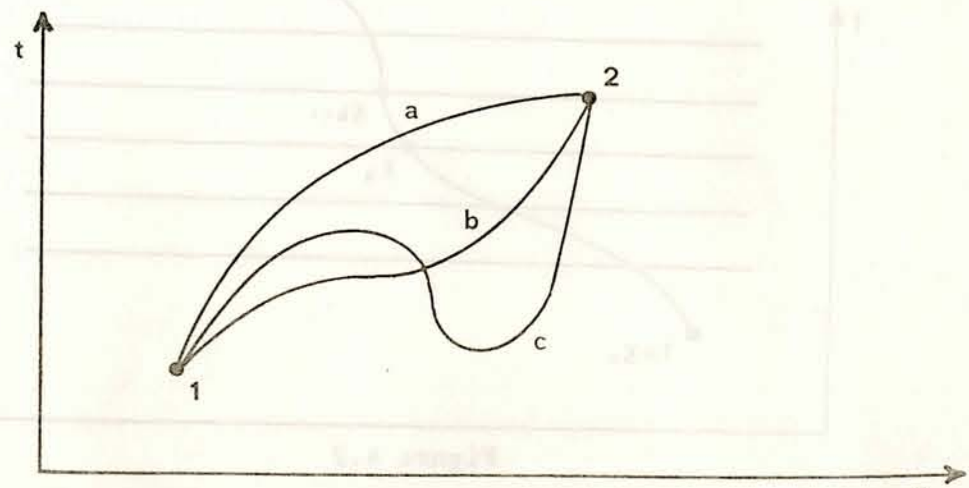

Figure 4.1

Rigorous mathematical treatments of this integral over paths are still in their early stages, but Feynman showed that the correct physical answers can be obtained if one is sufficiently clever! We can now see how the classical prescription $\delta S = 0$ arises. In a macroscopic system, $S >> \hbar$, and a small change in the path will result in a huge change in the phase. For most paths there will be many others around it with very different phases, and these will tend to all add up to zero. This will be the case except for those paths very near the classical path $\delta S = 0$. In this region, the action is stationary for a small change of path, so these paths make a finite contribution. This idea is due to Dirac originally, and is described in great detail by Feynman and Hibbs (1965). Let us consider the example of a free particle with the action given by (1.3). We proceed by dividing the space up into small time steps of length ε, with $N\varepsilon = t_2 - t_1$. We will approximate the path between two neighbouring points $\underline{x}_{k+1}$ and $\underline{x}_k$ by a straight line. For a particular path, we add up the contributions from each little interval to get

$$S \simeq \frac{1}{2} m \sum_{k=0}^{N-1} \left[\frac{\underline{x}_{k+1} - \underline{x}_k}{t_{k+1} - t_k} \right]^2 (t_{k+1} - t_k) \qquad (4.3)$$

$$\simeq \frac{1}{2} m \sum_{k=0}^{N-1} \frac{(\underline{x}_{k+1} - \underline{x}_k)^2}{\varepsilon}$$

We must sum this over all paths. To do this, we integrate over each of the points $\underline{x}_k$ $(k = 1, \ldots, N-1)$ to generate all paths. Finally, we will pass to the limit $\varepsilon = 0$. Thus

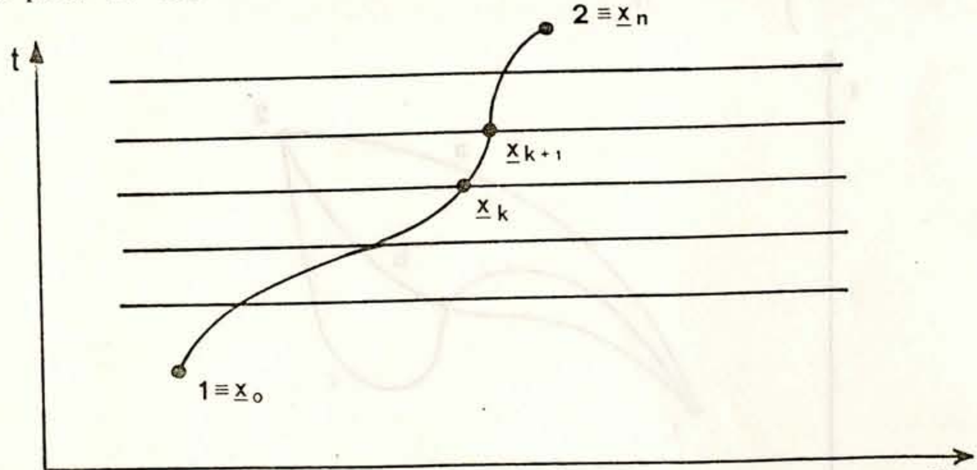

Figure 4.2

$$K(2,1)=\lim_{\varepsilon\to o}\int..\int A^{-N}\exp\left\{\frac{i}{\hbar}\frac{1}{2}m\sum_{k=o}^{N-1}\frac{(\underline{x}_{k+1}-\underline{x}_k)^2}{\varepsilon}\right\}d^3\underline{x}_1..d^3\underline{x}_{N-1}. \quad (4.4)$$

The factor A^{-N} represents the difficulties with the integration over paths. Feynman showed how to find this factor for various special cases. For this case it is

$$A = \left(\frac{m}{2\pi\hbar i\varepsilon}\right)^{-3/2} .$$

If we carry out all the integrations and take the limit $\varepsilon\to o (N\to\infty)$, we get

$$K(2,1)=\left[\frac{m}{2\pi\hbar i(t_2-t_1)}\right]^{3/2}\exp\frac{im(\underline{x}_2-\underline{x}_1)^2}{2\hbar(t_2-t_1)} \quad \text{for } t_2>t_1$$

$$= o \text{ for } t_2<t_1. \quad (4.5)$$

You will recognize this to be the *propagator* for the Schrödinger equation, and recall that it satisfies the equation

$$\left[-\frac{\hbar^2}{2m}\nabla_2^2 - i\hbar\frac{\partial}{\partial t_2}\right]K(2,1) = \delta^4(2,1). \quad (4.6)$$

