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Event horizon: Magnifying glass for Planck length physics

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An attempt is made to describe the 'thermodynamics' of semiclassical spacetime without specifying the detailed 'molecular structure' of the quantum spacetime, using the known properties of blackholes. I give detailed arguments, essentially based on the behaviour of quantum systems near the event horizon, which suggest that event horizon acts as a magnifying glass to probe Planck length physics even in those contexts in which the spacetime curvature is arbitrarily low. The quantum state describing a blackhole, in any microscopic description of spacetime, has to possess certain universal form of density of states which can be ascertained from general considerations. Since a blackhole can be formed from the collapse of any physical system with a low energy Hamiltonian H, it is suggested that when such a system collapses to form a blackhole, it should be described by a modified Hamiltonian of the form $H^2_{\rm mod} = A^2 \ln(1 + H^2/A^2)$ where $A^2 \propto E_P^2$. I also show that it is possible to construct several physical systems which have the blackhole density of states and hence will be indistinguishable from a blackhole as far as thermodynamic interactions are concerned. In particular, blackholes can be thought of as one-particle excitations of a class of *nonlocal* field theories with the thermodynamics of blackholes arising essentially from the asymptotic form of the dispersion relation satisfied by these excitations. These field theoretic models have correlation functions with a universal short distance behaviour, which translates into the generic behaviour of semiclassical blackholes. Several implications of this paradigm are discussed.

I. INTRODUCTION

It is generally believed that the spacetime continuum will give way to a more fundamental level of description at length scales smaller than Planck length $L_P \equiv (G\hbar/c^3)^{1/2}$, corresponding to energy scales larger than $E_P = L_P^{-1}$. Approaches to quantum gravity based on strings or Ashtekar variables [1,2] strengthens such a belief. If this is the case, then spacetime continuum — described by a solution to Einstein equations — is an approximate, coarse grained concept, similar to the continuum description of a fluid or gas. Einstein equations have a status similar to that of equations of fluid mechanics and are of limited validity.

Where exactly does the description based on Einstein's equations fail? It seems reasonable that the continuum description will fail whenever the length scale associated with the spacetime curvature is comparable to that of Planck length. While this may be a *sufficient* criterion, it need not be *necessary.* It may be possible to see the effects of the underlying theory in special circumstances, even if the curvature of the spacetime is arbitrarily small. I will argue in this paper that physics near event horizons can give glimpses of the structure of the underlying theory, even if the corresponding spacetime curvature is arbitrarily small, and show how this information can be utilized.

The key reason which prompts me towards this point of view is the following. If the spacetime has certain quantum mechanical micro-structure at Planck scales, and the continuum description based on Einstein's equations is a coarse-grained one, then the *necessary* criterion for the breakdown (or otherwise) of the continuum description should not be based on the approximate theory. An analogy might make this point of view clearer. In the study of a fluid system, one can obtain solutions to hydrodynamic equations describing the coarse-grained behaviour. In special circumstances, like in the case of shock waves, one can also obtain a *sufficient* condition for the breakdown of hydrodynamic description by studying these solutions. But if a very high energy beam of photons propagates through the fluid probing length scales comparable to those of constituent particles, then the smooth fluid description will necessarily breakdown around the region where the external influence interacts with the microscopic structure of the fluid. Similarly, if the modes of an external field can probe scales comparable to Planck length in some region of spacetime, then we cannot describe physics around that region using Einstein's equations. Normally, virtual modes of arbitrarily high energy of matter fields do interact strongly with the quantum micro-structure of spacetime; but this is of no consequence unless such a virtual process can manifest as a real one in some way. This is exactly what happens in spacetimes with an event horizon, like that of a blackhole. I shall elaborate on this theme in this paper.

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This paper is organized as follows: Section II gives a detailed conceptual foundation of this attempt and compares it with other approaches. Section III presents an argument indicating why the formulation of statistical mechanics of *any* system runs into trouble in spacetimes with horizon and suggests a possible transmutation of the hamiltonian of a system, if the system collapses to form a blackhole. The role of horizon in semiclassical gravity (ie., quantum fields interacting with classical gravity) is discussed next, in section IV. A simple derivation of Hawking evaporation is given, highlighting the role of infinite redshift surface. This analysis shows that event horizon is likely to play a vital role in the quantum description of spacetime itself. This role is discussed in section V, where it is shown that all of blackhole thermodynamics can arise from a particular form of density of states. In section VI, I discuss the consequences of such a universal density of states for the blackhole and present a simple field theoretic model designed in such a way that the quanta of this field have this density of states. In this approach, blackhole represents the one-particle state of a given energy of this system. The last section summarizes the paper. The key new results are in section VI and readers impatient with descriptions of points of view may to go directly to that section, though it is not a recommended procedure.

II. PROLOGUE

Since I want to attribute very special status to spacetimes with infinite redshift surfaces, I will begin by recalling certain well known features about them. To focus the ideas, I will discuss the case of a Schwarzschild blackhole and use the concepts of infinite redshift surface and event horizon interchangeably.

At the classical level, the horizon of a blackhole (which is the simplest infinite redshift surface) blocks out certain degrees of freedom from the outside observers. Dynamically it allows one to describe a blackhole by just three parameters irrespective of the initial configuration of matter which collapsed to form the blackhole. This idea has led Bekenstein [3] and later workers to suggest that "blackholes have entropy". Historically, an entropy was attributed to the blackhole before a temperature was assigned, essentially due to the role played by the event horizon.

Since then many workers have tried [4] to understand the origin of blackhole entropy and temperature but no clear consensus has emerged. The discussion usually centers around the the question of which degrees of freedom are contributing to blackhole entropy. Is the blackhole entropy a property of the matter fields which were outside the incipient blackhole ? Or, does it represent the degrees of freedom (both gravitational and matter)on the horizon or inside, hidden by the horizon?

The view that the entropy originates due to modes hidden by the horizon is more in conformity with the idea that the entropy arises from integrating over the degrees of freedom not relevant to the outside observer. On the other hand, the degrees of freedom of matter field outside the horizon are the ones which are of direct concern in any quantum field theory calculations carried out in Schwarzschild metric. The existence of event horizon or the specific nature of the metric enters only through boundary conditions imposed on the event horizon. Another way of stating this result is the following: The Feynman's Greens function for the quantum field contains all the information about the free fields which are outside the blackhole. Since it satisfies a local partial differential equation, it is independent of the detailed dynamics of the degrees of freedom hidden inside the horizon. As far as the solution to that equation is concerned, we only need to impose a boundary condition on the event horizon.

A comparison of the above two arguments suggest that, in either case it is the *surface* of the blackhole, viz.the event horizon, which will play a vital role. In the first line of thinking, this surface blocks out degrees of freedom and provides the justification for tracing them out. In the second approach, it is necessary to impose boundary conditions on the surface in order to understand the dynamics of the fields in the outside. Again, in the first approach, it is very clear that this surface must be an event horizon to play this role; otherwise it cannot block out the degrees of freedom. In the second approach, this is not so self-evident but arises from the nature of the conical singularity at the horizon and the structure of the corresponding Euclidian extension. There has also been attempts in literature to attribute the entropy to the properties of this surface itself [5].

In all the approaches outlined above, quantum gravitational effects do not play any role and one need not leave the comforts of the spacetime continuum as a backdrop for studying physics. (This is usually justified by arguing that the curvature at the event horizon for a stellar mass black hole is quite small and quantum gravity should not play a role; as I will argue in this paper, this is not sufficient justification.) A third point of view, which will be advocated here, is to treat the blackhole entropy as arising due to quantum structure of spacetime itself. In such an approach, one would like to think of classical spacetime as a very coarse level description (like that of a macroscopic gaseous system). The true Planckian level description of spacetime will involve certain degrees of freedom which are excited only when Planck energy processes take place. Such a description, of course, should be universally applicable irrespective of the nature of the semiclassical or classical metric which eventually arises in a particular context. Since certain microscopic degrees of freedom of spacetime are traced out, one may think that *any* spacetime will have a non zero entropy directly related to the number of micro-states which are consistent with certain macroscopic parameters which identify the classical spacetime. The question then arises as to what is special about a blackhole spacetime. The answer, as I will argue below, has to do with the existence of an infinite redshift surface. Classically, the infinite redshift of an event horizon acts as a great equalizer and arranges matters in such a way that very many collapsing configurations can all be described by the same kind of metric. Semi-classically, the existence of infinite redshift immediately leads to blackhole radiance and thermal spectrum. At the next level, quantum gravitationally, it is possible that the infinite redshift stretches out the subplankian degrees out of freedom and makes them 'visible', to us at low energies.

