16-2-93

3054

IUCAA-5/93 Feb'93

Electric charge from spontaneous breaking of symmetry

Patrick Das Gupta

Inter-University Centre for Astronomy and Astrophysics, Pune 411 007, INDIA

e-mail : pdg@iucaa.ernet.in

Abstract

The duality symmetry present in the Maxwell equations admitting magnetic charges is gauged using a complex scalar field. In this new model, electromagnetic charge arises because of a "Yukawa" type coupling of the matter with the scalar field. Considering an interacting Dirac field we show that spontaneous symmetry breaking in the scalar field sector leads at low energy to the reduction of this theory to the usual electrodynamics without magnetic charges. The notion of electric charge as a coupling constant arises very naturally in this model. Higgs mechanism in this model gives rise to the existence of a scalar and a vector field, both real and massive.

December 1992

Inclusion of magnetic charges in classical electrodynamics leads to the so-called duality symmetry that is exhibited by the extended Maxwell equations [1,2]. In an earlier work [3] (henceforth to be referred to as Paper I), we had shown that duality symmetry is meaningful only in 3+1 dimensional space-times, making thereby a connection between the concept of a magnetic charge and space-time dimensionality. Furthermore, in Paper I, we had also attempted to gauge the duality symmetry, in the context of interacting classical point particles carrying electric as well as magnetic charges, by introducing a non-dynamical real scalar field. However, novel features emerge when the duality symmetry is gauged with the help of a dynamical complex scalar field and, in what follows, constitutes the subject of this letter.

In the presence of magnetic charges, it is convenient to express the equations of motion in terms of a complex notation [for example, see Paper I]. In such a formalism, extended Maxwell-Lorentz equations take the following form,

$$\partial_\mu G^{\mu\nu} = \frac{4\pi}{c} J^\nu, \tag{1}$$

and,

$$\frac{dp^\mu}{d\tau} = \frac{1}{2c}\left[Q^* G^{\mu\nu} + Q G^{*\mu\nu}\right]\frac{dx_\nu}{d\tau}, \tag{2}$$

where the complex electromagnetic field tensor, complex charge and current density are defined, respectively, as

$$G_{\mu\nu} = F_{\mu\nu} + i\tilde{F}_{\mu\nu}, \tag{3}$$

$$Q = q_e + iq_m, \tag{4}$$

and,

$$J^\mu = j_e^\mu + ij_m^\mu. \tag{5}$$

The subscripts "e" and "m" refer to electric and magnetic, respectively, while $\tilde{F}_{\mu\nu}$ is just the dual of $F_{\mu\nu}$.

Under a global duality rotation, physical quantities transform in the following manner,

$$G_{\mu\nu} \to G'_{\mu\nu} = e^{i\theta} G_{\mu\nu}, \tag{6}$$

$$Q \to Q' = e^{i\theta} Q, \tag{7}$$

and,

$$J^{\mu} \to J'^{\mu} = e^{i\theta} J^{\mu}. \tag{8}$$

Ofcourse, (8) follows from (7).

It is obvious that the equations of motion (1) and (2) are invariant under the transformation (6) - (8). We wish to extend the hitherto global duality symmetry to a local one, by making the duality angle θ depend on space-time coordinates. With $\theta = \theta(x^{\mu})$, although the generalized Lorentz force equation (2) is invariant, the field equations (1) are not. It is evident that gauging the duality symmetry requires modification of the field equations. There is yet another curious point - local duality transformation makes the electromagnetic charge Q space-time dependent. This is an unusual feature that prompts one to view the concept of electromagnetic charge in a totally different manner. In the next paragraph we develop a new picture to accommodate the above points.

Consider a complex scalar field $\phi(x)$ which under a local duality transformation changes as follows,

$$\phi(x) \to \phi'(x) = e^{i\theta(x)} \phi(x) \tag{9}$$

We postulate that the electromagnetic charge of a particle arises due to the interaction between the particle and the scalar field ϕ so that,

$$Q(x) = \alpha\phi(x) \quad , \tag{10}$$

where α is a coupling parameter that solely depends on the particle, and is truly a constant. The transformation (7) then is guarranteed for Q because of (9) and (10). This way of looking at the electromagnetic charge is reminiscent of the origin of fermionic mass in electroweak theories [4-7] through Higgs field. In fact, shortly we will incorporate most of the features associated with the Higgs sector in the dynamics of ϕ, and demonstrate that spontaneous symmetry breaking gives rise to space-time independent electric charge, in conformity with experimental results.

The same field ϕ can be used to define a gauge covariant derivative,

$$\mathcal{D}_\mu \equiv \partial_\mu - \frac{\partial_\mu \phi}{\phi}, \tag{11}$$

so that under a local duality rotation we have,

$$\mathcal{D}_\mu G^{\alpha\beta} \rightarrow e^{i\theta(x)} \mathcal{D}_\mu G^{\alpha\beta}. \tag{12}$$

Modifying (1) to ,

$$\mathcal{D}_\mu G^{\mu\nu} = \frac{4\pi}{c} J^\nu, \tag{13}$$

and making use of (12), it is easy to see that the field equations given by (13) are invariant under local duality transformations.

