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30. MACH'S PRINCIPLE IN ELECTRODYNAMICS AND INERTIA

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1. Introduction

The concept of inertia, which originated with Galileo, found a mathematical expression in Newton's laws of motion. According to the second law of motion, the force acting on a particle is proportional to the acceleration of the particle. The constant of proportionality measures the inertia of the particle and is called its inertial mass. Thus, if $\mathbf{P}$ is the force, $\mathbf{f}$ the acceleration, and m the inertial mass, we have

$$\mathbf{P} = m\mathbf{f}. \tag{1}$$

However, when formulating this law, Newton faced a fundamental difficulty : How to fix the reference frame relative to which $\mathbf{f}$ is measured? Clearly, Eq.(1) cannot hold in all reference frames. If it is valid in frame Σ, then in another frame Σ', with an acceleration $\mathbf{a}$ relative to Σ, the law of motion becomes

$$\mathbf{P}' \equiv \mathbf{P} - m\mathbf{a} = m\mathbf{f}'. \tag{2}$$

where $\mathbf{f}'$ is the acceleration of the particle with respect to Σ', and $\mathbf{P}'$ is the force measured in it. Thus the force is modified from $\mathbf{P}$ to $\mathbf{P} - m\mathbf{a}$. The extra term $-m\mathbf{a}$ that has to be added to $\mathbf{P}$ depends on the mass of the particle and hence is called the inertial force. The frames in which Eq.(1) holds without the necessity of such a modification are called inertial frames. All these frames (including Σ) are unaccelerated relative to one another.

What distinguishes inertial frames from noninertial ones? A priori Newton could see no physical reason to make such a distinction. That such a distinction exists in nature can be seen or demonstrated in numerous ways. Newton has discussed the so-called bucket experiment. The rotation of a

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bucket is a relative term. If one observer sees the bucket rotating, another (e.g., one sitting on the bucket) can claim that it is nonrotating. In Newton's experiment the distinction can be made absolute. If the bucket is suspended by a string, the string is given a twist and then the bucket let go, it spins as the twisted string unwinds. If the bucket contains some water, its surface will become curved and dip toward the centre. This absolute effect demonstrates the presence of inertial forces and distinguishes between inertial and noninertial frames.

Why such a distinction should exist, Newton could not understand. However he used this distinction to postulate so-called absolute space. This is, in effect, a specific inertial frame in which Eq.(1) holds. All accelerations relative to this are detectable by the inertial forces.

Mach, in the last century, indicated that a clue to the identification of absolute space is indeed provided by the universe: a clue that was not available to Newton. Astronomical observations have shown that the frame of reference of the local observer, in which the distant parts of the universe appear to be nonrotating, is an inertial frame. This is a remarkable result! An observer can measure the rotation of the Earth relative to Newton's absolute space by observing the motion of a Foucault pendulum that is driven by the inertial forces. Or, he can measure the rotation relative to distant galaxies. In either case he gets the same answer! In other words, cosmology appears to provide a handle on the question of why certain frames enjoy a special status.

This was the point made by Mach in his critique of Newtonian mechanics. As one who believed in formulating all scientific deductions on directly observable quantities (and not on abstract theories), Mach strongly attacked the concept of absolute space: and sought to replace it by a background space of the remote parts of the universe. He went even further. The Newtonian concept of inertia and its measure in terms of mass were to him unsatisfactory. If mass is the quantity of matter in a body how does one set about measuring it? To Mach, mass and inertia were not the intrinsic properties of the body but the consequences of the existence of the body in a universe containing other matter. To measure mass one has to use Eq.(1): measure the force and divide it by the acceleration produced. But Eq.(1) itself depends on the use of absolute space, which has now been identified with the background space of distant matter. So, according to Mach's reasoning, mass is somehow determined by the distant matter.

Although initially impressed by Mach's ideas, Einstein later abandoned his efforts to incorporate them in his theories, for, Mach's principle and Newtonian gravitation were examples of an action-at-a-distance theories. Electromagnetism began as an action-at-a-distance theory but proved inadequate to describe all the observed phenomena including radiation. Einstein

was impressed by Maxwell's field theory, not only because it successfully described electromagnetism but also because it had internal beauty. The theory already contained the Lorentz invariance required by special relativity. Action at a distance, on the other hand, was instantaneous and hence inconsistent with special relativity. Although, unlike electromagnetism, there was no experimental data against Newtonian gravitation (with the esoteric exception of the perihelion precession of Mercury!) its logical inconsistency was already apparent. And Mach's principle seemed to imply a similar idea of action at a distance.