The $\delta^4(2,1)$ comes from the discontinuities of $K(2,1)$ at $2 = 1$. Thus, we ended up with the Schrödinger equation rather than having begun with it, as in the conventional formulations of the quantum theory. Now, if I do not know where the particle came from, but I have given the amplitude $\psi(\underline{x}_1,t_1)$ that the particle is at a point $\underline{x}_1$ at time t_1, I can calculate the amplitude $\psi(\underline{x}_2,t_2)$ that it will be at a point $\underline{x}_2$ at time t_2:

$$\psi(\underline{x}_2,t_2) = \int K(2,1)\psi(\underline{x}_1,t_1)d^3\underline{x}_1. \quad (4.7)$$

A more general situation would be represented by a Lagrangian

$$L = \frac{1}{2}m\dot{\underline{x}}^2 - V(\underline{x}).$$

The propagator then will give a wave function which satisfies

$$-\frac{\hbar^2}{2m}\nabla_2^2\psi + V(2)\psi = i\hbar\frac{\partial\psi}{\partial t_2} . \quad (4.8)$$

Now, if we use the principles of quantum mechanics, we can write down the expression for the transition amplitudes between states $\psi_i(1)$ and $\psi_f(2)$

$$\langle\psi_f|\psi_i\rangle = \iint \bar{\psi}_f(2)K(2,1)\psi_i(1)d^3\underline{x}_1 d^3\underline{x}_2$$

$$= \iiint^{P} \bar{\psi}_f(2)\exp\left[\frac{iS(\Gamma_{2,1})}{\hbar}\right]\psi_i(1)d^3\underline{x}_1 d^3\underline{x}_2 \mathcal{D}^3\Gamma_{2,1}, \qquad (4.9)$$

and for the transition probability

$$|\langle\psi_f|\psi_i\rangle|^2 = \iiiint\!\!\int^{P}\!\!\int^{P} \bar{\psi}_f(\underline{x}_2,t_2)\bar{\psi}_i(\underline{x}_1',t_1)\psi_i(\underline{x}_1,t_1)\psi_f(\underline{x}_2',t_2)$$

$$\times \exp\left[\frac{i}{\hbar}\{S(\Gamma)-S(\Gamma')\}\right]d^3\underline{x}_1 d^3\underline{x}_1' d^3\underline{x}_2 d^3\underline{x}_2' \mathcal{D}^3\Gamma \mathcal{D}^3\Gamma'. \qquad (4.10)$$

In general, we would like to be able to calculate the transition probability for a particle (a) under the influence of all the other particles (b) in the universe. However, we are not interested in the final states of the particles (b) and will sum over them. These effects are represented by the influence functional. For a simple case we have the action divided up in the following way:

$$S = S_o\left[q(t)\right] + S_E\left[Q(t)\right] + S_I\left[q(t),Q(t)\right]. \qquad (4.11)$$

S_o is the action for the free particle with coordinates $q(t)$; $S_E(Q(t))$ is the free action for the particles of coordinates $Q(t)$ which cause the influence, $S_I\left[q(t),Q(t)\right]$ represents the interaction between them. We are now interested in calculating the transition probability between a state $\psi_i(q_i,t_i)$ and a state $\psi_f(q_f,t_f)$ regardless of what happens to the other particles. Substituting in our previous expressions we get

$$P(\psi_i\to\psi_f) = \iiiint\!\!\int^{P}\!\!\int^{P} \bar{\psi}_f(q_f)\psi_f(q_f')\psi_i(q_i)\bar{\psi}_i(q_i') \qquad (4.12)$$

$$\times \exp\left[\frac{i}{\hbar}\{S_o(q)-S_o(q')\}\right] F(q,q')dq_i dq_i' dq_f dq_f' \mathcal{D}^3 q \mathcal{D}^3 q',$$

where q and q' represent paths from q_i to q_f, and the influence functional $F(q,q')$ is defined as:

$$F(q,q') = \sum \int^{P} \int^{P} \exp\left[\frac{i}{\hbar}\{S_E(Q)-S_E(Q')\}\right]$$

$$\times \ \exp\left[\frac{i}{\hbar}\{S_I(q,Q)-S_I(q',Q')\}\right] \mathcal{D}^3Q\mathcal{D}^3Q', \qquad (4.13)$$

where the sum is over the final states of Q.
This is what we will need for our calculation of the quantum response of the universe in various cosmological models.

For the classical case we showed that the effect of all other particles (b) on the particle (a) is given by the expression (2.7), where the first term is the normal retarded field and the second term is the effect of all the advanced fields. I will show that in the quantum mechanical case the retarded effects of the rest of the universe give rise to the induced transitions, and the advanced effects give rise to spontaneous transitions. Classically, a particle in an energy level above the ground state might stay there forever since there is no field to induce a transition. However, in our formulation, there are paths to the lower state which must be taken into account. Since the particle changes its energy on these paths, it is accelerated and the advanced field of the universe responds with a field at the particle which can be viewed as causing the transition. One might then ask the question:"What about spontaneous upward transitions?". We will be able to see later why these cannot happen. Let us now write down the transition probability for a particle to go from a state m of higher energy to a state n of lower energy. We make use of our previous results to get

$$P(m\to n)=\iiiint \bar{\psi}_n(\underline{a}_f)\psi_n(\underline{a}_f')K\psi_m(\underline{a}_i)\bar{\psi}_m(\underline{a}_i')d^3\underline{a}_i d^3\underline{a}_i' d^3\underline{a}_f d^3\underline{a}_f' , \qquad (4.14)$$

