NONLINEAR SIGMA MODELS FOR INFLATION SCENARIOS

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Abstract

In this paper new implementation from nonlinear sigma models are presented for cosmological inflationary scenarios. It is shown that the scalar field models with any form of a potential of self-interaction can be derived from the appropriate nonlinear sigma model. The complete set of new exact solutions for the two- component sigma model in the spatially flat Friedmann-Robertson-Walker and de Sitter universes is obtained.

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1. Introduction

The theory of self-gravitating nonlinear scalar field (or fields) is used in modern cosmological inflation scenarios as an effective tool to avoid some cosmological puzzling questions (see, for instance, the review articles [1] and references quoted therein). The arena of inflation scenarios is the Friedmann-Robertson-Walker (FRW) and de Sitter universes [1,2] or a space-time possessing the symmetry [2,3].The scalar field is described by the standard lagrangian density containing the potential of self-interaction $V(\phi)$. The various forms of the potential are exploited in the cosmological inflation models but as a rule $V(\phi)$ takes the form of a polynomial or an exponent function from the field ϕ. The self-gravitating scalar field with non-minimal interaction to gravity is used in the extended inflation models together with the scalar-tensor theory of gravity [4]. The last-mentioned theories can be reduced to the general relativity ones with a scalar field. The recent models such as hyper- and eternal extended inflation [5,6], double-field inflation [7-9] include two scalar fields with the potential of self-interaction. The self-interacting N- scalar fields theory coupled to gravity is also used in the "chaotic" inflationary models [11]. Recently N-fields model with an exponential potential coupled to anisotropic gravitation fields was investigated in ref.[3].

In this paper it will be shown that the self-gravitating nonlinear sigma models [10] are also a suitable tool for the inflation scenarios. It is proved that the above mentioned theories of self-interacting scalar fields (SSFs) can be derived from the appropriate nonlinear sigma models (NSMs). The two-component NSM, associated with SSF theory, in the FRW and de Sitter open, flat and closed universes is analysed.The new complete set of the exact solutions for a spatially flat universe is obtained.

In section 2 we show the way of construction of an appropriate NSM corresponding to the SSF theory and we present the self-gravitating NSMs as a generalization for the theory of a single scalar field, double-field model and for the SSF theory with non-minimal coupling to gravity (extended inflation models).

In section 3 we make a general analysis of the system of equations for the two-component NSM in the FRW and de Sitter open, flat and closed universes. We find that for all type spaces under consideration the dynamics of the chiral fields do not affect the gravitational "true", in this case, background.

Sections 4 and 5 contain the exact solutions for spatially flat FRW and de Sitter universes respectively. These solutions are written doun in a form sutable for further physical analysis. Finally, in section 6 we discuss our approach based on the above mentioned exact solutions for cosmological inflation scenarios.

2. The chiral model for the self-interacting scalar field

Let us consider the formalism of the nonlinear sigma model (or chiral model) based on the action

$$S_{NSM} = \int_{\mathcal{M}} d^m x \sqrt{g} \{ \frac{\alpha}{2} h_{AB} \varphi^A_i \varphi^B_k g^{ik} \}, \tag{1}$$

where $x = (x^1, \ldots, x^m)$ are the local coordinates of base space-time, Riemannian manifold $(\mathcal{M}, g_{ik})$; $\varphi = (\varphi^1, \ldots, \varphi^n)$ is the multiplet of scalar fields which lies in a target or chiral space, Riemannian manifold $(\mathcal{N}, h_{AB})$; α is a coupling constant; $g = |det(g_{ik})|$; $\varphi_k := \varphi_{,k} := \partial_k \varphi$. Where repeated indices occur the summation convention is assumed. By varying the action (1) with respect to φ^A we obtain the equations of motion

$$\frac{1}{\sqrt{g}} \partial_i (\sqrt{g} g^{ik} h_{AB} \varphi^B_k) - \{ h_{CD} \Gamma^D_{AB} \varphi^B_i \varphi^C_k g^{ik} \} = 0. \tag{2}$$

