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Entropy changes in the clustering of galaxies in an expanding universe

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ABSTRACT

In the present work the approach-thermodynamics and statistical mechanics of gravitating systems is applied to study the entropy change in gravitational clustering of galaxies in an expanding universe. We derive analytically the expressions for gravitational entropy in terms of temperature T and average density n of the particles (galaxies) in the given phase space cell. It is found that during the initial stage of clustering of galaxies, the entropy decreases and finally seems to be increasing when the system attains virial equilibrium. The entropy changes are studied for different range of measuring correlation parameter b. We attempt to provide a clearer account of this phenomena. The entropy results for a system consisting of extended mass (non-point mass) particles show a similar behaviour with that of point mass particles clustering gravitationally in an expanding universe.

Keywords: Gravitational Clustering; Thermodynamics; Entropy; Cosmology

1. INTRODUCTION

Galaxy groups and clusters are the largest known gravitationally bound objects to have arisen thus far in the process of cosmic structure formation [1]. They form the densest part of the large scale structure of the universe. In models for the gravitational formation of structure with cold dark matter, the smallest structures collapse first and eventually build the largest structures; clusters of galaxies are then formed relatively. The clusters themselves are often associated with larger groups called super-clusters. Clusters of galaxies are the most recent and most massive objects to have arisen in the hiearchical structure formation of the universe and the study of clusters tells one about the way galaxies form and evolve. The average density n and the temperature T of a gravitating system discuss some thermal history of cluster formation. For a better larger understanding of this thermal history it is important to study the entropy change resulting during the clustering phenomena because the entropy is the quantity most directly changed by increasing or decreasing thermal energy of intracluster gas. The purpose of the present paper is to show how entropy of the universe changes with time in a system of galaxies clustering under the influence of gravitational interaction.

Entropy is a measure of how disorganised a system is. It forms an important part of second law of thermodynamics [2,3]. The concept of entropy is generally not well understood. For erupting stars, colloiding galaxies, collapsing black holes - the cosmos is a surprisingly orderly place. Supermassive black holes, dark matter and stars are some of the contributors to the overall entropy of the universe. The microscopic explanation of entropy has been challenged both from the experimental and theoretical point of view [11,12]. Entropy is a mathematical formula. Standard calculations have shown that the entropy of our universe is dominated by black holes, whose entropy is of the order of their area in planck units [13]. An analysis by Chas Egan of the Australian National University in Canberra indicates that the collective entropy of all the supermassive black holes at the centers of galaxies is about 100 times higher than previously calculated. Statistical entropy is logrithmic of the number of microstates consistent with the observed macroscopic properties of a system hence a measure of uncertainty about its precise state. Statistical mechanics explains entropy as the amount of uncertainty which remains about a system after its observable macroscopic properties have been taken into account. For a given set of macroscopic quantities like temperature and volume, the entropy is a function of the probability that the system is in various quantumn states. The more states available to the system with higher probability, the greater the disorder and thus greater the entropy [2]. In real experiments, it is quite difficult to measure the entropy of a system. The technique for doing so is based on the thermodynamic definition of entropy. We discuss the applicability of statistical mechanics and thermodynamics for gravitating systems and explain in what sense the entropy change $S - S_0$ shows a changing behaviour with respect to the measuring correlation parameter $b = 0 - 1$.

2. THERMODYNAMIC DESCRIPTION OF GALAXY CLUSTERS

A system of many point particles which interacts by Newtonian gravity is always unstable. The basic instabilities which may occur involve the overall contraction (or expansion) of the system, and the formation of clusters within the system. The rates and forms of these instabilities are governed by the distribution of kinetic and potential energy and the momentum among the particles. For example, a finite spherical system which approximately satisfies the viral theorem, contracts slowly compared to the crossing time ~ $(G\rho)^{-1/2}$ due to the evaporation of high energy particles [3] and the lack of equipartition among particles of different masses [4]. We consider here a thermodynamic description for the system (universe). The universe is considered to be an infinite gas in which each gas molecule is treated to be a galaxy. The gravitational force is a binary interaction and as a result a number of particles cluster together. We use the same approximation of binary interaction for our universe (system) consisting of large number of galaxies clustering together under the influence of gravitational force. It is important to mention here that the characterization of this clustering is a problem of current interest. The physical validity of the application of thermodynamics in the clustering of galaxies and galaxy clusters has been discussed on the basis of N-body computer simulation results [5]. Equations of state for internal energy U and pressure P are of the form [6]:

$$U = \frac{3NT}{2}(1-2b) \tag{1}$$

$$P = \frac{NT}{V}(1-b) \tag{2}$$

b defines the measuring correlation parameter and is dimensionless, given by [8]

$$b = -\frac{W}{2K} = 2\tau G m^2 \frac{n}{3T}\int_0^\infty \xi(\bar{n},T,r)\, r\, dr \tag{3}$$

W is the potential energy and K the kinetic energy of the particles in a system. $\bar{n} = N/V$ is the average number density of the system of particles each of mass m, T is the temperature, V the volume, G is the universal gravitational constant. $\xi(\bar{n},T,r)$ is the two particle correlation function and r is the inter-particle distance. An overall study of $\xi(\bar{n},T,r)$ has already been discussed by [7]. For an ideal gas behaviour $b = 0$ and for non-ideal gas system b varies between 0 and 1. Previously some workers [7,8] have derived b in the form of:

$$b = \frac{\beta\bar{n}T^{-3}}{1+\beta\bar{n}T^{-3}} \tag{4}$$

Eq.4 indicates that b has a specific dependence on the combination $\bar{n}T^{-3}$.

3. ENTROPY CALCULATIONS

Thermodynamics and statistical mechanics have been found to be equal tools in describing entropy of a system. Thermodynamic entropy is a non-conserved state function that is of great importance in science. Historically the concept of entropy evolved in order to explain why some processes are spontaneous and others are not; systems tend to progress in the direction of increasing entropy [9]. Following statistical mechanics and the work carried out by [10], the grand canonical partition function is given by

$$Z_N(T,V) = \frac{1}{N!}\left(\frac{2\pi mkT}{\Lambda^2}\right)^{\frac{3N}{2}} V^N \left[1+\beta\bar{n}T^{-3}\right]^{N-1} \tag{5}$$

Where *N!* is due to the distinguishability of particles. Λ represents the volume of a phase space cell. N is the number of paricles (galaxies) with point mass approximation. The Helmholtz free energy is given by:

$$A = -T\ln Z_N \tag{6}$$

Thermodynamic description of entropy can be calculated as:

$$S = -\left(\frac{\partial A}{\partial T}\right)_{N,V} \tag{7}$$

The use of **Eq.5** and **Eq.6** in **Eq.7** gives

$$S - S_0 = \ln\left(\bar{n}^{-1}T^{\frac{3}{2}}\right) - \ln(1-b) - 3b \tag{8}$$

Where S_0 is an arbitary constant. From **Eq.4** we write

$$\bar{n} = \frac{b}{(1-b)\beta T^{-3}} \tag{9}$$

Using **Eq.9**, **Eq.8** becomes as

$$S - S_0 = -\left[3b + \ln bT^{\frac{3}{2}}\right] \tag{10}$$

Again from **Eq.4**

$$T^{\frac{3}{2}} = \left[\frac{\beta\bar{n}(1-b)}{b}\right]^{\frac{1}{2}} \quad (11)$$

With the help of **Eq.11, Eq.10** becomes as

$$S - S_0 = -\left[\frac{1}{2}\ln\bar{n} + \frac{1}{2}\ln\left[b(1-b)\right] + 3b\right] \quad (12)$$

This is the expression for entropy of a system consisting of point mass particles, but actually galaxies have extended structures, therefore the point mass concept is only an approximation. For extended mass structures we make use of softening parameter ε whose value is taken between 0.01 and 0.05 (in the units of total radius). Following the same procedure, **Eq.8** becomes as

$$S - S_0 = N\ln\left[\frac{N}{V}T^{\frac{3}{2}}\right] - N\ln(1-b_\varepsilon) - 3Nb_\varepsilon \quad (13)$$