where the a's are particle coordinates and

$$K = \int^{P}\int^{P} \exp\left\{\frac{i}{\hbar}\left[S_o\{\underline{a}(t)\}-S_o\{\underline{a}'(t)\}\right]\right\} F\{\underline{a}(t),\underline{a}'(t)\}\mathcal{D}^3\underline{a}\mathcal{D}^3\underline{a}',$$

with

$$F\{\underline{a}(t),\underline{a}'(t)\}=\prod_{b\neq a} F^{(b)}\{\underline{a}(t),\underline{a}'(t)\},$$

$$F^{(b)}\{\underline{a}(t),\underline{a}'(t)\}=\sum\iiiint \bar{\psi}_f(\underline{b}_f)\psi_f(\underline{b}_f')J^{(b)}\psi_i(\underline{b}_i)\bar{\psi}_i(\underline{b}_i')d^3\underline{b}_i d^3\underline{b}_i' d^3\underline{b}_f d^3\underline{b}_f',$$

where the sum is over the final states of (b), and

$$J^{(b)}=\int^{P}\int^{P} \exp\left\{\frac{i}{\hbar}\left[S_o(\underline{b})+S_I(\underline{a},\underline{b})-S_I(\underline{a}',\underline{b}') - S_o(\underline{b}')\right]\right\}\mathcal{D}^3\underline{b}\mathcal{D}^3\underline{b}'.$$

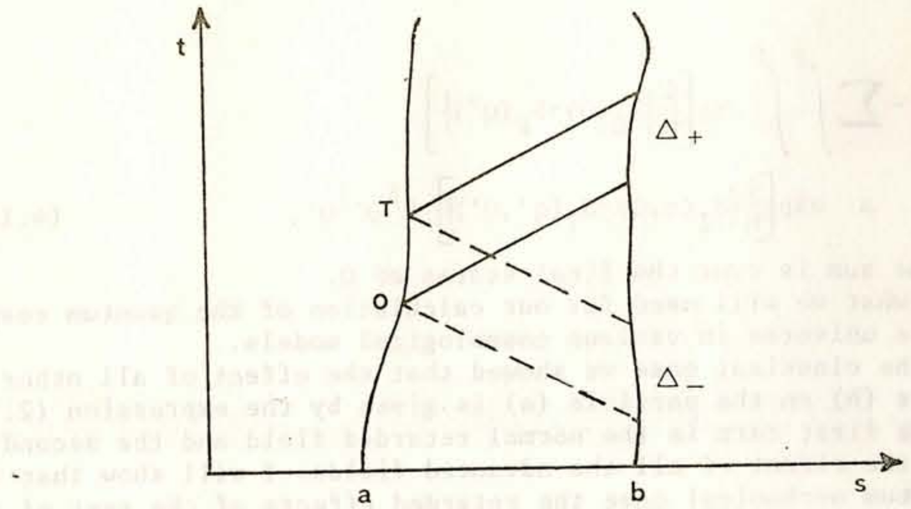

Figure 4.3

We are interested in the same sort of self-consistent calculation as before so we take the retarded interaction

$$S_I\left(\underline{a},\underline{b}\right) = -\int_{\Delta_+} \underline{A}^{(a)}_{ret}(\underline{b})\dot{\underline{b}}dt. \qquad (4.15)$$

Since we are interested in the behaviour of particle (a) between t = o and t = T, the integral is over Δ_+ which is the part of the world-line of particle (b) that is between the forward light cones from particle (a) at t = o and t = T. It is at this point that we must consider the role played by the universe, and the distinction that it makes between upward and downward transitions. In order to evaluate $F\{\underline{a}(t),\underline{a}'(t)\}$ we consider a Fourier decomposition of the path just as we did in the classical case. As a wave of frequency ω from particle (a) travels into the future in the expanding universe, it is red-shifted. When it reaches an absorber particle it will cause it to make a transition. We shall assume that an expanding universe is cold in the future. That is, all of the absorber particles are in their ground states, and the wave from particle (a) will induce only upward transitions. With this in mind, the calculation of $F\{\underline{a}(t),\underline{a}'(t)\}$ is performed by resolving a(t) and a'(t) into Fourier frequencies. If we assume perfect absorption in the future, and combine the contributions from all absorber particles, the parameters of the absorber again drop out, and we get:

$$F\{\underline{a}(t), \underline{a}'(t)\} = \exp\left[\frac{e^2}{4\pi^2\hbar}\int d\Omega\int_o^\infty kdk \sum_{j=1,2}\left\{\int_o^T (\underline{\alpha}_k^{(j)}\cdot\dot{\underline{a}}) \times \right.\right.$$

$$\times \exp(-i\underline{k}\cdot\underline{a} + ikt)dt \int_o^T (\underline{\alpha}_k^{(j)}\cdot\dot{\underline{a}}') \exp(i\underline{k}\cdot\underline{a}' - ikt')dt' -$$

$$-\int_o^T (\underline{\alpha}_k^{(j)}\cdot\dot{\underline{a}}) \exp(-i\underline{k}\cdot\underline{a} - ikt)dt \int_o^t (\underline{\alpha}_k^{(j)}\cdot\dot{\underline{a}}) \exp(i\underline{k}\cdot\tilde{\underline{a}} + ik\tilde{t})d\tilde{t} -$$

$$-\int_o^T (\underline{\alpha}_k^{(j)}\cdot\dot{\underline{a}}') \exp(-i\underline{k}\cdot\underline{a}' + ikt)dt \int_o^t (\underline{\alpha}_k^{(j)}\cdot\dot{\underline{a}}') \exp(i\underline{k}\cdot\underline{a}' -$$

$$\left.\left. - ik\tilde{t})d\tilde{t}\right\}\right], \qquad (4.16)$$

where $\int d\Omega$ is the solid angle summation of the type I did in the classical calculation. The $\underline{\alpha}_k^{(j)}$ are the polarization vectors of the k Fourier component of the field. Note that the expression is not symmetric in k and -k. When it is substituted back into the rest of the expression for P(m→n), it gives the correct downward spontaneous transitions and no spontaneous upward transitions.