Nevertheless, it is worth asking now, whether the Machian ideas are really irreconcilable with a theory of gravitation. Cannot the Newtonian action at a distance be reformulated to give expression to Mach's ideas without offending relativity? We first turn our attention to the action at a distance approach and see how this approach can be made to work in electromagnetism. We then formulate a theory of gravity on similar lines, which also incorporates Mach's ideas, and demonstrate how this theory approaches general relativity in the case of many particle systems.

2. The Absorber Theory of Radiation

In a letter to Weber on March 19, 1845, Gauss wrote:

I would doubtless have published my researches long since were it not that at the time I gave them up I had failed to find what I regarded as the keystone. Nil actum reputans si quid superesset agendum, namely, the derivation of the additional forces - to be added to the interaction of electrical charges at rest, when they are both in motion - from an action which is propagated not instantaneously but in time as is the case with light.

Gauss's attempts came some three decades before the Maxwellian field theory and six decades before special relativity. The success of these two theories shifted the emphasis from action at a distance to fields and it was not until well into the present century that the problem posed by Gauss was solved.

A beginning was made by Schwarzschild [1], Tetrode [2], and Fokker [3, 4, 5], who independently formulated the concept of delayed action at a distance. The action principle as formulated by Fokker may be written in the following form:

$$J = -\sum_a \int m_a da - \sum_{a<b}\sum \int\int e_a e_b \delta(s^2_{1B})\eta_{ik} da^i db^k. \qquad (3)$$

In the above expression the charged particles are labeled, $a, b, \ldots$ with e_a and m_a the charge and mass of particle a. The worldline of a is given by the coordinate functions $a^i(a)$ of the proper time a. The spacetime is

Minkowskian, so that

$$da^2 = \eta_{ik} da^i da^k, \tag{4}$$

with η_{ik}= diag $(-1,-1,-1,1)$. The first term of J therefore describes the inertial term. The second term describes the electromagnetic interaction between the worldlines of a typical pair of particles a, b. The delta function shows that the interaction is effective only when s^2_{AB} , the invariant square of distance between typical world points A, B on the worldlines of a and b, vanishes. This implies delayed action: $s^2_{AB}= 0$ means that world points A and B are connected by a light ray.

Although this formulation met the requirement of relativistic invariance, it gave rise to other difficulties. The major difficulty is as follows. For a typical point A on the worldline of a there are two points B_+ and B_- on the world line of b for which $s^2_{AB} = 0$. The effect of A is felt at B_+ (at a later time) and at B_- (at an earlier time). Similarly, since the action principle guarantees the equality of action and reaction, the reaction from B_- and B_- is felt at A. Thus there are influences propagating with the speed of light, not only into the future but also into the past. This led to a conflict with the principle of causality, which seems to hold in everyday life. The other difficulties were of a less serious nature although not ignorable. For example, there was no "self-action" ($a = b$ is avoided in the double sum) and so there did not appear to be any obvious way of accounting for radiation damping.

These difficulties were removed by Wheeler and Feynman [6] by bringing into the discussion the important role of the absorber. In our above example, the reactions from B arrive at A instantaneously, whatever be the spatial separation of a and b. So it becomes necessary to take into account the reaction from the entire universe to A. Although the remote particles are expected to contribute less, their total number is large enough to make the calculation nontrivial. The essence of the argument given by Wheeler and Feynman is described below.

To begin with, define the 4-potential at X due to particle b by

$$A_i^{(b)}(X) = e_b \int \delta(s^2_{XB}) \eta_{ik} db^k, \tag{5}$$

and the corresponding direct-particle field by

$$F_{ik}^{(b)} = A_{k;i}^{(b)} - A_{i;k}^{(b)}. \tag{6}$$

A direct particle field is not an ordinary field, because it does not have any independent degrees of freedom. The 4-potential *identically* satisfies the relations

$$A^{(b)k}_{\ ,k} \equiv 0, \quad \Box A^{(b)k} \equiv \eta^{mn} A^{(b)k}_{\ ,mn} = 4\pi j^{(b)k}. \tag{7}$$

where $j^{(b)k}(X)$ is the current density vector of the particle b at a typical point X, defined in the usual way. Thus although Eq.(7) resembles the Maxwell wave equation (and the gauge condition) it represents identities.

The equation of motion of a typical charge a is obtained by varying its worldline and requiring $\delta J = 0$. We get the analogue of the Lorentz force equation in which the charge a is acted on by all other charges in the universe.

