

# Doing it with Mirrors: Classical analogues for Black Hole radiation

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We construct analogues for the quantum phenomena of black hole radiation in the context of *classical field theory*. Hawking radiation from a (radially) collapsing star is mathematically equivalent to radiation from a mirror moving along a specific trajectory in Minkowski spacetime. We construct, in this paper, a classical analogue for this flat spacetime quantum phenomenon and use it to construct a classical analogue for black hole radiation.

The radiation spectrum in quantum field theory has the power spectrum as its classical analogue. Monochromatic light is continually reflected off a moving mirror or the silvered surface of a collapsing star. The reflected light is Fourier analysed by the observer and the power spectrum is constructed. For a mirror moving along the standard black hole trajectory, it is seen that the power spectrum consists of three terms: (i) a factor  $(1/2)$  that is typical of the ground state energy of a quantum oscillator, (ii) a Planckian distribution  $N(\Omega)$  and—most importantly—(iii) a term  $\sqrt{N(N+1)}$ , which is the root mean square fluctuations about the Planckian distribution. It is the appearance of the root mean square fluctuations which suggests that we attribute a ‘thermal’ nature to the power spectrum just like its quantum counterpart (which is truly thermal).

Mirror-observer configurations like an inertial mirror observed in an accelerated observer’s frame and an uniformly accelerated mirror observed in a Rindler frame are investigated and conditions under which a “thermal” power spectrum is obtained are derived. The corresponding results in the black hole case are then elucidated. It is seen that a “thermal” spectrum can arise either due to the collapse of the star or due to the motion of the observer in the Schwarzschild spacetime. The “temperature” of the Planckian spectrum seen is therefore dependent on whether the star is in motion or the observer is in motion. In the latter case it is possible to obtain a “temperature” which is entirely independent of the mass of the star that is being observed.

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## I. INTRODUCTION AND SUMMARY

Hawking radiation from black holes is a quantum gravitational effect which is derived using the framework of quantum field theory in curved spacetime. A radially collapsing star of mass  $M$  is studied close to its event horizon  $r = 2M$ . The system is explicitly time-dependent and standard quantum field theoretic methods, which are applicable in the semi-classical limit, can be used to study it. Due to the collapse of the star, the in-vacuum is not the same as the out-vacuum and hence the Bogoliubov coefficients connecting the two vacua are non-zero implying particle production. The radiation essentially arises due to the redshifting of vacuum modes near the event horizon. It is shown in [4] that the process by which Hawking radiation is derived is mathematically equivalent to the radiation spectrum seen due to the motion of mirrors in  $(1+1)$ -dimensional Minkowski spacetime. The mirror moves along a late time trajectory of the form

$$x(t) = A - t - Be^{-Dt} \quad (1.1)$$

where  $A$ ,  $B$  and  $D$  are arbitrary constants with  $D > 0$ . We shall refer to trajectories of the above form as black hole trajectories since the motion of the surface of a collapsing star near its event horizon follows a similar trajectory. In this case too, the acceleration of the mirror causes the redshifting of vacuum modes giving rise to non-zero Bogoliubov coefficients and hence particle production. This equivalence suggests that radiation from black holes can be modelled using simpler concepts from standard flat space quantum field theory [2–6].

In this paper we ask whether there exist analogues to the above quantum phenomena in the context of classical field theory. In classical field theory, concepts such as vacuum and quantum fluctuations are absent. To construct a viable classical model in concrete terms, we adopt the following procedure. An observer shines monochromatic

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light on a object (which can be a mirror in flat spacetime or the silvered surface of a star in Schwarzschild spacetime) which is moving along a specified trajectory in the observer's rest frame. The reflected light is fourier analysed by the observer and the power spectrum is calculated. We will regard the power spectrum as the classical analogue of the radiation spectrum that arises in quantum field theory in flat and curved spacetime. The crucial difference between the two is that the radiation spectrum is a function of the photon energy (expressed as  $\hbar\Omega$  with  $\Omega$  being the mean photon frequency) which explicitly contains  $\hbar$  while the power spectrum is a function of the fourier transform frequency  $\Omega$  alone and does not contain  $\hbar$ . As it stands, the power spectrum too is a well defined and measurable quantity though in the limit of low frequencies (like radio frequencies) or a large number of photons where the classical limit is applicable. In order to state that a particular classical power spectrum is the analogue of the corresponding quantum radiation spectrum, similarities in the *form* of the two spectra are considered. The specific terms of the power spectrum are then suitably interpreted based on the properties of its quantum analogue. The focus, in this paper, will be on the thermal radiation spectrum obtained in the study of collapsing stars and moving mirrors in quantum field theory. Its classical analogue is described in the following paragraph.

It is a well known result [2,4] that, for mirrors or a collapsing star travelling along a black hole trajectory given in Eqn. (1.1), a thermal radiation spectrum is obtained. Not only is the mean occupation number in any mode Planckian in form, but the fluctuations about the mean are also characterised by standard thermal noise. We look at classical scenarios in which the asymptotic trajectory is that given in Eqn. (1.1) and whose *power spectrum* consists of terms that motivate us to attribute a "thermal" nature [1] to it. The "thermal" spectrum consists of three terms, none of which have a classical meaning. These three terms are: (i) a factor (1/2) that is typical of the ground state energy of a quantum oscillator, (ii) a Planckian distribution  $N(\Omega)$  and—most importantly—(iii) a term  $\sqrt{N(N+1)}$ , which is the root mean square fluctuations about the Planckian distribution. It is due to the appearance of the root mean square fluctuations that the power spectrum is considered to have a 'thermal' nature attributed to it. It must be emphasised again that the systems we are considering have no fluctuations or temperature in the sense of statistical physics. Being classical systems, no quantum fluctuations are present either. But the three terms obtained above have a natural interpretation in terms of notions such as thermal spectrum and its fluctuations. It was shown in Ref. [1] that such a "thermal" power spectrum also arises when a real, monochromatic plane wave of frequency  $\omega$  is fourier analysed in the frame of a uniformly accelerating observer. Even in the limit of  $\omega \rightarrow 0$ , where the original wave tends to a constant, the power spectrum still remains "thermal". A constant field is the closest to what one can call a 'classical' vacuum. The effect of the thermal property of the power spectrum surviving even in the case of a wave with infinitesimal frequency can clearly be interpreted as the classical analogue of the quantum phenomenon in which a Rindler observer sees a thermal spectrum when the field is in the Minkowski vacuum state. However, in the case of the moving mirror and collapsing star systems studied here, the power spectrum is found to be "thermal" strictly for non-zero incident frequencies. The limit of  $\omega \rightarrow 0$  cannot be applied to the power spectrum as in [1] and consequently one cannot envisage the existence of a "classical" vacuum state for a black hole or moving mirror. The work presented here is a logical extension of that in Ref. [1]. For related work see Ref. [7–9]

We study two specific mirror-observer systems in Minkowski spacetime and their analogues in the Schwarzschild spacetime (which are star-observer systems) for which the power spectrum (of the reflected light) in the observer's frame of reference is "thermal" in the sense explained in the previous paragraph. The two mirror-observer configurations that we consider are (i) an inertial mirror viewed in an accelerated observer's frame and (ii) an uniformly accelerated mirror viewed in an uniformly accelerated observer's frame of reference. In case (i), the mirror always moves along a geodesic in Minkowski spacetime while the observer does not. This is analogous to the quantum system consisting of a Rindler observer in the Minkowski vacuum with the mirror as a boundary. Two situations relevant to a collapsing star scenario are considered. The first is that of an observer who is uniformly accelerating only for times greater than an initial proper time  $t_i > 0$  and is inertial before that. Such a situation in Schwarzschild spacetime would correspond to a star that starts collapsing at an initial time  $t_i$  and thereafter collapses to form a black hole. The second is that of an observer who is uniformly accelerating only between times  $t_i$  and  $t_f > t_i$  and is inertial otherwise. This system is more physically relevant since the observer accelerates only for a finite time interval. In Schwarzschild spacetime, such a situation would correspond to a star which starts collapsing at an initial time  $t_i$  and stops at time  $t_f$ . In other words, it does not collapse all the way to form a black hole. For all these cases, definite and, in principle, observable conditions are derived such that the power spectrum is "thermal".