We will now consider an actual cosmological model. As we have seen, the only requirements are that the universe should be perfectly absorbing in the future, and that it should have a cold environment in the future. We have seen classically that the steady-state model meets these requirements. The numbers which follow refer to this model. One should not worry too much exactly what Hubble constant or particle density I put in; it won't make much difference. Collisional damping has been taken as a typical absorbing process. The collisional frequency for ionized hydrogen is given by

$$\nu_{eff} \simeq 2\pi N v (e^2/mv^2)^2 \ln(mv^2/\hbar\omega) \qquad (4.17)$$

If we take $\rho = 3H^2/4\pi G$ to be $\sim 4\times 10^{-29}$ gm/cm^3, then $N \sim$ $\sim 2\times 10^{-5}$ atoms/cm^3. The average electron velocity v/c should be $\sim$1/300. ω is the frequency of the wave when it is absorbed. We will anticipate the result and take $\omega \gg \nu_{eff}$ in which case the attenuation factor is $\sim (2\pi Ne^2/m\omega^2)\nu_{eff}$. The attenuation over a range $0 \leq r < R$ is

$$\exp\left[-\frac{2\pi Ne^2\nu_{eff}}{mk^2}\int_0^R \frac{dr}{(1-Hr)^3}\right] \tag{4.18}$$

where we have ignored the dependence of ν_{eff} on ω. The factor (1-Hr) is a redshift factor similar to those that appeared in the classical calculation, and k is the frequency which appeared in our previous expression for the influence functional and arose from a transition $E_i \rightarrow E_f = E_i + \hbar k$. We are measuring this frequency with respect to the time of the conformal space, so the proper frequency is redshifted:

$$\omega = (1 - Hr)k \tag{4.19}$$

Appreciable absorption from the integral requires that

$$(1 - Hr)^2 \simeq \frac{2\pi Ne^2}{mk^2 H}\nu_{eff} \tag{4.20}$$

or, from (4.19)

$$\omega^2 \simeq \frac{2\pi Ne^2}{mH}\nu_{eff} \tag{4.21}$$

If we solve this equation along with our other equation between ω and ν_{eff} we get $\omega \sim 10^6$ sec^{-1} and $\nu_{eff} \sim 10^{-10}$ sec^{-1}, justifying the $\omega \gg \nu_{eff}$ assumption. These are just some numbers to show where the absorption is taking place. If the frequency emitted in the laboratory is $> 10^6$ sec^{-1}, the radiation is first redshifted until it is below that critical frequency, and is then absorbed over a distance of 10^{28} cm. If it is emitted at $< 10^6$

sec^{-1} it is absorbed right away.

I will conclude this lecture with a discussion of what is meant by the concept of a photon in the Wheeler-Feynman theory. You will recall that the photon normally comes from quantizing the free Maxwell field. In the Wheeler-Feynman theory, there is no electromagnetic field to quantize, so one would expect there to be no such thing as a photon. One can introduce the concept of a photon in a straight forward way having obtained the transition probability formula. You are used to talking about the photon as a solution of the source-free equations. In practice, a photon is always emitted somewhere and absorbed somewhere else. The source of a photon is a particle in an upper energy state. As we have seen, the universe offers a cold environment and the particle tends to jump to a lower state and lose some energy. We can take that spontaneous downward transitions as a unit and speak in the following way. For $E_m > E_n$, we can always write

$$\frac{P(E_m \rightarrow E_n)}{P(E_n \rightarrow E_m)} = \frac{n + 1}{n} \qquad (4.22)$$

and call n the number of photons of energy $E_m - E_n$ present. We are able to do this because of the asymmetry between upward and downward transitions which, as we have seen, is due to the asymmetry between future and past of the universe.

Lecture 5

WHEELER-FEYNMAN WITHOUT WHEELER-FEYNMAN

I now wish to continue the discussion of the Quantum aspects of the Wheeler-Feynman theory. Whenever I talk to Professor Wheeler about the Wheeler-Feynman theory he says that his view is like that of a converted alcoholic who started out pro action at a distance but realized the errors of his ways and is now strongly pro local field theory. Professor Feynman's view is that it is something that ought to be done, but he is not going to do it. Hence Wheeler-Feynman without Wheeler-Feynman.

In the last lecture I discussed the non-relativistic treatment of Wheeler-Feynman theory and showed how the universe played a role in atomic transitions. In this lecture I will discuss the treatment of relativistic electrons. The first difficulty we run into is writing down the path integral formulation of spin $\frac{1}{2}$ particles. There is presently no path integral formulation for spin $\frac{1}{2}$ particles which fits into the Wheeler-Feynman description. As Professor Hoyle will discuss this topic in his lectures, I will just treat it briefly.