We now turn to the difficulty introduced by the time symmetry of this formulation. Instead of being the retarded solution of Eq.(7), Eq.(5) is the time-symmetric half-advanced and half-retarded solution. The same applies to the direct-particle fields. Suppressing the indices i, k, we may write

$$F^{(b)} = \frac{1}{2}[F^{(b)}_{\text{ret}} + F^{(b)}_{\text{adv}}]. \tag{8}$$

This field is present in the past as well as the future light cone of B.

Wheeler and Feynman argued in the following way. If we move the charge b, it generates a disturbance that affects all other charges in the universe. Their reaction arrives back instantaneously. Wheeler and Feynman showed how to calculate such a reaction in a universe of static Minkowski type with a uniform distribution of electric charges. They showed that the reaction to the motion of charge b can be calculated in a consistent fashion and comes out to be

$$R^{(b)} = \frac{1}{2}[F^{(b)}_{\text{ret}} - F^{(b)}_{\text{adv}}]. \tag{9}$$

Thus a test particle in the neighbourhood of charge b experiences a net total "field"

$$F^{(b)}_{\text{tot}} = F^{(b)} + R^{(b)} = F^{(b)}_{\text{ret}}. \tag{10}$$

This is the pure retarded field observed in real life! The self-consistency of the argument follows from the fact that the reaction $R^{(b)}$ has been calculated by adding the $\frac{1}{2}F^{(a)}$ fields of all particles $a \neq b$ that have been excited by this total field $F^{(b)}$. Thus only the future light cone of B comes into play. The reaction from the future cancels the advanced component of $F^{(b)}$ and doubles its retarded component.

Also according to the Lorentz force equation, $R^{(b)}$ is the force arising from all *other* particles in the universe experienced by the particle B. This is nothing but the radiative reaction to the motion of b as obtained earlier by Dirac [7] on empirical grounds. Earlier, Dirac's rule was difficult to understand within the context of the field theory, although it was known to give the right answer. Here the Dirac formula is understood as the consequence of a response of the universe to the local motion of the charge. Thus the theory not only gets round the problem of causality but it also accounts for the radiation damping formula.

Physically, what happens is the following. To the motion of b the future half of the universe acts as an absorber. It "absorbs" all the "energy" radiated by b, and in this process sends the reaction $R^{(b)}$, which does the trick! For this reason Wheeler and Feynman called this theory the *absorber theory of radiation*. The presence of the absorber is essential for the calculation to work. For example, it will not work in an empty universe surrounding the electric charge.

In the above self-consistent derivation there was still one defect: it was not unique. Another self-consistent picture was possible in which the nett field near every particle was the pure advanced field and the radiative reaction was of opposite sign to that of Eq.(9). The two solutions are compared thus. In (a) we have the retarded solution while in (b) we have the advanced solution. In (a) absorption in the future light cone is responsible while in (b) it is the absorption in the past that plays the crucial role. The important role of the absorbers is that they convert the time-symmetric situation of an isolated charge, to a time-asymmetric one of (a) or (b) type. It is, however, not possible to distinguish between (a) and (b) without reference to some other independent type of time asymmetry.

Wheeler and Feynman realized this and linked the choice of (a) to thermodynamics. Given the usual thermodynamic time asymmetry, they argued that the situation (b) would be highly unlikely (under the probability arguments of statistical mechanics) and that the usual asymmetry of initial conditions will favour (a) to (b).

It was, however, pointed out by Hogarth [8] that it is not necessary to bring thermodynamics into the picture at all. If one takes account of the fact that the universe is expanding, its past and future are naturally different. The reaction from the absorbing particles in the future light cone (designated by Hogarth collectively as the *future absorber*) does not automatically come out equal and opposite to the reaction from the past absorber. Thus the two pictures (a) and (b) do not always follow in an expanding universe. Hogarth found that for (a) to hold but not (b), the future absorber must be perfect and and past absorber imperfect; and vice versa for (b) to hold but not (a).

An absorber is perfect if it entirely absorbs the radiation emitted by a typical charge. In the static universe discussed by Wheeler and Feynman both the past and future absorbers are perfect; and this leads to the ambiguity mentioned earlier. However, Hogarth found that the ambiguity is resolved if the cosmological time asymmetry is taken into account. He found, for example, that in most big-bang models which expand forever, (b) is valid and not (a). In the steady-state model (Bondi and Gold [9]; Hoyle [10]), (a) is valid and not (b). In the big-bang models that expand and contract both the absorbers are perfect and the outcome is ambiguous.

Though interesting, Hogarth's work was incomplete in two aspects. First, he had not shown how to generalize the Fokker action to curved spacetimes needed to describe the expanding world models. Second, he had used collisional damping to decide upon the nature of absorbers, past and present; and this brought in thermodynamics by the back door!