For extended structures of galaxies, **Eq.4** gets modified to

$$b_\varepsilon = \frac{\beta\bar{n}T^{-3}\alpha(\varepsilon/R)}{1+\beta\bar{n}T^{-3}\alpha(\varepsilon/R)} \quad (14)$$

where α is a constant, R is the radius of a cell in a phase space in which number of particles (galaxies) is N and volume is V. The relation between b and b_ε is given by:

$$b_\varepsilon = \frac{b\alpha}{1+b(\alpha-1)} \quad (15)$$

b_ε represents the correlation energy for extended mass particles clustering gravitationally in an expanding universe. The above **Eq.10** and **Eq.12** take the form respectively as;

$$S - S_0 = -\left[\ln\frac{bT^{\frac{3}{2}}}{1+b(\alpha-1)} + \frac{3b\alpha}{1+b(\alpha-1)}\right] \quad (16)$$

$$S - S_0 = -\left[\frac{1}{2}\ln\bar{n} + \ln\frac{\left[b(1-b)\right]^{\frac{1}{2}}}{1+b(\alpha-1)} + \frac{3b\alpha}{1+b(\alpha-1)}\right] \quad (17)$$

where

$$\alpha\left(\frac{\varepsilon}{R}\right) = \sqrt{1+\left(\frac{\varepsilon}{R}\right)^2} + \left(\frac{\varepsilon}{R}\right)^2 \ln\frac{\frac{\varepsilon}{R}}{1+\sqrt{1+\left(\frac{\varepsilon}{R}\right)^2}} \quad (18)$$

If $\varepsilon = 0$, $\alpha = 1$ the entropy equations for extended mass galaxies are exactly same with that of a system of point mass galaxies approximation. **Eq.10**, **Eq.12**, **Eq.16** and **Eq.17** are used here to study the entropy changes in the cosmological many body problem. Various entropy change results $S - S_0$ for both the point mass approximation and of extended mass approximation of particles (galaxies) are shown in (**Figures 1** and **2**). The results have been calculated analytically for different values of