Consider a relativistic, spin $\frac{1}{2}$ particle going from point 1 to point 2. Feynman has considered this problem non-relativistically. He has shown that the probability amplitude of a particle going from 1 to 2 can be understood as a product of a chain of propagators between the point 1 and 2. If K(i, i + 1) represents the propagator between two neighboring points i, i + 1, then the probability amplitude for the particle going from 1 to 2 is

$$\text{Probability Amplitude } (1 \rightarrow 2) = \prod_{i=1}^{N-1} K(i, i+1)\, A. \qquad (5.1)$$

Here A is a factor of dimensionality $(\text{length})^3$ put in to reconcile the dimensionless probability amplitude and the propagator whose dimensionality is $(\text{length})^{-3}$. We let $N \rightarrow \infty$ and the probability amplitude is the equivalent to that defined by the sum over paths. Feynman applied this procedure to relativistic particles by using the propagator, K, appropriate to the Dirac equation. He used a propagator which was zero for motion into the past and non-zero for motion into the future. The difficulty with this approach is the same one which occurs for the Dirac equation, namely the existence of negative energy states. In the non-relativistic approach one need only consider positive energy solutions propagating forward in time, while in the Dirac theory both positive and negative solutions propagate forward in time. To get around this difficulty it is necessary to postulate a sea of electrons which fill the negative energy states.

Relativistic Electrons

I will now discuss a somewhat different approach to the problem. Feynman's treatment, with the filled sea of negative energy states is intrinsically a many body problem. To avoid this problem we use another idea of Feynman, treating the negative energy solutions as going backwards in time, or positrons.

From the Dirac equation

$$(\not{\nabla} + im)\psi = 0 \tag{5.2}$$

one gets an equation for the propagator

$$(\not{\nabla}_2 + im)K_o^{\pm}(2, 1) = \delta_4(2, 1). \tag{5.3}$$

The solutions for the propagators are

$$K_o^{+} = \Theta(t_2 - t_1)\sum_n U_n(2)\,\bar{U}_n(1) \tag{5.4a}$$

$$K_o^{-} = -\Theta(t_1 - t_2)\sum_n U_n(2)\,\bar{U}_n(1) \tag{5.4b}$$

Here $\Theta(x)$ is the Heaviside function $\Theta(x) = 1$ for $x > 0, \Theta(x) =$ $= 0$ for $x < 0$; U_n are just the free particle solutions of the Dirac equation. We see that K_o^- is just the time reversed solution of K_o^+. Thus K_o^+ describes propagation forward in time and K_o^- is used to describe those paths which propagate backwards in time. Thus we can define the probability amplitude for a path going forward in time as

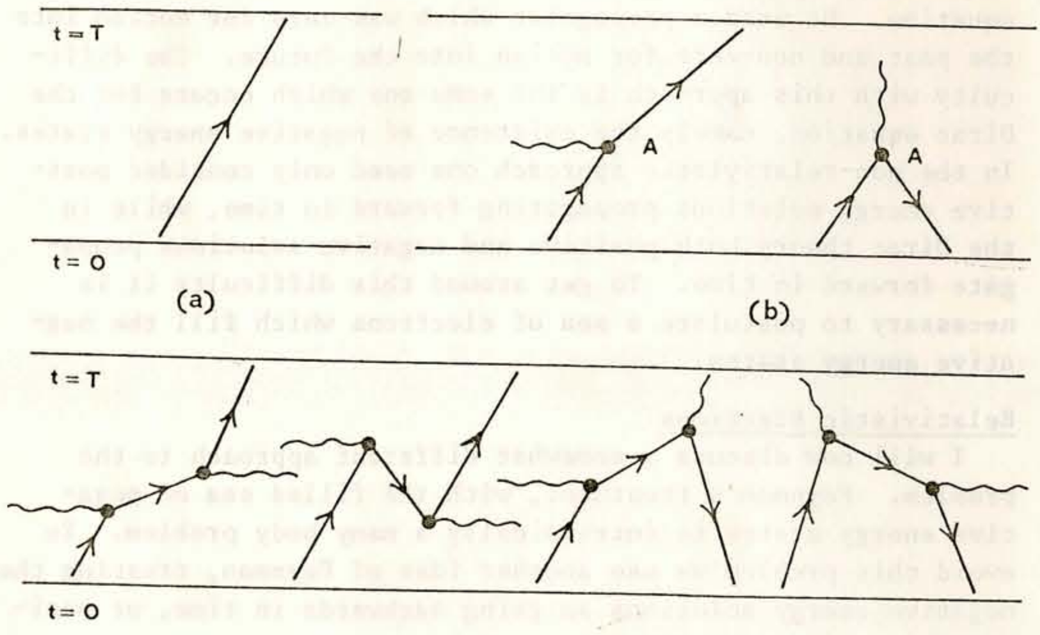

Figure 5.1

$$P(\Gamma^{+}_{21}) = \prod_{i=1}^{N-1} A_i \, K^{+}_{o}(i, i + 1) \qquad (5.5)$$

where $\Gamma^{\pm}$ represents paths going forward (backward) in time. Thus, electron wavefunctions are built up of Γ^{+} paths and of positron wavefunctions of Γ^{-} paths.

We now come to the more complicated case where an external potential can cause reversals in the paths of the particle. Consider the motion in a time slab between t = 0 and t = T. Looking at Fig. 5.1 you can see the various probabilities that must be computed for zero, one and two interactions of the electron field. In order to calculate the wavefunction at time T from that at time t = 0 we must take into account the various possibilities in calculating the propagator. If the propagator from t = 0 to t = T is $K^{A}(2, 1)$ we have

$$K^A(2, 1) = K_+(2, 1) - ie \int K_+(2, 3)\, K_+(3, 1) d^3x_3 + \ldots \quad (5.6)$$

Here the first term describes a path which goes through without interacting with the electromagnetic field, and the second term describes a single interaction. As seen from Fig. 5.1(b) we have two possibilities: the particle can propagate forward, or can scatter off the external field and travel backward in time. So K_+ must include the two possibilities

$$K_+(2, 1) = \sum_{E_n > 0} U_n(2)\, \bar{U}_n(1) \quad t_2 > t_1$$

$$= - \sum_{E_n < 0} U_n(2)\, \bar{U}_n(1) \quad t_2 < t_1 \quad (5.7)$$

which you recognize as the Feynman propagator for electrons and positrons.

