

Constraints on Dirac-Born-Infeld type dark energy models from varying alpha

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We study the variation of the effective fine structure constant alpha for Dirac-Born-Infeld (DBI) type dark energy models. The DBI action based on string theory naturally gives rise to a coupling between gauge fields and a scalar field responsible for accelerated expansion of the universe. This leads to the change of alpha due to a dynamical evolution of the scalar field, which can be compatible with the recently observed cosmological data around the redshift $\tilde{z} \lesssim 3$. We place constraints on several different DBI models including exponential, inverse power-law and rolling massive scalar potentials. We find that these models can satisfy the varying alpha constraint provided that mass scales of the potentials are fine-tuned. When we adopt the mass scales which are motivated by string theory, both exponential and inverse power-law potentials give unacceptably large change of alpha, thus ruled out from observations. On the other hand the rolling massive scalar potential is compatible with the observationally allowed variation of alpha. Therefore the information of varying alpha provides a powerful way to distinguish between a number of string-inspired DBI dark energy models.

I. INTRODUCTION

One of the most remarkable discoveries in cosmology is attributed to the late time acceleration of universe which is supported by supernova observations [1] and receives independent confirmation from CMB and galaxy clustering studies. The current acceleration may be accounted for by supplementing an exotic form of energy density with a negative pressure, popularly known as dark energy (see Refs. [2] for reviews). It was earlier thought that this could originate from a cosmological constant, but the idea is fraught with an extreme fine-tuning problem.

This problem is alleviated in scalar-field models in which the energy density of dark energy dynamically changes such that it remains sub-dominant during the radiation and matter dominant eras and becomes dominant at present. In recent years, a wide variety of dark energy models have been proposed, including Quintessence [3], K-essence [4], Chaplygin gas [5], modifications of gravity [6], Born-Infeld scalars (rolling tachyon [7], massive scalars [8]), with the last one being originally motivated by string theoretic ideas [9]. The common feature of these models is that they operate through an undetermined field potential which in principal can incorporate any priori assigned cosmological evolution, thus lacking predictive power at the fundamental level [7]. These models should be judged by their physical implication and by the generic features which arise in them. There is tremendous degeneracy in this description and it is therefore important to find other physical criteria which can constrain these models. One such criterion can be

provided by the variation of the effective fine structure constant alpha on cosmological scales.

The old idea of time-varying fundamental physical constants [10] has recently attracted much attention in cosmology (see Ref. [11] for review). In fact the Oklo natural fission reactor [12] found the variation of alpha with the level $-0.9 \times 10^{-7} < \Delta\alpha/\alpha < 1.2 \times 10^{-7}$ at the redshift $\tilde{z} \sim 0.16$. The absorption line spectra of distance quasars [13, 14, 15] suggests that $\Delta\alpha/\alpha = (-0.574 \pm 0.102) \times 10^{-5}$ for $0.2 < \tilde{z} < 3.7$ [16]. The recent detailed analysis of high quality quasar spectra [17] gives the lower variation of alpha, $\Delta\alpha/\alpha = (-0.06 \pm 0.06) \times 10^{-5}$ over the redshift range $0.4 < \tilde{z} < 2.3$.

It is well known that the interaction of scalar fields with gauge fields can lead to the variation of the effective fine structure constant. Typically the coupling is chosen to be arbitrary and ad hoc in most of the scalar-field dark energy models [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33], reflecting the fact that the coupling of a quintessence field ϕ to matter and radiation is not fixed by the standard model of particle physics (see Refs. [34] on the proposal of the least coupling principal). Assuming specific forms of the interaction and tuning the coupling parameters along with the appropriate choice of field potentials may lead to a desired change in the effective fine structure constant. However it looks unsatisfactory to work with several arbitrary functions and parameters to produce a physically well-motivated result; such models have a limited predictive power. It is, therefore, necessary to look for the possibilities of obtaining these couplings from a fundamental theory which could fix the above mentioned arbitrariness in the models. Such couplings can be consistently derived from an

effective D-brane action.

D-branes are extended dynamical objects in string theory on which the end points of open strings live. Their tree level action is given by the Dirac-Born-Infeld (DBI) type action which contains gauge fields and scalar fields (tachyons [9], massless scalars [35] and massive scalars [36]). The DBI action naturally gives rise to the coupling of the Born-Infeld scalars with gauge fields, which can account for the variation of the electromagnetic coupling over cosmological time scales. As extensively studied in Refs. [7, 37, 38, 39] the tachyon field might be responsible for accelerated expansion of the universe at late times (see Refs. [40] for the cosmological general dynamics of tachyon). In this paper we study the variation of alpha in the presence of the Born-Infeld scalar coupled to gauge fields and place constraints on the model parameters. In fact this provides us a powerful way to distinguish several different tachyon potentials.

The paper is organized as follows. In section II we examine the DBI action to derive the form of the coupling between the tachyon and gauge fields and set up the frame work for our analysis. Then we study the variation of alpha together with the background cosmological evolution for several different tachyon potentials—(i) exponential (Sec. III), (ii) inverse power-law (Sec. IV) and (iii) rolling massive scalar (Sec. V). We investigate the evolution of alpha both analytically and numerically for arbitrary mass scales of the potentials and then proceed to the case in which the mass scale is constrained by string theory. We show that the information of varying alpha provides important constraints on tachyon potentials. Section VI concludes our results.

II. DBI MODEL

We start with a Dirac-Born-Infeld type effective 4-dimensional action

$$\mathcal{S} = - \int d^4x \tilde{V}(\varphi) \sqrt{-\det(g_{\mu\nu} + \partial_\mu\varphi\partial_\nu\varphi + 2\pi\alpha'F_{\mu\nu})}, \quad (1)$$

where $\tilde{V}(\varphi)$ is the potential of a scalar field φ and $F_{\mu\nu} \equiv 2\nabla_{[\mu}A_{\nu]}$ is a Maxwell tensor with A_μ the four-potential. α' is related to string mass scale M_s via $\alpha' = M_s^{-2}$. We are interested in a situation in which brane is located in a ten-dimensional spacetime with a warped metric [8]

$$ds_{10}^2 = \beta g_{\mu\nu}(x)dx^\mu dx^\nu + \beta^{-1}\tilde{g}_{mn}(y)dy^m dy^n, \quad (2)$$

where β is a warped factor. Note that the first term on the r.h.s. of Eq. (2) corresponds to the metric on the brane.