Case (ii) consists of a system in which both the mirror and the observer are moving along non-inertial uniformly accelerating trajectories. The trajectories, which are both hyperbolae in Minkowski spacetime, are chosen so that their centers (the center of a hyperbola is defined as the point where the asymptotes of the hyperbola cross) are shifted with respect to each other. The motion of the mirror in the observer's frame is therefore explicitly time-dependent and in certain situations of relevance gives rise to "thermal" power spectra. The quantum analogue of such a system would possibly be that a Rindler observer sees particles in the vacuum of a shifted Rindler frame. This quantum aspect will be dealt with suitably in a future publication. If the centers coincide, however, the mirror remains stationary

with respect to the observer for all time and the power spectrum reduces to a constant. The quantum analogue of this is the obvious result that a Rindler observer sees no particles in the vacuum of the same Rindler frame. The corresponding system in the Schwarzschild spacetime is that of a static star viewed by an observer moving away along a specific non-geodesic trajectory. This trajectory is such that the observer's frame of reference appears to possess a event horizon that is dependent entirely on the observer's motion and not on the star. Consequently, the "thermal" nature of the power spectrum arises solely due to the presence of this apparent horizon.

This paper is organised as follows. In section (II) we outline the solution for a scalar field in  $(1+1)$ -dimensions in the presence of an arbitrarily moving mirror. In section (III) we discuss the system of an inertial mirror viewed in an uniformly accelerated observer's frame while in section (IV) we study the system of an accelerated mirror viewed in an accelerated observer's frame. Finally in section (V) we study the analogues of the mirror configurations in the Schwarzschild spacetime.

## II. MOVING MIRRORS IN $(1+1)$ -DIMENSIONS

In this section we describe the purely classical problem of light reflected off an arbitrarily moving mirror in a  $(1+1)$ -dimensional conformal spacetime. We work in  $(1+1)$ -dimensions since, in this case, the line element can always be made conformally flat for any arbitrary spacetime metric. This conformal property simplifies the problem greatly since the mode functions of a massless scalar field in conformal co-ordinates are plane waves.

We first briefly mention the solution to the massless scalar field with the reflection boundary condition imposed on it (see, for instance Ref [2]). A minimally coupled massless scalar field satisfies the Klein-Gordon equation

$$\square\Phi \equiv \Phi^{i\mu}{}_{;i\mu} = 0. \quad (2.1)$$

In any conformally flat spacetime, the basis solutions to the above Klein-Gordon equation in the coordinates  $(t, x)$  can be taken to be plane waves labeled by the frequency  $\omega$ . Let the mirror move along an arbitrary trajectory  $x_m(t)$ . The solution to the above equation can be written in the form

$$\Phi(t, x) = e^{-i\omega(t+x)} + f((t-x)). \quad (2.2)$$

This particular solution describes a situation where left moving plane waves, denoted by the waves  $\exp(-i\omega(t+x))$ , are incident on the mirror which acts as a boundary, and the resulting reflected wave, which consists of a right moving wave, is denoted by the as yet unknown function  $f(t-x)$ . Therefore, the solution to the right of the mirror is the superposition of the left moving and right moving waves while to the left of the mirror, the solution is identically zero. The reflection boundary condition is imposed by demanding that

$$\Phi(t, x_m(t)) = 0 \quad (2.3)$$

This implies that

$$f(t-x_m(t)) = -e^{-i\omega(t+x_m(t))} \quad (2.4)$$

Using the definitions  $u = t-x$  and  $v = t+x$ , the solution for an arbitrarily moving mirror can be written as

$$\Phi(u, v) = e^{-i\omega v} - e^{-i\omega(2\tau-u)}, \quad (2.5)$$

where  $\tau$  is determined by solving the equation

$$\tau - x_m(\tau) = u \quad (2.6)$$

Since, in classical field theory, only real waves are of significance, the reflected light can be written in the form

$$\Phi_R = \cos(\omega(2\tau-u)). \quad (2.7)$$

where we have omitted the contribution from the incident waves. Note that, in three dimensions, or for a massive scalar field, one cannot use the above method to solve for the field with the mirror trajectory acting as the boundary. However, in the special case of the three dimensional wave vector  $\mathbf{k}$  of the massless field being restricted to any one space dimension, say  $\mathbf{k} = (\omega, 0, 0)$ , then the above solution is obviously still valid.

The above result can be rederived in an equivalent manner by studying the redshift of the incoming modes caused due to the movement of the mirror. Let the mirror move in an arbitrary trajectory  $x_m(t)$  in  $(1+1)$  dimensional

Minkowski (or conformal) spacetime. Consider an observer positioned at the origin and shining a beam of light with the frequency  $\omega$  at the mirror. To be specific, let the mirror be situated to the *left* of the observer. The incident beam of light is represented as  $(\omega, k)$  with the wave vector  $k = -\omega$ . Let the light beam reach the mirror at the time  $t_0$ . At this time, the mirror has a velocity  $v_m(t_0)$  and therefore, in a Lorentz frame moving with the velocity  $v_m(t_0)$  in which the mirror is instantaneously at rest, the incoming light beam appears to be doppler shifted with a new frequency  $\omega'$  and wave vector  $k'$  given by the relations

$$\omega' = \omega \sqrt{\frac{1 + v_m(t_0)}{1 - v_m(t_0)}} \quad ; \quad k' = -\omega \sqrt{\frac{1 + v_m(t_0)}{1 - v_m(t_0)}} \quad (2.8)$$

After reflection, the reflected wave has the same frequency  $\omega'$  in the instantaneous Lorentz frame, but the wave vector changes sign to  $-k'$ . Therefore, the quantities  $(\omega_R, k_R)$ , which refer to the reflected light in the observer's frame of reference, are

$$\omega_R = \omega' \sqrt{\frac{1 + v_m(t_0)}{1 - v_m(t_0)}} \quad ; \quad k_R = \omega' \sqrt{\frac{1 + v_m(t_0)}{1 - v_m(t_0)}} \quad (2.9)$$

where the corresponding *inverse* Lorentz transformations have been used to transform back to the observer's frame. Substituting for  $\omega'$  from Eqn. (2.8) into Eqn. (2.9), one obtains,

$$\omega_R = \omega \left( \frac{1 + v_m(t_0)}{1 - v_m(t_0)} \right) \quad ; \quad k_R = \omega \left( \frac{1 + v_m(t_0)}{1 - v_m(t_0)} \right) \quad (2.10)$$