These possibilities describe whether the electron has scattered off the external field and remains an electron or if the situation is one of pair annihilation. Of course, the description can be carried out to higher order of scattering

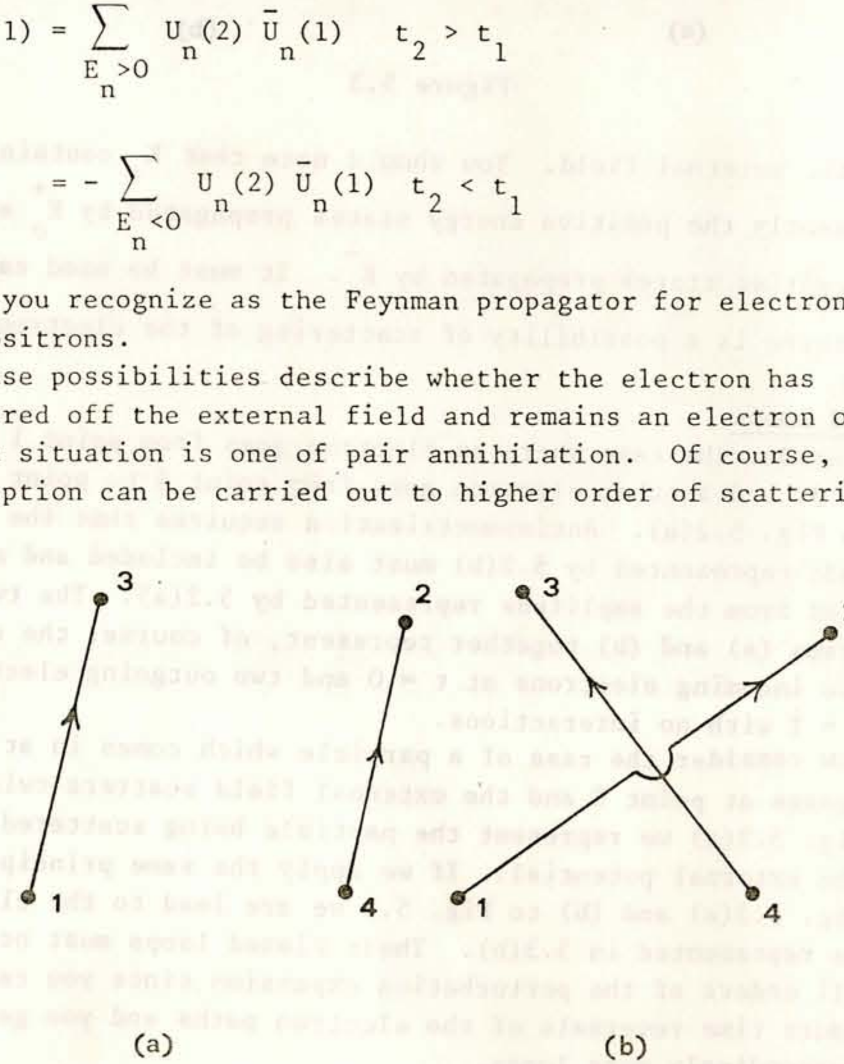

Figure 5.2

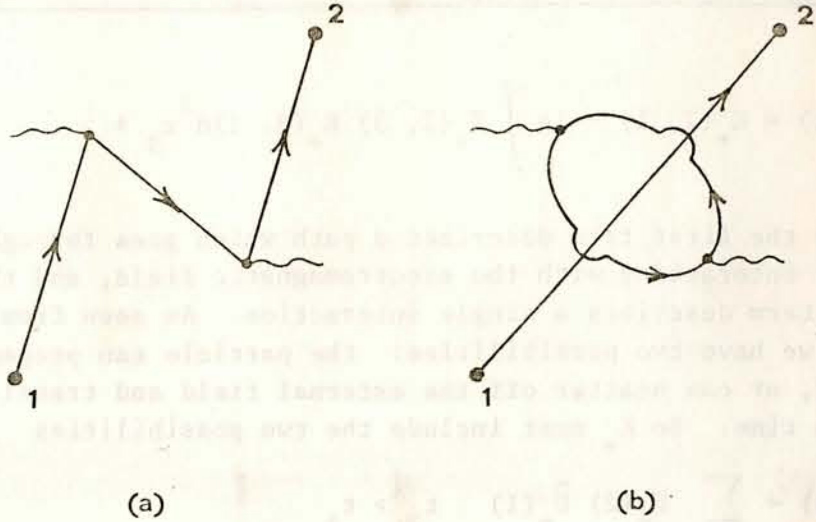

Figure 5.3

with the external field. You should note that K_+ contains very neatly the positive energy states propagated by K_o^+ and the negative states propagated by K_o^-. It must be used each time there is a possibility of scattering of the electron.

Closed Loops

Consider the case where an electron goes from point 1 to point 3, and another electron goes from point 4 to point 2 as in Fig. 5.2(a). Antisymmetrization requires that the amplitude represented by 5.2(b) must also be included and subtracted from the amplitude represented by 5.2(a). The two diagrams (a) and (b) together represent, of course, the case of two incoming electrons at t = 0 and two outgoing electrons at t = T with no interactions.

Now consider the case of a particle which comes in at point 1, leaves at point 2 and the external field scatters twice. In Fig. 5.3(a) we represent the particle being scattered twice by the external potential. If we apply the same principle as in Fig. 5.2(a) and (b) to Fig. 5.3 we are lead to the closed loops represented in 5.3(b). These closed loops must occur in all orders of the perturbation expansion since you can allow for more time reversals of the electron paths and you get correspondingly more loops.

I would now like to consider the question of self-interaction.