The above program has two parts: kinematical and dynamical. The kinematical part attributes certain substructure to spacetime and assumes that any given macroscopic spacetime can be consistent with very many different configurations of the underlying – as yet unspecified – substructure. This is, of course, similar to what is being done in attempts to derive blackhole entropy from strings or Ashtekar variables [6,7]. But while these attempts are from "bottom-up" and tries to arrive at the entropy from a microscopic theory of quantum gravity, I want to proceed in a "top-down" fashion and try to arrive at some broad characterization of the microscopic theory from the known classical and semiclassial properties of the blackholes. In fact, the true spirit of the above approach will require the final result to be independent of the detailed nature of the substructure and depend only on the existence of *some* substructure. For example, both strings and Ashtekar variables could lead to the same kind of blackhole entropy.

One clear (negative ?) aspect of this point of view, which seems to me inevitable, is that blackhole entropy is not going to help us in deciding the final version of quantum gravity. Attempts to derive laws of quantum gravity using insights of blackhole entropy will be somewhat similar to attempts in deriving microscopic laws of physics from the laws of equilibrium thermodynamics. It should, however, be remembered that original hypothesis of Max Planck, $E = \hbar\omega$, did arise from an attempt to understand the behaviour of radiation in thermodynamic equilibrium. Something similar should be certainly possible. This could amount to having an insight over the existence of certain internal degrees of freedom, their density of states etc. Given the equilibrium thermodynamics of radiation field, one could have never obtained the full quantum theory of radiation, but could only infer the existence of quanta. Similarly, given the equilibrium thermodynamics of blackhole, we will not be able to obtain the dynamical equations of quantum gravity but may be led to the inevitability of the existence of certain microscopic degrees of freedom which (eventually) leads to the continuum spacetime in the coarse grained point of view. On the positive side, such an approach has the advantage that it is independent of the detailed dynamics of the microscopic theory about which we have no consensus. We should only require the minimal assumption that the microscopic structure of spacetime has certain degrees of freedom which are traced out in the macroscopic limit. The behaviour of a solid in terms of elastic constants or that of gas in terms of thermodynamic variables should not depend on our knowing the atomic physics; historically, it did not. The question I want to address is similar: Can we develop a working theory for 'thermodynamics of spacetime' without knowing the 'molecular basis of spacetime'?

The dynamical aspect of the above program, which I will concentrate on in this paper, is closely related to the task of understanding why spacetimes with infinite redshift are special. It is well known that frequencies of outgoing waves at late times in blackhole evaporation correspond to super Planckian energies of the in-going modes near the horizon. One must then consider the possibility that these in-going modes interact with the microscopic degrees of freedom of the spacetime located near the event horizon. This ties up the two separate ideas: (i) There are internal degrees of freedom in spacetime which are ultimately relevant for blackhole entropy. (ii) There is a dynamical mechanism for exciting these degrees of freedom in spacetimes with infinite redshift surface. The second feature is absolutely essential. Existence of degrees of freedom which are unexcited in a given physical context does not contribute to thermodynamics. For example, the degrees of freedom of atomic nuclei or quarks will not contribute to the specific heat of a solid at ordinary temperatures. This, in fact, is the reason why the entropy of an ordinary spacetime is effectively zero. In a normal spacetime, energy scales above Planck energies are hardly excited and unexcited levels do not contribute to entropy. Infinite redshift surfaces provide a way of magnifying Planck level physics and make it visible. Dynamically, this arises from the fact that virtual modes of Planckian energy are converted into real modes of subplankian energies in such spacetimes. I stress the fact that having spacetime micro-structure is only one aspect of any entropy calculation. One should have reasonable grounds to believe that dynamical process which excite these micro levels are also present in the spacetime. This happens naturally when an infinite redshift surface is available.

It is also easy to see how certain well known features will arise naturally in this picture. Since one cannot operationally measure length and time scales below Planck values, any complete theory necessarily has to accommodate this feature in a fundamental manner. In the naivest level, this is equivalent to assuming that the number of independent micro-cells which are present in a region of spacetime of volume V will be about $N \approx V/L_P^3$. All these states will *not*, however, contribute to entropy in the semiclassical limit. In order to make these states contribute it is necessary to have a physical process which excites them. Though virtual processes of Planckian energies and above will be constantly exciting and de exciting them, to get *observable* entropy, we need a situation in which the *virtual* excitation of super Planckian energies are converted to *real* excitations of sub Planckian energies. This is possible in a spacetime with an infinite redshift surface, by the in-going modes of super Planckian energies. But it occurs in a thin shell of thickness L_P around the event horizon, i.e., in a region of volume of the size $V \cong AL_P$ where A is the area of the event horizon. Clearly the entropy will now turn out to be proportional to $N \cong (V/L_P^3) = (A/L_P^2)$, that is, to the area of the infinite redshift surface.

III. PHASE VOLUME AND ENTROPY IN SPACETIMES WITH HORIZONS

Since the event horizon seems to play an important role in the study of statistical concept like entropy, I will begin by discussing a thought experiment related to the statistical description of any system in a region surrounding an incipient blackhole. The statistical mechanics of such a system will require computation of the phase volume available for the system when its energy is E. I will show that certain difficulties arise in formulating the statistical mechanics of the system even at the classical level when an event horizon forms in the spacetime.

Consider a system of N relativistic point particles located in a static spacetime with line element

$$ds^2 = g_{00}(\mathbf{x})dt^2 - \gamma_{\alpha\beta}(\mathbf{x})dx^\alpha dx^\beta \qquad (\alpha, \beta = 1, 2, 3) \tag{1}$$

Any particle with four-momentum p^a possess the conserved energy (scalar) $E = \xi_a p^a$ in such a spacetime where $\xi^a = (1, \mathbf{0})$ is the time-like killing vector field. Standard statistical mechanics of this system is based on the density of states $g_N(E)$ which, in turn, can be built out of the density of state $g(E) \equiv g_{N=1}(E)$ for individual particles;

$$g_N(E) = \int \Pi_i^N \, g(E_i)\, dE_i\, \delta(E - \sum E_i) \tag{2}$$

The volume of phase space available for a single particle with energy *less than* E, is given by

$$\Gamma(E) = \int dx^\alpha dp_\alpha \; \Theta(E - \xi^i p_i) = \frac{4\pi}{3} \int \sqrt{\gamma}\, d^3x^\alpha \,(E^2/g_{00} - m^2)^{3/2} \tag{3}$$

where Θ is the Heavside theta function. The first form of the expression shows that $\Gamma(E)$ is generally covariant; the second expression is obtained by integrating over the momentum variables. This result is identical to the one which would have been used by any locally inertial observer at an event $\mathcal{P}(t, x^\alpha)$ who attributes a local value of energy $E_{\rm loc} = u^i p_i = E(g_{00})^{-1/2}$ to the particle. (For more details, see [8].) The density of states is given by $g(E) = (d\Gamma(E)/dE)$ and the entropy of the system is $S(E) = \ln g(E)$. All of the standard statistical mechanics of the system in an external gravitational field described by the metric in (1) will follow from the expression for the space volume.

Consider now the spacetime of a spherical star of mass M and radius $R > 2M$. The phase volume available for a particle located outside the star is given by

$$\Gamma(E) = \frac{16\pi^2}{3} \int_R^{R_{\rm max}} \frac{r^2 dr}{(1 - 2M/r)^2} \left[E^2 - m^2 \left(1 - \frac{2M}{r} \right) \right]^{3/2} \tag{4}$$

where $R_{\rm max} = 2M(1 - E^2/m^2)^{-1}$ is the maximum radius allowed for energy $E < m$. For $E > m$, we shall assume that the system is confined inside a large volume V as is usual in statistical mechanics. This expression, as well as the statistical mechanics developed by using it, remains well defined as long as $R > 2M$ but diverges when $R \to 2M$. Let us suppose that the star now starts to collapse eventually forming a blackhole. At late times $t \gtrsim 2M$ the radius of the star will follow the trajectory (see e.g., ref.[9]):

$$R(t) \cong 2M + 2M\, e^{-t/2M} \tag{5}$$

making the phase volume $\Gamma(E)$ and density of states $g(E)$ diverge exponentially in time; for example,

$$g(E) = \frac{d\Gamma}{dE} \cong 16\pi^2 E^2 (2M)^3 \exp\left(\frac{t}{2M}\right) \tag{6}$$

for $t \gtrsim 2M$. In other words, the entropy of *any* system of particles located outside a collapsing star diverges as the star collapses to form a blackhole due to the availability of infinite amount of phase space.