The action (1) for this metric yields

$$\mathcal{S} = - \int d^4x \beta^2 \tilde{V}(\varphi) \times \sqrt{-\det(g_{\mu\nu} + \beta^{-1}\partial_\mu\varphi\partial_\nu\varphi + 2\pi\alpha'\beta^{-1}F_{\mu\nu})}. \quad (3)$$

Introducing new variables

$$\phi = \varphi/\sqrt{\beta}, \quad V(\phi) = \beta^2\tilde{V}(\sqrt{\beta}\phi), \quad (4)$$

the action (3) can be written as

$$\mathcal{S} = - \int d^4x V(\phi) \sqrt{-\det(g_{\mu\nu} + \partial_\mu\phi\partial_\nu\phi + 2\pi\alpha'\beta^{-1}F_{\mu\nu})}. \quad (5)$$

The warped metric (2) makes the mass scale on the brane from the string mass scale $M_s = 1/\sqrt{\alpha'}$ to an effective mass which is $m_{\text{eff}} = \sqrt{\beta}M_s$. In what follows we shall consider cosmological dynamics and the variation of alpha for the action (5).

We adopt a spatially flat Friedmann-Robertson-Walker (FRW) metric on the brane with a scale factor $a(t)$ (t is cosmic time). We also account for the contributions of non-relativistic matter and radiation, whose energy densities are ρ_m and ρ_r , respectively. Then the background equations of motion are

$$\dot{H} = -\frac{1}{2} \left(\frac{\dot{\phi}^2 V}{\sqrt{1-\dot{\phi}^2}} + \rho_m + \frac{4}{3}\rho_r \right), \quad (6)$$

$$\frac{\ddot{\phi}}{1-\dot{\phi}^2} + 3H\dot{\phi} + \frac{V_\phi}{V} = 0, \quad (7)$$

$$\dot{\rho}_m + 3H\rho_m = 0, \quad (8)$$

$$\dot{\rho}_r + 4H\rho_r = 0, \quad (9)$$

together with a constraint equation for the Hubble rate $H \equiv \dot{a}/a$:

$$3M_p^2 H^2 = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} + \rho_m + \rho_r, \quad (10)$$

where M_p is the reduced Planck mass ($M_p^{-2} = 8\pi G$). In the above equations we assumed the condition, $|F_{\mu\nu}| \ll m_{\text{eff}}^2$.

If the energy density of the field ϕ dominates at late times and it leads to the acceleration of the universe, we can employ the following slow-roll approximation:

$$3M_p^2 H^2 \simeq V(\phi), \quad 3H\dot{\phi} \simeq -\frac{V_\phi}{V}. \quad (11)$$

The slow-roll parameter for the DBI-type scalar field is defined by [40]

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{M_p^2}{2} \left(\frac{V_\phi}{V} \right)^2 \frac{1}{V}. \quad (12)$$

We rewrite the above equations in an autonomous form. Following Ref. [41] we define the following dimensionless quantities:

$$x \equiv \dot{\phi} = H\phi', \quad y \equiv \frac{\sqrt{V(\phi)}}{HM_p}, \quad z \equiv \frac{\sqrt{\rho_r}}{\sqrt{3}HM_p}, \quad (13)$$

where a prime denotes the derivative with respect to the number of e -folds, $N = \ln a$. Then we obtain the following equations (see Refs. [38, 39] for related works):

$$x' = -(1 - x^2)(3x - \lambda y), \quad (14)$$

$$y' = \frac{y}{2} \left(-\lambda xy - \sqrt{1 - x^2} y^2 + z^2 + 3 \right), \quad (15)$$

$$z' = z \left(-2 - \frac{1}{2} \sqrt{1 - x^2} y^2 + \frac{1}{2} z^2 + \frac{3}{2} \right), \quad (16)$$

$$\lambda' = -\lambda^2 xy \left(\Gamma - \frac{3}{2} \right), \quad (17)$$

where

$$\lambda = -\frac{M_p V_\phi}{V^{3/2}}, \quad \Gamma = \frac{V V_{\phi\phi}}{V_\phi^2}. \quad (18)$$

We note that λ is related to ϵ by the relation $\lambda^2 = 2\epsilon$. Therefore one has $|\lambda| \ll 1$ when the slow-roll condition $\epsilon \ll 1$ is satisfied. We also define

$$\Omega_\phi \equiv \frac{\rho_\phi}{\rho_{\text{cr}}} = \frac{y^2}{3\sqrt{1 - x^2}}, \quad (19)$$

$$\Omega_r \equiv \frac{\rho_r}{\rho_{\text{cr}}} = z^2, \quad (20)$$

$$\Omega_m \equiv \frac{\rho_m}{\rho_{\text{cr}}} = 1 - \frac{y^2}{3\sqrt{1 - x^2}} - z^2, \quad (21)$$

where $\rho_{\text{cr}} \equiv 3M_p^2 H^2$ is a critical energy density. Note that these satisfy the constraint equation $\Omega_\phi + \Omega_r + \Omega_m = 1$.

The expansion of the action (5) to second order of the gauge field, for arbitrary metric, becomes

$$\mathcal{S} \simeq \int d^4x \left[-V(\phi) \sqrt{-\det(g_{\mu\nu} + \partial_\mu \phi \partial_\nu \phi)} + \frac{(2\pi\alpha')^2 V(\phi)}{4\beta^2} \sqrt{-g} \text{Tr}(g^{-1} F g^{-1} F) \right]. \quad (22)$$

There are many other terms that are the second order of the gauge field. They involve the derivative of the field ϕ which we are not interested in. Comparing the above action with the standard Yang-Mills action, one finds that the effective fine-structure 'constant' α is

$$\alpha \equiv g_{\text{YM}}^2 = \frac{\beta^2 M_s^4}{2\pi V(\phi)}. \quad (23)$$

The present value of fine structure constant is $\alpha = 1/137$. Since the potential energy of ϕ at present is estimated as $V(\phi_0) \simeq 3H_0^2 M_p^2$, one finds

$$\beta^2 \simeq \frac{6\pi}{137} \left(\frac{H_0}{M_s} \right)^2 \left(\frac{M_p}{M_s} \right)^2. \quad (24)$$

If the string mass scale is the same order as the Planck mass ($M_s \simeq M_p$), we obtain

$$\beta \simeq 10^{-61}, \quad (25)$$

where we used $H_0 \sim 10^{-42}$ GeV. This property holds independent of the form of the scalar-field potential.

The variation of α compared to the present value α_0 is given as

$$\frac{\Delta\alpha}{\alpha} \equiv \frac{\alpha - \alpha_0}{\alpha_0} = \frac{V(\phi_0)}{V(\phi)} - 1, \quad (26)$$

where ϕ_0 is the present value of the field. In the case of the exponential potential $V(\phi) = V_0 e^{-\mu\phi}$, we obtain

$$\frac{\Delta\alpha}{\alpha} = e^{\mu(\phi - \phi_0)} - 1 \simeq \mu(\phi - \phi_0), \quad (27)$$

where the last equality is valid for $|\mu(\phi - \phi_0)| \ll 1$. This corresponds to the choice (2.14) in Ref. [27]. The authors in Ref. [27] chose the coupling of the type $\Delta\alpha/\alpha \propto \Delta\phi$ phenomenologically for *any* potentials of the field ϕ . In our case, however, this is dependent on the form of $V(\phi)$.