Here the Γ^D_{AB} are the Christoffel symbols of the chiral manifold $(\mathcal{N}, h_{AB})$. Now let us turn our attention to the self-interacting scalar field models which are described by the action

$$S_{SSF} = \int_{\mathcal{M}} d^m x \sqrt{g} \{ \frac{1}{2} \phi_i \phi_k g^{ik} + V(\phi) \}. \tag{3}$$

Here $V(\phi)$ can be regarded as a sum of a self-interacting potential and a massive term

$$V(\phi) = \epsilon m^2 \phi^2 + V_1(\phi). \tag{4}$$

By introducing the sign $\epsilon = \pm 1$ we have taken into consideration higgs scalar fields. The standard procedures of variation of the action (3) lead to the scalar field equation

$$\frac{1}{\sqrt{g}}\partial_i(\sqrt{g}g^{ik}\phi_k) - \left\{\frac{dV}{d\phi}\right\} = 0. \tag{5}$$

It is important to note that a nonlinear additional term (the expressions marked by $\{\ldots\}$ in (2) and (5)) has been introduced in a different way for the models under consideration. In the equations of motion for NSM (2)the nonlinear additional term arises from the variational procedure in a natural way. Unfortunately in the scalar field equation (5) the nonlinear additional term has an arbitrary character and is selected in a phenomenological way. However it was proved earlier [12] that SSF theory can be derived from an appropriate NSM. To get the model (3) let us choose the metric tensor for the two- component NSM (1) in the next form

$$h_{AB} = diag\{1,\ V(\phi)\},\ \varphi^1 = \phi,\ \varphi^2 = \chi \tag{6}$$

and introduce the constraint

$$\chi_i\chi^i = \epsilon \tag{7}$$

on the auxiliary scalar field χ. Substituting (6) and (7) into (2), it is easy to obtain the equation (5). The constraint (7) clearly may be viewed as a new type of NSM with the restriction on gradients of chiral fields. But in our case it is sufficient to have in mind the relation (7) as a bridge to the SSF theory. In this way it is not difficult to obtain an appropriate NSM for other types of inflation models. For example the double-field models [7,9] with the lagrangian density

$$L_{D-F} = \sqrt{g}\ \{\frac{1}{2}\phi_i\phi^i + \frac{1}{2}\psi_k\psi^k + V_{tot}(\phi,\psi)\}, \tag{8}$$

may be presented by a three-component NSM with the metric

$$h_{AB} = diag\{\ 1,\ 1. V_{tot}(\phi,\psi)\},\ \varphi^1 = \phi,\ \varphi^2 = \psi,\ \varphi^3 = \chi \tag{9}$$

and the constraint (7). For the SSF theory with non-minimal coupling to gravity, the associate NSM looks like

$$S_1 = \int_{\mathcal{M}} d^m x\sqrt{g}\{\zeta R h_{AB}\varphi^A\varphi^B\} + S_{NSM} + S_G. \tag{10}$$

where ζ is a coupling constant and

$$S_G = \int_{\mathcal{M}} d^m x \sqrt{g} \Big\{ \frac{R}{2\kappa} \Big\} \tag{11}$$

is the standard action for a gravitational field, R is the scalar curvature and κ is Einstein's constant. The model (10) leads to the conclusion that for extended inflation type models we must take into consideration the direct interaction between the potential $V(\phi)$ and gravity [12]

$$S_2 = \int_{\mathcal{M}} d^m x \sqrt{g} \{ \zeta [\phi^2 + V(\phi) \chi^2] R \} + S_{SSF} + S_G. \tag{12}$$

3. The self-gravitating two-component chiral model in the inflation arena

To demonstrate the possibilities of self-gravitating NSM for inflation scenarios let us consider the two-component chiral model (1),(6) associated with the SSF theory in the standard inflation arena

$$dS^2_{\mathcal{M}} = (dx^4)^2 - e^{a(x^4)} \{ (dx^1)^2 + A^2(x^1) [(dx^2)^2 + \sin^2 x^2 (dx^3)^2] \}. \tag{13}$$

