# Higher dimensional dust collapse with a cosmological constant

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The general solution of the Einstein equation for higher dimensional (HD) spherically symmetric collapse of inhomogeneous dust in presence of a cosmological term, i.e., exact interior solutions of the Einstein field equations is presented for the HD Tolman-Bondi metrics imbedded in a de Sitter background. The solution is then matched to exterior HD Scwarschild-de Sitter. A brief discussion on the causal structure singularities and horizons is provided. It turns out that the collapse proceed in the same way as in the Minkowski background, i.e., the strong curvature naked singularities form and that the higher dimensions seem to favor black holes rather than naked singularities.

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## I. INTRODUCTION

That the results coming from the analysis of high redshift Type Ia supernovae [1, 2, 3] indicate the Universe is accelerating. This suggests the possibility that a nonzero cosmological constant ($\Lambda$) may dominate the total energy of our Universe. The cosmological implications of the existence of a $\Lambda$ term are enormous, concerning not only the evolution of the Universe, but also structure formation and age problems. If $\Lambda$ term must be restored to the Einstein equations, surprises may turn up in other physical applications of Einstein's equations as well. For example, Markovic and Shapiro [4] generalized the Oppenheimer-Snyder model (which describes the gravitational collapse of a spherical homogeneous dust ball initially at rest in exterior vacuum to a Schwarzschild black hole) taking into account the presence of a positive $\Lambda$. They showed that $\Lambda$ may affect the onset of collapse and decelerate the implosion initially. The results of the Markovic and Shapiro were qualitatively generalized to the inhomogeneous dust and degenerate cases by Lake [5] for both $\Lambda > 0$ and $\Lambda < 0$. It was shown explicitly by Cissoko *et al.* [6] that the cosmological term slows down the collapse of matter, limiting the size of black holes.

The Tolman-Bondi metric [7], which is asymptotically flat, has been extensively used to study the formation of naked singularities in spherical collapse. It is seen that the Tolman-Bondi metric admits both naked and covered singularities depending upon the choice of initial data and there is a smooth transition from one phase to the other [8, 9, 10, 11, 12, 13, 14, 15]. However, according to the cosmic censorship conjecture (CCC) [17], the singularities that appear in gravitational collapse are always surrounded by an event horizon. Moreover, according to the strong version of the CCC, such singularities are not even locally naked, i.e., no non-spacelike curve can emerge from such singularities (see [18], for reviews on the CCC). The study in the inhomogeneous dust collapse with a positive $\Lambda$, from the viewpoint of CCC, was examined in [6, 19, 20, 21]. Deshingkar *et al.* [19] showed that the presence of a $\Lambda$ can cover a part of the singularity spectrum which is visible in the corresponding dust collapse models for the same initial data. Whereas Wagh and Maharaj [22] showed that in spherically symmetric radiation collapse (Vaidya collapse), the effect of adding a positive $\Lambda$ does not radically alter the description. Lemos [16] arrived at the same conclusion for a negative $\Lambda$. The result in both these cases are the same as in the case of collapsing radiation in the Minkowskian background. Therefore, at least, in the case of spherical radiation collapse, the asymptotic flatness is not essential for the development of a naked singularity.

While gravitational collapse has been originally studied in four dimensions (4D), there have been several attempts, mainly motivated by string theory, to study it in HD space-time [23, 24, 25, 26, 27, 28, 29, 30, 31]. Since, current experimental results involving tests of the inverse square law do not rule out extra dimensions even as large as a tenth of a millimeter. It is now important to consider the evolution of the extra dimensions since the observed strength of the gravitational force is directly dependent on the size of the extra dimensions. As a consequence, there is a renewed interest towards understanding of the general relativity in more than four dimensions, as growing volume of recent literature indicates. In particular, several solutions to the Einstein equations of localized sources in higher dimensions have been obtained in the recent years [32].

In this paper, we shall study spherical inhomogeneous dust collapse with a positive $\Lambda$ in HD theory of gravity, and present solutions in closed form. This is HD analogous of 4D Tolman-Bondi-de Sitter solutions and for definiteness we shall call it HD Tolman-Bondi-de Sitter solutions. Then, we show that HD Tolman-Bondi-de Sit-

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ter admits strong curvature naked singularity. However, the presence of a positive $\Lambda$ does not radically alter the established picture of Inhomogeneous dust collapse.