Similar disaster occurs even for a field. The wave equation describing a scalar field can be written in the form

$$\left(\frac{\partial^2\phi}{\partial t^2} - \frac{1}{4M^2} \frac{\partial^2 \phi}{\partial x^2} \right) + e^x \left(-\frac{1}{4M^2} \nabla^2_{(2)}\phi + m^2\phi \right) = 0 \tag{7}$$

near the horizon if we use the spatial coordinate x defined through $r - 2M = 2M\exp(x)$. (Here $\nabla^2_{(2)}$ is the Laplacian on the $r =$ constant surface). As $x \to -\infty$, the field becomes free and solutions are simple plane waves propagating all the way to $x = -\infty$. The existence of such a continuum of wave modes will lead to infinite phase volume for the system. More formally, the number of modes $N(E)$ for a scalar field ϕ with vanishing boundary conditions at two radii $r = R$ and $r = L$ is given by

$$N(E) \cong \frac{1}{\pi}\int_R^L \frac{dr}{(1-2M/r)} \int dl\,(2l+1)\left[E^2 - \left(1-\frac{2M}{r}\right)\left(m^2 + \frac{l(l+1)}{r^2}\right)\right]^{1/2}$$
$$= \frac{2}{3\pi}\int_R^L \frac{r^2 dr}{(1-2M/r)^2}\left[E^2 - \left(1-\frac{2M}{r}\right)m^2\right]^{3/2} = \frac{\Gamma(E)}{(2\pi)^3} \tag{8}$$

in the WKB limit [10,11]. This expression diverges as $R \to 2M$ showing that a scalar field propagating in a blackhole spacetime has infinite phase volume and entropy. The divergences described above occur around any infinite redshift surface and is a geometric (covariant) phenomenon.

This result, which shows that even conventional statistical mechanics does not exist in a spacetime with infinite redshift surface, need to be taken seriously. Physically, blackholes are fairly strange kind of entities when viewed from the point of view of statistical mechanics; statistical physics is based on *mutual* interactions of constituents leading to energy exchange where any subsystem can influence *and* be influenced by the rest of the system. Regions of spacetime covered by infinite redshift surfaces break this symmetry, since the material has fallen inside the horizon cannot causally influence the outside.

The key *mathematical* reason for all the above divergences is the behaviour of locally defined energy $E_{\rm loc} = E/(g_{00})^{1/2}$ at the surface of the star $r = R$ as $R \to 2M$; in this limit, $E_{\rm loc} \to \infty$ and the available phase volume becomes infinite essentially due to the behaviour of the system near the horizon. This feature suggests that the solution to this problem lies in physics at arbitrarily high energies. If true, there is hope for turning around this argument in order to catch a glimpse of Planck scale physics by demanding that the breakdown of statistical mechanics should not occur.

It is very likely that the classical divergence of phase volume signals the necessity of a new physical principle in dealing with blackholes. Recall that in classical relativity a Schwarzschild blackhole can be formed out of any spherically symmetric system provided sufficient amount of energy is confined to a small enough radius. Consider an arbitrary physical system with a Hamiltonian H which is collapsing in a spherically symmetric manner to form a blackhole. If the initial energy of the configuration is E, which will be conserved during a spherically symmetric collapse, the resulting blackhole will have a mass $M = E$. Initially, the system has access to a phase volume $\Gamma(E)$ which depends only on the form of the Hamiltonian H and the energy E. At sufficiently late times, the radius of the system approaches $R \to 2M$ with the energy remaining the same. If we assume that the Hamiltonian describing a system is the same irrespective of whether the system becomes a blackhole or not, it follows that the phase volume available to the system diverges as $R \to 2M$ but the formal dependence of $\Gamma(E)$ on E remains the same. (In the example studied above, $\Gamma(E) \propto E^2 f(t)$ with $f(t) \to \infty$ for $R \to 2M$; the E dependence is a power law.). We shall, however, see in section 5 that blackholes should be described by a universal form of phase volume $\Gamma_{\rm BH}(E) = \exp(4\pi E^2/E_P^2)$ quantum mechanically. [This phase volume is very large for macroscopic blackholes with $E \gg E_P$ but is finite due to quantum mechanical considerations. We see the historically familiar phenomena of a classical divergence being regularized by quantum mechanics, with the replacement of infinity by a large number.] But the important point to note is that the energy dependence of the phase volume has to change completely when the system collapses to form a black hole with $R \to 2E$. If the energy is conserved, then such a change can occur only if the Hamiltonian of the system gets transformed (by, as yet unknown, quantum gravitational process) when the system forms a blackhole. If I postulate that the formation of a blackhole, from a conventional physical system with hamiltonian $H_{\rm conv}$, changes the hamiltonian to the form $H_{\rm mod}$, with

$$H^2_{\rm mod} = A^2 \ln\left(1 + \frac{H^2_{\rm conv}}{A^2}\right); \qquad A \propto E_P \tag{9}$$

then, the formation of the blackhole will lead to a universal form of phase volume $\Gamma_{\rm BH}(E)$ at the conserved value of the energy $H_{\rm conv} = E$. The arguments given later in section 6 suggest that such a transformation has certain inevitability and leads to interesting consequences.

IV. EVENT HORIZON IN THE SEMICLASSICAL LIMIT

As shown above, the existence of event horizon leads to difficulties in formulating statistical mechanics for *any* system even classically. At the next level, when one studies quantum filed theory in the blackhole spacetime, the event horizon assumes a greater role: If the event horizon is formed due to collapse of matter, then it radiates like a black body of temperature $T = (8\pi M)^{-1}$ at late times $t \gg 2M$. It is precisely the existence of an infinite redshift surface which leads to the characteristic Planckian form of the spectrum and distinguishes the blackhole of mass M from, say, a neutron star of mass M. While this feature is apparent in the primary derivation of blackhole radiation by Hawking, it is worthwhile to establish the connection between infinite redshift surface and the Planck radiation in a simple and direct manner which I shall now do.

Consider a radial null geodesic in the Schwarzschild spacetime which propagates from $r = 2M + \epsilon$ at $t = t_{in}$ to the event $\mathcal{P}(t, r)$ where $\epsilon \ll 2M$ and $r \gg 2M$. The trajectory can be determined from the Hamilton-Jacobi equation for the action $\mathcal{A}(r, t) \equiv -Et + S(r)$, written in the form

$$g^{ik}\partial_i \mathcal{A}\, \partial_k \mathcal{A} = \frac{E^2}{(1 - \frac{2M}{r})} - \left(1 - \frac{2M}{r}\right)\left(\frac{dS}{dr}\right)^2 = 0 \tag{10}$$

The trajectory, determined by the condition $(\partial\mathcal{A}/\partial E) =$ constant, with the required initial conditions is

$$r \cong t - t_{\rm in} + 2M \ln\left(\frac{\epsilon}{2M}\right) \qquad (\epsilon \ll 2M, \quad r \gg 2M) \tag{11}$$

The frequency of a wave will be redshifted as it propagates on this trajectory. The frequency ω at r will be related to the frequency ω_{in} at $r = 2M + \epsilon$ by

$$\omega \cong \omega_{\rm in}\, [g_{00}(r = 2M + \epsilon)]^{1/2} \cong \omega_{\rm in}\left(\frac{\epsilon}{2M}\right)^{1/2} = \omega_{\rm in} \exp\left(-\frac{t - t_{\rm in} - r}{4M}\right) \tag{12}$$

where we have used (11). If the wave packet, $\Phi(r, t) \simeq \exp(i\theta(t, r))$, centered on this null ray has a phase $\theta(t, r)$, then the instantaneous frequency is related to the phase by $(\partial\theta/\partial t) = \omega$. Integrating (12) with respect to t, we get the relevant wave mode to be

$$\Phi(t, r) \propto \exp i \int \omega\, dt \propto \exp\left[-4M\omega i\ \exp\left(-\frac{t - t_{\rm in} - r}{4M}\right)\right] \tag{13}$$

(This form of the wave can also be obtained by directly integrating the wave equation in Schwarzschild geometry with appropriate boundary conditions; the above derivation, however, is simpler.) An observer using the time coordinate t will Fourier decompose these modes with respect to the frequency ν defined using t:

$$\Phi(t, r) = \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} f(\nu) e^{-i\nu t} \tag{14}$$

where

$$f(\nu) = \int_{-\infty}^{\infty} dt\, \Phi(t, r) e^{-i\nu t} \propto \int_0^{\infty} dx\, x^{-4M\nu i - 1} \exp(-4M\omega_{\rm in} i x) \tag{15}$$

The integral can be evaluated by rotating the contour to the imaginary axis. The corresponding power spectrum is

$$|f(\nu)|^2 \propto (\exp(8\pi M\nu) - 1)^{-1} \tag{16}$$

which is Planckian at temperature $T = (8\pi M)^{-1}$.