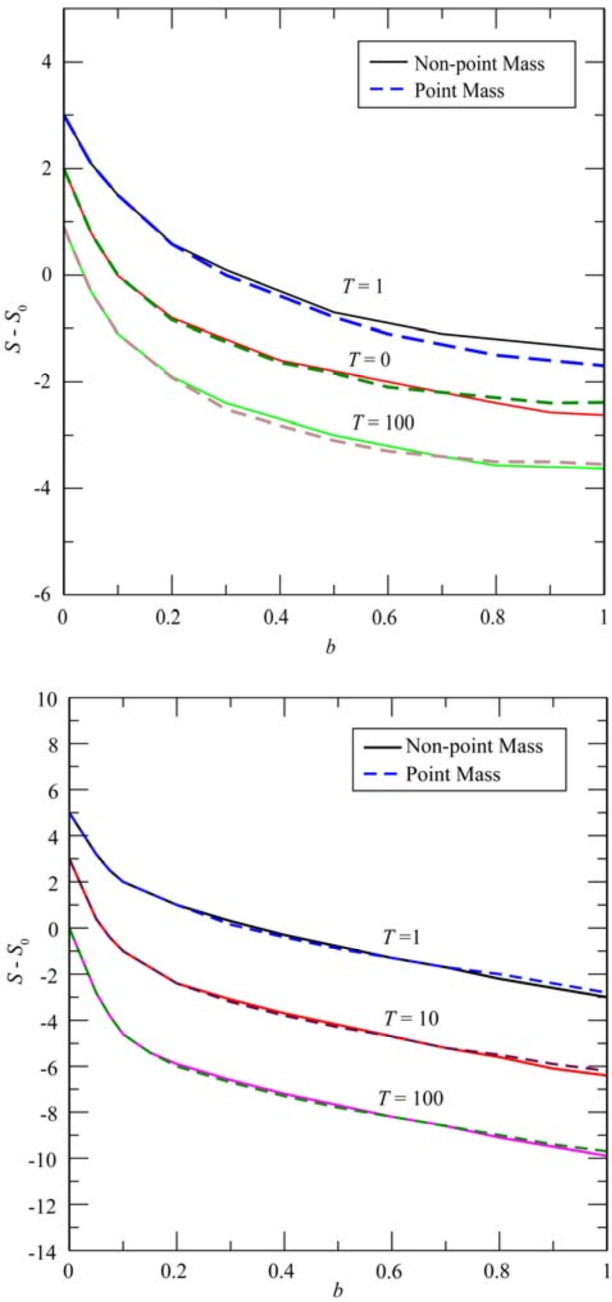

Figure 1. (Color online) Comparison of isothermal entropy changes for non-point and point mass particles (galaxies) for an infinite gravitating system as a function of average relative temperature T and the parameter b. For non-point mass ε = 0.03 and R = 0.06 (left panel), ε = 0.04 and R = 0.04 (right panel).

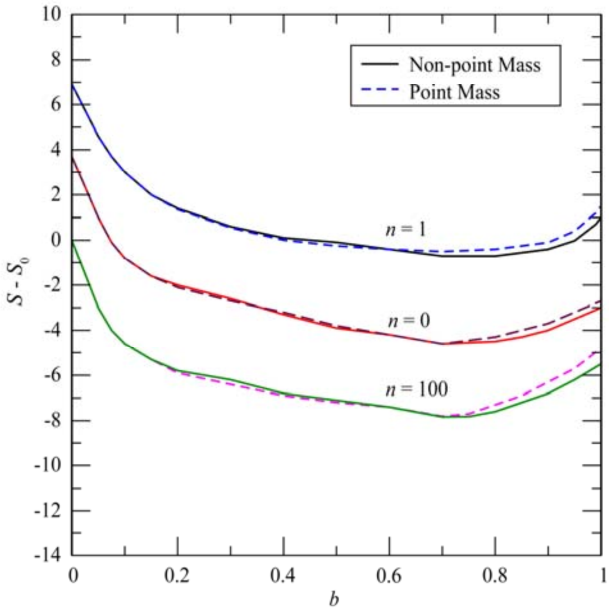

Figure 2. (Color online) Comparison of equi-density entropy changes for non-point and point mass particles (galaxies) for an infinite gravitating system as a function of average relative density n and the parameter b. For non-point mass ε = 0.03 and R = 0.04.

R (cell size) corresponding to different values of softening parameter ε. We study the variations of entropy changes $S - S_0$ with the changing parameter b for different values of n and T. Some graphical variations for $S - S_0$ with b for different values of n = 0, 1, 100 and average temperature T = 1, 10 and 100 and by fixing value of cell size R = 0.04 and 0.06 are shown. The graphical analysis can be repeated for different values of R and by fixing values of ε for different sets like 0.04 and 0.05. From both the figures shown in 1 and 2, the dashed line represents variation for point mass particles and the solid line represents variation for extended (non-point mass) particles (galaxies) clustering together. It has been observed that the nature of the variation remains more or less same except with some minor difference.

4. RESULTS

The formula for entropy calculated in this paper has provided a convenient way to study the entropy changes in gravitational galaxy clusters in an expanding universe. Gravity changes things that we have witnessed in this research. Clustering of galaxies in an expanding universe, which is like that of a self gravitating gas increases the gases volume which increases the entropy, but it also increases the potential energy and thus decreases the kinetic energy as particles must work against the attractive gravitational field. So we expect expanding gases to cool down, and therefore there is a probability that the entropy has to decrease which gets confirmed from our theoretical calculations as shown in **Figures 1** and **2**. Entropy has remained an important contributor to our understanding in cosmology. Everything from gravitational clustering to supernova are contributors to entropy budget of the universe. A new calculation and study of entropy results given by **Eqs.10**, **12**, **16** and **17** shows that the entropy of the universe decreases first with the clustering rate of the particles and then gradually increases as the system attains viral equilibrium. The gravitational entropy in this paper furthermore suggests that the universe is different than scientists had thought.

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