Let this reflected light reach the observer at time  $t > t_0$ . The phase of the reflected wave  $\theta(t)$  reaching the observer as a function of  $t$  is given by

$$\theta(t) = \int dt \omega_R(t_0(t)) = \omega \int dt \left( \frac{1 + v_m(t_0)}{1 - v_m(t_0)} \right) \quad (2.11)$$

The relationship between  $t$  and  $t_0$  is quite obviously

$$t - t_0 = -x_m(t_0) \quad (2.12)$$

where we have taken into account the fact that the mirror is situated to the left of the observer and  $x_m(t_0)$  is the distance between the mirror and observer at time  $t_0$ . The above equation is exactly the same equation as in Eqn. (2.5) with  $u$  replaced by  $t$  (the observer is seated at the origin and so  $u = t - x \equiv t$ ) and  $\tau$  replaced by  $t_0$ . If the observer were situated at some other position  $x$ , then it is easy to see that the above equation still holds with  $t$  replaced by  $t - x$  since the wave reaching the observer is a right moving wave. From Eqn. (2.12) we have,

$$\frac{dt_0}{dt} = \frac{1}{1 - v_m(t_0)} \quad (2.13)$$

Using this in Eqn. (2.11), one has

$$\theta(t) = \omega \left[ 2 \int \frac{dt}{1 - v_m(t_0)} - t \right] = \omega(2t_0 - t) \quad (2.14)$$

with  $t_0$  being given in terms of  $t$  by solving Eqn. (2.12). Thus, the phase of the reflected wave given in the above equation matches that in Eqn. (2.4) with  $u$  replaced by  $t$  and  $\tau$  replaced by  $t_0$ .

With the basic formalism outlined in this section, we move on to describing scenarios where the mirror trajectory assumes the trajectory given in Eqn. (1.1) which gives a power spectrum that is "thermal" in form.

### III. INERTIAL MIRROR VIEWED IN AN ACCELERATED OBSERVER'S FRAME

In this section, we study a system consisting of a mirror at rest in Minkowski spacetime viewed in an uniformly accelerated observer's frame of reference. The mirror moves along a geodesic while the motion of the observer is non-geodesic. Consider an observer who is accelerating uniformly in Minkowski spacetime with an acceleration

$g > 0$ . Let the co-ordinates of the observer's frame of reference be denoted by the conformal co-ordinates  $(t, x)$ . The transformations [11] between the Minkowski co-ordinates  $(T, X)$  and the Rindler co-ordinates  $(t, x)$  are

$$T = T_o + g^{-1}e^{gx} \sinh(gt) \quad ; \quad X = X_o + g^{-1}e^{gx} \cosh(gt) \quad (3.1)$$

where  $X_o$  and  $T_o$  are arbitrary constants. The trajectory of an observer at rest in the accelerated frame is a hyperbola whose center (the center of a hyperbola is defined as the point at which the asymptotes of the hyperbola cross) is shifted from the Minkowski spacetime origin by the vector  $(T_o, X_o)$ . Let the mirror be at rest at a spatial distance  $X_m$  from the origin of co-ordinates. The trajectory of the mirror as viewed in the Rindler frame is

$$X_m = X_o + g^{-1}e^{g x_m(t)} \cosh(gt) \quad (3.2)$$

which implies

$$g x_m(t) = \ln [g(X_m - X_o)] - \ln [\cosh(gt)] \quad (3.3)$$

Note that  $X_m$  has to be chosen so that  $X_m - X_o > 0$ . This is necessary because otherwise the reflected light will never reach the Rindler observer. (If  $X_m = X_o$ , the mirror is always at the event horizon  $x_m = -\infty$  for all time and hence not of interest.) Since only the combination  $(X_m - X_o)$  appears, we can set  $X_o = 0$  without any loss of generality. Therefore, as observed in the Rindler frame, the mirror advances from the event horizon (for times  $t \rightarrow -\infty$ ) to a point  $x_m = g^{-1} \ln(gX_m)$  (at  $t = 0$ ) and recedes back into the horizon (for times  $t \rightarrow \infty$ ). Let the observer be seated at a distance  $x_0$  (with  $u = t - x_0$ ). To avoid a collision with the mirror one must have  $x_0 > g^{-1} \ln(gX_m)$ . The Rindler observer continuously shines monochromatic light of frequency  $\omega$  on the moving mirror and receives the reflected light. Solving Eqn. (2.6), the reflected wave (see Eqn. (2.7)) is

$$\Phi_R = \cos [\omega g^{-1} \ln(2gX_m e^{gu} - 1) - \omega u] \quad (3.4)$$

The exact fourier transform of this real wave with respect to  $t$  along the entire trajectory of the mirror yields a fourier amplitude given in terms of Beta functions which is not of interest here. In a collapse scenario, such a model would correspond to a star expanding from the event horizon to a finite radius and then collapsing back again into the horizon. This is clearly not a physically relevant situation. Instead, we focus on the mirror trajectory at late times  $t > (1/g)$  when it is receding into the event horizon. Notice that this late time condition is similar to the condition  $t > (\text{surface gravity})^{-1} = (4M)^{-1}$  for Hawking radiation to arise for a collapsing star. The role of the "surface gravity" is played by  $g$  in this case. So, for late times, one has,

$$g x_m(t) = \ln(2gX_m) - gt - e^{-2gt} + \mathcal{O}(e^{-4gt}) \quad (3.5)$$

where  $\mathcal{O}$  is the order symbol which indicates the order of the terms being neglected. In an expression of the form  $Q = R + \mathcal{O}(x^\alpha)$ , the term  $\mathcal{O}(x^\alpha)$  indicates that  $Q = R$  to order  $x^\alpha$ . All powers of  $x$  greater than or equal to  $\alpha$  are ignored. The reflected wave reduces to,

$$\Phi_R = \cos [\omega A - \omega B e^{-gt} + \mathcal{O}(e^{-2gt})] \quad (3.6)$$

where

$$A = g^{-1} \ln(2gX_m) \quad ; \quad B = \frac{1}{2g^2 X_m} e^{g x_0} \quad (3.7)$$