In the next section, we give exact HD spherically symmetric solution of Einstein field equation for a collapsing inhomogeneous dust with a cosmological constant $\Lambda$. This is followed by junction conditions between a static and a non-static HD spherically symmetric space-time in section III. The nature of singularities of such a space-time, and the consequence of cosmological constant $\Lambda$ is a subject of section V. This is preceded by detailed analysis on apparent horizon in section IV.

We have used units which fix the speed of light and the gravitational constant via $8\pi G = c^4 = 1$.

## II. HIGHER DIMENSIONAL TOLMAN-BONDI DE SITTER SPACE-TIMES

The standard 4D Tolman-Bondi solution [7] represents an interior of a collapsing inhomogeneous dust sphere. The solution we seek is - collapse of a spherical dust with a positive $\Lambda$ in HD space-time. We choose a spherically symmetric comoving metric in HD [23, 25], which has form

$$ds^2 = dt^2 - e^{\lambda(t,r)}dr^2 - R(t,r)^2 d\Omega^2, \tag{1}$$

where

$$\begin{aligned} d\Omega^2 = \quad & \sum_{i=1}^{n}\left[\prod_{j=1}^{i-1}\sin^2\theta_j\right]d\theta_i^2 = d\theta_1^2 + sin^2\theta_1 d\theta_2^2 \\ & + sin^2\theta_1 sin^2\theta_2 d\theta_3^2 + \ \ldots \\ & + sin^2\theta_1 sin^2\theta_2 \ldots sin^2\theta_{n-1} d\theta_n^2, \end{aligned} \tag{2}$$

is the metric on an $n$-sphere and $n = D - 2$ (where $D$ is the total number of dimensions), together with the stress-energy tensor for dust:

$$T_{ab} = \zeta(t,r)\delta_a^t \delta_b^t, \tag{3}$$

where $u_a = \delta_t^a$ is the $(n+2)$-dimensional velocity. The coordinate $r$ is the co-moving radial coordinate, $t$ is the proper time of freely falling shells, and $R$ is a function of $t$ and $r$ with $R > 0$ and $\lambda$ is also a function of $t$ and $r$. With the metric (1), the Einstein equations are

$$\begin{aligned} G_t^t = \quad & \frac{n(n-1)}{2R^2}(e^{-\lambda}R'^2 - \dot{R}^2 - 1) - \\ & \frac{n}{2}\frac{1}{R}(\dot{R}\dot{\lambda} + e^{-\lambda}R'\lambda') + ne^{-\lambda}\frac{R''}{R} = \zeta - \Lambda, \end{aligned} \tag{4}$$

$$G_r^r = \frac{n(n-1)}{2R^2}(e^{-\lambda}R'^2 - \dot{R}^2 - 1) - n\frac{\ddot{R}}{R} = -\Lambda, \tag{5}$$

$$\begin{aligned} & G_{\theta_1}^{\theta_1} = G_{\theta_2}^{\theta_2} = \ \ldots \ = G_{\theta_n}^{\theta_n} = \frac{(n-1)(n-2)}{2}\frac{1}{R^2} \\ & \times(e^{-\lambda}R'^2 - \dot{R}^2 - 1) - \frac{n-1}{2}\frac{1}{R}(\dot{R}\dot{\lambda} + e^{-\lambda}R'\lambda') - \\ & (n-1)\frac{1}{R}(\ddot{R} - e^{-\lambda}R'') - \frac{1}{2}(\ddot{\lambda} + \frac{\lambda^2}{2}) = -\Lambda, \end{aligned} \tag{6}$$

$$G_r^t = \frac{n}{2R}(2\dot{R}' - \dot{\lambda}R') = 0, \tag{7}$$

where the overdot and the prime denote the partial derivative with respect to t and r, respectively. Integration of Eq. (7) gives

$$e^{\lambda(t,r)} = \frac{R'^2}{1 + f(r)}, \tag{8}$$

which can be substituted into Eq. (5) to yield

$$\dot{R}^2 = \frac{\mathcal{M}(r)}{R^{n-1}} + \frac{2\Lambda R^2}{n(n+1)} + f(r). \tag{9}$$