In our model, (13) and (2) constitute the equations of motion as far as classical electrodynamics is concerned. To see how standard electrodynamics emerges from this model, we consider a Dirac field coupled to the electromagnetic field as well as the scalar field.

Let $a_\mu(x)$ be a complex 4-vector field that under duality transformation behaves in the following way :

$$a_\mu(x) \to a'_\mu(x) = e^{i\theta(x)} a_\mu(x). \tag{14}$$

The complex electromagnetic field tensor $G_{\mu\nu}$ is related to a_μ in the following way,

$$G_{\mu\nu} = (\partial_\mu + \frac{\partial_\mu \phi^*}{\phi^*})a_\nu - (\partial_\nu + \frac{\partial_\nu \phi^*}{\phi^*})a_\mu \quad . \tag{15}$$

It is easy to see from (14) and (15) that under a local duality rotation, $G_{\mu\nu} \to e^{i\theta(x)} G_{\mu\nu}$. However, not all the components of a_μ are independent. This is because of the definition (3) for $G_{\mu\nu}$ that requires the following constraint to be satisfied,

$$G_{\mu\nu} = \frac{i}{2} \epsilon_{\mu\nu}{}^{\alpha\beta} G_{\alpha\beta}. \tag{16}$$

The Lagrangian density for the electromagnetic field is given by,

$$\mathcal{L}_0 = -\frac{1}{16\pi} G^*_{\mu\nu} G^{\mu\nu} \quad . \tag{17}$$

Charged fermionic matter may be described in terms of a Dirac field $\psi(x)$ that has the following Lagrangian density,

$$\mathcal{L}_1 = \frac{1}{2}\bar{\psi}\gamma^\mu[i\partial_\mu - \frac{\alpha}{2}(\phi^* a_\mu + c.c.)]\psi + \frac{1}{2}[-i\partial_\mu - \frac{\alpha}{2}(\phi^* a_\mu + c.c.)]\bar{\psi}\gamma^\mu\psi - m\bar{\psi}\psi \quad . \tag{18}$$

$\alpha\phi(x)$ plays the role of the generalized charge in (18). Under a local duality rotation, ϕ and a_μ transform according to (9) and (14), respectively, while $\psi \to \psi$. Clearly, both $\mathcal{L}_0$ as well as $\mathcal{L}_1$ are invariant under local duality transformation.

$\mathcal{L}_0 + \mathcal{L}_1$ has another U(1) gauge symmetry, namely,

$$a_\mu \rightarrow a_\mu - \frac{1}{\alpha\phi^*}\partial_\mu\beta(x), \tag{19}$$

and,

$$\psi(x) \rightarrow e^{i\beta(x)}\psi(x), \tag{20}$$

where $\beta(x)$ is a real function. It is easy to guess that (19) and (20) give rise to the usual U(1) gauge symmetry of the standard electrodynamics.

The Lagrangian densities (17) and (18) lead to the following equations of motion,

$$\gamma^\mu[i\partial_\mu - \frac{\alpha}{2}(\phi^* a_\mu + c.c.)]\psi - m\psi = 0, \tag{21}$$

and,

$$\mathcal{D}_\mu G^{\mu\nu}(x) = \frac{4\pi}{c}\alpha\phi(\bar{\psi}\gamma^\nu\psi). \tag{22}$$

We now come to the scalar field ϕ. To make the scalar field sector invariant under local U(1) group (see (9)) we need an abelian gauge field χ_μ, so that the corresponding gauge covariant derivative then can be written as,

$$\nabla_\mu = \partial_\mu - ig\chi_\mu \quad , \tag{23}$$

where g is the gauge coupling constant. Under local duality transformation, the abelian gauge field transforms as,

$$\chi_\mu \rightarrow \chi'_\mu = \chi_\mu + \frac{1}{g}\partial_\mu\theta \quad . \tag{24}$$

The U(1) invariant action for the scalar field sector is taken to be,

$$\mathcal{A} = \int d^4x[\mathcal{L}_\phi - \frac{1}{16\pi}\Sigma_{\mu\nu}\Sigma^{\mu\nu}], \tag{25}$$

where,

$$\mathcal{L}_\phi = \frac{1}{2}(\nabla_\mu \phi)^*(\nabla^\mu \phi) - \frac{\lambda}{4}(\phi^* \phi - \eta^2)^2, \tag{26}$$

and,

$$\Sigma_{\mu\nu} = \partial_\mu \chi_\nu - \partial_\nu \chi_\mu \quad , \tag{27}$$

The ground state of this sector is described by the following well known solutions,

$$\phi_{vac}(x) = \eta e^{i\zeta(x)} \quad , \tag{28}$$

and,

$$(\chi_\mu)_{vac} = 0 \quad , \tag{29}$$

where η and $\zeta(x)$ are real.