Here $x^1 = r, x^2 = \theta, x^3 = \varphi, x^4 = t$ as in ordinary notation [1] and

$$A^2(x^1) = \left\{ \begin{array}{c} \sin^2 x^1 \\ (x^1)^2 \\ \sinh^2 x^1 \end{array} \right\} \tag{14}$$

for a closed, flat and open universe, respectively. For the sake of completness we also shall use the cosmological constant Λ in Einstein's equations. Thus the theory under consideration is

$$S_3 = \int_{\mathcal{M}} d^m x \sqrt{g} \Big\{ \frac{R}{2\kappa} + \frac{\alpha}{2} h_{AB} \varphi^A_i \varphi^B_k g^{ik} + \Lambda \Big\}, \tag{15}$$

where h_{AB} is defined by (6) and g_{ik} is defined by (13). Einstein's equations for the model (15) with the energy momentum tensor

$$T_{ik} = \alpha\{h_{AB}\varphi_i^A\varphi_k^B - \frac{1}{2}g_{ik}\varphi_j^A\varphi_l^B h_{AB}g^{jl}\} \tag{16}$$

can be reduced to the next form

$$R_{ik} = \kappa\alpha h_{AB}\varphi_i^A\varphi_k^B - \Lambda g_{ik}. \tag{17}$$

Thus equations (17) and (2) give us the complete system of differential equations (for chiral and gravitational fields) wich determines the so-called self-gravitating non-linear sigma model [10]. Substituting (6) and (13) into (2) and (17), and then doing some simplification we get the system of equations

$$e^a[\frac{1}{2}a_{44} + \frac{3}{4}a_4^2] = \kappa\alpha 2P(\xi)[\xi_1^2 + \chi_1^2] + \Lambda e^a - 2k, \tag{18.1}$$

$$[\xi_1^2 + \chi_1^2]A^2\sin^2 x^2 = [\xi_2^2 + \chi_2^2]A^2 = [\xi_3^2 + \chi_3^2], \tag{18.2}$$

$$\frac{3}{2}[a_{44} + \frac{1}{2}a_4^2] = -\kappa\alpha 2P(\xi)[\xi_4^2 + \chi_4^2] + \Lambda, \tag{18.3}$$

$$\xi_i\xi_k + \chi_i\chi_k = 0, \quad (i \neq k), \tag{18.4}$$

$$\frac{1}{\sqrt{g}}\partial_i(\sqrt{g}g^{ik}\phi_k) - \frac{dP}{d\phi}\chi_i\chi^i = 0, \tag{18.5}$$

$$\frac{1}{\sqrt{g}}\partial_i(\sqrt{g}2P(\phi)g^{ik}\chi_k) = 0, \tag{18.6}$$

where $k = +1, 0, -1$ for closed, flat and open universes, respectively. Here we have used the representation for the metric (6) in the next form

$$dS_{\mathcal{N}}^2 = d\phi^2 + 2P(\phi)d\chi^2 = 2P(\xi)[d\xi^2 + d\chi^2], \tag{19}$$

where

$$\xi = \int \frac{d\phi}{2P(\phi)}.$$

The analysis of the equations (18.2),(18.4) and their differential consequences give us the next

STATEMENT: The necessery and sufficient conditions to satisfy the equations (18.2) and (18.4) are

$$\xi_1 = \xi_2 = \xi_3 = 0, \quad \chi_1 = \chi_2 = \chi_3 = 0. \tag{20}$$

It should be noted here that the relation (20) is a special one for NSM, not for the SSF theory for which the analogue of equation (18.1) takes the form independent from ξ_1 and χ_1. Introducing the new fields

$$\Phi(t) = \phi_t, \quad X(t) = \chi_t, \quad \mu(t) = a_t, \quad (x_4 = t), \tag{21}$$

and using (20) we can reduce the complexity of the remaining equations and (18) take the form

$$\frac{1}{2}\mu_t + \frac{3}{4}\mu^2 = \Lambda - 2ke^{-a}. \tag{22.1}$$

$$\frac{3}{2}[\mu_t + \frac{1}{2}\mu^2] = -\kappa\alpha[\Phi^2 + 2PX^2] + \Lambda, \tag{22.2}$$