Later Hoyle and Narlikar [11] completed the work by first rewriting the Fokker action Eq.(3) in curved space as is necessary for any cosmological discussion. They also deduced conclusions similar to Hogarth's but by using the radiative damping for producing absorption. This kept the asymmetry entirely within electrodynamics and cosmology.

Finally they extended the entire picture to quantum theory. Thus it is possible to discuss the entire range of phenomena of quantum electrodynamics without recourse to field theory (Hoyle and Narlikar [12, 13]). This therefore removes any possible objection to the concept of action at a distance in so far as it is applicable to electrodynamics.

The crucial role played in the whole calculation is that of the response of the universe. In the classical calculation the steady-state universe generates the "correct" response so that the local electric charges interact through retarded signals. The response from the big-bang models is of the wrong type. We are thus able to distinguish between the different cosmological models and decide on their validity or otherwise on the basis of the Wheeler-Feynman theory. We also see why charges interact through retarded signals: they do so because of the response of the universe. In the Maxwell field theory the choice of retarded solutions of Maxwell's equations is by an arbitrary fiat.

In the quantum calculation also it can be shown that the asymmetric phenomenon with respect to time, like the spontaneous downward transition of an atomic electron, is caused by the response of the universe. By contrast, the quantization of the Maxwell electromagnetic field ascribes these asymmetries to the so-called vacuum and to the rather abstract rules of quantization.

In a recent paper Hoyle and Narlikar [14] have shown that with suitable cosmological boundary conditions, like the de Sitter horizon, there is a cut off on high frequencies that lead to divergent integrals in the standard field theory. Thus the electron self energy problem and the various radiative corrections of quantum electrodynamics can be handled without subtraction of one infinity from another. In this sense the direct particle theory fares better than the field theory.

The direct-particle approach therefore achieves for electrodynamics what Mach sought to achieve for inertia. By bringing in the response of the universe to a local experiment in electrodynamics we have essentially incorporated Mach's principle into electromagnetic theory. Given the correct

response of the universe, we can almost decouple our local system from it, although, strictly speaking, the theory would not be possible without the universe.

Can the same prescription be applied to inertia and gravitation? We discuss this problem in the following section.

3. Inertia as a Direct-Particle Field

We now return to the problem of achieving a "reconciliation" between general relativity and Mach's principle. To this end we shall look for a theory with the following properties:

(a) It has Mach's principle built into one of its postulates.

(b) It is conformally invariant.

(c) It does not have the conceptual difficulties associated with the case of a single particle in an otherwise empty universe.

(d) For a universe containing many particles the theory reduces to general relativity for most physical situations.

Some discussion is needed as to why the theory should be conformally invariant. The reasons are twofold. First, when we take note of the local Lorentz invariance of special relativity, the natural units to use are those in which the fundamental velocity $c=1$. The quantum theory, with which our theory should be consistent throws up another fundamental constant, the Planck constant related to the uncertainty principle. Thus it is natural to use units in which $\hbar=1$. This makes the classical action J, for example, dimensionless and the natural unit of mass the Planck mass which we shall quantify by $\sqrt{3c\hbar/4\pi G}$.

All masses are then expressible as numbers in units of this mass. With our choice of units, only one independent dimension out of the three length, mass and time, remains. Taking it as the dimension of mass, length goes as reciprocal of mass.

However, in a Machian theory, we expect the particle masses to be functions of space and time and as such not necessarily constant. Therefore, the standards of lengths and time intervals may also vary from one point to another. We therefore need laws of physics which are invariant with respect to this variation. Conformal invariance guarantees this.

Our second reason is based on the nature of action-at-a-distance. Given that, as in electrodynamics, the interaction propagates principally along null rays, we need an invariance that preserves the global structure of light cones. This again is guaranteed by the conformal invariance. Just as Lorentz invariance identifies the light cone as an invariant structure *locally*, so does conformal invariance identify it *globally*.

A theory following these guidelines was developed by Hoyle and Narlikar [15, 16] and its broad features are described next. We begin by a second look at the Fokker action for electrodynamics, this time rewritten in a curved Riemannian space-time:

$$J = -\sum_a \int m_a da - \sum_{a<b}\sum 4\pi e_a e_b \int\int \bar{G}_{i_A k_B} da^{i_A} db^{k_B}. \tag{11}$$

Here, in going from Eq.(3) to Eq.(11) the first term of J needs a trivial modification: da is now computed with a Riemannian metric. The modification of the second term of J requires considerable thought. The $\delta(s_{AB}^2)\eta_{ik}$ is now replaced by $\bar{G}_{i_A k_B}$, a bivector propagator between A and B. It is the symmetric Green's function for the wave equation

$$\Box \bar{G}_{ik_B} + R_i^l \bar{G}_{lk_B} = [-\bar{g}(X,B)]^{-1/2} \bar{g}_{ik_B} \delta_4(X,B). \tag{12}$$