When taking the fourier transform of the reflected wave, we integrate, not from  $(-\infty, \infty)$ , but from  $(t_i, \infty)$  where  $t_i \sim (1/g)$ . (In calculating the order in Eqns. (3.6), (3.5) the quantity  $x_0$  in  $u = t - x_0$  has been assumed to be either negative or small compared to  $(1/g)$  if positive. If  $x_0 \sim (1/g)$  say, the reflected wave can assume the form given in Eqn. (3.6) only if the order of the terms neglected is  $\mathcal{O}(e^{-2gt})$ . This in turn implies that the late time trajectory has to hold for  $t > x_0 + (1/g)$  and not just for  $t > (1/g)$ .) It is important to note that we have assumed that the trajectory for times  $t < t_i$  is inertial. Consequently the contribution to the reflected wave from times  $t < t_i$  can be safely ignored. In a realistic collapse scenario, this model would correspond to a stationary star that starts collapsing at a finite time and which collapses all the way to form a black hole and thus is physically feasible. To compute the fourier transform, it is convenient to decompose the reflected wave as follows:

$$\Phi_R = \psi_R + \psi_R^* \quad (3.8)$$

where

$$\psi_R = \frac{1}{2} e^{i\omega A} e^{-i\omega B e^{-gt}} \quad (3.9)$$

with a corresponding expression for  $\psi_R^*$ . The fourier transform of  $\psi_R$  is given by the expression,

$$\tilde{\psi}_R = \int_{t_i}^{\infty} dt e^{-i\Omega t} \psi_R = \frac{1}{2g} e^{i\omega A} (\omega B)^{-i\Omega/g} \int_0^{\omega B e^{-gt_i}} \frac{dy}{y} y^{i\Omega/g} e^{-iy} \quad (3.10)$$

where we have made the change of variable  $y = \omega B e^{-gt}$ . Evaluating  $\tilde{\psi}_R$  by appropriately rotating in the complex plane, one obtains,

$$\tilde{\psi}_R(\Omega) = \frac{1}{2g} e^{i\omega A} (\omega B)^{-i\Omega/g} e^{\pi\Omega/2g} \left\{ \Gamma[i\Omega/g] - \mathcal{O}\left(\frac{e^{-\omega B e^{-gt_i}}}{\omega B e^{-gt_i}}\right) \right\} \quad (3.11)$$

where  $\omega$  must be chosen appropriately so that the above relation is correct to  $\mathcal{O}(e^{-\omega B e^{-gt_i}}/\omega B e^{-gt_i})$ . This implies that  $\omega$  cannot be arbitrarily small. It must at the very least satisfy the inequality  $\omega > B e^{-gt_i}$ . Evaluating the fourier transform of  $\psi_R^*$  in a similar manner and adding it to the expression for  $\tilde{\psi}_R$ , the fourier transform of  $\Phi_R$  is given by,

$$\tilde{\Phi}_R(\Omega) = \frac{1}{2g} (\omega B)^{-i\Omega/g} \Gamma[i\Omega/g] \left( e^{i\omega A} e^{\pi\Omega/2g} + e^{-i\omega A} e^{-\pi\Omega/2g} \right) \quad (3.12)$$

The power spectrum per unit logarithmic frequency interval [1] is,

$$\mathcal{P}(\Omega) \equiv \Omega |\tilde{\Phi}(\Omega)|^2 = \left(\frac{\pi}{g}\right) \left\{ \frac{1}{2} + N + \sqrt{N(N+1)} \cos(2\beta) \right\} \quad (3.13)$$

where  $N$  is defined to be

$$N(\Omega) = \left( \frac{1}{\exp(\Omega/\Omega_0) - 1} \right). \quad (3.14)$$

and

$$\Omega_0 = g/2\pi \quad ; \quad \beta = \omega A = \omega g^{-1} \ln(2gX_m) \quad (3.15)$$

where we have substituted the expressions for  $A$  and  $B$  from Eqn. (3.7). This power spectrum, as explained in section (I), has the appearance of a ‘‘thermal’’ spectrum. It can be therefore regarded as a fit classical analogue of its quantum counterpart, the radiation spectrum. (For a detailed discussion see Ref. [1].) The incident frequency  $\omega$  must at least satisfy an inequality of the form

$$\omega > \frac{2g^2 X_m}{c^3} e^{g(t_i - x_0)/c} \quad (3.16)$$

where the correct factors of  $c$  have been put in, for the power spectrum to have the form above.

Note that since the trajectory (3.3) is symmetric in the time co-ordinate, considering the *early* time behaviour of the mirror trajectory gives the same power spectrum per unit logarithmic frequency interval as the late time behaviour did. This, however, is not of physical relevance since such a model would correspond to a star expanding outwards from the event horizon to a finite radius.

We now analyse a situation in which the mirror has the trajectory given in Eqn. (3.5) for a given time interval and ask under what conditions would a spectrum of the form in Eqn. (3.13) be obtained. To make things more specific, consider a mirror trajectory of the form

$$gx_m(t) = \ln(2gX_m) - gt - e^{-2gt} \quad t_i < t < t_f \quad (3.17)$$

for  $t_i < t < t_f$  and inertial otherwise. In Minkowski space, this corresponds to the observer accelerating uniformly only between the times  $t_i$  and  $t_f$  (and moving along an inertial trajectory for  $t < t_i$  and  $t > t_f$ ) with the mirror placed as before at the spatial distance  $X_m$ . The mirror trajectory in Eqn. (3.17) is a good approximation to the exact trajectory in Eqn. (3.3) if  $t_i \sim (1/g)$ . The reflected waveform is of the same form as in Eqn. (3.6). Decomposing it into the form given in Eqn. (3.8), the fourier transform of  $\psi_R$  is now given by

$$\tilde{\psi}_R(\Omega) = \int_{t_i}^{t_f} dt e^{-i\Omega t} \psi_R = \frac{1}{2g} e^{i\omega A} (\omega B)^{-i\Omega/g} \int_{\omega B e^{-gt_f}}^{\omega B e^{-gt_i}} \frac{dy}{y} y^{i\Omega/g} e^{-iy} \quad (3.18)$$

where the usual change of variable  $y = \omega B e^{-gt}$  has been made. Setting  $\omega B e^{-gt_f} = \epsilon$ , the above integral can be evaluated to give,

$$\tilde{\psi}_R(\Omega) = \frac{1}{2g} e^{i\omega A} (\omega B)^{-i\Omega/g} \left\{ e^{\pi\Omega/2g} \left[ \Gamma[i\Omega/g] - \mathcal{O}\left(\frac{e^{-\omega B e^{-gt_i}}}{\omega B e^{-gt_i}}\right) \right] - \frac{\epsilon^{i\Omega/g}}{i\Omega/g} + \mathcal{O}(\epsilon) \right\} \quad (3.19)$$

Evaluating  $\tilde{\psi}_R^*$  in a similar manner and neglecting the correction terms, the expression for  $\tilde{\Phi}_R(\Omega)$  can be written as,

$$\tilde{\Phi}_R(\Omega) = \frac{1}{2g} (\omega B)^{-i\Omega/g} \left( e^{i\omega A} e^{\pi\Omega/2g} + e^{-i\omega A} e^{-\pi\Omega/2g} \right) \Gamma[i\Omega/g] + \frac{i}{\Omega} \cos(\omega A) e^{-i\Omega t_f} \quad (3.20)$$

where we have substituted for  $\epsilon$  in terms of  $t_f$ . The power spectrum per unit logarithmic frequency interval is easily calculated to be

$$\mathcal{P}(\Omega) \equiv \Omega |\tilde{\Phi}_R(\Omega)|^2 = \left(\frac{\pi}{g}\right) \left\{ \frac{1}{2} + N + \sqrt{N(N+1)} \cos(2\beta) \right\} + \frac{1}{\Omega} \cos^2(\omega A) + \sqrt{\frac{\pi}{2g\Omega}} \cos(\omega A) \left\{ \cos(\omega A) \sin(\alpha) \sqrt{1+2N} + \sin(\omega A) \cos(\alpha) \frac{1}{\sqrt{1+2N}} \right\} \quad (3.21)$$

where  $N$  is defined as in Eqn. (3.14) and the quantity  $\alpha$  is given by

$$\alpha = \Omega (t_f - g^{-1} \ln(\omega B)) + \arg(\Gamma[i\Omega/g]) \quad (3.22)$$