The functions $\mathcal{M}(r)$ and $f(r)$ are arbitrary and referred to as the mass and energy functions, respectively. Since in the present discussion we are concerned with gravitational collapse, we require that $\dot{R}(t,r) < 0$. The energy density $\zeta(t,r)$ is calculated as

$$\zeta(t,r) = \frac{n\mathcal{M}'}{2R^n R'}, \tag{10}$$

For physical reasons, one assumes that the energy density $\zeta(t,r)$ is everywhere non-negative. The special case $f(r) = 0$ corresponds to the marginally bound case which is of interest to us in this paper. Substituting Eqs. (8) and (9) into Eq. (4) yields

$$\mathcal{M}' = \frac{2}{n}\zeta R^n R'. \tag{11}$$

Integrating Eq. (11) leads to

$$\mathcal{M}(r) = \frac{2}{n}\int \zeta R^n dR, \tag{12}$$

where constant of integration is taken as zero since we want a finite distribution of matter at the origin $r = 0$. The function $\mathcal{M}(r)$ must be positive, because $\mathcal{M}(r) < 0$ implies the existence of negative mass. This can be seen from the mass function $\mathbf{M}(t,r)$, which is given by

$$\begin{aligned} \mathbf{M}(t,r) \ &= \ \frac{n-1}{2}R^{n-1}\left(1 - g^{ab}R_{,a}R_{,b}\right) \\ &= \frac{n-1}{2}R^{n-1}\left(1 - \frac{R'^2}{e^{\lambda}} + \dot{R}^2\right). \end{aligned} \tag{13}$$

Now Eqs. (8), (9) and (13) implies that

$$\mathbf{M}(t,r) = \frac{n-1}{2}\mathcal{M}(r) + \frac{n-1}{n(n+1)}\Lambda R^{n+1}. \tag{14}$$

The quantity $\mathcal{M}(r)$ can be interpreted as energy due to the energy density $\zeta(t,r)$ given by Eq. (12), and since it is measured in a comoving frame, $\mathcal{M}$ is only $r$ dependent. Cissoko *et al.* [6] have derived marginally bound ($f=0$) dust solution in the presence of $\Lambda>0$. Here we derive the analogous HD solutions. Equation (9), for vanishing $\Lambda$, in 4D as well as in HD, has three types of solutions, namely, hyperbolic, parabolic and elliptic solutions depending on whether $f(r)>0$, $f(r)=0$ or $f(r)<0$, respectively. The condition $f(r)=0$ and $\Lambda=0$ is the marginally bound condition, limiting the situations where the shell is bounded from those it is unbounded. In the presence of a cosmological constant, the situation is more complex, and $f(r)=0$ leads to an unbounded shell. The assumption allows for analytical solutions in closed form

$$R(t,r)=\left[\frac{n(n+1)\mathcal{M}}{2\Lambda}\right]^{1/n+1}\sinh^{2/n+1}\alpha, \tag{15}$$

$$R'(t,r) = \left[\frac{n(n+1)\mathcal{M}}{2\Lambda}\right]^{1/n+1}\Big[\frac{\mathcal{M}'}{(n+1)\mathcal{M}}\sinh\alpha + \sqrt{\frac{2\Lambda}{n(n+1)}}t_0'\cosh\alpha\Big]\sinh^{(1-n)/(1+n)}\alpha, \tag{16}$$

where $\alpha=\alpha(t,r)$ has the form

$$\alpha(t,r)=\sqrt{\frac{(n+1)\Lambda}{2n}}[t_0(r)-t], \tag{17}$$

where $t_0(r)$ is an arbitrary function of integration which represents the proper time for the complete collapse of a shell with coordinate $r$. It follows from the above that there is a space-time singularity at $R=0$ and at $R'=0$.

It is easy to see that as $\Lambda\to 0$ the above solution reduces to the HD Tolman-Bondi solutions [25]:

$$\lim_{\Lambda\to 0}R(t,r)=\left[\frac{(n+1)^2\mathcal{M}}{4}(t_0-t)^2\right]^{1/n+1}, \tag{18}$$

$$\lim_{\Lambda\to 0}R'(t,r)=\frac{\mathcal{M}'(t_0-t)+2\mathcal{M}t_0'}{\left[\frac{(n+1)^{n-1}}{4}\mathcal{M}^n(t_0-t)^{n-1}\right]^{1/n+1}}. \tag{19}$$