Here $\bar{G}_{ik_B}$ behaves as a vector at X and B, respectively, with the indices i and k_B (the subscript X on i is suppressed for the convenience of writing). $\bar{g}_{ik_B}$ is the parallel propagator between X and B [see Synge [17] for details] and $\bar{g}(X,B)$ its determinant. In the limit $g_{ik} \to \eta_{ik}$, $\bar{G}_{i_A k_B} \to \delta(s_{AB}^2)\eta_{ik}$. The detailed structure of this propagator has been studied by DeWitt and Brehme [18].

The electromagnetic part of J is conformally invariant but the mechanical part (the first term) is not. We now compare Eq.(11) with the action for field theory of Maxwell and for general relativity. This action, denoted by $J^{(F)}$ is given by

$$J^{(F)} = \frac{1}{16\pi G}\int R(-g)^{1/2} d^4x - \sum_a \int m_a da - \frac{1}{16\pi}\int F^{lm}F_{lm}(-g)^{1/2}d^4x$$

$$- \sum_a \int A_i da^i. \tag{13}$$

The third and fourth term of $J^{(F)}$ represent, respectively, the free-field term and the field-particle interaction term.

In the direct-particle theory the second term of J replaces these two terms of the field theory. The fields as such lose their independent status and are replaced by propagators connecting particle world lines. What can we do about the first two terms of Eq.(13)? The second term already exists in Eq.(11) and it is tempting to simply insert the first term into Eq.(11) as representing gravity.

This procedure, however, is contrary to the spirit of the direct-particle picture. The first term of Eq.(13), although containing geometrical information, has also the character of a field. Hence it is out of place. We have

already commented on the non-Machian character of the second term of Eq.(13). For these reasons the approach suggested above is not desirable.

The clue to the correct procedure that needs to be adopted is provided by a comparison of the last term of Eq.(13) with the second term of Eq.(11). If in the former we replace the potential A_i by a sum over the direct-particle potentials defined by a relation analogous to Eq.(5) for a curved space, we shall recover something that looks like the latter! In the same way we now replace the masses m_a by direct-particle fields defined in the following manner:

$$m^{(b)}(X) = \int \lambda_b G(X, B) db. \quad \lambda_b = \text{a coupling constant.} \tag{14}$$

$$m_a(A) = \lambda_a \sum_{b \neq a} m^{(b)}(A), \quad \lambda_a = \text{a coupling constant.} \tag{15}$$

The propagator $G(X, B)$ has to be a biscalar since masses are scalars and we wish to preserve a symmetry between X and B. The action Eq.(11) is now changed to

$$J = -\sum_{a<b}\sum \int\int \lambda_a \lambda_b G(A, B) da\, db - \sum_{a<b}\sum 4\pi e_a e_b \int\int \bar{G}_{i_A k_B} da^{i_A} db^{k_B}. \tag{16}$$

What should be the exact form of $G(A, B)$? Taking a clue from electromagnetism, we expect it to be a symmetric Green's function of a scalar wave equation. However, we also want the equation to be conformally invariant. These two requirements fix the form of the scalar propagator uniquely to within a multiplicative factor. We shall take $G(A, B)$ to satisfy the scalar wave equation

$$\Box G(X, B) + \frac{1}{6} R(X) G(X, B) = [-\bar{g}(X, B)]^{-1/2} \delta_4(X, B). \tag{17}$$

The wave operator is uniquely fixed by the requirement of conformal invariance.

Turning from these purely formal aspects to those of interpretation we note that Eq.(14) and Eq.(15) are essentially Machian ideas on inertia expressed mathematically. The mass of a particle a at its world point A is the sum of the contributions of all other particles in the universe. Thus requirement (a) has been met. Requirement (c) is also met, because for a single particle in an otherwise empty universe there is no action! The minimum number of particles required to define J is two. Thus for each of the two particles the other provides the "background" in the Machian sense. The requirement of conformal invariance is also met by our choice of the propagator. It therefore remains to examine requirement (d).

So far we have concentrated on inertia and ignored gravity. The action Eq.(16) does not contain the gravitational term $(1/16\pi G) \int R(-g)^{1/2} d^4x$ explicitly. Yet, as we shall see in the following section, the theory is fully capable of describing gravitational phenomena.