To neglect the correction terms in Eqn. (3.19),  $\omega$  must at least satisfy the inequality given in Eqn. (3.16) while  $t_f$  should be chosen so that  $t_f - t_i > (1/g)$ . If  $t_f$  is chosen to be large enough,  $\alpha \rightarrow \infty$  and so the second term in curly braces can be set to zero. But there is still a term proportional to  $1/\Omega$  which diverges for small  $\Omega$ . However, notice that when  $\Omega > g$ ,  $N(\Omega) \simeq e^{-\Omega/\Omega_0}$ . When this is satisfied, both the extra terms can be neglected and one obtains the usual thermal spectrum. Summing up, one sees that for a mirror moving along the trajectory (3.17) for times  $t_i < t < t_f$ , the power spectrum matches that given in Eqn. (3.13) for  $\Omega > g$  when the conditions  $t_i \sim (1/g)$ ,  $t_f - t_i > (1/g)$  and the inequality in Eqn. (3.16) are satisfied.

#### IV. ACCELERATED MIRROR VIEWED IN AN ACCELERATED OBSERVER'S FRAME

We next consider the system of an uniformly accelerated mirror viewed in an uniformly accelerated observer's frame. The trajectories of both the observer and the mirror are hyperbolae in Minkowski spacetime. However, the centers<sup>1</sup> of both these hyperbolae are shifted. Such a situation is an explicitly time-dependent one with the mirror moving from in from the event horizon of the observer and receding back. We will concentrate as usual only on the receding part of the mirror's trajectory.

The transformations between the Minkowski co-ordinates  $(T, X)$  and the Rindler co-ordinates  $(t, x)$  of the observer's frame are given as usual by

$$T = T_o + g^{-1} e^{gx} \sinh(gt) \quad ; \quad X = X_o + g^{-1} e^{gx} \cosh(gt) \quad (4.1)$$

where  $g$  is the acceleration of the observer and  $T_o$  and  $X_o$  are arbitrary constants. We assume  $X_o > 0$  here and discuss the case  $X_o < 0$  later. Thus, the center of the hyperbolic trajectory of an observer seated at a distance  $x_0$  is at  $(T_o, X_o)$ . The trajectory of the mirror in Minkowski space is chosen to be the hyperbola

<sup>1</sup>The center of a hyperbola is the point at which its asymptotes cross. For the usual Rindler observer, the center is just the Minkowski spacetime origin.

$$X^2 - T^2 = g_m^{-2} \quad (4.2)$$

where  $g_m$  is the acceleration of the mirror and whose center is at the origin. This can always be done without any loss of generality since only the relative motion between the mirror and the observer is of consequence. We assume that the mirror is confined to the right Rindler sector with  $X > 0$  always. The trajectory of the mirror in the observer's frame is obtained by substituting for  $T$  and  $X$  from Eqn. (4.1) into Eqn. (4.2). Doing this one obtains the following quadratic equation in the variable  $e^{gx_m(t)}$ .

$$g^{-2}e^{2gx_m} + 2g^{-1}e^{gx_m} [X_o \cosh(gt) - T_o \sinh(gt)] = g_m^{-2} - (X_o^2 - T_o^2) \quad (4.3)$$

We now set  $T_o = 0$  for simplicity. The case  $T_o \neq 0$  offers no new possibilities and will not be considered further. The above equation is easily solved to give

$$e^{gx_m(t)} = -gX_o \cosh(gt) \pm \sqrt{g^2 X_o^2 \cosh^2(gt) + g^2(g_m^{-2} - X_o^2)} \quad (4.4)$$

Notice that only the positive sign leads to real  $x_m(t)$  since  $X_o > 0$ . If  $X_o = 0$ , then the solution reduces very simply to  $x_m(t) = g^{-1} \ln(g/g_m)$ . The mirror remains at a constant distance in the observer's frame for all time. In Minkowski space, the centers of the hyperbolic trajectories of the observer and mirror coincide and one gets the trivial result of a constant power spectrum. The quantum analogue of this result merely states that a Rindler observer sees no particles in the Rindler vacuum. This can be made more precise by constructing the Wightman function in the Rindler frame (which is identical to that in Minkowski space since the Rindler frame is conformal to it) for a trajectory that is inertial in the observer's frame. The detector response is determined by fourier transforming the Wightman function. The expected answer is zero. Hence, this classical result does have a corresponding quantum analogue.

For a non-trivial time-dependent solution for  $x_m(t)$  to exist, one must have  $(g_m^{-2} - X_o^2) > 0$ . (The case  $(g_m^{-2} - X_o^2) = 0$  just corresponds to a situation in which the mirror is at the event horizon of the observer's frame for all times since  $x_m(t) = -\infty$ .) Since a collision between the observer sitting at the point  $x_0$  and the mirror is unwanted,  $x_0$  should satisfy the inequality,

$$x_0 > g^{-1} \ln(g[g_m^{-1} - X_o]) \quad (4.5)$$

For late times  $t > 0$  such that

$$\cosh(gt) \gg \sqrt{\frac{g_m^{-2} - X_o^2}{X_o^2}}, \quad (4.6)$$

we have,

$$e^{gx_m(t)} = -gX_o \cosh(gt) + gX_o \cosh(gt) \sqrt{1 + \frac{g_m^{-2} - X_o^2}{X_o^2 \cosh^2(gt)}} \approx \frac{g(g_m^{-2} - X_o^2)}{2X_o \cosh(gt)} \quad (4.7)$$

which implies that

$$gx_m(t) = \ln \left[ \frac{g(g_m^{-2} - X_o^2)}{2X_o} \right] - \ln[\cosh(gt)] \quad (4.8)$$

Therefore, one sees that the trajectory of the mirror as seen by the observer resembles that of a inertial mirror viewed in an uniformly accelerated observer's frame which was treated in the last section. Such a trajectory has exactly the same form as that in Eqn. (3.3). Hence, the late time power spectrum per unit logarithmic frequency interval has the same form as that in Eqn. (3.13) but with  $\beta$  given by

$$\beta = \omega g^{-1} \ln \left( \frac{g(g_m^{-2} - X_o^2)}{X_o} \right) \quad (4.9)$$

The same condition on  $\omega$  in Eqn. (3.16) must hold for the spectrum to be thermal. Knowing the classical result we can attempt to predict the analogous quantum result. The classical result appears to indicate that the vacua of two Rindler frames whose centers in Minkowski spacetime are shifted are inequivalent. This means that the Bogolibov coefficients between the two vacua must be non-zero implying particle production. Since the classical power spectrum is "thermal", the quantum radiation spectrum can also be expected to be thermal in nature. An observer present in one Rindler frame therefore sees a thermal distribution of photons when the scalar field is in the vacuum of the

shifted frame. This result is characteristic of the Minkowski-Rindler system and the connection does not appear to be an obvious one. We will explore this quantum analogue more fully in a future publication.

