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Gravitational waves from mini-creation events

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ABSTRACT

This paper examines the possibility of testing the hypothesis recently proposed by some authors that, instead of the one-shot creation of the entire Universe in a big bang, creation of matter takes place in finite bursts at random in a Universe that is without a beginning and whose expansion on the large scale is as given by the steady-state model. It is argued that if the creation events are anisotropic then they generate gravitational waves. Calculations are given to show that a laser interferometric detector of the LIGO type would easily detect an event that generates ~100–1000 $M_\odot$. Further, the cosmological gravitational wave background generated by the mini-creation events is compared with the limits set by the analysis of the arrival time of pulses from millisecond pulsars. The existing data place severe constraints on the mass and anisotropies of the mini-creation events.

Key words: radiation mechanisms: gravitational – cosmology: observations – cosmology: theory – diffuse radiation.

1 INTRODUCTION

As a serious alternative to the standard hot big bang cosmology, Arp et al. (1990) and subsequently Hoyle, Burbidge & Narlikar (1993) proposed a variation on the old steady-state model by Bondi & Gold (1948) and Hoyle (1948) that combines some features of both these models. Instead of a single (and primordial) creation event that launched the big bang Universe, the alternative envisages an endless chain of creation events randomly distributed in space and time. Each event creates a finite mass which may vary from event to event, but the average rate of creation of matter per unit volume remains steady over the cosmological time-scale H_0^{-1}, H_0 being the Hubble constant. We will call such an event a mini-creation event (MCE). Hoyle et al. (1993) have suggested that MCEs may create masses of varying sizes, ranging from the supercluster-size mass $\sim 10^{16}$ $M_\odot$ to masses of the order of 10–10^6 $M_\odot$ within galaxies which manifest themselves as explosive phenomena.

Hoyle et al. have claimed that the primordial single creation event of the big bang is observationally undetectable, whereas the MCEs are more amenable to detection. It may well be possible to detect differences in the predictions of the two cosmologies through measurements of primordial abundances, the anisotropies of the microwave background, the ages of stars and galaxies, etc. We consider here two new methods that can in principle tell us whether MCEs are taking place, their frequencies, and what mass limits are allowed. The methods are 'new' in the sense that they deal with the gravitational waves emerging from the MCEs rather than the more common and conventional electromagnetic waves; and they illustrate the potential role of gravitational wave astronomy in the years to come.

Specifically, we shall calculate two measurable quantities. The first is the gravitational wave amplitude generated by an MCE of mass M at a typical cosmological distance r. We shall then compare this with the magnitudes that are detectable by laser interferometric detectors available today and planned for the future. Next we shall estimate the background spectrum of gravitational waves produced by the existing population of MCEs and compare it with the limits set by the timing measurements of millisecond pulsars.

We shall begin by outlining the mini-creation scenario of Arp et al. (1990) with further details supplied by Hoyle et al. (1993). Although previous authors have not specifically discussed the shapes of the MCEs, in order to establish their relevance to the gravitational wave problems we make the (not unreasonable) assumption that the MCEs possess a small degree of anisotropy, and discuss a simple model for anisotropic expansion of the local region around a typical MCE.

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2 THE DYNAMICS OF A MINI-CREATION EVENT

2.1 Isotropic events

The equations determining the expansion of the steady-state Universe were given by Hoyle & Narlikar (1966a, b), and are summarized below.

The line element for space–time has the standard Robertson–Walker form:

$$ds^2 = c^2\,dt^2 - S^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\,d\phi^2)\right], \tag{1}$$

where $k = 0, 1, -1$ and $S(t)$ is the scale factor for expansion.

The field equations are those of Einstein, with a scalar field C which is linked to the creation phenomenon. For the above line element, the field equations become

$$2\frac{\ddot{S}}{S} + \frac{\dot{S}^2 + kc^2}{S^2} = -\frac{8\pi Gp}{c^2} + 4\pi Gf\dot{C}^2, \tag{2}$$

$$3\frac{\dot{S}^2 + kc^2}{S^2} = 8\pi G\rho - 4\pi Gf\dot{C}^2, \tag{3}$$

where f is the coupling constant of the C-field, G being the gravitational constant. The pressure p and density ρ have their standard cosmological interpretations.

The steady-state model that is dominated by dust-like matter is given by

$$k = 0, \qquad \dot{C} = m, \qquad S \propto \exp H_0 t, \qquad p = 0,$$

$$\rho = fm^2 = \frac{3H_0^2}{4\pi G}, \tag{4}$$

where m is the mass of a typical created particle. For the radiation-dominated model we have

$$\rho = \frac{3p}{c^2} = \frac{9H_0^2}{8\pi G}. \tag{5}$$

If we take m to be large compared to the baryon mass and argue that a fraction ε of the created energy mc^2 goes into matter and the remaining fraction $1-\varepsilon$ into radiation, we then obtain

$$\rho_{\rm matter} = \varepsilon fm^2, \qquad \rho_{\rm radiation} = \frac{3}{4}(1-\varepsilon)\,fm^2, \tag{6}$$