4. Conformal Gravity

Returning to the action Eq.(13) we note that when we try to derive the Einstein field equations by the Hilbert action principle, we get the Einstein tensor from the first term. This term does not exist any more in the direct particle action Eq.(16). Shall we get any gravitational term at all from Eq.(16) if we sought to perform the metric variation $g_{ik} \to g_{ik} + \delta g_{ik}$? A look at the electromagnetic part of Eq.(13) does not inspire confidence that the answer to this question should be in the affirmative. There it is the third rather than the fourth term that contributes the energy tensor of electrodynamics, and it is the fourth term that was used in going over to Eq.(16). Nevertheless, a closer examination shows that the terms in Eq.(13) do give nontrivial answers when the metric variation is performed.

The reason for this is understood as follows. Consider the electromagnetic propagator $\bar{G}_{i_A k_B}$ connecting A and B respectively, on the world lines of a and b. Suppose we perform a variation in the space-time metric of a compact region Ω. Since the propagator is a global property of space-time structure, it will change because of this change in structure of Ω. The change in the propagator is therefore expressible, in a first order calculation, as a functional of δg_{ik} over the volume Ω.

In the electromagnetic case the answer may be expressed in the following form

$$-\delta \sum_{a<b}\sum 4\pi e_a e_b \int\int \bar{G}_{i_A k_B} da^{i_A} db^{k_B} = -\frac{1}{2} \int T^{ik} \delta g_{ik} (-g)^{1/2} d^4x, \tag{18}$$

where

$$T^{ik} = \frac{1}{8\pi} \sum_{a<b}\sum \left[\frac{1}{2} g^{ik} F^{(a)mn}_{\text{ret}} F^{(b)}_{mn\ \text{adv}} - F^{(a)i}_{l\ \text{ret}} F^{(b)kl}_{\text{adv}} - F^{(b)i}_{l\ \text{ret}} F^{(a)kl}_{\text{adv}}\right]. \tag{19}$$

The details of this derivation are given by Narlikar [19].

It is interesting to note in passing that this derivation resolves an ambiguity about the energy tensors of direct-particle electrodynamics. Wheeler and Feynman [20] had discussed two tensors for this theory. Of these one was the canonical tensor given above by Eq.(19) and the other was the Frenkel tensor whose form differs from that given in Eq.(19) in having all the direct particle fields $F^{(a)lm}$ as the symmetric half-advanced-plus-half-retarded fields. Wheeler and Feynman had concluded:

From the standpoint of pure electrodynamics it is not possible to choose between the two tensors. The difference is of course significant for the general theory of relativity, where energy has associated with it a gravitational mass. So far we have not attempted to discriminate between the two possibilities by way of this higher standard.

As mentioned above, the usual prescription of metric variation uniquely yields the canonical tensor. The fact that we could get a nontrivial answer to the variational problem and that this resolves a long-standing ambiguity, reinforces our belief that we are proceeding along the correct path toward a theory of gravitation.

We now consider the variation of the first term of Eq.(16) as $g_{ik} \to g_{ik} + \delta g_{ik}$. We shall ignore the second term and concentrate on gravitation alone. Also for simplicity we begin by putting $\lambda_a = 1$ for all a. This does not alter the essential features of the theory.

The method is similar to that adopted for electromagnetism. We compute the change in the propagator $G(A, B)$ as the geometry changes in any compact region Ω. The details of this somewhat lengthy calculation are given elsewhere [see Hoyle and Narlikar [21]]. We simply quote the result. The field equations turn out to be

$$\frac{1}{2}\phi(R_{ik} - \frac{1}{2}g_{ik}R) = -T_{ik} + \frac{1}{6}[g_{ik}\Box\phi - \phi_{;ik}] + \frac{1}{2}[m_i^{\rm ret}m_k^{\rm adv} + m_k^{\rm ret}m_i^{adv}$$

$$-g_{ik}m^{l\ {\rm ret}}m_l^{\rm adv}], \quad \text{where} \tag{20}$$

$$m(X) = \sum_a \int G(X, A)da. \quad m_i = \partial m/\partial x^i, \tag{21}$$

$$\phi(X) = m^{\rm adv}(X)m^{\rm ret}(X). \tag{22}$$

and $m^{\rm ret}$ and $m^{\rm adv}$ denote twice the retarded and advanced parts of $m(X)$, respectively. The energy tensor T_{ik} is the familiar energy tensor for a system of particles $a, b, \ldots$ with masses as defined by the Machian preciption, Eq.(14) and Eq.(15). Note that the masses are time symmetric. The function $m(X)$ satisfies the conformally invariant wave equation

$$\Box m + \frac{1}{6}Rm = N. \quad \text{where} \tag{23}$$

$$N(X) = \sum_a \int \delta_4(X, A)[-\bar{g}(X, A)]^{-1/2}da \tag{24}$$

is the invariant particle number density.