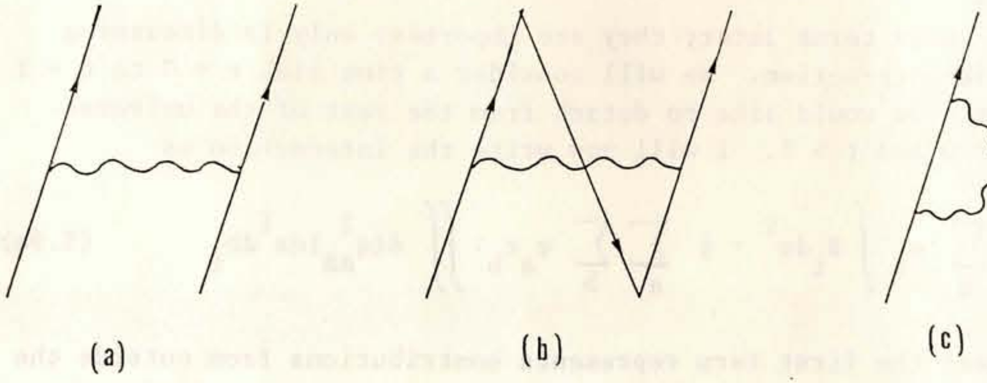

Figure 5.4

We saw that there was no self-action in the classical Wheeler-Feynman theory and we would like to see to what extent this is true in the relativistic quantum theory. An interaction between two worldlines is drawn in Fig. 5.4(a). The relativistic theory allows worldlines which go backwards so we might expect that the picture in Fig. 5.4(b) would also be possible. However, this represents self-interaction and we might want to eliminate it. Experimentally we cannot determine if such an interaction has taken place, so we have no reason to eliminate the possibility. If we are going to include graphs of that type, then ones of the type in Fig. 5.4(c) should be possible also.

In the classical theory all paths are time-like and we would not get a contribution because the s^2 in the δ-function would never be zero. An exception to this would arise if the points 1 and 2 coincide. This problem is removed in the quantum theory by putting in a requirement which does not allow the two points to coincide.

I turn now to the relativistic quantized Wheeler-Feynman theory. The interaction is now written

$$R = -\tfrac{1}{2} \sum_a \sum_b e_a e_b \iint \delta(q_{AB}^2)\, da_i\, db^i \qquad (5.8)$$

R is what we have previously called S_I and q^2_{AB} was s^2_{AB}. The restriction $a \neq b$ has been dropped. I will look at these terms later; they are important only in discussing self-interaction. We will consider a time slab $t = 0$ to $t = T$ which we would like to detach from the rest of the universe $t < 0$ and $t > T$. I will now write the interaction as

$$-\sum_a e_a \int B_i da^i - \tfrac{1}{2} \sum_a \sum_b e_a e_b \iint \delta(q^2_{AB}) da^i db_i \qquad (5.9a)$$

where the first term represents contributions from outside the slab and the second term is the contribution from inside the slab. The potential B_i will have two parts, $B_i(t < 0)$ and $B_i(t > T)$ arising from contributions before and after the slab respectively.

My aim is to determine what the universe must do in order to give results consistent with local experiments.

First, look at the classical case. The potential produced by particle b is

$$A^{i(b)}(X) = e_b \int \delta(q^2_{XB}) db^i \qquad (5.10)$$

The total potential which acts on the particle a is

$$A^i_{(a)}(X) = \sum_{b \neq a} A^{i(b)}(X)$$

$$= \tfrac{1}{2}\left[A^i_{(a)\,ret}(X) + A^i_{(a)\,adv}(X)\right] \qquad (5.11)$$

The classical action can then be rewritten as

$$-\sum_a e_a \int B^i_{t>T}\, da_i - \tfrac{1}{2} \sum_a e_a \int \left[A^i_{(a)\,ret} + A^i_{(a)\,adv}\right] da_i \qquad (5.9b)$$

The $B^{i}_{t<0}$ part has not been included because I am only interested in considering the contributions which might lead to spontaneous transitions.

If there is complete absorption in the future, then the action would become

$$-\sum_a e_a \int \left\{ A^{i}_{(a)\,ret} + \tfrac{1}{2}\left[A^{i(a)}_{ret} - A^{i(a)}_{adv}\right]\right\} da_i$$

What we can do is subtract this from our action and require that the difference be zero. The result is that we must have

$$B_i(X)_{t>T} = \tfrac{1}{2}\sum_a \left[A_{i\,ret}^{(a)} - A_{i\,adv}^{(a)}\right] \qquad (5.12)$$

For the quantum mechanical case we must multiply the action by $i/\hbar$ and exponentiate it, and, if you recall the form of a transition probability, then you will see that the expression

$$-\sum_a e_a \left[\int \left\{ \tfrac{1}{2}\left[A^{i}_{(a)\,ret} + A^{i}_{(a)\,adv}\right] + B^{i}(t>T)\right\} da_i - \right.$$

$$\left. - \int \left\{ \tfrac{1}{2}\left[A'^{i}_{(a)\,ret} + A'^{i}_{(a)\,adv}\right] + B'^{i}(t>T)\right\} da'_i \right]$$

will appear in the exponent. The primes indicate conjugate paths and the sum is over all a. We must now lay down a condition corresponding to the perfect absorption condition that will give us the right kind of QED and that will reduce to the classical result when $\hbar\to 0$. The result is that we must have

$$B_i(t>T) = B'_i(t>T) = \tfrac{1}{2}\sum_b \left[A_{i\,ret+}^{(b)} - A_{i\,adv+}^{(b)}\right] + \tfrac{1}{2}\sum_b \left[A'_{i\,ret-}{}^{(b)} - A'_{i\,adv-}{}^{(b)}\right] \qquad (5.13)$$