The simple derivation given above strips the process of Hawking evaporation to its bare bones and establishes the following: (i) The key input which leads to the Planckian spectrum is the exponential redshift given by equation (12) of modes which scatter off the blackhole and travel to infinity at late times. This requires an infinite redshift surface and — in fact — the derivation can be easily generalized to other cases which have infinite redshift surfaces (Rindler, de Sitter ...). (ii) The analysis up to equation (16) is entirely classical and no $\hbar$ appears anywhere. In normal units, $8\pi M\nu$ becomes $8\pi GM\nu/c^3$. It is only our desire to introduce an energy $E = \hbar\nu$ which makes this factor being written as $8\pi GM\nu/c^3 = (8\pi GM/\hbar c^3)\hbar\nu = E/k_BT$. The mathematics of Hawking evaporation is purely classical and lies in the Fourier transform of an exponentially redshifted wave mode (for a discussion of the classical versus quantum features see ref.[12]).

It is clear from the above derivation that the existence of infinite redshift surface allows modes with frequencies $\omega \gg E_P$ near the horizon to appear as sub Planckian radiation at future null infinity. Any non trivial part of the Planck spectrum arising out of Hawking evaporation will correspond to a mode which originated with virtual energies much higher than Planck energy near the horizon. For example, frequencies near the peak of the Planck spectrum, $\omega \approx (8\pi M)^{-1}$ will arise from incoming modes with frequencies $\omega > E_P$ at all retarded times $u > t_Q \equiv 4M \ln(8\pi M/E_P)$. For a stellar mass blackhole $(t_Q/2M) \approx 10^2$. Since the Planck spectrum gets established and the transients die down only for $t \gtrsim 2M$, most of the Hawking radiation originates from modes with $\omega \gg E_P$ near the horizon. These modes also approach the event horizon arbitrarily closely; for $u > t_Q$ the modes have originated from $2M < r < 2M + L_P$. Clearly, the existence of a horizon allows one, through the process of blackhole evaporation, to probe the physics at sub Planckian length scales near the horizon.

We have now seen two examples of how arbitrarily high energies near the event horizon plays a vital role. In the case of phase volume this creates difficulties in formulating a viable statistical mechanics and makes the entropy of any system formally infinite. In the case of Hawking evaporation, these modes are primarily responsible for most of the Planck spectrum seen at late retarded times. These two features suggest that any attempt to obtain a correct description of blackhole evaporation will be incomplete without addressing the arbitrarily high frequency virtual excitations near the horizon. Clearly, the relevant issue is not whether the gravitational field described by Einstein's equation has arbitrarily high curvature; the curvature at the event horizon of a stellar mass blackhole can be quite low. The relevant question, it appears to me, is whether processes in any local region involves energy higher than Planck energies. If they do, the description in terms of continuum spacetime breaks down and we have to worry about Planck scale physics.

Let us ask what this paradigm implies for the divergence of the phase volume derived earlier. To begin with, it is clear that the problem is "local" and confined to a region around the event horizon. Since the energy of the modes are higher than Planck energy as one approaches the horizon, it is likely that the excitation of microscopic degrees of freedom of spacetime (whatever they may be) will occur in this region. Let us suppose that the modes of the scalar field excites the microscopic degrees of freedom of spacetime in a shell-like region between $r_1 = 2M + h$ and $r_2 = 2M + H$. The *proper* distance l of these surfaces from $r = 2M$ will be taken to be some definite multiple of Planck length with $l_1 \equiv (L_P/c_1)$ and $l_2 \equiv c_2 L_P$ where c_1 and c_2 are, at present, unknown numerical constants. (We expect both c_1 and c_2 to be larger than unity so that $l_1 < L_P$ and $l_2 > L_P$.) It is easy to relate the proper and coordinate distances and obtain, for $L_P \ll 2M$,

$$h \cong (L_P^2/8M)\, c_1^{-2}; \qquad H \cong (L_P^2/8M)\, c_2^2 \tag{17}$$

The modes of the scalar field in this region interacts with the microscopic degrees of freedom and I will now assume that this interaction drives the system to a state of maximum probability. In other words local thermodynamic equilibrium exists between the degrees of freedom of the spacetime and the modes of the scalar field leading to some local temperature $T/(g_{00})^{1/2}$ (The corresponding value of temperature at infinity is T). The entropy of the scalar modes confined to this region at some given temperature is straightforward to compute using the standard relations:

$$S = \beta \left[\frac{\partial}{\partial \beta} - 1\right] F; \qquad F = -\int_0^\infty dE \frac{N(E)}{e^{\beta E} - 1}, \tag{18}$$

and the expression for $N(E)$ in equation (8) (see ref [10,11]). We get

$$S = \frac{32\pi^3}{45}(2MT)^3 \left(\frac{2M}{L_P}\right)^2 \left\{c_1^2 - \frac{1}{c_2^2}\right\} \tag{19}$$

When the modes of the scalar field propagate to infinity, they are redshifted to sub Planckian energies and appears with the spectrum in (16), corresponding to the temperature at infinity to be $T = 1/8\pi M$. Using this in (19) we get

$$S = 4\pi \frac{M^2}{L_P^2} \left[\frac{1}{90\pi}\left(c_1^2 - \frac{1}{c_2^2}\right)\right] \tag{20}$$

There are several features to note in this expression. (i) To begin with it shows that c_2 is relatively unimportant and we could as well take $c_2 \to \infty$; this is to be expected since most of the divergent contribution arises from the lower limit of the integration. Hereafter, we shall set $c_2^{-1} = 0$. (ii) Second, the result diverges when $c_1 \to \infty$; this is just the old result of the divergence of phase volume for $R \to 2M$. Arguments I gave above, however, suggests that we are not allowed to take this limit since it makes the locally defined energy, $E_{\rm loc}$, of the mode to exceed the Planck energy by an arbitrary amount. I will discuss the actual value of c_1 later on. (iii) Third, expression (20) gives an entropy which is proportional to the area of the event horizon. This is non trivial and two key mathematical features of the spacetime geometry have conspired to produce this result: (a) The metric coefficient g_{00} is the reciprocal of g_{11} and g_{00} goes as $\mathcal{R}(r-r_H)$ near the event horizon $r_H = 2M$. It is easy to show that any such spacetime will lead to a thermal state with temperature $T = (\mathcal{R}/4\pi)$. (This is proved, for example, in ref.[13] and is directly verifiable from the periodicity in Euclidean time.) (b) The coordinate distance and the proper distance to the event horizon scales as in equation (17). This, in turn, is a consequence of the metric having a simple zero at the horizon. The proportionality between the entropy and the area will fail except when the above two mathematical criteria are met. In other words, the result holds only around an infinite redshift surface and not at any arbitrary radius.

The physical reason for this proportionality to area is simple and more important. In the approach I have outlined, any volume L^3 in space is assumed to be made of $N = (L/L_P)^3$ Planck sized cells. The microscopic degrees of freedom in these cells can be excited *only* when (i) sufficiently high virtual energies are available and (ii) some mechanism exists for these modes to convert themselves as low energy propagating waves. The first condition is met only in a small region of thickness of the order of L_P around an infinite redshift surface. Hence the number of microscopic cells which can contribute to the entropy is reduced from the total number of cells inside the blackhole, $N_{\rm in} = (2M/L_P)^3$ to those residing in a thin shell of radius $r = 2M$ and thickness L_P; the latter is about $N_{\rm shell} \cong (2M)^2(L_P)/L_P^3 = (2M/L_P)^2$.