The standard 4D Tolmon-Bondi solution can be now recovered by setting $n=2$. The three arbitrary functions $\mathcal{M}(r)$, $f(r)$ and $t_0(r)$ completely specify the behavior of shells with radius $r$. It is possible to make an arbitrary relabeling of spherical dust shells by $r\to g(r)$, without loss of generality, we fix the labeling by requiring that, on the hypersurface $t=0$, $r$ coincides with the radius

$$R(0,r)=r. \tag{20}$$

This corresponds to the following choice of $t_0(r)$:

$$t_0(r)=\sqrt{\frac{2n}{(n+1)\Lambda}}\sinh^{-1}\left[\sqrt{\frac{2\Lambda}{n(n+1)\mathcal{M}}}r^{(n+1)/2}\right]. \tag{21}$$

The central singularity occurs at $r=0$, the corresponding time being $t=t_0(0)=0$. We denote by $\rho(r)$ the initial density:

$$\rho(r)=\zeta(0,r)=\frac{n\mathcal{M}'}{2r^n}\Rightarrow\mathcal{M}(r)=\frac{2}{n}\int\rho(r)r^n dr. \tag{22}$$

## III. JUNCTION CONDITIONS

In order to study the gravitational collapse of a finite spherical body we have to match the solution along the time like surface at some $R=R_\Sigma$ to a suitable HD exterior. We consider a spherical surface with its motion described by a time-like (n+1)-surface $\Sigma$, which divides space-times into interior and exterior manifolds $\mathcal{V}_I$ and $\mathcal{V}_E$. According to the generalized Birkoff theorem [33] the vacuum space-time outside is HD Schwarzschild-de Sitter space-time:

$$ds^2=F(Y)dT^2-\frac{1}{F(Y)}dY^2-Y^2d\Omega^2, \tag{23}$$

where $F$ is a function of $Y$ given by

$$F(Y)=1-\frac{2M}{(n-1)Y^{n-1}}-\frac{2\Lambda Y^2}{n(n+1)}, \tag{24}$$

and $M$ is a constant. In accordance with Darmois junction condition, we have to demand when approaching $\Sigma$ in $\mathcal{V}_I$ and $\mathcal{V}_E$

$$(ds_-^2)_\Sigma=(ds_+^2)_\Sigma=(ds^2)_\Sigma, \tag{25}$$

where the subscript $\Sigma$ means that the quantities are to be evaluated on $\Sigma$ and let $K_{ij}^\pm$ is extrinsic curvature to $\Sigma$, defined by

$$K_{ij}^\pm=-n_\alpha^\pm\frac{\partial^2\chi_\pm^\alpha}{\partial\xi^i\partial\xi^j}-n_\alpha^\pm\Gamma_{\beta\gamma}^\alpha\frac{\partial\chi_\pm^\beta}{\partial\xi^i}\frac{\partial\chi_\pm^\gamma}{\partial\xi^j}, \tag{26}$$

and where $\Gamma_{\beta\gamma}^\alpha$ are Christoffel symbols, $n_\alpha^\pm$ the unit normal vectors to $\Sigma$, $\chi^\alpha$ are the coordinates of the interior and exterior space-time and $\xi^i$ are the coordinates that defines $\Sigma$. The intrinsic metric on the hypersurface $r=r_\Sigma$ is given by

$$ds^2=dt^2-R^2(r_\Sigma,t)d\Omega^2, \tag{27}$$

with coordinates $\xi^a=(t,\ \theta_1,\ \theta_2,\ \theta_3,\ \ldots,\ \theta_n)$. In this coordinate the surface $\Sigma$, being the boundary of the matter distribution, will have the equation

$$r-r_\Sigma=0, \tag{28}$$

where $r_\Sigma$ is a constant. The first fundamental form of $\Sigma$ can be written as $g_{ij}d\xi^i d\xi^j$. Then the exterior metric, on $\Sigma$, becomes:

$$ds_\Sigma^2=\left[F(Y_\Sigma)-\frac{1}{F(Y_\Sigma)}\left(\frac{dY_\Sigma}{dT}\right)^2\right]dT^2-Y_\Sigma^2d\Omega^2, \tag{29}$$

where we assume that the coefficient of $dT^2 > 0$ so that $T$ is time like coordinate. From the first junction condition we obtain

$$R(r_\Sigma, t) = Y_\Sigma,$$
$$\left[F(Y_\Sigma) - \frac{1}{F(Y_\Sigma)}\left(\frac{dY_\Sigma}{dT}\right)^2\right]^{1/2} dT = dt. \tag{30}$$