We now briefly discuss the case when  $X_o < 0$ . Setting  $X_o = -|X_o|$  in Eqn. (4.4), one finds that

$$e^{g x_m(t)} = g|X_o| \cosh(gt) \pm \sqrt{g^2|X_o|^2 \cosh^2(gt) + g^2(g_m^{-2} - |X_o|^2)} \quad (4.10)$$

The simplest case above is when  $g_m^{-2} - |X_o|^2 = 0$ . The only non-trivial solution is (the trivial solution is  $x_m(t) = -\infty$  which is uninteresting since it implies that the mirror remains at the event horizon for all time)

$$x_m(t) = g^{-1} \ln\left(\frac{2g}{g_m}\right) + \ln[\cosh(t)] \quad (4.11)$$

The crucial difference between the above solution and that in Eqn. (4.8) is the presence of a positive sign before the  $\ln[\cosh(t)]$  term. Note that the mirror is always to the *right* of the observer. To avoid a collision between the mirror and the observer, the observer has to be stationed at points  $x_0 < g^{-1} \ln(2g/g_m)$ . Modifying the formalism developed in section (II) appropriately (replacing  $u = t - x$  by  $v = t + x$ ) to take into account this case, one finds that the late time power spectrum (for  $t > 1/g$  as usual and making the usual approximations made in section (III)) is exactly the same as that in Eqn. (3.13) with  $\beta = \omega g^{-1} \ln(g/g_m)$ . This shows that as long as the trajectory is of the form in Eqn. (1.1), one obtains a thermal power spectrum regardless of the presence of an event horizon or not. The above solution naturally cannot be used as a model since its Schwarzschild analogue would be an observer seated *inside* an expanding star watching its surface expand out to infinity. In the case  $g_m^{-2} - |X_o|^2 > 0$ , only the positive sign in Eqn. (4.10) should be chosen for the solution to be physically relevant and it is easy to convince oneself that the late time behaviour corresponds exactly with that for the case  $g_m^{-2} - |X_o|^2 = 0$ . When  $g_m^{-2} - |X_o|^2 < 0$ , the minus sign can also be chosen to provide a valid solution (choosing the positive sign gives a solution that is qualitatively the same as that considered above). But, it is easy to check that in this case, the mirror must collide with the observer when the late time behaviour is considered. This is not a physically suitable solution.

## V. SCHWARZCHILD SPACETIMES: A CLASSICAL CALCULATION

We now study the system consisting of a star whose surface is silvered and an observer situated well outside it. The observer shines monochromatic light of frequency  $\omega$  on the star and fourier transforms the reflected light. We consider two specific star-observer configurations in analogy to the mirror-observer ones studied in sections (III,IV). The first configuration deals with a system in which the star is collapsing with the observer being stationary while the second deals with a system in which the star is static with the observer moving away along a specified trajectory. Since these are two distinct cases, they will be dealt with in two separate sections. In the first section, the trajectory of a radially collapsing surface near the event horizon will be derived using the Hamilton-Jacobi formalism for black holes [10] and the power spectrum is computed accordingly. In the next section, the Schwarzschild analogy of the results in section (IV) will be derived using the Kruskal extension.

### A. Thermal spectrum from collapsing star

We consider here a collapsing star of mass  $M$  whose surface moves along an ingoing radial geodesic. The spacetime outside the star is just the Schwarzschild geometry. The observer is stationed at a constant radius  $r_0$  and is positioned outside the star for all time. We work in  $(1+1)$ -dimensions as done in the case of the mirrors. It is also assumed that the surface of the star is silvered. The observer shines light along radial geodesics on the surface and fourier analyses the reflected light and subsequently calculates the power spectrum.

We will do the following calculations in a more general Schwarzschild-like spacetime and then specialise to the usual Schwarzschild metric. Consider the Schwarzschild-like metric in  $(1+1)$ -dimensions.

$$ds^2 = B(r)dt^2 - \frac{dr^2}{B(r)} \quad (5.1)$$

We consider spacetimes that have a horizon at  $r = r_h$  such that  $B(r)$  has the form

$$B(r) = B_1(r - r_h) + \mathcal{O}([r - r_h]^2) \quad (5.2)$$

near the surface  $r = r_h$ .  $B_1$  is the first derivative of  $B(r)$  evaluated at  $r = r_h$  and is assumed to be non-zero. Define a new variable  $r^*$ ,

$$r^* = \int \frac{dr}{B(r)} \quad (5.3)$$

to obtain the conformal metric

$$ds^2 = B(r^*) [dt^2 - d(r^*)^2] \quad (5.4)$$

The equation of motion of the surface of the star can be evaluated by solving the Hamilton-Jacobi equation in the spacetime [10]

$$g^{ij}(\partial_i S)(\partial_j S) = m^2 \quad (5.5)$$

where  $S$  is Hamilton's characteristic function and  $m$  is the mass of a particle moving along with the surface of the star. Solving for  $S$ , one has,

$$S = -Et \pm \int \frac{dr}{B(r)} \sqrt{E^2 - m^2 B} \quad (5.6)$$

where  $E$  is the energy of the particle. The equation of motion of the surface of the star as a function of time  $t$  is obtained by solving the equation

$$\frac{\partial S}{\partial E} = \text{constant} = t_0 \quad (5.7)$$

Defining  $\xi = E/m$ , the trajectory of the star's surface is,

$$t - t_0 = \pm \xi \int \frac{dr}{B(r) \sqrt{\xi^2 - B}} \quad (5.8)$$

Near the horizon, we use the condition in Eqn. (5.2). Substituting for  $B(r)$  into the above equation and defining a new variable  $x = B_1(r - r_h)$ , one obtains,

$$t - t_0 = \pm \frac{\xi}{B_1} \int \frac{dx}{x \sqrt{\xi^2 - x}} \quad (5.9)$$

Noting that, close to the horizon, the approximation,

$$(\xi^2 - x)^{-1/2} \approx \xi^{-1} \left( 1 + \frac{x}{2\xi^2} + \mathcal{O}(x^2) \right) \quad (5.10)$$

is accurate to  $\mathcal{O}(x^2)$  and choosing the minus sign for infalling geodesics, we obtain after performing the integration over  $x$ ,

$$t - t_0 = -\frac{1}{B_1} \left( \ln(x) + \frac{x}{2\xi^2} + \mathcal{O}(x^2) \right) \quad (5.11)$$

Substituting for  $x$  in terms of  $r$ , one has,

$$\frac{1}{B_1} \ln(B_1(r - r_h)) + \frac{r - r_h}{2\xi^2} + \mathcal{O}([r - r_h]^2) = -(t - t_0) \quad (5.12)$$

Since, near the horizon,

$$r^* \approx \int \frac{dr}{B_1(r - r_h)} = \frac{1}{B_1} \ln(B_1(r - r_h)) - \mathcal{O}(r - r_h) \quad (5.13)$$

it is easy to see that, neglecting all powers of  $(r - r_h)$  greater than the first power, the trajectory assumes the form,

$$r^* + \frac{1}{2B_1\xi^2} e^{B_1 r^*} + \mathcal{O}(e^{2B_1 r^*}) = -(t - t_0) \quad (5.14)$$

which, for late times, has the approximate solution

$$r^* = -(t - t_0) - \frac{1}{2B_1\xi^2} e^{-B_1(t-t_0)} - \mathcal{O}(e^{-2B_1(t-t_0)}) \quad (5.15)$$

This trajectory, in the conformal co-ordinate  $r^*$ , has exactly the same form as given in Eqn. (3.5).