For the inverse power-law potential $V(\phi) = M^{4-n} \phi^{-n}$ considered in Refs. [7, 37, 38, 39], one gets

$$\frac{\Delta\alpha}{\alpha} = \left(\frac{\phi}{\phi_0} \right)^n - 1, \quad (28)$$

and for the massive rolling scalar potential $V(\phi) = V_0 e^{\frac{1}{2} M^2 \phi^2}$ considered in Refs. [8], we have

$$\frac{\Delta\alpha}{\alpha} = e^{-\frac{1}{2} M^2 (\phi^2 - \phi_0^2)} - 1. \quad (29)$$

In subsequent sections we shall study the variation of alpha for several different tachyon potentials and place constraints on the parameters of the models. Recently tachyon potentials are classified in three ways [39]– (i) $\lambda = \text{const}$, (ii) $\lambda \rightarrow 0$ asymptotically and (iii) $|\lambda| \rightarrow \infty$ asymptotically. The case (i) corresponds to an inverse square potential $V(\phi) \propto \phi^{-2}$, which gives the border of acceleration and deceleration (see Refs. [7, 37, 42]). The case (ii) gives rise to an accelerated expansion at late times. An example is provided by inverse power-law potential $V(\phi) \propto \phi^{-n}$ with $n < 2$. The potential $V(\phi) = V_0 e^{\frac{1}{2} M^2 \phi^2}$ of a rolling massive scalar field also belongs to this class. The case (iii) corresponds to a deceleration at late times, but with a possible stage of transient acceleration. The exponential potential $V(\phi) = V_0 e^{-\mu\phi}$ belongs to this class.

In Sec. III we shall consider the exponential potential as an example of the case (iii). In Sec. IV the inverse power-law potential $V(\phi) \propto \phi^{-n}$ with $0 \leq n \leq 2$ is studied as examples of the cases (i) and (ii). The rolling massive scalar potential $V(\phi) = V_0 e^{\frac{1}{2} M^2 \phi^2}$ is discussed in Sec. V, since this has somewhat different property compared to other potentials in which the field rolls down toward infinity.

III. EXPONENTIAL POTENTIALS

We first consider the exponential potential

$$V(\phi) = V_0 e^{-\mu\phi}, \quad (30)$$

where μ has a dimension of mass. The exponential potential may appear in the context of the D_3 anti- D_3 cosmology [43]. The tachyon potential for the coincident D_3 anti- D_3 is twice the potential for non-BPS D_3 -brane [44]. The latter is given by $V(\phi) = 2\beta^2 T_3 / \cosh(\sqrt{\beta} M_s \phi)$ [45], where β is a warp factor at the position of the D_3 anti- D_3 in the internal compact space, T_3 is the tension of branes. Then the potential behaves as $V(\phi) \sim \beta^2 T_3 e^{-\sqrt{\beta} M_s \phi}$ at sufficiently late times ($\phi \rightarrow \infty$), which has a correspondence

$$V_0 = \beta^2 T_3, \quad \mu = \sqrt{\beta} M_s, \quad (31)$$

in Eq. (30). Therefore V_0 and μ are not free parameters if we restrict to string theory. In what follows we will consider cosmological constraints on the exponential potential (30) in general and then proceed to the case in which the potential is motivated by string theory.

By Eq. (12) the slow-roll parameter is given by $\epsilon = \mu^2 M_p^2 e^{\mu\phi} / 2V_0$. The accelerated expansion occurs for $\epsilon < 1$, but this stage eventually ends for $\phi > \phi_f \equiv (1/\mu) \log(2V_0/\mu^2 M_p^2)$. Therefore the current acceleration of the universe corresponds to transient quintessence for exponential potentials. Let us employ the slow-roll approximation (11) around the redshift $\tilde{z} \lesssim \mathcal{O}(1)$ under the condition that the energy density of the universe is dominated by ρ_ϕ . Then the slow-roll parameter ϵ is simply written as

$$\epsilon \simeq \frac{\mu^2}{6H_0^2}. \quad (32)$$

This means that accelerated expansion occurs for $\mu \lesssim H_0$.

The evolution of the field ϕ is given by [48]

$$e^{-\mu\phi_0/2} - e^{-\mu\phi/2} = \frac{\mu^2 M_p}{2\sqrt{3}V_0} (t - t_0), \quad (33)$$

where ϕ_0 and t_0 are the present values. Under the condition with $|\mu(\phi - \phi_0)| \ll 1$, which is actually required from the observational constraint $|\Delta\alpha/\alpha| \ll 1$ in Eq. (27), we have that $\phi - \phi_0 \simeq \mu M_p e^{\mu\phi/2} / \sqrt{3}V_0 (t - t_0)$. Since the redshift is given by $\tilde{z} \simeq -H_0(t - t_0)$ for small \tilde{z} , we find that the time-variation of alpha is approximately written as

$$\frac{\Delta\alpha}{\alpha} \simeq -\frac{\mu^2}{3H_0^2} \tilde{z}. \quad (34)$$

In order to obtain $|\Delta\alpha/\alpha| = 10^{-6}$ - 10^{-5} around $\tilde{z} = \mathcal{O}(1)$, the mass scale μ is constrained to be $\mu/H_0 = 10^{-3}$ - 10^{-2} . In this case the accelerated expansion actually occurs, since ϵ is much smaller than 1 by Eq. (32).