where the + and - indicate the positive and negative frequency parts respectively. This expression is relativistically invariant. In the classical theory, there is a unique path and the conjugate path is thus equal to the path. In this case, the above expression for $B_i(t>T)$ reduces to the classical condition. This expression for $B_i(t>T)$ leads to the influence functional

$$F(\underline{a}, \underline{a}') = \exp\left\{ ie_a^2 \left[\iint_{t_{A'}>t_A} \delta_+(q^2_{A'A}) da'_i da^i - \iint_{t_A>t_{A'}} \delta_-(q^2_{AA'}) da'_i da^i \right.\right.$$

$$\left.\left. - \tfrac{1}{2}\iint \delta_+(q^2_{A\tilde{A}}) da^i d\tilde{a}_i + \tfrac{1}{2}\iint \delta_-(q^2_{A'\tilde{A}'}) da'^i d\tilde{a}'_i \right] \right\} =$$

$$= \exp\left[\frac{e_a^2}{4\pi^2} \int d\Omega \int_0^\infty kdk \left\{ -\iint \exp\left[ik(t_A - t_{A'}) + \right.\right.\right.$$

$$\left. + i\underline{k}\cdot(\underline{x}_{A'} - \underline{x}_A)\right] da^i da'_i + \iint_{t_A>t_{\tilde{A}}} \exp\left[ik(t_{\tilde{A}} - t_A) + \right.$$

$$\left. + i\underline{k}\cdot(\underline{x}_{\tilde{A}} - \underline{x}_A)\right] da^i d\tilde{a}_i + \iint_{t_{A'}>t_{\tilde{A}}} \exp\left[ik(t_{A'} - t_{\tilde{A}'}) + \right.$$

$$\left.\left.\left. + i\underline{k}\cdot(\underline{x}_{\tilde{A}'} - \underline{x}_{A'})\right] da'^i d\tilde{a}'_i \right\} \right] \tag{5.14}$$

which is equal to the corresponding expression for the non-relativistic case. This gives the interaction of a single worldline with the rest of the universe. It would be applicable for the calculation of processes such as pair creation or annihilation. For the interaction of two particles we require the influence functional

$$
\begin{aligned}
F(\underline{a}, \underline{b}; \underline{a}', \underline{b}') = \exp \Bigg\{ & i e_a e_b \Bigg[\iint_{t_{A'} > t_B} \delta_+ (q^2_{A'B}) \\
& da_i' db^i - \iint_{t_B > t_{A'}} \delta_- (q^2_{BA'}) da_i' db^i + \iint_{t_{B'} > t_A} \delta_+ (q^2_{B'A}) \\
& da^i db_i' - \iint_{t_A > t_{B'}} \delta_- (q^2_{AB'}) da^i db_i' - \iint \delta_+ (q^2_{AB}) da^i db_i + \\
& + \iint \delta_- (q^2_{A'B'}) da_i' db'^i \Bigg] \Bigg\} = \exp \left[\frac{e_a e_b}{4\pi^2} \int d\Omega \int k dk \right\} \iint \exp \Big[-ik |t_A - \\
& - t_B| + i\underline{k} \cdot (\underline{x}_B - \underline{x}_A) \Big] da_i db^i + \iint \exp \Big[ik |t_{A'} - t_{B'}| + i\underline{k} \cdot (\underline{x}_{A'} - \underline{x}_{B'}) \Big] \\
& da_i' db'^i - \iint \exp \Big[ik(t_A - t_{B'}) + i\underline{k} \cdot (\underline{x}_{B'} - \underline{x}_A) \Big] da_i db'^i - \\
& - \iint \exp \Big[ik(t_B - t_{A'}) + i\underline{k} \cdot (\underline{x}_{A'} - \underline{x}_B) \Big] da_i' db^i \qquad (5.15)
\end{aligned}
$$

This expression contains the information necessary for calculating many processes in electrodynamics including scattering and energy level shifts. One should note that it is given as an exponential in closed form. In principle, one could work out the answers for various processes in closed form. In practice, we expand the exponential and this leads to the standard perturbation expansion. We can now consider any model of the universe and check in detail whether it satisfies the condition on $B^i(t>T)$ that we have derived. The necessary condition for this to work is first of all that the classical theory should work and second that the model should provide a cold environment in the future.

I would now like to return to the discussion of the self-interaction problem. We saw before that in a self energy graph we did not want the points 1 and 2 to get too close together. A close look at this condition reveals that we must know what shape the worldline takes in the quantum sense. It turns out that the line is not smooth but is made up of small null steps. This comes from the fact that we wrote K(2, 1) as a product of propagators for very short steps, and that for short steps the main contribution to such a propagator comes from null directions. So, we must decide how small we will allow the steps to get. We can get a finite answer for $\delta m/m$ if we do not allow the step size to go to zero. If we take the size equal to the gravitational radius of the electron we get a result like Prof. Salam showed. Similar ideas apply to the calculation of closed loops.

In their book, Feynman and Hibbs (1965) showed in a problem that for an electron in one space and one time dimension, the Dirac equation comes out of the following consideration. The electron is assumed to do a random walk with velocity plus or minus c. The amplitude for a particular path is $(im\varepsilon)^R$, where R is the number of velocity reversals or corners in the path. If N(R) is the number of paths with R corners, the propagator is proportional to

$$\sum (im\varepsilon)^R \, N(R)$$

and leads to the Dirac equation when $\varepsilon \to 0$. One wonders what sort of magic leads to this result. Dr. Hoyle will talk more about the role of the mass as a scatterer.

This result, along with the considerations that arose in the self energy problem, suggests those problems should be looked into more carefully. We saw that in the interaction part of the action a $\delta(q^2)$ was converted into a $\delta_\pm(q^2)$ by the response of the universe. One wonders whether the universe plays some role in converting the $K_0^\pm$ propagators that we began with to the K_+ propagator.

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