We specialise henceforth to the usual Schwarzschild spacetime with  $B(r) = 1 - 2M/r$  which implies  $B_1 = 1/2M$ . The trajectory for the Schwarzschild case is

$$r^* = t_0 - t - \frac{M e^{t_0/2M}}{\xi^2} e^{-t/2M} \quad (5.16)$$

Since the trajectory given in Eqn. (5.16) is not physical for all times, we have to set the limits of integration from  $(t_i, \infty)$  where  $t_i > (4M)^{-1}$  is chosen so that the above trajectory is a good approximation to the actual trajectory. Using the results of section (III), the power spectrum per unit logarithmic frequency interval is,

$$\mathcal{P}(\Omega) = (4\pi M) \left\{ \frac{1}{2} + N + \sqrt{N(N+1)} \cos(2\beta) \right\}, \quad (5.17)$$

where  $N$  is given as usual by Eqn. (3.14) but with  $\Omega_0$  and  $\beta$  given by

$$\Omega_0 = \frac{1}{8\pi M} \quad ; \quad \beta = \omega t_0 \quad (5.18)$$

The inequality that the initial frequency  $\omega$  should satisfy such that the above spectrum is thermal can be determined along the lines of Eqn. (3.16),

$$\omega > \frac{\xi^2}{M} e^{-t_0/4M} e^{(t_i - r_0^*)/4M} \quad (5.19)$$

where  $r_0^*$  is related to  $r_0$  by the formula in Eqn. (5.3). This is the usual ‘‘thermal’’ power spectrum obtained in the frequency  $\Omega$  and is analogous to the radiation spectrum obtained in the quantum case. In conformal co-ordinates, the solution of the massless scalar field are just ordinary plane waves. These co-ordinates essentially indicate the obvious point that the net redshift caused by the curvature of spacetime when light moves from one point to another and back is zero. The redshift that is observed is caused solely by reflection off the moving surface of the star and that is what gives rise to the non-trivial thermal power spectrum.

If we were to consider a star that collapses for a certain time interval  $t_i \sim (4M)^{-1}$  to  $t_f$ , then, by applying the results of section (III), one obtains a thermal spectrum for frequencies  $\Omega > 4M$  only when the following inequalities are satisfied.

$$\omega > \frac{\xi^2}{M} e^{-t_0/4M} e^{(t_i - r_0^*)/4M} \quad ; \quad (t_f - t_i) > 1/4M \quad (5.20)$$

Therefore, in this way, the radiation spectrum that arises from the collapse of a star to form a black hole has a viable classical analogue.

## B. Thermal spectrum from motion of the observer

Here, we attempt to make a correspondence with the results of section (IV) in the black hole case. To do this, we now use the Kruskal co-ordinates in order to make a formal correspondence with the Minkowski-Rindler system. Let the Kruskal co-ordinates be denoted by  $(T, X)$  and the Schwarzschild conformal co-ordinates by  $(t, r^*)$ . The usual transformations between these two sets of co-ordinates are

$$T = e^{r^*/4M} \sinh(t/4M) \quad ; \quad X = e^{r^*/4M} \cosh(t/4M) \quad (5.21)$$

The above transformations are relevant only for the first sector of the full Kruskal manifold which represents the standard Schwarzschild spacetime outside the event horizon. This is the region of interest here.

Consider a static, spherically symmetric star of mass  $M$  with a radius  $r_0 > 2M$ . The star is at rest in the usual Schwarzschild spacetime. In the Kruskal co-ordinates, however, the motion of the surface of the star is given by the equation,

$$X^2 - T^2 = e^{r_0^*/2M} \quad (5.22)$$

Now, consider an observer moving in a "frame of reference" whose transformations with respect to the Kruskal co-ordinates are given by

$$T = e^{x^*/4M'} \sinh(\tau/4M') \quad ; \quad X = X_o + e^{x^*/4M'} \cosh(\tau/4M') \quad (5.23)$$

where  $(\tau, x^*)$  are the (conformal) co-ordinates in the observer's frame,  $X_o > 0$  is an arbitrary constant and  $M' \neq M$  is also a constant. The observer's frame of reference, in the  $(\tau, x)$  co-ordinates ( $x$  is related to  $x^*$  in the same way as  $r$  is related to  $r^*$ ), resembles the Schwarzschild frame in that it appears to possess a event horizon located at  $x = 2M'$ . This observer hence has a event horizon corresponding to a star of mass  $M'$ . We will discuss the motion of the observer in the standard Schwarzschild spacetime a little later.

The trajectory of the silvered star's surface as seen from the observer's frame of reference is given by substituting Eqn. (5.23) into Eqn. (5.22) and solving the resulting quadratic equation in the variable  $e^{x^*/4M'}$ . One therefore obtains,

$$e^{x_m^*(t)/4M'} = -X_o \cosh(\tau/4M') \pm \sqrt{X_o^2 \cosh^2(\tau/4M') + (e^{r_0^*/2M} - X_o^2)} \quad (5.24)$$

By construction, the above equation has the same form as that in Eqn. (4.4). Hence we can use all the results of that section in an identical manner. If  $X_o = 0$ , it is clear that the observer's frame of reference and the standard Schwarzschild spacetime are identical and so the star's surface appears to be at rest. This is a trivial solution. Further, to obtain a physically viable trajectory for the star, one must have  $e^{r_0^*/2M} - X_o^2 > 0$ . In this case, it is clearly seen that for late times  $\tau > 4M'$ , the power spectrum seen by the observer upon observing the star is of the same form as in Eqn. (5.17) with  $M$  replaced by  $M'$  and with

$$\Omega_0 = \frac{1}{8\pi M'} \quad ; \quad \beta = \omega \ln \left[ \frac{e^{r_0^*/2M} - X_o^2}{X_o} \right] \quad (5.25)$$

Thus we see that the spectrum seen by the observer is independent of the mass of the star and is dependent only on  $M'$ . Classically, at least, we have a situation where the motion of the observer produces the redshift required to obtain a thermal power spectrum.

The motion of the observer (sitting at a constant  $x_0^*$  in his/her frame) in the standard Schwarzschild spacetime can be determined by eliminating the Kruskal co-ordinates from Eqn (5.21) and Eqn (5.23) to give

$$e^{r^*/2M} = e^{x_0^*/2M'} + X_o^2 + 2X_o e^{x_0/4M'} \cosh(\tau/4M') \quad ; \quad \tanh(t/4M) = \frac{e^{x_0/4M'} \sinh(\tau/4M')}{X_o + e^{x_0/4M'} \cosh(\tau/4M')} \quad (5.26)$$

For late times  $\tau > 4M'$  which is of interest here, the trajectory of the observer in the standard Schwarzschild spacetime has the form

$$r^* = 2M \ln \left[ X_o e^{x_0/4M'} \right] + \frac{M}{2M'} \tau + 2M e^{-\tau/2M'} \quad ; \quad t = \frac{M}{M'} \tau \quad (5.27)$$

Thus, we see that if the observer moves along the above trajectory, then the power spectrum seen has a thermal nature. This result is interesting since it implies the equivalence of the motion of the star vis a vis the observer. The phenomenon of the collapse of a star to form an event horizon is equivalent to a situation where the star is static and the observer is moving away from it along a trajectory given above. Both situations produce the same thermal spectrum in the observer's frame of reference.