We need to caution that slow-roll analysis we used is not necessarily valid in presence of the background energy density. In order to confirm the validity of the above analytic estimation, we numerically solved the background equations (14)-(17) together with Eq. (26). Figure 1 shows one example of the variation of alpha for

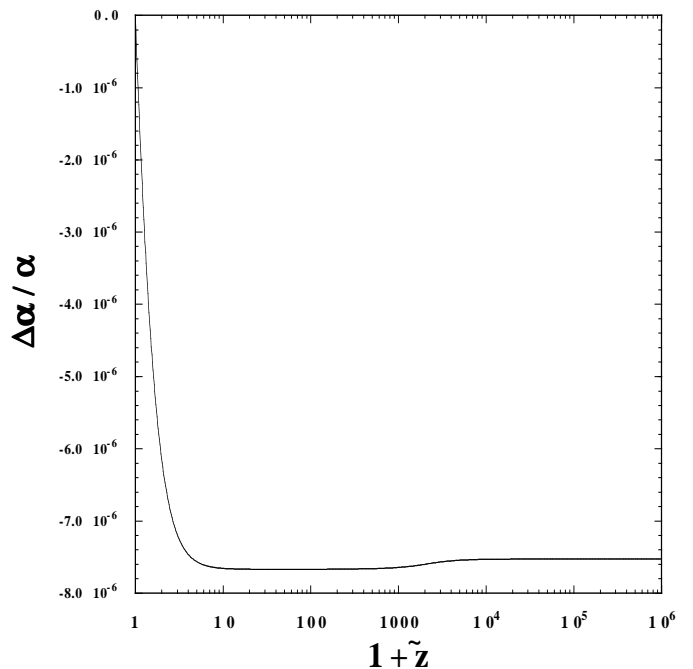


FIG. 1: The variation of $\Delta\alpha/\alpha$ as a function of the redshift \tilde{z} for the exponential potential (30) with $\mu = 8.65 \times 10^{-3} H_0$. Initial conditions are chosen to be $x_i - 1 = -1.0 \times 10^{-12}$, $y_i = 2.5 \times 10^{-10}$, $z_i = 0.99$ and $\lambda = 5.0 \times 10^{-3}$ at $\tilde{z} = 10^6$. The present epoch ($\tilde{z} = 0$) corresponds to $\Omega_\phi \simeq 0.7$ and $\Omega_m \simeq 0.3$.

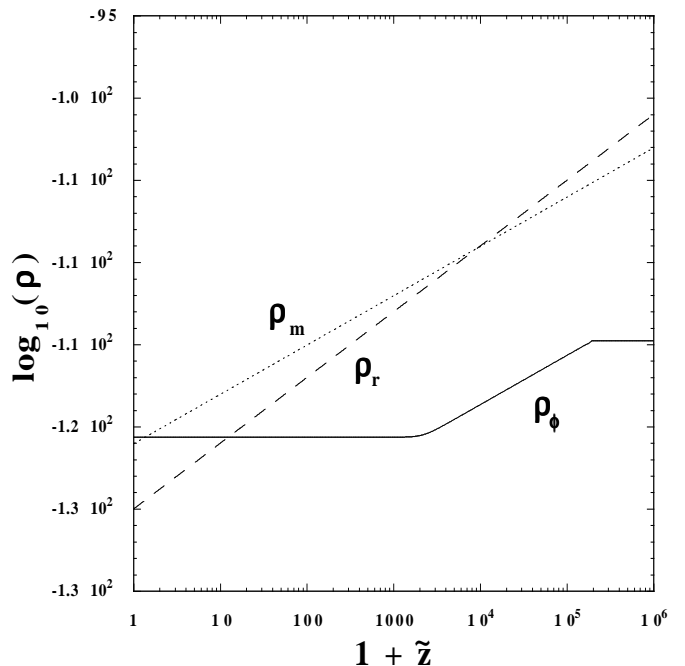


FIG. 2: The evolution of the energy densities ρ_ϕ , ρ_r and ρ_m for the exponential potential (30) with $\mu = 8.65 \times 10^{-3} H_0$. Initial conditions are the same as in Fig. 1.

$\mu = 8.65 \times 10^{-3} H_0$. Initial conditions of the variables x , y and z are chosen so that a viable cosmological evolution can be obtained starting from the radiation dominant era. We begin integrating around from the redshift $\tilde{z} = 10^6$ with $\dot{\phi}^2$ very close to 1. In Fig. 2 we find that ρ_ϕ decreases similarly to ρ_m around $10^3 \lesssim \tilde{z} \lesssim 10^5$, which comes from the fact that the tachyon behaves as a pressureless dust for $\dot{\phi}^2 \sim 1$. This is followed by a stage with slowly changing energy density ρ_ϕ that behaves as an effective cosmological constant. The tachyon evolves very slowly during this stage ($\dot{\phi}^2 \ll 1$), which leads to the accelerated expansion once the energy density ρ_ϕ becomes dominant. In the case of Fig. 1 we have $\Omega_\phi = 0.7$ and $\Omega_m = 0.3$ at present ($\tilde{z} = 0$) with $\epsilon = \lambda^2/2 \ll 1$. The universe will eventually enter a phase with a decelerated expansion in future after λ grows of order unity.

In Fig. 1 we find $|\Delta\alpha/\alpha| \simeq 10^{-6}-10^{-5}$ around the redshift $\tilde{z} = \mathcal{O}(1)$, thus showing the validity of our analytic estimation based on the slow-roll approximation. Since one has $\Delta\alpha/\alpha \simeq -2\epsilon\tilde{z}$ by Eqs. (32) and (34), the slow-roll condition, $\epsilon \ll 1$, is crucially important to provide an appropriate level of the variation of the effective fine structure constant ($|\Delta\alpha/\alpha| \ll 1$). It is intriguing that the condition for an accelerated expansion is compatible with that for varying alpha around $\tilde{z} \lesssim \mathcal{O}(1)$. We note that pure cosmological constant does not give rise to any variation of alpha. In this sense the information of varying alpha is important to distinguish between the cosmological constant and the DBI dark energy model.

If we consider the exponential potential motivated by string theory, the mass μ may be replaced by $\mu = \sqrt{\beta}M_s$ [see Eq. (31)]. Then by Eqs. (24) and (34) we find

$$\frac{\Delta\alpha}{\alpha} \simeq -\sqrt{\frac{2\pi}{411}} \frac{M_p}{H_0} \tilde{z} \simeq -10^{59} \tilde{z}. \quad (35)$$

This gives $|\Delta\alpha/\alpha| \gg 1$ for $\tilde{z} = \mathcal{O}(1)$, which completely contradicts with observational values. Therefore the mass scale μ given in Eq. (31) is too large to account for the small variation of alpha ($|\Delta\alpha/\alpha| \ll 1$). This means that exponential potentials of the tachyon are ruled out from the information of varying alpha if we adopt the mass scale μ motivated by string theory.

IV. INVERSE POWER-LAW POTENTIALS

The model based upon exponential potentials involves very small mass scales ($\mu \ll H_0$) and suffers from a severe fine tuning problem. We now consider a class of inverse power-law potentials for which the problem of the fine tuning may be considerably reduced. The potentials are of the form:

$$V(\phi) = M^{4-n} \phi^{-n}, \quad (36)$$

where M has a dimension of mass.

Here again we should emphasize that M is not necessarily a free parameter if we restrict to string theory. If

ones takes the tachyon potential $\tilde{V}(\varphi) = M_s^{4-n} \varphi^{-n}$ in the original DBI action (1), we obtain the potential (36) in the action (5) with

$$M = \sqrt{\beta} M_s. \quad (37)$$

We shall first consider arbitrary values of M and then proceed to the case in which the relation (37) is used.