The non-vanishing components of extrinsic curvature $K^{\pm}_{ij}$ of $\Sigma$ can be calculated and the result is

$$K^+_{t\,t} = \left[\dot{Y}\ddot{T} - \dot{T}\ddot{Y} - \frac{F}{2}\frac{dF}{dY}\dot{T}^3 + \frac{3}{2F}\frac{dF}{dY}\dot{T}\dot{Y}^2\right]_\Sigma, \tag{31}$$
$$K^+_{\theta_n\,\theta_n} = \left[FY\dot{T}\right]_\Sigma, \tag{32}$$
$$K^-_{t\,t} = 0, \tag{33}$$
$$K^-_{\theta_n\,\theta_n} = \left[\frac{RR'}{e^\lambda}\right]_\Sigma. \tag{34}$$

With the help of Eqs. (30) - (34) and (9) , the total energy entrapped within the surface $\Sigma$ can be given by [34]

$$M = \frac{n-1}{2}\mathcal{M}(r). \tag{35}$$

Thus, the junction conditions demand that the HD Schwarzschild mass $M$ is given by Eq. (35).

## IV. HORIZONS

The apparent horizon is formed when the boundary of trapped $n$ spheres are formed. In spherical dust collapse, the event horizon coincides with the apparent horizon at the boundary of the spherical mass distribution. The apparent horizon is the solution of

$$g^{ab}R_{,a}R_{,b} = -\dot{R}^2 + f(r) + 1 = 0. \tag{36}$$

Upon using Eqs. (8) and (9), we have

$$\Lambda R^{n+1} - \frac{n(n+1)}{2}R^{n-1} + \frac{n(n+1)}{2}\mathcal{M} = 0. \tag{37}$$

For $\Lambda = 0$ we have the Schwarzschild horizon $R^{n-1} = \mathcal{M}$, and for $\mathcal{M} = 0$ we have the de Sitter horizon $R = \pm\sqrt{n(n+1)/2\,\Lambda}$. The approximate solutions of Eq. (37)to first order are

$$\begin{aligned} R_{bh} &= R^{(0)}_{bh} + R^{(1)}_{bh} + \ldots \\ &= \mathcal{M}^{1/n-1} + \frac{2\Lambda}{n(n-1)(n+1)}\mathcal{M}^{3/n-1}\ldots, \end{aligned} \tag{38}$$

where $R_{bh}$ is the radius of the black hole event horizon.

$$\begin{aligned} R_{ch} &= R^{(0)}_{ch} + R^{(1)}_{ch} + \ldots \\ &= \left[\frac{n(n+1)}{2\Lambda}\right]^{1/2} - \frac{1}{2}\left[\frac{2\Lambda}{n(n+1)}\right]^{n/2}\mathcal{M}\ldots, \end{aligned} \tag{39}$$

where $R_{ch}$ is the radius of the cosmological event horizon [35]. There exist a critical solution of Eq. (37), where two roots coincides and there is only one horizon. The time for the formation of apparent horizon, from Eq. (15) is

$$t_{AH} = t_0(r) - \sqrt{\frac{2n}{(n+1)\Lambda}}\,\sinh^{-1}\left[\sqrt{\frac{2\Lambda}{n(n+1)}}\,\mathcal{M}^{1/n-1}\right]. \tag{40}$$

In the limit as $\Lambda \to 0$, using Eq. (18), we obtain:

$$t_{AH} = t_0(r) - \frac{2}{n+1}\mathcal{M}^{1/n-1}. \tag{41}$$

Equation (41), gives the time for the formation of event horizon in HD Tolman-Bondi-de Sitter space-time. A necessary condition for the singularity to be globally naked is $t_{AH} \geq t_0(0)$.

## V. CAUSAL STRUCTURE OF SINGULARITIES

It has been shown [13] that Shell-crossing singularities are characterized by $R' = 0$ and $R > 0$. On the other hand the singularity at $R = 0$ is, where all matter shells collapses to a zero physical radius and hence known as shell focussing singularity.