$$\frac{3}{2}\mu\Phi + \Phi_t - \frac{dP}{d\phi}X^2 = 0, \tag{22.3}$$

$$\frac{3}{2}\mu X + X_t - \frac{d(lnP)}{d\phi}\Phi X = 0. \tag{22.4}$$

It is clear from the equations (22) that the dynamics of the chiral fields governed by (22.2–22.4) do not affect the gravitational field which can be defined from equation (22.1). The last equation can be reduced to the first order ordinary differential equation

$$zp\frac{dp}{dz} + 2p^2 - \Lambda z^2 + 2k = 0, \quad (p = z_t, z = e^{a/2}), \tag{22.*}$$

which will be solved for the case $k = 0$ in the next two sections. When $k \neq 0$ (closed and open universes) even the simplest case of the FRW space-time ($\Lambda = 0$) leads to the solutions for (22.*) in terms of elliptic functions and needs detailed physical analysis. Thus the equation (22.*) gives us the solutions for the FRW and de Sitter open, flat and closed universes which can be considered as a "true" background for phase transitions and other phenomenon in GUTs effectively described by the scalar field potential of self-interaction. In such a way our model gives a good opportunity for the inflation producers. Namely, there exist the exact solutions (see sections 4 and 5) for the gravitational field, while the chiral fields are respectively free: the potential-like term is not fixed.

4. Exact solutions for the Friedmann-Robertson-Walker universe

In the case of the FRW universe we must take into account $\Lambda = 0$ in equations (22). Then from (22.1) we immediately obtain

$$a = \frac{2}{3}\ln(t + t_0) + \ln a_0, \; e^a = a_0(t + t_0)^{2/3}, \; a_0 = const. \tag{23}$$

The solution (23) is the exact solution for the gravitational field (13) for any type of "potential" $2P(\phi)$. The remaining equations can be reduced to

$$t\Psi_t + \frac{1}{2}F_1(\phi)[\Psi^2 - \beta^2] = 0, \tag{24.1}$$

$$tZ_t + F_1(\phi)\Psi Z = 0, \tag{24.2}$$

where

$$t := t + t_0, \; \Psi = \Phi t, \; Z = Xt, \; \beta^2 = 2(3\kappa\alpha)^{-1}, \; F_1(\phi) = \frac{d(\ln P)}{d\phi}. \tag{25}$$

The formal solution of (24) is given by

$$\Psi = \beta \tanh(\frac{1}{2}\beta I_1), \quad I_1 = \int F_1(\phi) t^{-1} dt; \tag{26.1}$$

$$Z = e^{-\beta I_2}, \quad I_2 = \int F_1(\phi) \tanh(\frac{1}{2}\beta I_1) dt. \tag{26.2}$$

This formal solution is equivalent to equation (24) and presents an integro- differential relation between the chiral fields, their derivatives and a potential $2P(\phi)$. It should be noted here that for every type of solutions, including the general one (26), the only non-vanishing component of energy momentum tensor is

$$\varepsilon = T_4^4 = \frac{1}{3\kappa t^2}. \tag{27}$$

From (27) it is clear that there exists a physical singularity for the solutions presented in this section as $t \to 0$.

4.1 Solution A.

Let us suppose that $\Phi = \Phi(\phi)$ and $X = X(\phi)$,then the equations (24) can be reduced to

$$\Psi \frac{d\Psi}{d\phi} + \frac{1}{2} F_1(\phi)[\Psi^2 - \beta^2] = 0, \tag{28.1}$$

$$\Psi[\frac{dZ}{d\phi} + F_1(\phi) Z] = 0. \tag{28.2}$$

The system of equations (28) may be solved exactly. The result is

$$\Psi^2 = \beta^2 - \frac{2}{c_0^2 P(\varphi)}, \quad Z = \frac{1}{c_0 P(\varphi)}, \quad c_0 = const. \tag{29a}$$

In terms of φ and χ the solution is defined from the relations

$$\ln t = \int \frac{d\phi}{\sqrt{\beta^2 - 2[c_0^2 P(\phi)]^{-1}}},$$

$$\chi = \int [P(\phi)]^{-1} dt. \tag{29b}$$

The simple solution

$$\phi = \phi_0 = const, \quad P = P_0 = const$$

$$\chi = \chi_0 + \beta (2P_0)^{-1/2} \ln t \tag{29c}$$

corresponds to the case $\Psi = 0$ in equations (28). The chiral metric (19) is degenerate for this solution which is similar to the "pure" scalar field solution (30).