As shown in Ref. [37, 39] the accelerated expansion occurs at a later stage for $0 \leq n \leq 2$. Employing the slow-roll approximation (11), we get the following solution

$$\begin{aligned} A_n(t-t_0) &= \phi^{(4-n)/2} - \phi_0^{(4-n)/2}, \\ A_n &= \frac{n(4-n)M^{(n-4)/2}M_p}{2\sqrt{3}}, \end{aligned} \quad (38)$$

where t_0 and ϕ_0 are present values. Around the region $t \sim t_0$ we can expand the solution as

$$\phi = \phi_0 [1 + B_n(t-t_0)], \quad B_n = \frac{nM_p}{\sqrt{3}(\phi_0 M)^{(4-n)/2}}. \quad (39)$$

In order for this expansion to be valid, we require $|B_n(t-t_0)| \ll 1$.

Then Eq. (28) gives

$$\frac{\Delta\alpha}{\alpha} = nB_n(t-t_0) = -\frac{n^2 M_p}{\sqrt{3}(\phi_0 M)^{(4-n)/2} H_0} \tilde{z}. \quad (40)$$

Using the slow-roll approximation $3H_0^2 M_p^2 \simeq M^{4-n} \phi_0^{-n}$ around $\tilde{z} \lesssim \mathcal{O}(1)$, we find

$$\frac{\Delta\alpha}{\alpha} = -\frac{n^2}{3^{1-2/n}} \left(\frac{M}{M_p}\right)^{2-8/n} \left(\frac{H_0}{M_p}\right)^{4/n-2} \tilde{z}. \quad (41)$$

For example one has $\Delta\alpha/\alpha = -4(M/M_p)^{-2} \tilde{z}$ for $n = 2$. By Eq. (41) we can constrain the mass scale M by using the information of varying alpha:

$$\frac{M}{M_p} = \left[\left| \frac{\Delta\alpha}{\alpha} \right| \frac{1}{n^2 \tilde{z}} \left(\frac{M_p}{\sqrt{3} H_0} \right)^{(4-2n)/n} \right]^{n/(2n-8)}. \quad (42)$$

Let us first consider the case of $n = 2$ whose cosmological evolution was investigated in Ref. [7, 38]. When we use the constraint $|\Delta\alpha/\alpha| \lesssim 10^{-6}$ for $\tilde{z} = \mathcal{O}(1)$, we have $M \gtrsim 10^3 M_p$ by Eq. (42). Such a large mass is obviously problematic since we expect general relativity itself to break down in such a regime. In order to obtain the mass scale M that is smaller than M_p , $\Delta\alpha/\alpha$ needs to be much greater than unity, thus incompatible with observations.

The problem of the super-Planckian mass scale can be circumvented by considering the class of less steeper potentials with n smaller than 2. In Fig. 3 we plot M/M_p as a function of n for several different values of $|\Delta\alpha/\alpha|$ at $\tilde{z} = 1$. The allowed mass scale corresponds to the region which is above the borders shown in this figure. We then have $M \lesssim M_p$ for $n \lesssim 1.9$ and $M/M_p \sim 10^{-19}$

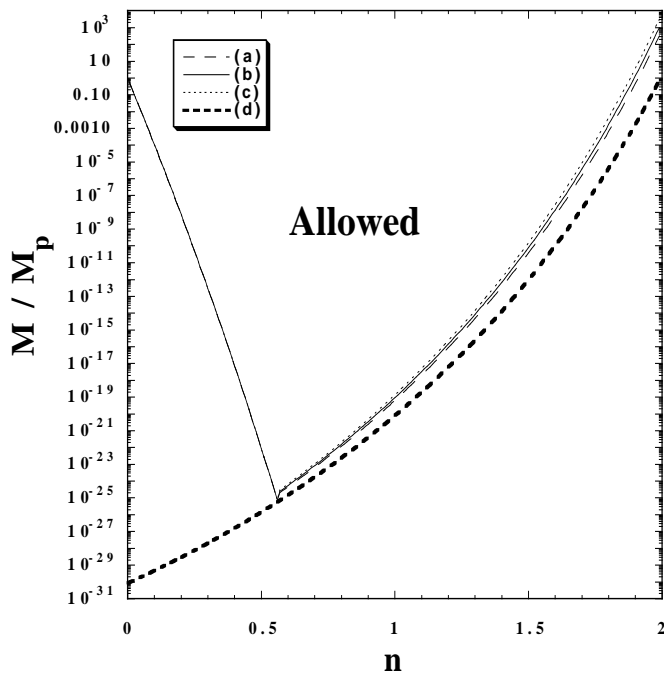


FIG. 3: The mass scale M/M_p determined by the varying alpha constraint in terms of the function of the power n . Each case corresponds to (a) $|\Delta\alpha/\alpha| = 10^{-5}$, (b) $|\Delta\alpha/\alpha| = 10^{-6}$, and (c) $|\Delta\alpha/\alpha| = 10^{-7}$ at $\tilde{z} = 1$. The curve (d) is the bound which is determined by the condition for accelerated expansion, see Eq. (45).

for $n = 1$. The mass M has a minimum around $n = 0.57$. In the limit $n \rightarrow 0$ one has $M \rightarrow M_p$ by Eq. (42). This corresponds to a cosmological constant M^4 with a Planck energy density. Therefore the constraint from varying alpha gives high energy scales around $n = 0$, which does not provide a realistic dark energy scenario.

Let us consider the evolution of $\Delta\alpha/\alpha$ together with the background cosmological dynamics. The slow-roll parameter for the field ϕ is

$$\epsilon = \frac{n^2}{2} \left(\frac{M_p}{M} \right)^2 \frac{1}{(\phi M)^{2-n}}. \quad (43)$$

This is constant for $n = 2$, i.e., $\epsilon = 2(M_p/M)^2$. In order to get an accelerated expansion at late times, we require $M > \sqrt{2}M_p$ in this case. The mass scale which is constrained from the information of varying alpha ($M/M_p \gtrsim 10^3$) satisfies the condition for acceleration.