We shall consider the case $t \geq t_0$. In the context of the Tolman-Bondi models the shell crossings are defined to be surfaces on which $R' = 0$ ($R > 0$) and where the density $\zeta$ diverges. A regular extremum in $R$ along constant time slices may occur without causing a shell crossing, provided $\zeta(t, r)$ does not diverge. By Eq. (10), this implies $\mathcal{M}' = 0$ where ever $R' = 0$ and also that the surface $R' = 0$ remain at fixed $R$. Now Eq. (16) implies $t'_o = 0$. Thus the condition for a regular maximum in $R(t, r)$ is that $\mathcal{M}' = 0$, $t'_o = 0$ hold at the same $R$. It has been shown [13] that shell crossing singularities are gravitationally weak and hence such singularities cannot be considered seriously.

Next, we turn our attention to shell-focusing singularities. Christodoulou [36] pointed out in the 4D case that the non-central singularities are not naked. Hence, we shall confine our discussion to the central shell focusing singularity. The energy density diverge at $t = t_0(r)$ indicating the presence of curvature singularity [37]. It is known that, depending upon the inhomogeneity factor, the 4D Tolman-Bondi solutions admits a central shell focusing naked singularity in the sense that outgoing geodesics emanate from the singularity. Here we wish to investigate the similar situation in our HD Tolman-Bondi-de Sitter space-time. We consider a class of models such that

$$\mathcal{M}(r) = \gamma r^{n-1}, \tag{42a}$$
$$t_0(r) = Br. \tag{42b}$$

This class of models for 4D space-time is discussed in [5, 10, 14]. The parameter $B$ gives the inhomogeneity of the

collapse. For $B = 0$ all shells collapse at the same time. For higher $B$ the outer shells collapse much later than the central shell. We are interested in the causal structure of the space-time when the central shell collapses to the center ($R = 0$). From Eqs. (10) and (42a), the energy density at the singularity

$$\zeta = \frac{n(n-1)\gamma}{2r^2}, \tag{43}$$

and the equation of general density becomes

$$\zeta = \frac{\Lambda}{\sinh^2\left[\sqrt{\frac{(n+1)\Lambda}{2n}}\,\frac{(B-y)}{y}\,t\right]\left[1+\sqrt{\frac{2(n+1)\Lambda}{n}}\,\frac{t}{y}\,\frac{B}{n-1}\,\coth\left[\sqrt{\frac{(n+1)\Lambda}{2n}}\,\frac{t}{y}\,(B-y)\right]\right]}. \tag{44}$$

As $t \to t_0(r)$ i.e. in approach to singularity, we have $\sinh\alpha \approx \alpha$ and $\coth\alpha \approx 1/\alpha$ then Eq. (44) reduces to:

$$\zeta = \frac{n(n-1)y^2}{(n+1)(B-y)\left[\frac{n+1}{2}B - \frac{n-1}{2}y\right]t^2} = \frac{C(y)}{t^2}. \tag{45}$$

The nature (a naked singularity or a black hole) of the singularity can be characterized by the existence of radial null geodesics emerging from the singularity. The singularity is at least locally naked if there exist such geodesics, and if no such geodesics exist, it is a black hole. The critical direction is the Cauchy horizon. This is the first outgoing null geodesic emanating from $r = t = 0$. The Cauchy horizon of the space-time has $y = t/r = $ const [10, 15]. The equation for outgoing null geodesics is

$$\frac{dt}{dr} = R'. \tag{46}$$

Hence along the Cauchy horizon, we have

$$R' = y, \tag{47}$$

and using Eqs. (47) and (16), with our choice of the scale, we obtain the following algebraic equation:

$$y\left(1-\frac{y}{B}\right)^{\frac{n-1}{n+1}} = \left(\frac{n+1}{2}B\sqrt{\gamma}\right)^{\frac{2}{n+1}}\left[1-\frac{n-1}{n+1}\frac{y}{B}\right]. \tag{48}$$

To facilitate comparison with Ghosh and Beesham [25], we introduce a a new relation between $B$ and $\gamma$ as,

$$B = \frac{2}{n+1}\frac{1}{\sqrt{\gamma}}, \tag{49}$$

with the help of above relation and after some rearrangement, Eq. (48) takes the exactly the same form as in Ghosh and Beesham [25]

$$y\left[1-\frac{n+1}{2}\sqrt{\gamma}\,y\right]^{\frac{n-1}{n+1}} + \frac{n-1}{2}\sqrt{\gamma}\,y - 1 = 0. \tag{50}$$