## VI. CONCLUSIONS

In conclusion, it is seen that viable classical models that mimic the phenomena of black hole radiation can be constructed. We have discussed various mirror trajectories and the conditons that give rise to a "thermal" power spectrum. For the system consisting of an inertial mirror viewed by an accelerated observer, only situations relevant

to the Schwarzschild spacetime are studied. The mirror moves along a geodesic in Minkowski spacetime with the observer in non-geodesic motion. For an observer moving along an accelerated trajectory for a finite interval, a thermal spectrum is obtained only if the time interval is large compared to  $(1/g)$  where  $g$  is the acceleration. Such a result has an analogy in the Schwarzschild spacetime in which a star collapses for a finite time interval *without* forming a black hole. The time interval of collapse must be large compared to the inverse of the surface gravity of the star which is  $(4M)^{-1}$ . In this case too, the star collapses along a geodesic of the spacetime while the observer, who is stationary, is in non-geodesic motion.

The reverse situation in which a uniformly accelerated mirror is viewed by an inertial observer does *not* yield a thermal power spectrum (see Appendix (A) for details). This is analogous to the well known quantum result in Ref. [13]. In this situation, it is the observer who is moving along a geodesic with the mirror moving along a non-geodesic trajectory. Therefore, it appears that the thermality of the power spectrum seems to be dependent on whether the mirror or the observer is in geodesic motion.

However, a thermal power spectrum also arises in the case of a uniformly accelerated mirror viewed in a uniformly accelerated observer's frame for late times. In this case, both the mirror and the observer move in non-geodesic motion. This situation has a simple analogy in the Schwarzschild spacetime in which an observer moving along a late time trajectory given Eqn. (5.27) sees a thermal spectrum when viewing a static non-collapsing star. The spectrum seen is independent of the mass  $M$  of the star and depends only on the parameters defining the observer's frame of reference. We will discuss the quantum analogue of the classical system of an accelerated mirror viewed in an accelerated observer's frame and its corresponding Schwarzschild system in a future publication.

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#### APPENDIX A: ACCELERATED MIRROR VIEWED IN INERTIAL FRAME

We will now consider the reverse case of an inertial observer and an accelerating mirror. The trajectory of a uniformly accelerating "receding" mirror is

$$gx(t) = 1 - \sqrt{1 + g^2 t^2} \quad (\text{A1})$$

where we have assumed the initial conditions that the position and velocity are zero at  $t = 0$ . The reflected wave is easily found to be given by the relation

$$\Phi_R(u) = -\exp\left(-i\frac{\omega u}{gu + 1}\right) \quad (\text{A2})$$

Assume that the observer is sitting at a distance  $x_0$  from the origin of co-ordinates. The FT of the reflected wave is

$$\tilde{\Phi}_R(\Omega) = -\int_{-\infty}^{\infty} dt e^{-i\Omega t} \exp\left(-i\frac{\omega(t-x_0)}{g(t-x_0)+1}\right) \quad (\text{A3})$$

where it is assumed that  $\Omega > 0$ . Making the change of variable  $y = g(t - x_0) + 1$ , we obtain

$$\begin{aligned} \tilde{\Phi}_R(\Omega) &= -\frac{e^{(\Omega-\omega)/g}}{g} e^{-i\Omega x_0} \int_{-\infty}^{\infty} dy \exp\left(-\frac{i\Omega}{g}x + \frac{i\omega}{g}\frac{1}{x}\right) \\ &= -\frac{e^{(\Omega-\omega)/g}}{g} e^{-i\Omega x_0} \left[ \int_0^{\infty} dx \exp\left(-\frac{i\Omega}{g}x + \frac{i\omega}{g}\frac{1}{x}\right) + \int_0^{\infty} dx \exp\left(\frac{i\Omega}{g}x - \frac{i\omega}{g}\frac{1}{x}\right) \right] \\ &= -\frac{e^{(\Omega-\omega)/g}}{g} e^{-i\Omega x_0} \left[ \frac{\sqrt{2\omega\Omega}}{i\Omega} + \frac{\sqrt{2\omega\Omega}}{-i\Omega} \right] K_1(\sqrt{2\omega\Omega}/g) \equiv 0 \quad \text{for } \Omega > 0 \end{aligned} \quad (\text{A4})$$

where we have used the formula in Ref. [12]

$$\int_0^{\infty} dy \exp\left(-Ay - \frac{B}{y}\right) = \frac{\sqrt{2AB}}{A} K_1(\sqrt{2AB}) \quad (\text{A5})$$

with  $K_1(x)$  being the modified Bessel function of order 1. We see that the FT taken over the entire trajectory is identically zero. If instead of the trajectory given above in Eqn. (A1), we assume that the mirror is inertial to start with and then accelerates continuously, we do get a non-zero spectrum. Assume that the trajectory of the mirror is now

$$gx(t) = \begin{cases} 0 & t \leq 0 \\ 1 - \sqrt{1 + g^2 t^2} & t > 0 \end{cases} \quad (\text{A6})$$

The reflected wave is now given by

$$\Phi_R(u) = \begin{cases} -e^{-i\omega u} & t \leq 0 \\ -\exp\left(-i\frac{\omega u}{gu+1}\right) & t > 0 \end{cases} \quad (\text{A7})$$

Assuming as usual that the observer is seated at a distance  $x_0$  from the origin, the FT of the reflected wave for  $t > 0$  yields a non-zero result. Therefore, we have, after making the previous change of variable,

$$\begin{aligned} \tilde{\Phi}_R(\Omega) &= \int_{-\infty}^{\infty} dt e^{-i\Omega t} \Phi_R(t - x_0) \\ &= -e^{i\omega x_0} \int_0^{\infty} dt e^{i(\Omega+\omega)t} - \frac{e^{i(\Omega-\omega)/g}}{g} e^{-i\Omega x_0} \int_0^{\infty} dx \exp\left(-\frac{i\Omega}{g}x + \frac{i\omega}{g}\frac{1}{x}\right) \\ &= -e^{i\omega x_0} \frac{i}{\Omega + \omega} - \frac{e^{i(\Omega-\omega)/g}}{g} e^{-i\Omega x_0} \frac{\sqrt{2\omega\Omega}}{i\Omega} K_1(\sqrt{2\omega\Omega}/g) \end{aligned} \quad (\text{A8})$$

Taking the modulus square of the above fourier amplitude we obtain the required power spectrum. The result is definitely not a Planck spectrum of the form given in Eqn. (3.13). Thus, a scenario where an accelerated observer looks at an inertial detector is not the same as an inertial detector looking at an accelerated detector. It appears to be important as to which is moving along an inertial trajectory or in geometric terms, which of the two is moving along a geodesic of the spacetime.

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