4.2 Solution B

Let $X = 0$ or $\chi_i \chi^i = 0$, that is χ is an isotropic field. Then we have the standard solution for "pure" scalar field for any type of potential-like term $2P(\phi)$

$$\phi = \phi_0 + \beta \ln t, \quad \chi = \chi_0 = const \tag{30}$$

The solution (30) confirms that the model under consideration is the direct generalization for a massless scalar field one. The non-vanishing component of the energy momentum tensor is also determined by (27).

4.3 Solution C

Let $X = 1$. This case corresponding to SSF theory (3) is described by relations

$$\phi = 2f_1(t) + \beta \ln \left| \frac{\beta - f_1(t)}{\beta + f_1(t)} \right| + \phi_0,$$

$$f_1(t) = \sqrt{|\beta^2 - 2c_1 t|}, \; c_1 = const, \tag{31}$$

$$\chi = t + t_0, \quad P(\phi) = \frac{c_1}{t}.$$

4.4 Solution D

Let us consider the case of the exponential potential

$$2P(\phi) = me^{\lambda\phi}, \; m = const, \; \lambda = const. \tag{32}$$

The function $F_1(\phi)$ is eliminated from (28) by the assumption (32). The resulting solution for the case may be reduced to

$$\phi = -\beta \ln t + 2\lambda^{-1} \ln(1 + c_2 t^{\lambda\beta}), \; c_2 = const,$$

$$\chi = 2\beta m^{-1} \int t^{(3/2)\lambda\beta - 1}[1 + c_2 t^{\lambda\beta}]^{-3} dt \tag{33}$$

It is clear that for some relations between λ and β (for example $\lambda\beta = n \in \mathbf{Z}$) the last integral in (33) leads to the analytical form.

5. Exact solutions for de Sitter universe

Solving the equation (22.1) with $\Lambda \neq 0$ we can obtain the exact solution for the gravitational field (13) in the form

$$e^a = c_3[\cosh(\xi/2)]^{2/3}, \; \xi = 2\sqrt{3\Lambda}t, \; c_3 = const. \tag{34}$$

The remaining equations can be written as the first order differential equation system

$$uQ_\xi + \frac{1}{4\sqrt{3\Lambda}} F_1(\phi)[8\Lambda + \kappa\alpha Q^2] = 0,$$

$$uY_\xi + \frac{1}{2\sqrt{3\Lambda}} F_1(\phi)QY = 0, \tag{35a}$$

where

$$u = 2\cosh(\xi/2), \; Q = \Phi u, \; Y = Xu. \tag{35b}$$

The formal integration of equations (35) give us

$$Q = -\sqrt{\frac{8\Lambda}{\kappa\alpha}}\Big\{\frac{1}{4}\sqrt{\frac{2\kappa\alpha}{3}}I_3\Big\}, \quad I_3 = \int F_1(\phi)\frac{2d\xi}{u},$$

$$Y = exp\{-\frac{1}{4\sqrt{3\Lambda}}I_4\}, \quad I_4 = \int F_1(\phi)Q\frac{2d\xi}{u}. \tag{36}$$

The non-vanishing component of the energy momentum tensor is

$$\varepsilon = T_4^4 = -\frac{\Lambda}{\kappa\cosh^2(\xi/2)}. \tag{37}$$

In this way we have got non-singular solutions as $t \to 0$ with the energy density $\varepsilon = -\frac{\Lambda}{\kappa}$ at $t = 0$. Let us now take into account some special cases leading to analytical solutions.

5.1 Solution A

Let $Q = Q(\phi)$ and $Y = Y(\phi)$. The equations (35) can be reduced to

$$Q\frac{dQ}{d\phi} + \frac{1}{2}F_1(\phi)[8\Lambda + \kappa\alpha Q^2] = 0,$$

$$Q[\frac{dY}{d\phi} + F_1(\phi)Y] = 0. \tag{38}$$

By integrating the equations (38) we can find the next solutions:

$$Q = 0,$$

$$\chi = [2\sqrt{3\Lambda}P_0c_4]^{-1}\arctan(e^{\xi/2}) + \chi_0, \tag{39a}$$

$$P(\phi) = P_0 = const, \phi = \phi_0 = const, c_4 = -\frac{\kappa\alpha}{4\Lambda P_0};$$

$$Q \neq 0,$$

$$Q^2 = -[\frac{2}{P(\phi)c_5^2} + \frac{8\Lambda}{\kappa\alpha}], \qquad (39b)$$

$$Y = |P(\phi)c_5|^{-1}, \quad c_5 = const.$$

The last solution in terms of ϕ and χ is defined from the relations

$$4\sqrt{3\Lambda}\int\frac{d\phi}{Q(\phi)} = \arctan(e^{\xi/2}),$$

$$\chi = 2\sqrt{3\Lambda}\int|P(\phi)c_5|^{-1}\frac{d\xi}{u}. \qquad (39c)$$

5.2 Solution B

Let $X = 0$, then we can find

$$\phi = \frac{1}{4}\sqrt{-\frac{2}{3\kappa\alpha}}\arctan(e^{\xi/2}) + \phi_0,$$

$$\chi = \chi_0 = const. \qquad (40)$$

5.3 Solution C

Considering the case of the exponential potential (30) one can obtain the next solution

$$\phi = \phi_0 - \frac{4}{\lambda\kappa\alpha}\ln\cos(\frac{v}{4}\sqrt{2\kappa\alpha/3}),$$

$$v = \frac{\lambda}{2}\arctan(e^{\xi/2}), \quad c_6 = const, \qquad (41)$$

$$\chi = c_6 \int \left\{ \cos(v\sqrt{2\kappa\alpha/3}) \right\}^{2/\kappa\alpha} \frac{dv}{\lambda.}$$

The energy momentum tensor for all the solutions presented here is defined by (36).

6. Discussion

In this paper it was shown that nonlinear sigma models can be regarded as a generalization or an alternative for self-interacting scalar field theories when cosmological inflation scenarios are considered. To demonstrate this we have studied the two-component NSM inspired by the SSF theory in the standard inflation arena. We have obtained a complete set of exact solutions for a spatially flat FRW and de Sitter universes which lead us to the next geometrical scenario. The chiral fields ϕ and χ lying in the chiral (intrinsic) space (19) are the source of the gravitational fields (13,23) or (13,34). The changing of the scalar potential $V(\phi)$, predicted by particle physics [1], imply the changing of the potential-like term $2P(\phi)$ and therefore the deformation of the geometry of the intrinsic space. This deformation is proved to occur without any changing of a gravitational fields corresponding to the inflation arena ("true" background). While for the self-interacting scalar field are known [1,2] any changing of $V(\phi)$ leads to a changing of a gravitational field. Thus, in our case, the phase transition and restoration of symmetry predicted by GUTs may be considered in the true background. Namely the solution of section 4 (23) gives us the model with the power law expansion $e^a = a_0 t^{2/3}$ and with a physical singularity as $t \to 0$. In the case of de Sitter universe (section 5) we have got a nonsingular universe (34) with the exponential expansion

$$e^a = c_3[\cosh(\xi/2)]^{2/3}, \ \xi = 2\sqrt{3\Lambda}t$$

and the Minkowskian space-time at the beginning of the universe ($t = 0$). The energy density at the beginning of the universe was $\varepsilon = -\Lambda/\kappa$.

The detailed physical analysis of the obtained solutions and investigations for open and closed universe will be presented in the next paper.

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