When $n < 2$ the condition for accelerated expansion, $\epsilon < 1$, yields

$$\phi M > \left(\frac{n}{\sqrt{2}} \frac{M_p}{M} \right)^{2/(2-n)}. \quad (44)$$

The initial value of the field in the radiation dominant epoch needs to be chosen so that it satisfies the condition (44) at late times. For example one has $\phi M > 5.0 \times 10^{37}$

for $n = 1$ and $M/M_p = 10^{-19}$. We can place another constraint on the mass scale M . The present potential energy is approximated by $V(\phi_0) = M^4/(\phi_0 M)^n \simeq \rho_c \simeq 10^{-47} \text{GeV}^4$ with ϕ_0 satisfying Eq. (44). Then we obtain the relation

$$\frac{M}{M_p} > \left[\left(\frac{\rho_c}{M_p^4} \right)^{1-\frac{n}{2}} \left(\frac{n}{\sqrt{2}} \right)^n \right]^{1/(4-n)}. \quad (45)$$

In Fig. 3 this bound is plotted as a curve (d). We find that the varying alpha bound (42) provides a severer constraint compared to (45) coming from the condition of accelerated expansion. Therefore it is important to take into account the information of varying alpha when we constrain the inverse power-law potential.

We shall numerically study the evolution of $\Delta\alpha/\alpha$ in order to confirm the analytic estimation based on the slow-roll approximation. First of all, we checked that the inverse square potential ($n = 2$) can explain the required variation of α ($|\Delta\alpha/\alpha| = 10^{-6}-10^{-5}$) provided that the mass M is much larger than the Planck mass. However this mass scale is unacceptably large from the viewpoint of the validity of general relativity.

Let us proceed to the case with $n < 2$. By Eq. (43) the slow-roll parameter decreases as the field evolves toward larger values. We can consider two situations which lead to the acceleration of the universe at late times. The first is the case in which the slow-roll condition $\epsilon \ll 1$ is satisfied even for $\tilde{z} > \mathcal{O}(1)$ [we call this the case (a)]. In this case the field ϕ evolves slowly ($\dot{\phi}^2 \ll 1$) during the transition from the matter-dominant era to the scalar field dominant era. The second corresponds to the case in which the slow-roll parameter is larger than 1 for $\tilde{z} \gtrsim 1$ but becomes less than 1 for $\tilde{z} \lesssim 1$ [we call this the case (b)]. Since the transition from the non slow-roll phase to the slow-roll stage occurs around $\tilde{z} \sim 1$ in this case, one can not necessarily employ the approximation (11) in this region.

In both cases one can obtain a viable cosmological evolution which reaches $\Omega_\phi \simeq 0.7$ and $\Omega_m \simeq 0.3$ at present. In Fig. 4 we plot the evolution of Ω_ϕ , Ω_m and Ω_r for $n = 1$ and $M = 3.79 \times 10^{-19}$ with the present value $\phi_0 M = 1.50 \times 10^{44}$ [corresponding to the case (a)]. In this case the slow-roll parameter ϵ is much smaller than unity and is nearly constant even for $\tilde{z} > \mathcal{O}(1)$, as plotted in Fig. 5. When ϵ becomes less than unity only near to the present [the case (b)], we numerically checked that a viable cosmological solution that leads to the late-time acceleration can be obtained as well.

We can distinguish the above two different cases by having a look at the evolution of the effective fine structure constant. Figure 6 shows that $|\Delta\alpha/\alpha|$ is kept to be small ($|\Delta\alpha/\alpha| \sim 10^{-6}$) even for $\tilde{z} < \mathcal{O}(1)$ in the case (a). This reflects the fact that the field ϕ evolves very slowly because of a small slow-roll parameter ($\epsilon \ll 1$). On the other hand the situation is different in the case (b). Since the evolution of the field ϕ is not described by a slow-roll for $\tilde{z} \gtrsim 1$, this leads to a larger variation of alpha com-

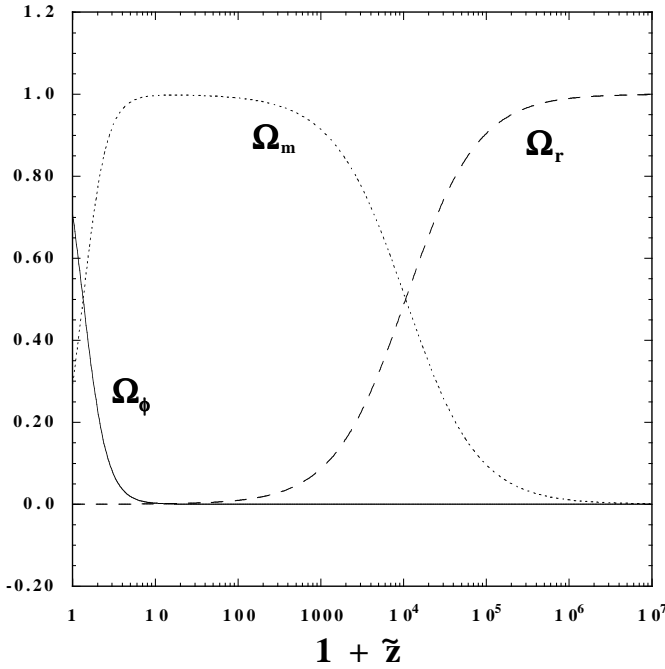


FIG. 4: The evolution of Ω_ϕ , Ω_m and Ω_r as a function of the redshift \tilde{z} for the inverse power-law potential with $n = 1$ and $M = 3.79 \times 10^{-19}$. This case satisfies $\Omega_\phi = 0.7$ and $\phi_0 M = 1.50 \times 10^{42}$ at $\tilde{z} = 0$. This cosmological evolution corresponds to the case (a) in which the slow-roll condition $\epsilon \ll 1$ is satisfied for $\tilde{z} > \mathcal{O}(1)$.

pared to the one derived by the slow-roll analysis. This property is clearly seen in Fig. 6.

The above results indicate that the field ϕ needs to satisfy the condition (44) in the matter-dominant era prior to the accelerating phase in order for the variation of α to be compatible with observations. Together with the minimum mass scale M determined by Eq. (42), the information of varying alpha places a constraint on the initial condition of ϕ . In the case (a) which gives $|\Delta\alpha/\alpha| = 10^{-6}$ - 10^{-5} around $\tilde{z} = \mathcal{O}(1)$ the variation of alpha is small even for $\tilde{z} \gg 1$, thus satisfying the nucleosynthesis constraint $|\Delta\alpha/\alpha| < 2 \times 10^{-8}$ around $\tilde{z} = 10^8$ - 10^{10} [46] (see Fig. 6).

Let us finally consider the case in which the mass scale is constrained by Eq. (36) from string theory. Figure 3 indicates that $M/M_p > 10^{-25}$ from the requirement of varying alpha. Using Eqs. (25) and (36) we find that the string mass scale is unacceptably large, i.e., $M_s/M_p \gtrsim 10^5$. In order to obtain $M_s \lesssim M_p$ we require the condition $|\Delta\alpha/\alpha| \gg 1$, which is incompatible with observations. Therefore the inverse-power law potential is disfavored from the information of varying alpha when we use the mass scale M motivated by string theory.