There are 10 equations in Eq.(20) and one equation Eq.(23) for the 11 unknowns g_{ik} and m. However, the divergence and trace of Eq.(20)

identically vanish, showing that there are in fact five fewer independent equations. This is hardly surprising since four of these five are due to the general coordinate invariance (as in general relativity) while the fifth identity (the vanishing of trace) is due to conformal invariance. It is easy to verify that if $[g_{ik}, m]$ is a solution of these equations then so is $[\zeta^2 g_{ik}, \zeta^{-1}m]$ for an arbitrary well-behaved (i.e., of type C^2) nonvanishing finite function ζ. This arbitrary function is nothing but the expression of the arbitrariness of mass-dependent units discussed in Section 3.

Suppose now that it is possible to choose ζ such that

$$m^{\rm ret}\zeta^{-1} = m_0 = \text{constant}. \tag{25}$$

Suppose also that the response of the universe is such as to cancel all advanced components and double the retarded ones so that the effective mass function is $m^{\rm ret}$. Then the field equations are simplified to

$$R_{ik} - \frac{1}{2}g_{ik}R = -\kappa T_{ik}, \quad \kappa \equiv \frac{8\pi G}{c^4} = 6/m_0^2. \tag{26}$$

We shall later identify m_0 with the mass of the Planck particle. However, as seen above, we have arrived at the familiar equations of general relativity! The conformal frame for which Eqs.(25) and (26) hold will be called the *Einstein frame*. We have thus completed the remaining part of the programme outlined at the beginning of Section 3.

The following points are worth emphasizing in the above derivation of Einstein's equations, which is so radically different from the standard ones (used for example, by Einstein in 1915 and by Hilbert later the same year):

(I) The approach to Einstein's equations is via the wider framework of a conformally invariant gravitation theory. Only in the limit of many particles in a suitably responding universe do we arrive at Einstein's equations. In the other limit of zero or no particles there is no theory! Thus it brings out the reason why the Machian paradox of one particle in an empty universe is not valid in the context of Einstein's equations. This reason does not emerge in the standard derivations of Einstein's equations.

(II) It is significant that the coupling constant κ is positive in this approach. This conclusion is unaffected by the change of sign of the coupling constants λ_a, λ_b, etc. (taken here as unity); nor it is affected by the choice of signature (i.e., $----$ instead of $+++-$) of the spacetime metric. The choice of the conformally invariant scalar propagator leads to the coupling constant being positive, i.e. to gravity being "attractive". In the standard deviation the coupling constant is fixed (in sign as well as magnitude) by a comparison with Newtonian gravity.

(III) A considerable discussion has gone on regarding the admissibility of the so-called λ-term in Einstein's equations. This is because this term

could be accommodated in Einstein's heuristic derivation or in Hilbert's action principle. It is worth emphasizing that the direct-particle approach to gravity given so far does not permit the λ-term. As we shall see later, this term does arise in a Machian way in the direct particle theory, *provided* we allow the wave equation (23) for inertia to be nonlinear. The present cosmological observations generally seem to require the cosmological constant (Bagla, et al, [22]).

(IV) The condition Eq.(25) that leads to Einstein's equations needs to be re-examined carefully under two special circumstances. Near a typical particle a, we expect the mass function $m^{(a)}(X)$ to "blow up" so that $m(x) \to \infty$, as $X \to A$, on the worldline of a. In order to make $\zeta^{-1}m(X)$ finite at A, we therefore require $\zeta \to \infty$, as $X \to A$. However, we have already ruled out such conformal functions by restricting ζ to finite values. Thus the transition to Einstein's equations is not valid as we tend to any typical source particle. The nature of the equations and their solutions near a particle in this theory have been discussed by Hoyle and Narlikar [16] and by Islam [23].

5. Cosmology and the Creation of matter

In recent years, this theory has been further generalized and applied to cosmology to include the cosmological constant as well as explicit description of creation of matter (see Hoyle *et al.* [24]).