The *actual* value of the entropy contributed by the scalar field cannot be computed without knowing the value of c_1. If we take $c_1 = (90\pi)^{1/2}$, then the entropy turns out to be one-fourth of the area of the event horizon which is the Bekenstein-Hawking result. In the conventional discussions of blackhole entropy, a sharp distinction is made as to whether the entropy is contributed by (a) non gravitational degrees of freedom external to blackhole or (b) those internal to the blackhole or whether (c) the entropy is a basic property of the gravitational field of the blackhole itself. In the approach advocated here, this question becomes somewhat irrelevant. Near the event horizon, high energy excitations of the scalar field interact strongly with some unspecified degrees of freedom of quantum spacetime. The entropy contained in this region of strong coupling, cannot be properly separated as entropy of either the scalar field or that of the blackhole. The blackhole constantly scatters the incoming modes to outgoing modes which should be thought of as excitations followed by a radiative decay of the microscopic degrees of freedom of spacetime around the event horizon.

In the thermodynamic interaction between any two systems, temperatures will be equalized when the system is driven to the state of maximal space volume. I will argue in the next section that the density of state of microscopic spacetime degrees of freedom in the case of a blackhole has the form $g(E) = \exp[4\pi(E/E_P)^2]$. Given this, we are assured that any other field which come into interaction with these degrees of freedom near the horizon will behave as though it has a temperature $T = (8\pi M)^{-1}$. Actually this is *all* we know from the semiclassical analysis of quantum fields in curved spacetime. The total entropy of the system made of microscopic spacetime degrees of freedom as well as the matter fields is not an operationally well defined concept. If some fundamental theory of quantum spacetime provides the density of states $g(E) = \exp[4\pi(E/E_P)^2]$, rest of blackhole thermodynamics will follow from such a result. From this point of view, the actual value of c_1 is quite irrelevant and we need *not* arrange it so as to get any particular value for the entropy.

In this approach, the density of states of the microscopic degrees of freedom is a more fundamental construct than the so called entropy of blackhole. The role of a blackhole is limited to providing a simple context in which these microscopic degrees of freedom can be excited and de-excited due to the existence of an infinite redshift surface. In this sense blackholes act as a magnifying glass which allows us to see the microscopic quantum spacetime. [The mathematics of this analysis was first given in the 'brick-wall model' of t'hooft [10] and was generalized in ref. [11]; the interpretation given here, however, is quite different.]

V. EVENT HORIZON AND MICRO-STRUCTURE OF SPACETIME

Having shown that the interaction between quantum micro-structure of spacetime and the local high energy modes of a scalar field can occur near an event horizon, I now take up the question of the nature of these microscopic degrees of freedom. As emphasized several times before, there is no way one can obtain the detailed theory of quantum gravity just from the results derived so far. The hope is to obtain some general characteristics of blackhole spacetimes which is enough to provide a description of the results obtained above. I will first re-derive a result obtained earlier by t'hooft for the density of states of a blackhole and will then show how one can construct an effective field theory from which blackhole spacetime will emerge as an "one-particle" excitation.

There are several reasons to believe that event horizon will continue to play a special role in the next level of description — viz. in a microscopic theory of spacetime. Any classical (asymptotically flat) spacetime with energy $M \gg E_P$ has to arise as some nonperturbative quantum condensate of the true microscopic degrees of freedom (strings, membranes, spin networks....). The classical solutions for a spherically symmetric neutron star of mass M and for a blackhole of mass M belong to this class. But there is a fundamental difference between these two solutions:

A neutron star is stable against Hawking evaporation while a blackhole is not. The quantum states representing these two objects must reflect this feature in some manner. Naively, one would expect the energy of the state to pick up an imaginary part characterizing the decay. If $|M, \mathrm{star}>$ and $|M, \mathrm{bh}>$ are the two quantum states, then at some suitable limit, the time evolution of these states will be

$$|M,t,\mathrm{star}>= \exp(-iMt)|M,0,\mathrm{star}>; \quad |M,t,\mathrm{bh}>= \exp\left(-i[M-iQ(M)]t\right)|M,0,\mathrm{bh}> \tag{21}$$

where $Q(M) \propto (E_P/M)^2 M^{-1}$ is the adiabatic decay rate of the energy due to Hawking evaporation. How can two states, which are parametrized classically by the same quantity M have very different quantum descriptions ? This is possible *only if* the microscopic theory takes cognizance of the existence of the infinite redshift surface in one of the solutions, since these two classical solutions differ only in that aspect. Hence the microscopic theory must have a mechanism to take this feature into account even though the issue is not directly related to the existence of high curvature.

Consider one such quantum state, describing a blackhole of mass M (say). Whatever may be the microscopic degrees of freedom, we can try to characterize their configuration in such a state by their average energy $\bar{E}$ and density of states $g(\bar{E})$. Given $g(\bar{E})$, we can define the entropy $S(\bar{E}) \equiv \ln g(\bar{E})$ and inverse temperature $\beta(\bar{E}) = (\partial S/\partial \bar{E})$. Note that these are well defined, formal constructs defined using microcanonical ensemble, without using any external systems or heat bath. We are interested in the form of $\beta(\bar{E})$. Since $\bar{E}$ has to be determined by the spacetime geometry in the classical limit, it appears quite reasonable to take $\bar{E} = M$. (If the Hawking evaporation is thought of as the decay of an unstable quantum state, then it is *necessary* that $\bar{E} = M$.) The temperature is related to M by $T = (E_P^2/8\pi M)$, thereby leading to the result $\bar{E} = (\beta/8\pi)$.

I want to stress that a physical system with such a relation between mean energy and temperature must possess very unconventional features. Conventional statistical mechanics invariably leads to a mean energy which is an increasing function of temperature. This is trivially true for any system possessing canonical ensemble description since the specific heat in canonical ensemble must be positive definite. But even ordinary self gravitating systems have negative specific heats and do not allow a description in terms of canonical ensemble. In the micro-canonical ensemble, it is possible to have negative specific heats with mean energy decreasing with temperature. One simple consequence of this fact is that when two blackholes combine to form a larger one, the total energy (and density of states) increase but the temperature decreases. This suggests that the degrees of freedom of spacetime from which blackhole states are built should be described by micro-canonical ensemble and its partition function may exist only in a formal sense.

Let us next consider another feature of the quantum state describing a blackhole, namely, the density of states. Since the macroscopic description uses only the mean energy and does not specify the detailed configuration of the microscopic quantum degrees of freedom of the spacetime, it stands to reason that the quantum states will be highly degenerate and should be describable in terms of some density of state function $g(\bar{E})$. The fact that entropy S is proportional to the area ($\mathcal{A} \propto M^2$) alone suggests that the density of states for a blackhole of energy $E = M$ scales as $g(E) = \exp(S(E)) = \exp(\alpha_1 E^2)$ where α_1 is some constant. This is very different from the scaling which arises in statistical mechanics of normal systems. For a system made of massless quanta at temperature T, we have $S \propto T^3, E \propto T^4$ giving $S \propto E^{3/4}$; hence the density of states grows as $\exp[\alpha_2(LE)^{3/4}]$ where L is the linear dimension of the system and α_2 is some constant. If such a system (with energy E and confined in a spatial region of size L) should not become a blackhole, then we must have $E < (L/L_P^2)$. Using this bound we see that the density of states for such a system is bounded by a form $\exp[\alpha_3(L/L_P)^{3/2}]$. This increases more slowly than the form $\exp[(\mathrm{constant})\ L^2]$.

Such a rapid growth of the phase volume, $g \propto \exp(\alpha_1 E^2)$, immediately implies that one cannot obtain the partition function as a standard Laplace transform of the density of states. This, however, should not come as a surprise since the existence of a well defined partition will imply the existence of a description in terms of canonical ensemble and consequently positive specific heat. Since the blackhole state has negative specific heat, something has to give way and what breaks down is the conventional relation between $g(E)$ and $Z(\beta)$. We can, however, obtain a partition function by a different procedure as follows. We consider the density of states $g(E)$ of a blackhole to be a function of a complex variable z and study the behaviour of $g(z)$ along the imaginary axis $z = iy$. If $g(z) \propto \exp[4\pi(z/E_P)^2]$, it remains bounded along the purely imaginary axis with $g(y) \propto \exp[-4\pi(y/L_p)^2]$. Defining the partition function $Z(\beta)$ as Laplace transform along the imaginary axis (which becomes a Fourier transform), we get

$$Z(\beta) = \int_{-\infty}^{\infty} dy\, \rho(y) e^{-i\beta y} \propto \exp\left(-\frac{\beta^2 E_P^2}{16\pi}\right) \tag{22}$$

Such a partition function will correctly reproduce the relation between mean energy and temperature

$$M = \bar{E} = -\frac{\partial \ln Z(\beta)}{\partial \beta} = \frac{E_P^2}{8\pi}\beta = \frac{E_P^2}{8\pi T} \tag{23}$$

which justifies the formal manipulations a posteriori.

The above analysis shows that the key feature characterizing the microscopic quantum degrees of freedom of spacetime with event horizon is the peculiar density of states of the form $g(E) = \exp[4\pi(E/E_P)^2]$ [This form for $g(E)$ was earlier derived from scattering arguments by 't Hooft in ref.[10] and is implicit in several earlier works]. Given such a density of state, these degrees of freedom will possess a relation between mean energy and temperature which is appropriate for that of a blackhole. The excitation and subsequent de-excitation of these degrees of freedom with high energy modes of other fields will allow for the other fields to reach thermal equilibrium at the characteristic temperature. The entire picture seems to be quite consistent provided the density of state can be understood from some more fundamental considerations.

The situation is — in fact — reminiscent of the fact that a classical blackhole is describable by just three parameters, irrespective of the details of the original system which collapsed to form the it. Similarly, the quantum blackhole has a universal form of density of states which is independent of the form of the density of states of the original system which collapsed to form the blackhole. We will see in the next section that this feature allows us to construct several model systems with the correct form of density of states for the blackhole.

The above arguments notwithstanding, one may not be quite happy with any physical system which has a density of states which grows so fast. In judging such a prejudice, we have to keep the following caveats in mind: (i) There exists no really not a good argument against such density of states for self gravitating systems which are, anyway, not describable by canonical ensemble. The partition function for such systems cannot be obtained as a Laplace transform of density of states and has to be defined in a generalized manner using some suitable contour in complex plane. This is precisely what we have done above. (ii) The existence of a density of state for a dynamical system, by itself, does not mean much. Consider for example, the density of states for an ordinary solid. While formal integrations over E extending up to infinity will be done in defining variables like partition function, it is always assumed that high energy physics is irrelevant if those modes are not excited. So in studying a solid at room temperatures, we are not bothered by the form of $g(E)$ for say, $E > 1$ GeV. This works well for systems in which mean energy is comparable to the temperature. In the case of a stellar mass blackhole, the key difference is that the mean energy and temperature are widely different and hence it is not clear up to what energy scales one needs to know the density of states. (iii) The above comment is particularly relevant because one cannot use thermodynamic description for arbitrarily high energies. Eventually the details of the microscopic degrees of freedom, whatever they are, will be brought into picture. Of course, when this happens, even the concept of density of states may break down and we enter a truly unknown region of quantum gravity.

VI. MODELLING THE BLACKHOLE DENSITY OF STATES

Since $g(E)$ for the blackhole contains the essence of its behaviour, let us ask which physical systems possess such a density of states. While attempting to do this, it is necessary to note that the density of states obtained above can only be trusted for $E \gg E_P$. In general, the density of states can have a form

$$\rho(\bar{E}) \approx \exp[4\pi(\bar{E}/E_P)^2 + \mathcal{O}(\ln(\bar{E}/E_P)\cdots] \tag{24}$$

where the leading log corrections are unimportant for $\bar{E} \gg E_P$. I will now show how several toy field theoretical models can be constructed, which have such a density of state. While this shows that blackhole entropy by itself cannot provide more information regarding the quantum micro-structure, the study of the model systems will reveal some interesting features.

The most straight forward attempts to obtain such density of state are based on combinotrics and using expansions of the form:

$$g(E) = \exp\left(\frac{4\pi M^2}{L_P^2}\right) = \exp\left(\frac{\mathcal{A}}{4L_P^2}\right) = \sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{\mathcal{A}}{4L_P^2}\right)^n \approx \lim_{N\to\infty}\left(\frac{\mathcal{A}}{4L_P^2 N}\right)^N \tag{25}$$

where $\mathcal{A}$ is the area of the event horizon. These expansions readily suggest interpretations based on combinotrics in which the event horizon is divided into patches of Planck size. We shall not consider them since it seems to me that they do not have a direct dynamical content.

To obtain a model for blackhole density of states with some dynamical content, we can take a clue from the arguments presented in section 3. Consider any physical system with a Hamiltonian $H(\Gamma)$ which depends on some suitably defined phase space variables denoted symbolically by Γ. When the system has energy $\bar{E}$, let the density of states be given by

$$g(E) = \int d\Gamma\, \delta_D\left[E - H(\Gamma)\right] \propto E^N \tag{26}$$

where N is related to the degrees of freedom of the system. We now let this system collapse in a spherically symmetric manner to form a blackhole with the same energy E. Once the blackhole is formed, the density of states should have the characteristic form $g_{\rm BH}(E) = \exp(4\pi E^2/E_P^2)$ for $E \gg E_P$. If E remains conserved during the collapse, then this can only happen if we assume that the formation of the event horizon somehow changes the Hamiltonian of the system to the form

$$H^2_{\rm mod} = A^2 \ln\left(1 + \frac{H^2}{A^2}\right); \qquad A \propto E_P (N+1)^{1/2} \tag{27}$$

This is easy to verify: The density of states for $H_{\rm mod} \equiv f(H)$, where $f^2(x) = A^2 \ln[1 + (x^2/A^2)]$, is given by

$$\begin{aligned} g_{\rm mod}(E) &= \int d\Gamma\, \delta_D\left(f[H(\Gamma)] - E\right) = \int d\Gamma \int_0^\infty d\epsilon\, \delta_D\left(H(\Gamma) - \epsilon\right) \delta_D\left(f(\epsilon) - E\right) \\ &= \int_0^\infty d\epsilon\, g(\epsilon)\, \delta_D\left(f(\epsilon) - E\right) = \left[g(\epsilon)\left|\frac{df}{d\epsilon}\right|^{-1}\right]_{\epsilon = \epsilon_r(E)} \end{aligned} \tag{28}$$

where $\epsilon_r(E)$ is the root of the equation $f(\epsilon_r) = E$. We have,

$$\left[g(\epsilon)\left|\frac{df}{d\epsilon}\right|^{-1}\right]_{\epsilon=\epsilon_r} = \left(\frac{E}{A}\exp\frac{E^2}{2A^2}\right)\left[A^N\left(\exp\frac{E^2}{A^2} - 1\right)^{N/2}\right] \simeq \exp\left[\frac{N+1}{2A^2}E^2 + \mathcal{O}(\ln E)\right] \tag{29}$$

for $E \gg E_P$. This has the correct form for the blackhole density of states if we take $A^2 = [(N+1)/8\pi]E_P^2$.

If the effect of quantum structure of spacetime is to modify *any* low energy hamiltonian in such a form, then any physical system will have the density of states needed for blackholes for $E \gg E_P$. Such a modification will also ensure that all the symmetries of the H_{conv} will be preserved by H_{mod}. Consider, for example, the following question: Some physical system, say, a spherical cloud of dust has a certain hamiltonian H_{conv} and energy E at $t = 0$. It will have certain amount of phase volume available corresponding to this particular hamiltonian and energy. When this system collapses to form a blackhole, its energy is conserved but the density of states have to change to a particular form. One possible way of modeling this transition is to find a general principle which will modify the hamiltonian of any system which forms a black hole in the manner suggested above. We have no idea, right now, as to how this transformation takes place and as such, it should be thought of as a completely ad-hoc postulate. Right now I will merely use this postulate to construct model systems with the correct form of density of states for the blackhole. Towards the end of this section, I will discuss further consequences of this postulate. I stress that whether such a *universal* postulate is valid or not is immaterial for the *specific* models which I construct below.

The simplest *dynamical* system which has the required density of states can be constructed as follows: Consider some system with the Hamiltonian $H(p)$ which only depends on the magnitude of a D-dimensional momentum vector $\mathbf{p}$. The phase volume $\Gamma[E]$ bounded by the energy surface $H(p) = E$, in the momentum space is given by

$$\Gamma(E) = \int d^D p\, \Theta(E - H(p)) \propto p^D_{\rm max}(E) \tag{30}$$

where p_{max} is the momentum corresponding to the energy E. For $E \gg E_P$ we want this phase volume to grow as $\exp(4\pi E^2/E_P^2)$. [It does not matter whether we work with $\Gamma[E]$ or $g(E)$ for $E \gg E_P$ since they differ only by sub-dominant logarithmic terms mentioned in equation (24).]. It follows that $E^2 \to (E_P^2 D/8\pi)\ln p^2_{max}$ in this limit. Hence the Hamiltonian we need has the asymptotic form $H(p) \to (E_P^2 D/8\pi)\ln p^2$ in the same limit. The form of the hamiltonian for $E \ll E_P$ is not fixed by these considerations but we will expect it to reduce to some conventional form independent of L_P. One simple choice we will explore is

$$H^2(p) = \frac{E_P^2 D}{8\pi}\ln\left(1 + \frac{8\pi p^2}{D E_P^2}\right) \tag{31}$$

which reduces to that of a massless particle in the low energy limit. [The expression in (31) is of the form a relation between a modified Hamiltonian $H_{\rm mod}$ and the conventional low energy Hamiltonian $H_{\rm conv}$ postulated above. Using $(p^2 + m^2)$ in place of p^2 in the above expression we can arrange for the low energy limit to describe a *massive* particle; we will not bother to do this here since we anyway expect $m \ll E_P$.]

Classically, such a system describes particles which move with trajectories of the form

$$\mathbf{x} = \mathbf{v}t; \qquad \mathbf{v} = \frac{\partial H}{\partial \mathbf{p}} = \frac{\mathbf{p}}{E(\mathbf{p})}\frac{1}{[1 + (8\pi p^2/DE_P^2)]} \tag{32}$$

For $E \ll E_P$, this is just a null ray of massless particle; but for $E \gg E_P$ we have $v \rightarrow [1/(p \ln p)]$ which decreases with increasing momentum. One may interpret this as the dynamics slowing down significantly at high energies. Quantum mechanically, we get the modified Klein-Gordon equation for a scalar field to be

$$\frac{\partial^2 \phi}{\partial t^2} + A^2 \ln\left(1 - \frac{\nabla^2}{A^2}\right)\phi = 0; \quad A^2 = \frac{D}{8\pi}E_P^2 \tag{33}$$

The general solution is of the form

$$\phi(t, \mathbf{x}) = \int \frac{d^D k}{(2\pi)^D} a(\mathbf{k}) \exp(-i\omega_{\mathbf{k}} t + i\mathbf{k}\cdot\mathbf{x}) \tag{34}$$

with the dispersion relation

$$\omega^2(\mathbf{k}) = A^2 \ln\left(1 + \frac{k^2}{A^2}\right); \qquad A^2 = \frac{D}{8\pi}E_P^2 \tag{35}$$

[The $a(\mathbf{k})$ gives the initial amplitude of mode]. Any wave packet will, of course, disperse under such an evolution. The low-k modes move with the speed of light while the high-k modes do not propagate at all since their group velocity [given by equation (32)] grinds to zero at high-k.

The above attempts, of course, are based on "single particle" models and hence are probably too naive. It is however easy to construct a field theory such that the one-particle excited state of the theory can have the same density of states as the blackhole. To do this, we only have to put together a bunch of harmonic oscillators with the dispersion relation given by (35). More formally, let us assume that the transition to continuum spacetime limit of a blackhole can be described in terms of certain fields $\phi(t, \mathbf{x})$ which are to be constructed in some suitable manner from the fundamental microscopic variables q_j. I take the Lagrangian describing the effective field $\phi(t, \mathbf{x})$ to be

$$L = \frac{1}{2}\int d^D\mathbf{x}\, \dot{\phi}^2 - \frac{1}{2}\int d^D\mathbf{x} d^D\mathbf{y}\, \phi(\mathbf{x})F(\mathbf{x}-\mathbf{y})\phi(\mathbf{y}) = \int \frac{d^D\mathbf{k}}{(2\pi)^D}\frac{1}{2}\left[|\dot{Q}_{\mathbf{k}}|^2 - \omega_{\mathbf{k}}^2 |Q_{\mathbf{k}}|^2\right] \tag{36}$$

The Lagrangian is non local in the space coordinates $\mathbf{x}$ which is taken to be $D-$dimensional; the corresponding Fourier space coordinates are labelled by $\mathbf{k}$. The quadratic non locality allows us to describe the system in terms of free harmonic oscillators with a dispersion relation $\omega(\mathbf{k})$ related to $F(\mathbf{r})$ by

$$\omega^2(\mathbf{k}) = \int d^D\mathbf{r}\, F(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}} \tag{37}$$

The energy levels of the system are built out of elementary excitations with energy $\hbar\omega(\mathbf{k})$ and the density of states corresponding to the one particle state of the system is:

$$g(E) = \int d^D k\, \delta_D\left[\omega(\mathbf{k}) - E\right] \tag{38}$$

which is just the Jacobian $|d^D\mathbf{k}/d\omega|$. It is straightforward to see that if the dispersion relation $\omega(\mathbf{k})$ has the asymptotic form

$$\omega^2(\mathbf{k}) \rightarrow \frac{E_P^2 D}{8\pi}\ln k^2 \qquad (\text{for } k^2 \gg E_P^2) \tag{39}$$

then the density of states has the required form for that of a blackhole.

In this class of model, the blackhole is treated as the one particle state of a *nonlocal* field theory. The dispersion relation is arranged so as to give a normal massless scalar field at low energies. It is certainly of interest to explore the new features of this field theory. Since non-locality is the key new feature let us begin by studying the form of $F(\mathbf{x})$ in real space. We can evaluate it by inverting the Fourier transform in (37) and using (35). The logarithmic singularity can be regularized by using an integral representation for ln and interchanging the orders of integration. These steps lead to

$$F(\mathbf{x}) = \int \frac{d^D\mathbf{k}}{(2\pi)^D} A^2 \ln\left(1+\frac{k^2}{A^2}\right) e^{i\mathbf{k}\cdot\mathbf{x}} = -A^2 \int \frac{d^D\mathbf{k}}{(2\pi)^D} \int_0^\infty \frac{d\mu}{\mu} e^{-\mu - \frac{\mu}{A^2}k^2 + i\mathbf{k}\cdot\mathbf{x}}$$

$$= -\frac{A^{2+D}}{(4\pi)^{D/2}} \int_0^\infty \frac{d\mu}{\mu^{1+D/2}} e^{-\mu - \frac{A^2x^2}{4\mu}} = -\frac{2A^{2+D}}{(2\pi)^{D/2}} \left(\frac{1}{A^2x^2}\right)^{D/4} K_{D/2}(Ax); \quad A^2 = \frac{D}{8\pi}\frac{1}{L_P^2} \tag{40}$$

where $K_\nu(z)$ is the MacDonald function. This function vanishes exponentially for large values of the argument, showing that the effective correlation length of the nonlocal field is of the order of $A^{-1} = (8\pi/D)^{1/2}L_P$. For small z, we have $K_\nu(z) \to z^{-\nu}$ and the correlation function goes as a power law $F(x) \propto x^{-D}$. The short distance behaviour of the correlation function is universal and depends only on the asymptotic form of the dispersion relation. In fact, this is the only feature of $F(\mathbf{x})$ which is needed to reproduce the density of states leading to the correct theory of blackhole thermodynamics. When $L_P \to 0$, the function $F(x)$ is proportional to the second derivative of Dirac delta function as can be seen from the fact that, as $L_P \to 0, \omega^2(k) \to k^2$. In this limit, we recover the standard local field theory.

The expressions are more tractable for the simplest case of $D = 1$. I will now illustrate the above phenomena using the simplest possible choice, corresponding to $D = 1$ and a dispersion relation

$$\omega^2(k) = \frac{E_P^2}{8\pi} \ln\left(1+\frac{8\pi k^2}{E_P^2}\right) \tag{41}$$

The density of states corresponding to this dispersion relation is given by

$$g(\bar{E}) \cong \exp\left[4\pi\frac{\bar{E}^2}{E_P^2} + \mathcal{O}\left(\ln\frac{\bar{E}}{E_P}\right)\right] \equiv \exp S(\bar{E}) \tag{42}$$

The corresponding blackhole temperature is

$$T(\bar{E}) = \left(\frac{\partial S}{\partial \bar{E}}\right)^{-1} = \frac{E_P^2}{8\pi\bar{E}}\left[1+\mathcal{O}\left(\frac{E_P^2}{\bar{E}^2}\right)\right] \cong \frac{E_P^2}{8\pi M} \tag{43}$$

for $\bar{E} = M \gg E_P$.

The function $F(r)$ corresponding to the $\omega^2(k)$ in equation (41) is

$$F(x) = \frac{E_P^2}{8\pi}\int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{-ikx} \ln\left(1+\frac{8\pi k^2}{E_P^2}\right) = -\frac{E_P^3}{8\pi}\left(\frac{L_P}{|x|}\right)\exp\left(-\frac{|x|}{\sqrt{8\pi}L_P}\right) \tag{44}$$

for finite, nonzero, x. This can obtained by direct integration, or more simply, from (40) using the identity $K_{1/2}(z) = (\pi/2z)^{-1/2}\exp(-z)$. The functional form of $F(x)$ clearly illustrates the smearing of the fields over a region with correlation length $\sqrt{8\pi}\, L_P$.

The physical content of the above analysis can be viewed as follows. I start with certain loosely defined dynamical variables q_j describing the quantum micro-structure of the spacetime. The dynamical theory describing q_j's must lead, in suitable limit, to a continuum spacetime with quantum states having mean energies much larger than E_P. Among them are the classical spacetimes with compact, infinite redshift surfaces like that of a blackhole forming out of a stellar collapse. I describe these blackhole spacetimes in terms of an intermediate effective field theory in $(D+1)$ dimension. The existence of an infinite redshift surface allows the elementary excitations of this field with arbitrarily high energies to occur in such spacetimes. Such a theory is nonlocal in space and is based on smearing of fields over a correlation length of the order of L_P. The one particle excitations of this field has the correct density of states to describe the black hole.

The above approach also suggests that there is nothing mysterious in completely different microscopic models (like those based on strings or Ashtekar variables) leading to similar results regarding blackhole entropy. Any theory which has the correct density of states can do this; in fact, the models I have described are only a very specific subset of several such toy field theories which can be constructed. The situation is reminiscent of one's attempt to understand the quantum nature of light from blackbody radiation. The spectral form of blackbody radiation can be derived from the assumption that $E = \hbar\omega$ and is quite independent of the details of quantum dynamics of the electromagnetic field. Similarly, the blackhole thermodynamics can be explained if one treats spacetimes with event horizons as highly excited states of a toy, nonlocal field theory whose elementary excitations obey a dispersion relation with the asymptotic form given by (39).

Within the limited point of view of modeling a blackhole, one need not even identify the $(D+1)$ dimensional space as a superset of conventional spacetime. The $\mathbf{x}$ and t could represent variables in some abstract space and the spacetime structure could emerge in a more complicated manner in terms of the fields themselves. However, if we take the postulate of (27) seriously, then it is logical to think of the $(D+1)$ dimensional space to be connected with spacetime in some manner. In that case, the field theoretic model discussed above has no Lorentz invariance at length scales comparable to Planck length. This does not bother me in the least and, in fact, I consider the breakdown of Lorentz invariance at Planck scales almost inevitable. There are physical reasons to believe that one cannot measure length scales with an accuracy greater than Planck length which suggests that some of the basic postulates of special relativity like the propagation of light signals to identify spacetime events etc. will get modified at very small length scales. Lorentz invariance emerges as a symmetry of the continuum spacetime in the models constructed above and hence all the conventional experimental consequences of special relativity, of course, are preserved. The situation is analogous to the emergence of smooth continuum description of a solid from a discrete crystal lattice. The continuum system can possess translational and rotational invariance for infinitesimal translations and rotations; but the microscopic crystal lattice will not respect these symmetries. Similarly, the lattice structure of the quantum spacetime can break the continuum symmetry of Lorentz group. One, of course, needs to study the modified structure which emerges from the above postulate but I am not certain whether this will lead to any unique microscopic description. The ideas presented here also has connections with the principle of path integral duality [14] which I have discussed in detail elsewhere.

Let me now return to the principle of transmutation of Hamiltonian encoded in the equation (27). Such a postulate can be viewed at different levels depending on ones philosophical inclination. We know that, classically, blackhole spacetimes are characterized by very few parameters and — in the context of Schwarzschild blackhole — energy is the only relevant quantity. In the quantum mechanical context, the properties of such a blackhole is encoded in the density of states which has a universal form independent of the original physical system from which the black hole has formed. One way of incorporating this universality will be to postulate that quantum dynamics of blackholes leads to the transmutation of Hamiltonians as suggested by (27). In the case of spacetimes with a timelike Killing vector ξ^a, the Hamiltonian H can be defined as a generally covariant scalar as

$$H = \int T_{ab}\xi^a d\sigma^b \tag{45}$$

where T_{ab} is the stress tensor of matter and $d\sigma^b$ is the element of volume on a spacelike surface. According to equation (27), this H is modified to

$$H^2_{\text{mod}} = A^2 \ln\left(1 + \frac{1}{A^2}\left[\int T_{ab}\xi^a d\sigma^b\right]^2\right) \tag{46}$$

In this approach, H_{mod} remains a covariant scalar. I plan to discuss the consequences of this modification elsewhere.

VII. CONCLUSIONS

Since the point of view advocated in this paper has been described 'online' I shall merely summarize the key results in this section. We begin by assuming that:

(i) There exists certain microscopic degrees of freedom for the spacetime which manifest themselves only at length scales comparable to L_P. The continuum description of spacetime as a solution to Einstein's equation is an approximate one and is similar to the description of a solid by laws of elasticity. In the description of any macroscopic spacetime by some parameters, one is coarse-graining over a large number of microscopic configurations of the quantum spacetime.

(ii) Just as one can provide a thermodynamical description of matter without knowing the details of atomic physics, it is possible to provide a semiclassical description of spacetime which is reasonably independent of the microscopic theory.

Of these two assumptions, the first one seems to be generally accepted by most of the workers in the field. The second one should be treated as a working hypothesis at the moment. Given these two assumptions, one would like to investigate situations in which some properties of the microscopic spacetime structure will manifest itself. I have given detailed arguments in sections 3 and 4 as to why event horizons can help us in this task. The study of event horizons leads to the conclusion that the density of states of blackholes must have a particular form in order to provide the correct thermodynamic description. In fact, any physical system with such a density of state will be indistinguishable from a blackhole as far as thermodynamic interactions are concerned. Since a blackhole can be formed from the collapse of any physical system, the above result suggests the possible existence of some of new physical principle in the theory of quantum blackholes along the following lines: Physical systems characterized by a given Hamiltonian will have particular energy dependence for the density of states. When such a a system collapses to form a blackhole, conserving the energy, the density of state has to change to a universal form. This is possible only if systems which collape to form blackholes are described by an effective Hamiltonian which is related to the original hamiltonian in a particular manner. Using this principle, it is possible to construct several model systems which have the correct density of states for the blackhole. Most important among them are those based on non local field theories with particle states having the same density of states as a blackhole. Several features of such field theories are discussed in section 6.

While it may not be possible to obtain a unique quantum description of spacetime from our knowledge of semi-classical blackhole physics, it does give three clear pointers. First is the indirect, but essential, role played by the infinite redshift surface: It is the existence of such a surface which distinguishes the *star* of mass M from a *blackhole* of mass M. A stellar spacetime will not be able to populate high energy states of the toy field as required by the statistical description; in a blackhole spacetime, virtual modes of arbitrarily high energies near the event horizon will allow this to occur. (It may be possible to model such a process by studying the interaction of this toy field with a more conventional field near the event horizon.) Second one is the universal transformation of the hamiltonian of physical systems which collapses to form the blackhole. This transformation leads to a unique asymptotic form for the dispersion relation for the elementary excitations of the model field theories. The third is the fact that such a dispersion relation almost invariably leads to smearing of local fields over regions of the order of Planck length.

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