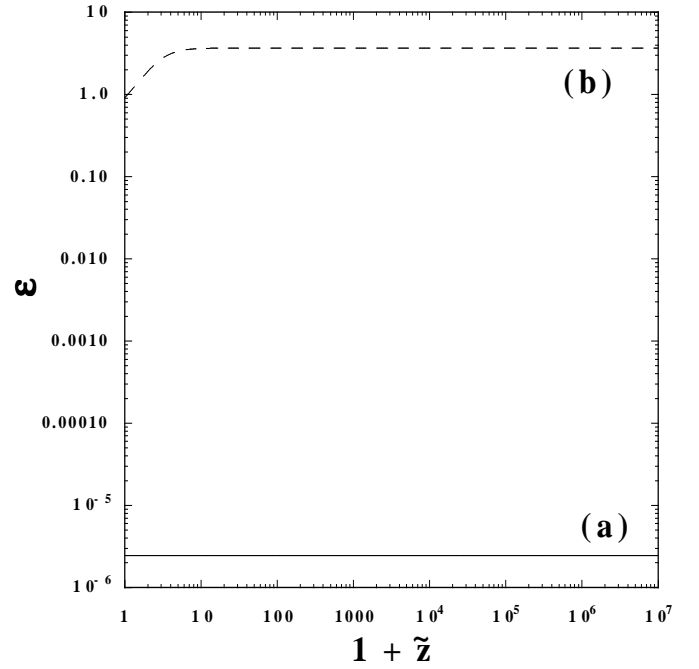


FIG. 5: The evolution of the slow-roll parameter ϵ for the inverse power-law potential with $n = 1$ and $M = 3.79 \times 10^{-19}$. Each case corresponds to (a) $\phi_0 M = 1.50 \times 10^{42}$ and (b) $\phi_0 M = 3.98 \times 10^{36}$ at $\tilde{z} = 0$. In the case (a) ϵ is nearly constant with $\epsilon \ll 1$. Meanwhile ϵ becomes smaller than 1 only near to the present in the case (b).

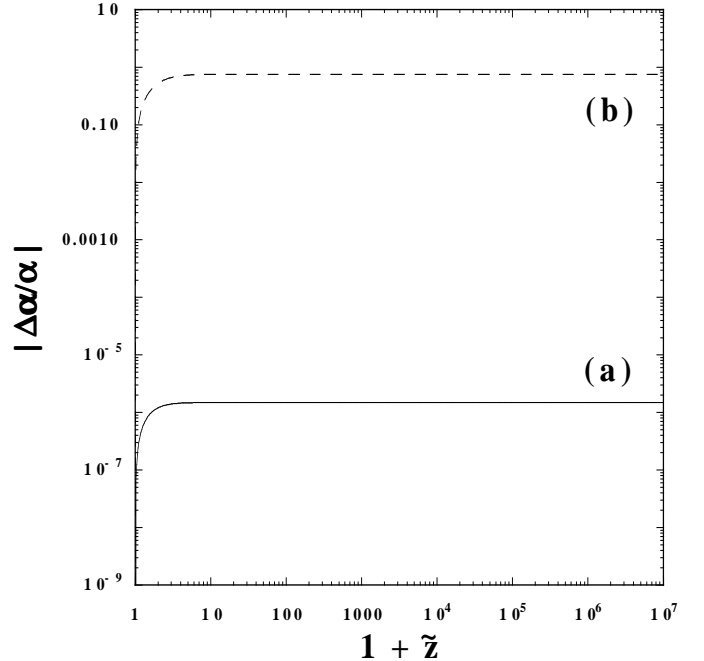


FIG. 6: The variation of alpha as a function of the redshift \tilde{z} with same model parameters and initial conditions as in Fig. 5. The case (a) shows a small variation of alpha, whereas the case (b) gives large values of $|\Delta\alpha/\alpha|$.

V. ROLLING MASSIVE SCALAR POTENTIAL

Let us finally study the rolling massive scalar potential [8]

$$V(\phi) = V_0 e^{\frac{1}{2} M^2 \phi^2}, \quad (46)$$

where V_0 and M are constants. If one considers the potential $V(\varphi) = T_3 e^{\frac{1}{2} M_s^2 \varphi^2}$ in the original action (1), one obtains the potential (46) in the action (5) with correspondence

$$V_0 = \beta^2 T_3, \quad M = \sqrt{\beta} M_s, \quad (47)$$

The cosmological dynamics for this potential was investigated in the context of inflation [8] and dark energy [39]. In both cases we require small warp factor β ($\ll 1$) to satisfy observational constraints.

In our scenario we recall that the warp factor is constrained by Eq. (24) from the present value of alpha. In this case it is easy to confirm that the potential (46) with $V_0 = \beta^2 T_3$ and $|\phi M| \ll 1$ can account for the the present value of the Hubble constant H_0 , provided that the string mass scale M_s is the same order as $T_3^{1/4}$. In what follows we shall study a situation in which the field evolves close to the potential minimum at present ($|\phi_0 M| \ll 1$).

The evolution equation for the massive DBI scalar is

$$\frac{\ddot{\phi}}{1 - \dot{\phi}^2} + 3H\dot{\phi} + M^2\phi = 0. \quad (48)$$

The Hubble rate at present is approximately given by $H_0 \simeq \sqrt{V_0/(3M_p^2)}$. We wish to consider the case with V_0 and M given by Eq. (47). Then the mass M is much larger than H_0 as long as M_s is not too much smaller than M_p , so the friction term is negligible around the potential minimum. When $\dot{\phi}^2 \ll 1$ the solution for Eq. (48) may be given by

$$\phi \simeq \Phi \cos(Mt), \quad (49)$$

where Φ is the field amplitude. In fact we numerically checked that the field ϕ oscillates as in Eq. (49) for $M \gg H_0$ and $\dot{\phi}^2 \ll 1$.

In the oscillatory regime, the condition for acceleration for the DBI scalar is

$$\langle \dot{\phi}^2 \rangle = M^2 \Phi^2 \frac{1}{T} \int_0^T \sin^2(Mt) dt < 2/3, \quad (50)$$

where T is the period of oscillations. This gives

$$M^2 \Phi^2 < 4/3. \quad (51)$$

Let us consider the situation with $M^2 \Phi^2 < 4/3$. Then by Eq. (29) the variation of α is approximately given by

$$\frac{\Delta\alpha}{\alpha} \simeq -\frac{1}{2} M^2 (\phi^2 - \phi_0^2). \quad (52)$$

Since $|\phi^2 - \phi_0^2| \lesssim \Phi^2$, we obtain

$$\left| \frac{\Delta\alpha}{\alpha} \right| \lesssim \frac{1}{2} M^2 \Phi^2. \quad (53)$$

It is possible to have $|\Delta\alpha/\alpha| \sim 10^{-6}-10^{-5}$ if $|M\Phi|$ is of order $10^{-3}-10^{-2}$. We recall that the condition for acceleration is satisfied in this case. When $M \gg H_0$ the time scale of the oscillation of the field ϕ is much smaller than the cosmological time corresponding to the redshift $\tilde{z} = \mathcal{O}(1)$. For the choice (47), M is much larger than the Hubble rate H around $\tilde{z} = \mathcal{O}(1)$, which means that the above estimation for alpha is valid even for $\tilde{z} = \mathcal{O}(1)$. The field oscillates coherently for many times while the universe evolves from $\tilde{z} = \mathcal{O}(1)$ to $\tilde{z} = 0$. In this case we have an interesting possibility to explain the oscillation of alpha which is actually seen in observational data [17, 32].