Taking the cosmological constant into account first, we may ask whether Eq.(23) is the most general conformally invariant wave equation satisfied by a scalar function $m(X)$. The answer is *No*. The most general such equation is

$$\Box m + 1/6Rm + \Lambda m^3 = N, \tag{27}$$

where Λ is a constant. This, of course, makes the scalar Machian interaction non-linear and more difficult to handle. However, it very naturally leads to the cosmological constant of the right magnitude at the present epoch. For, if we assume that for a single particle, the value of this constant is unity, then in the sum Eq.(21) leading to $m(X)$, because of the presence of a large number n of particles within the cosmological horizon, the cube term is less effective by the factor n^{-2}. Thus, the factor Λ in Eq.(27) is of this order. With the identification of m_0, with the only possible fundamental constant, viz. the Planck mass

$$m_0 = \sqrt{\frac{3\hbar c}{4\pi G}}, \tag{28}$$

one can write the familiar cosmological constant in the Einstein equations as

$$\lambda = -3\Lambda m_0^2. \tag{29}$$

With around $2\ X\ 10^{60}$ Planck particles in the horizon, we get the value of $\lambda \sim -2\ X\ 10^{-56}\text{cm}^{-2}$. It is of the right order, but *negative*. However, it leads to interesting and physically relevant cosmological models.

Let us consider matter creation next. The standard relativity theory starts with the assumption that matter can neither be created nor destroyed, that is, the worldlines of particles are endless. (Even where matter is converted into radiation the above assumption holds in the sense that the worldline of a matter particle is converted to a worldline of particle of radiation.) However, in the big bang singularity, all worldlines are incomplete, thus leading to a contradiction with the basic assumption. Indeed the presence of the singularity signifies the break-down of the basic rules such as the action principle from which the equations of general relativity are obtained.

To account for the creation of matter in a nonsingular fashion, we introduce the additional input into the theory that the particle worldlines are with ends. The same action as before then describes this theory but the endpoints generate extra terms in the field equations.

For details of this work we refer the reader to Hoyle *et al.* [24]. The field equations in the "constant mass" conformal frame then take the form:

$$R_{ik} - \frac{1}{2}g_{ik}R + \lambda g_{ik} = -\kappa[T_{ik} - \frac{2}{3}(c_i c_k - \frac{1}{4}g_{ik}c^l c_l)]. \tag{30}$$

The scalar c-field arises from the contribution to inertia from ends of particle worldlines. These are the contributions of the Planck particles created which last a very short time scale$\sim 10^{-43}$ second.

Sachs et al [25] have solved these field equations and obtained a series of cosmological models which are a combination of two kinds: (i) models with creation of matter and (ii) models without creation of matter. The generic solution is known as the *Quasi Steady State Cosmology*. It is a nonsingular model which has a de Sitter type expansion with short term oscillations superposed on it. The former represents the creative and the latter the noncreative mode. The QSSC is being proposed as an alternative to the standard hot big bang cosmology (Narlikar [26]).

6. Conclusions

To summarize, the ideas which go under the name Mach's Principle are capable of wider applications than thought earlier by Ernst Mach. One can use the Machian concept in electrodynamics where the response of the universe can play a key role in both classical and quantum electrodynamics. The action at a distance framework used here is consistent with special relativity as well as with causality. The formalism can be used to give an expression to inertia as a direct long range effect from the distant parts

of the universe. From inertia, one can arrive at a theory of gravity which is wider in its applications than general relativity. The theory can be extended to incorporate the cosmological constant and the concept of creation of matter without spacetime singularity. It leads to a viable cosmological model, known as the quasi-steady state cosmological model.

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31. THE EARLY HISTORY OF QUANTUM GRAVITY (1916-1940)

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It is with great affection that I dedicate this paper to Vishu, a dear friend and valued colleague for many years.

Most accounts of quantum gravity, insofar as they are concerned with its history at all, start with the post-World-War II period. Without attempting a full reconstruction of the pre-war history, I shall show that there was a lively discussion of some of the most important issues in the field, especially in the 1930s, which was cut off by the advent of the war and the untimely deaths of some of the main protagonists. When discussions of quantum gravity resumed after the war, they took place largely in ignorance of the fact that the main positions had been staked out earlier.

As so often the case in relativity, the story of quantum gravity begins with Einstein himself. Soon after the final formulation of general relativity, he pointed out the need for a quantum modification of the theory. In his first paper on gravitational radiation [1], Einstein argued that quantum effects must modify the general theory of relativity:

Due to the intra-atomic movement of electrons, atoms would have to radiate not only electromagnetic but also gravitational energy, if only in tiny amounts. As this is hardly true in nature, it appears that quantum theory would have to modify not only Maxwellian electrodynamics, but also the new theory of gravitation (p.209). Two years later, he reiterated this conclusion [2]:

As already emphasized in my previous paper, the final result of this argument, which demands a [gravitational] energy loss by a body due to its thermal agitation, must arouse doubts about the universal validity of the theory. It appears that a fully developed quantum theory must also bring about a modification of the theory of gravitation (p.164).

He soon began to speculate whether gravitation plays a role in the atomistic structure of matter [3]: