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Cosmology and Quantum Electrodynamics

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A fresh approach to quantum electrodynamics establishes a strong connexion between local electrodynamics and cosmology, and seems to support the steady state model of the universe.

It is usually claimed that electrodynamics is the best understood of all known interactions of physics today. Most of the problems of classical electrodynamics were solved soon after the arrival of Maxwell's field theory in the past century. Since then the theory has been successfully cast in the four dimensional language of special relativity. Attempts to integrate the theory with quantum mechanics have not been so successful, however. It is true that most predictions of quantum electrodynamics have been experimentally confirmed. The remarkable agreement with experiment, however, has been achieved in many cases at a cost of mathematical rigour in the theoretical framework. Ingenious renormalization techniques are required to make certain integrals converge.

This state of affairs indicates that the present description of electrodynamics is incomplete. Opinions among experts differ, however, as to where the fault lies. Various attempts to cure the theory of its divergences have been made but without success. It is important therefore to investigate an entirely different approach to quantum electrodynamics, especially because such an approach works well in classical electrodynamics.

This approach looks on electrodynamics as a theory of direct interparticle action rather than as a theory of fields. It was considered in the early part of the present century by Schwarzschild¹, Tetrode² and Fokker^{3,4}. In the form given by Fokker the laws of electrodynamics are derived from an action principle, the action being given by

$$S = - \sum_a \int m_a da - \sum_a \sum_b e_a e_b \iint \delta(S_{AB}^2) \eta_{ik} da^i db^k \quad (1)$$

Here a, b, \dots label the particles; e_a, m_a being the charge and mass of particle a . a^i are the Minkowski coordinates and da the element of proper time on the world line of a . η_{ik} is the space-time metric and S_{AB}^2 the square of the four dimensional distance between A and B , typical points on the world lines of a and b . The 4-potential generated by charge a at a general point X is defined by

$$A_i^{(a)}(x) = \int e_a \delta(S_{AX}^2) \eta_{ik} da^k \quad (2)$$

By virtue of this definition, Maxwell's equations and the gauge condition are satisfied identically. The equations of motion are obtained by the variation of the world lines, and are the usual Lorentz-force equations, but without self action.

The theory is completely time symmetric in its basic interactions. According to (2), the field produced by the charge a at X is not the usually observed retarded field $F_{ret}^{(a)}$ but is given by

$$F^{(a)} = \frac{1}{2} [F_{ret}^{(a)} + F_{adv}^{(a)}] \quad (3)$$

where $F_{adv}^{(a)}$ is the time reversed form of $F_{ret}^{(a)}$. The presence

of advanced fields was considered a drawback of the theory and prevented further progress until it was reconsidered by Wheeler and Feynman⁵. These authors pointed out that (3) was the case for a pair of particles alone. In the actual universe there is a large number of particles, and interference takes place. If the universe is a perfect absorber of all disturbances produced by the motion of charge a , then the net field acting on a can be shown to be

$$\sum_{b \neq a} F^{(b)} = \sum_{b \neq a} F_{ret}^{(b)} + \frac{1}{2} [F_{ret}^{(a)} - F_{adv}^{(a)}] \quad (4)$$

The first term denotes the usual (observed) retarded contribution from all other particles in the universe whereas the second term denotes the radiative reaction which damps the motion of a radiating charge. Unlike the usual field theory, the radiative reaction does not arise from self action but is the result of the response of the universe.

Although this description resolves the paradox of advanced potentials, it is incomplete without a specification of the arrow of time. In the static Euclidean universe considered by Wheeler and Feynman there was no way of distinguishing between (4) and the alternative solution obtained by time reversal. Wheeler and Feynman resolved this difficulty by introducing a thermodynamic arrow of time. Subsequent work^{6,7} has shown that this is unnecessary. Instead, an expanding universe provides an arrow of time in a more natural way. Furthermore, whether or not (4) is a self consistent solution depends on the cosmological model. Thus (4) is a self consistent solution in the steady state cosmology but not in the ever expanding Friedman cosmologies.

This approach has three attractive features which conventional field theory lacks. First, it establishes a strong connexion between local electrodynamics and cosmology, thereby providing us with a means of testing a cosmological model by a purely local experiment. Second, the choice of retarded potentials is not arbitrary as it is in the field theoretical approach. Instead it can be proved within the background of the right kind of cosmological model. Finally, this theory is free from the self action difficulties which beset the usual field theory.

Because of these successes of the theory in classical physics we have been led to consider its consequences in quantum physics. Here we summarize the salient aspects of this investigation which is reported in detail in an unpublished work.

First, because there are no fields with their own degrees of freedom the question of field quantization does not arise. Only the particle tracks are quantized. The theory therefore has to account for those results which are believed to follow from field quantization in the conventional theory. One such result is the formula for the

spontaneous transition probability of an atom between specified states.

The second apparent difference between the two theories relates to self action. The effects of self action are usually regarded as fully established by the phenomenon of the Lamb shift. It might therefore seem as if the direct particle approach cannot account for this phenomenon. This expectation is falsified by the detailed calculations. In fact both the spontaneous transition and the Lamb shift can be shown to owe their origin to the response of the universe.

The nature of atomic transitions in the present theory can be understood in terms of formula (4) applied to the electron a in the atom. The first term in the right-hand side is the prescribed field which causes induced transitions. The second term, however, is not prescribed, but owes its origin solely to the motion of a . It represents the radiative reaction from the universe generated by the motion of a . It is to this term that we must look in order to account for spontaneous transitions. The argument proceeds in a closed cycle which can be broken into four parts: (i) the electron a jumps down in the atom; (ii) this gives rise to radiation as in the classical version of the theory; (iii) the radiation generates a response from the universe; (iv) the response from the universe acts as a perturbation causing the transition.

This idea can be expressed in a quantitative form using the path integral approach to quantum mechanics. This was developed by Feynman⁸ and has been described in detail by Feynman and Hibbs⁹. This method is eminently suited for our purpose although it limits the present investigation to wave functions without spin. It is hoped to remove this limitation in a future investigation.

According to the path integral formulation if S is the action functional for the behaviour of particle a , the probability amplitude for a to go from 1 to 2 is given by

$$K(2,1) = \Sigma (\text{constant}) \exp \{iS/\hbar\} \quad (5)$$

where the summation is over all paths from 1 to 2. In classical physics $S \gg \hbar$ and $K(2,1)$ gets a significant contribution only along paths close to the classical path given by $\delta S = 0$. In this way we understand the principle of stationary action in classical physics.

Suppose now that $S_a[a(\tau)]$ denotes the action along a path $a(\tau)$ of a free particle a . The action for a pair of interacting charged particles a, b , is given by

$$S[a,b] = S_0[a] + S_0[b] + 4\pi e_a e_b \iint \bar{G}_{iA iB} da^i db^j \quad (6)$$

where the last term on the right-hand side of (6) is the generalized form of the Fokker interaction term of (1) in Riemannian space. We describe the model of the expanding universe by the Robertson-Walker line element. For example, the steady state universe has the line element

$$ds^2 = dt^2 - e^{2Ht} [dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)] \quad (7)$$

where H is Hubble's constant. Considerable simplification results from the circumstance that Robertson-Walker spaces are conformally flat and that the electromagnetic equations are conformally invariant. The conformally flat form of (7) is

$$ds^2 = (1 - H\tau)^{-2} [d\tau^2 - dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2)] \quad (8)$$

For the reasons just mentioned it is more convenient to use τ as the time coordinate. The quantum mechanical behaviour of a is given by the path integral

$$\iint_{b \neq a} \Pi \exp \{iS[a,b]/\hbar\} D^3a D^3b \quad (9)$$

We proceed to calculate (9) in the following way.

For a given path $a(\tau)$ calculate the full retarded field at b . Consider induced transitions of b under this direct particle field. The motion of b generates its own time-symmetric field, the advanced part of which is evaluated at a . Next the effect of summing over all $b \neq a$ is taken into account. Assuming that the universe is a perfect absorber along the future light cone we arrive at the probability of transition from a state with wave function $\varphi_m(a)$ and

energy E_m to a state with wave function $\varphi_n(a)$ and energy $E_n (< E_m)$ in the form

$$P(m \rightarrow n) = \iiint \varphi_n^*(a_f) \varphi_n(a_f) J \varphi_m^*(a_i) \varphi_m(a_i) d^3a_i d^3a_f d^3a' \quad (10)$$

where

$$J = \iint \exp [i\{S_0[a(\tau)] - S_0[a'(\tau')]\}/\hbar] F[a(\tau), a'(\tau')] D^3a(\tau) D^3a'(\tau') \quad (11)$$

The functional F includes the response of the universe, and can be evaluated by a Fourier resolution of paths $a(\tau), a'(\tau')$ over a pre-specified time interval. (a_i, a_f are the initial and final values of $a(\tau)$.)

The following results emerge. (i) For $m \neq n$ the spontaneous transition probability formula of the usual theory is obtained. The calculations are considerably simplified by the fact that most absorption takes place at cosmological distances which makes the wavelength of emitted radiation very long. Also it is important that dispersion in the intergalactic medium effectively decouples the different Fourier components of $a(\tau)$. These components can therefore be treated as independent. This is analogous to the random phasing of field oscillators in the quantum field theory although in the present case the randomness is shown to be a cosmological effect rather than postulated.

(ii) The formula (10) can also be applied in the case $m = n$. In this case the wave function φ_m is modified in its time dependence from $\exp [-iE_m t/\hbar]$ to

$$\exp [-i(E_m + \Delta E_m)t - \gamma t/2] \quad (12)$$

where ΔE_m and γ are real. Equation (12) shows that the free particle propagator is modified by the response of the universe. It can be readily shown that

$$\gamma = \Sigma_{E_n < E_m} P(m \rightarrow n) \quad (13)$$

Thus the probability that the atom remains in state φ_m decreases as $e^{-\gamma t}$ where γ is the spontaneous transition probability to any of the lower energy states. ΔE_m is the change in energy level E_m and has the same form as the non-relativistic Lamb shift formula (ref. 9, page 254). There is one important difference, however. The non-relativistic formula in the usual theory diverges at high frequencies, but in this case this formula does not remain valid at high frequencies. This is because the response of the universe at high frequencies to a relativistic motion is different from the extrapolation of the non-relativistic formula. The situation is similar to the difference between the Fourier components of synchrotron radiation emitted by relativistic charged particles and of radiation emitted by slow moving particles. This strongly suggests that the correct formula for ΔE_m will converge as $v \rightarrow \infty$ and without renormalization. This can be demonstrated once a theory of path integrals with spin is available.

We conclude by emphasizing the important part played by cosmology in these calculations. The results (i) and (ii) cannot be obtained in a static Euclidean universe, for example. Furthermore, even the expanding models of the universe do not automatically yield these results. Complete absorption along the future light cone is an essential requirement. Of the well known cosmological models only the steady state model meets the necessary conditions in an unambiguous manner. In our view this result goes far towards demonstrating that the steady state model is the correct form of cosmology.

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