In Ref. [8] the potential (46) was used for the inflation in early universe, in which case the warp factor is constrained to be $\beta \sim 10^{-9}$ from the COBE normalization (in this scenario the problem of reheating is overcome by accounting for a negative cosmological constant that may come from the stabilization of the modulus). This warp factor is very different from the one given in Eq. (25). In the so-called KKLT scenario [47], one may try to link the quintessential anti-D3 brane with the primordial inflationary anti-D3 brane. To this end, one should think of a mechanism which explains the reduction of β from $\beta \sim 10^{-9}$ to $\beta \sim 10^{-61}$ at some time after a reheating epoch. The warp factor at the tip of the Klebanov-Strassler throat in which the anti-D3 branes live is given by [49]

$$\beta \sim \exp\left(-\frac{4\pi\mathcal{N}}{3g_s\mathcal{M}}\right), \quad (54)$$

where g_s is the string coupling constant, and the integers \mathcal{M}, \mathcal{N} denote the R-R and NS-NS three form fluxes, respectively, in the Calabi-Yau manifold of the compact space. The warp factor has its minimum value at this point in the KS throat. This minimum can be extremely small for suitable choice of fluxes. The RR flux annihilation [50] may then explain the reduction of the warp factor at the tip of the KS throat.

We have found that the effective mass on anti-D3 branes is $m_{\text{eff}} = \sqrt{\beta} M_s \sim 10^{-12}$ GeV. This makes all massive excitations of the branes to be quite light. However, massive strings stretching between two nearly coincident such light branes which may play the role of W-bosons need not to be light. In fact the mass of stretched open string between two branes with separation ℓ and in the warped metric Eq. (2) is given by $M_W^2 = \ell^n \ell^m g_{nm} = \beta^{-1} \ell^2 \sim 10^{61} \ell^2$. For small enough ℓ one can find a suitable W-boson mass.

VI. CONCLUSIONS

In this paper we have studied the variation of the electromagnetic coupling in the frame work of Dirac-Born-

Infeld (DBI) type dark energy models. Since the Born-Infeld scalar field is generally coupled to gauge fields, the cosmological evolution of it naturally leads to the change of the effective fine structure constant. This can provide an interesting possibility to explain the observational data of the variation of alpha ($|\Delta\alpha/\alpha| = 10^{-5}$ - 10^{-6}) around the redshift $\tilde{z} \lesssim \mathcal{O}(1)$.

We adopted a configuration that branes are located in a ten-dimensional spacetime with a warp factor β . Then we find that the effective fine structure constant is given by $\alpha = \beta^2 M_s^4 / (2\pi V(\phi))$, where M_s is the string mass scale and $V(\phi)$ is the potential of the field ϕ . Since the potential energy at present is related with the Hubble constant $H_0 \sim 10^{-42}$ GeV, one can estimate the warp factor β by using the present value of alpha ($\alpha = 1/137$). This is found to be $\beta \simeq 10^{-61}$ when M_s is the same order as the Planck mass M_p . If one attempts to use the DBI field for the inflation in early universe, the COBE normalization gives $\beta \simeq 10^{-9}$ for exponential type potentials [8]. Therefore we can not use the single DBI field both for inflation and dark energy unless some specific mechanism makes the value of β smaller after inflation.

Since α is inversely proportional to $V(\phi)$, the variation of alpha is dependent on the forms of the DBI potentials. We have considered three different potentials—(i) exponential: $V(\phi) = V_0 e^{-\mu\phi}$ (Sec. III), (ii) inverse power-law: $V(\phi) = M^{4-n}\phi^{-n}$ (Sec. IV) and (iii) rolling massive scalar: $V(\phi) = V_0 e^{\frac{1}{2}M^2\phi^2}$ (Sec. V). We performed numerical calculations as well as analytic estimations in order to confirm the validity of slow-roll approximations.

For the exponential potential transient acceleration occurs when the quantity λ defined in Eq. (18) is less than unity [39], while the asymptotic solution is dust-like. We found that both conditions of varying alpha and accelerated expansion are satisfied for $\mu/H_0 = 10^{-3}$ - 10^{-2} . However if we adopt the mass scale μ motivated by string theory [see Eq. (31)], this gives unacceptably large variation of alpha ($|\Delta\alpha/\alpha| \gg 1$), which contradicts with observations.

The inverse power-law potential gives rise to an accelerated expansion at late times for $n \leq 2$. Note that the inverse square potential ($n = 2$) corresponds to the cosmological scaling solution that marks the border of acceleration and deceleration. We placed constraints on the mass scale M from the information of varying alpha, see Fig. 3. Although M needs to be much larger than M_p for $n = 2$, this problem is circumvented when n is less than 2. We have numerically confirmed that it is possible to have $|\Delta\alpha/\alpha| = 10^{-6}$ - 10^{-5} provided that the slow-roll parameter ϵ is nearly constant with $\epsilon \ll 1$ around the redshift $\tilde{z} = \mathcal{O}(1)$. Nevertheless this potential is again disfavored observationally if we use the mass scale M constrained by string theory.

The rolling massive scalar potential leads to the acceleration of the universe when the amplitude of the oscillation of ϕ is small. For the mass scale M constrained by string theory ($M = \sqrt{\beta}M_s$) the field ϕ oscillates coherently with a time scale M^{-1} much smaller than H_0^{-1} . If the amplitude Φ satisfies the condition $|M^\Phi| = 10^{-3}$ - 10^{-2} , it is possible to obtain $|\Delta\alpha/\alpha| = 10^{-6}$ - 10^{-5} for the redshift $\tilde{z} < \mathcal{O}(1)$ together with the condition for accelerated expansion. Then this potential is favoured observationally unlike exponential and inverse power-law potentials.

We thus found that the varying alpha provides a powerful tool to constrain several different DBI potentials motivated by string theory. It is of interest to extend our analysis to the case in which the constraints coming from CMB and structure formation are taken into account.

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