In the low energy limit, the scalar field takes the vacuum configuration (28) so that the gauge covariant derivative $\mathcal{D}_\mu$ takes the form,

$$\mathcal{D}_\mu = \partial_\mu - i\partial_\mu \zeta \quad . \tag{30}$$

By virtue of (9), under a local duality transformation the phase ζ transforms as,

$$\zeta(x) \rightarrow \zeta'(x) = \zeta(x) + \theta(x) \quad . \tag{31}$$

Since the entire theory is invariant under local duality transformation, we are free to choose a gauge $\theta(x) =$ - $\zeta(x)$ so that $\zeta'(x) = 0$ because of (31). This immediately makes the gauge covariant derivative (in the new gauge) reduce to ordinary partial derivative (see (30)),

$$\mathcal{D'}_\mu = \partial_\mu \quad . \tag{32}$$

Furthermore, in this gauge the generalized electromagnetic charge is given by,

$$\alpha\phi'(x) = \alpha\eta \equiv q_e \quad , \tag{33}$$

implying that the charge is a constant and is real (corresponding to electric charge alone). It is easy to see making use of (32) and (33) that (21) and (22) reduce to,

$$[\gamma^\mu(i\partial_\mu - q_e A_\mu) - m]\psi = 0, \tag{34}$$

$$\partial_\mu F^{\mu\nu} = \frac{4\pi}{c} q_e(\bar{\psi}\gamma^\nu\psi), \tag{35}$$

and,

$$\partial_\mu \tilde{F}^{\mu\nu} = 0, \tag{36}$$

where the real vector potential A_μ is defined to be,

$$A_\mu = \frac{1}{2}(a_\mu + a_\mu^*), \tag{37}$$

Equations (34) - (36) are the usual Dirac-Maxwell equations in the absence of magnetic monopoles. Using (37) one can also verify that the transformation (19) corresponds to the usual gauge transformation $A_\mu \rightarrow A_\mu - \frac{1}{q_e}\partial_\mu\beta(x)$. Thus, in the low energy region the electromagnetic sector of this theory is identical to the conventional electrodynamics.

The new physics arises out of the scalar field sector which has in it the entire machinary of theories incorporating spontaneous breaking of local U(1) symmetry. By the well known procedure of expanding the scalar field ϕ around the vacuum (28),

$$\phi(x) = [\eta + h(x)]\ e^{i\zeta} \quad , \tag{38}$$

and then invoking Higgs mechanism [8-11], one sees that the real fields $h(x)$ and χ_μ acquire masses $\sqrt{2\lambda\eta^2}$ and $g\eta$, respectively. For large η one expects high values of masses, however in this model there is no way of fixing the scales relevant to the scalar field sector naturally. The scalar field sector can also give rise to cosmic string like solutions [12-14] and is a promising candidate for inflationary scenarios [15].

To briefly summarize, gauging the duality symmetry necessitates a complex scalar field which couples with fermionic and vector fields to give rise to electromagnetic charges. The spontaneous breaking of the duality symmetry in the scalar field sector leads naturally to the standard Dirac-Maxwell equations, implying that this model reproduces the results of conventional electrodynamics with ease. Moreover,in this model, magnetic charge for every particle can be gauged away by a suitable choice of gauge. Work in the direction of blending this model with electroweak theories is in progress.

It is a pleasure to thank Professors R. Brandenberger, A. E. Everett, L. Ford and A. Vilenkin for useful discussions. The author gratefully acknowledges the hospitality of Tufts University, Brown University and the University of Vermont. This work was supported by the C.S.I.R. fellowship.

References

[1] For example, J. D. Jackson, *Classical Electrodynamics* (Wiley Eastern limited, New Delhi, 1988).

[2] B. Felsager, *Geometry, particles and fields* (Odense University Press, 1987).

[3] P. Das Gupta, P., submitted to Phys. Lett. A (1992) (Paper I).

[4] S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967).

[5] A. Salam, in *Elementary Particle Theory*, ed. N. Svartholm (Almquist and Wiksells, Stockholm, 1968).

[6] S. L. Glashow, J. Iliopoulos and L. Miani, Phys. Rev. D. **2**, 1285 (1970).

[7] G. 't Hooft, Nucl. Phys. B **33**, 173 (1971); Nucl. Phys. B **35**, 167 (1971).

[8] F. Englert and R. Brout, Phys. Rev. Lett. **13**, 321 (1964).

[9] P. W. Higgs, Phys. Lett. **12**, 132 (1964); Phys. Rev. Lett. **13**, 508 (1964); Phys. Rev. **145**, 1156 (1966).

[10] G. S. Guralnik, C.R. Hagen and T. W. B. Kibble, Phys. Rev. Lett. **13**, 585 (1964).

[11] T. Kibble, Phys. Rev. **155**, 1554 (1967).

[12] H. B. Nielsen and P. Olesen, Nucl. Phys. B **61**, 45 (1973).

[13] T. W. B. Kibble, J. Phys. A **9**, 1387 (1976).

[14] A. Vilenkin, Phys. Reports **121**, 265 (1985) and the references therein.

[15] A. Guth, Phys. Rev. D. **23**, 347 (1981).