This algebraic equation governs the behavior of the tangent vector near the singular point. The central shell focusing singularity is at least locally naked, if Eq. (50) admits one or more positive real roots. Hence in the absence of positive real roots, the collapse will always lead to a black hole. Since the Eq. (50) is the same as that obtained by Ghosh and Beesham [25] when the metric is asymptotically flat i.e., $\Lambda = 0$. Consequently the values of roots for the geodetic tangent and the condition for these values to be real and positive are the same as those obtained for the asymptotically flat situation in [25]. As a result, the Tolman-Bondi-de Sitter space-time has same singularity behavior as the Tolman-Bondi space-time in both 4D and HD. Thus the results of collapsing inhomogeneous dust in de Sitter background are similar to that of collapsing inhomogeneous dust in Minkowskian background, as it should have been expected, since when $t \to t_0(r)$ the cosmological term $2\Lambda R^2/n(n+1)$ is negligible. It is known that the Tolman-Bondi metric ($\Lambda = 0$), in the 4D case is extensively used for studying the formation of naked singularities in spherical gravitational collapse. It has been found that Tolman-Bondi metric admit both naked singularities and black holes form depending upon the choice of initial data. Indeed, both analytical [11]-[12] and numerical results [8] in dust indicate the critical behavior governing the formation of black holes or naked singularities. One can now safely assert that end state of 4D Tolman-Bondi collapse is now completely known in dependence of choice of initial data. A similar situation also occurs in HD Tolman-Bondi collapse [23, 25] for a class of model under discussion. Since the presence of $\Lambda > 0$ does not change the final fate of Inhomogeneous dust collapse. Hence, we can conclude both naked singularities and black holes can form in the HD spherical inhomogeneous dust collapse with a positive $\Lambda$.

Further, it is shown that, in the class models discussed here, the formation of a black hole is facilitated with introduction of the extra dimensions or, in other words, the naked singularity spectrum gets continuously covered in higher dimensional space-times. This is valid in the in

null fluid collapse [26] and scalar field collapse [28] in higher dimensional space-times. Thus it appears that the singularity will be completely covered for very large dimensions of the space-time. Recently, It has been shown that cosmic censorship can be restored in some classes of inhomogeneous dust gravitational collapse models when space-time dimension to be $N \geq 6$, i.e., the naked singularities of this model can be removed by going to higher dimensions [30]. However, this is valid with only smooth initial profiles.

## VI. CONCLUDING REMARKS

In this paper, We have shown that the 4D spherically symmetric solution describing inhomogeneous dust collapse with a cosmological term go over to $(n+2)$-dimensional spherically symmetric solution and essentially retaining its physical behavior and when $n = 2$, one recovers the 4D Tolman-Bondi-de Sitter solutions [6, 19, 21]. Thus we have obtained Tolman-Bondi-de Sitter metric in arbitrary dimensions and the junction condition for static and non-static space-times are deduced. We have also utilized this solution to study the end state of collapsing star and showed that there exists a regular initial data which leads to a naked singularity.

Our purpose was to investigate the collapse of inhomogeneous dust shells in an expanding de Sitter background, to find out if the naked singularity occurs in this situation and to compare any difference with the similar collapse in the asymptotically flat case. We have obtained a condition for the occurrence of a naked singularity in the collapse of dust shells in an expanding background which is the same as that obtained when the background is asymptotically flat. This very fact establishes that the space-time is asymptotically flat or not does not make any difference to the occurrence of a naked singularity. This is evident at least the in the class of models defined by Eq. (42).

Penrose [38] has conjectured that it seems unlikely that a $\Lambda$-term will really make much difference to the singularity structure in a collapse. The relevance of $\Lambda$ is really only at the cosmological scale. Our results are consistent with this. However, The introduction of a cosmological constant changes scenario in many ways. There are now several apparent horizons instead of one. However, only two apparent horizon are physical, namely the black hole horizon and the cosmological horizon. Other results derived in [6] do carry over to HD space-time essentially with same physical behavior hence not presented to avoid duplication.

Finally, the result obtained would also be relevant in the context of superstring theory which is often said to be next "theory of everything", and for an interpretation of how critical behaviour depends on the dimensionality of the space-time.

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