where

$$3H_0^2 = 2\pi G(1+\varepsilon)\,fm^2. \tag{7}$$

As discussed by Hoyle & Narlikar (1966b), the C-field works in two modes. The 'creative' mode drives the above steady-state solution, while the 'non-creative' mode leads to an adiabatic expansion with no injection of new energy. Hoyle & Narlikar envisaged a local region undergoing phase transitions with a switchover of the C-field from the creative to the non-creative mode. In such a locality the expansion is slowed down and is given by (for $k = 0$)

$$\frac{\dot{S}^2}{S^2} = \frac{8\pi Gfm^2}{3}\left[\varepsilon\frac{S_0^3}{S^3} + \frac{3}{4}(1-\varepsilon)\frac{S_0^4}{S^4} - \frac{1}{2}\frac{S_0^6}{S^6}\right], \tag{8}$$

$$\dot{C} = m\frac{S_0^3}{S^3}. \tag{9}$$

We have set S_0 to be the value of S at which the phase transition took place. Soon, with $S \gg S_0$, the first term on the right-hand side dominates the expansion and we obtain

$$S = S_0\left(\frac{t}{t_0}\right)^{2/3}, \qquad t_0 = (6\pi G\varepsilon fm^2)^{-1/2}. \tag{10}$$

These ideas of 1966 anticipated the inflationary model of Guth (1981), in which a GUT phase transition is responsible for a similar switchover of the Universe from the de Sitter state to the Friedman state. Hoyle & Narlikar (1966b) argued that our cosmological observations tend to be confined to the bubble expanding according to (10). In the later version of Arp et al. (1990), the bubbles are identified with mini-creation events. The creation process is of the on/off type, with the Universe accelerating (in the steady-state fashion) when the activity is 'on' and decelerating (in the Friedman way) when it is 'off'.

These ideas have been worked out in greater detail recently by Hoyle et al. (1993) in their 'quasi-steady-state cosmology'. We summarize the scenario here, highlighting the quantitative deductions that are relevant to this work.

In the quasi-steady-state cosmology, creation occurs in varying mass units up to $\sim 10^{15}$–10^{16} $M_\odot$ at epochs interspersed with periods of very little creation. During the creative phases the Universe is made to accelerate its expansion, while in the dormant phases the expansion tends to slow down. We ourselves happen to lie in a dormant phase, with the last creative phase being as far back as $z \sim 4$. These values are also related by Hoyle et al. to the observed degree of isotropy of the microwave background and the relative transparency of the Universe up to redshifts of this order.

In a typical mini-creation event of mass M, the C-field causes outwards motion starting from a density that is typical of a Schwarzschild sphere of radius $2GM/c^2$. Thus the initial density is given by

$$\rho_{\rm i} \cong 2 \times 10^{16}\left(\frac{M_\odot}{M}\right)^2 \ {\rm g\,cm^{-3}}. \tag{11}$$

The created particles are Planck particles of mass

$$M_{\rm p} = \left(\frac{3\hbar c}{4\pi G}\right)^{1/2},$$

which subsequently decay into baryons and photons leading to an equipartition of the energy density of matter $\rho_{\rm i}c^2$ with that of radiation $aT_{\rm i}^4$. This equality gives an initial temperature

$$T_{\rm i} \approx 7 \times 10^{12}\left(\frac{M_\odot}{M}\right)^{1/2} \ {\rm K}. \tag{12}$$

As the system expands, the radiation is initially confined to the object by its high opacity. A stage is reached, however, at which the radiation escapes, leaving the expansion to be matter-dominated. Hoyle et al. estimated the density at this stage to be

$$\rho \simeq \left[3 \times 10^{-9}\left(\frac{M_\odot}{M}\right)^{2/3}\kappa^{-1}\right]^{6/5} \ {\rm g\,cm^{-3}}, \tag{13}$$

where κ is the opacity, i.e. the mean absorption coefficient expressed in ${\rm cm^3\,g^{-1}}$.

Several such MCEs overlap in the course of expansion. By requiring the overlap density ρ_{overlap} to be such that at this stage the neighbouring objects touch one another, Hoyle et al. estimated this density as $\sim 10^{-27}$ g cm^{-3}. They argued that the present (closure) density of $\sim 10^{-29}$ g cm^{-3} implies that the Universe has expanded (with essentially no significant creation of matter) by a volume factor $\sim 10^2$. Setting this as $(1+z)^3$, the conclusion is that the overlap epoch was at a redshift of ~ 4.

Thus from the escape value of density given by (13) to the overlap value the temperature of the radiation field declines as the cube root of density, while thereafter it declines as the fourth root. Denoting by a factor β the fall in density in the escape-to-overlap stage, Hoyle et al. estimated the present radiation temperature as

$$T_0 = 2.6 \times 10^7 \left(\frac{M}{\mathrm{M}_\odot}\right)^{1/6} \beta^{1/12} \rho_{\mathrm{present}}^{1/3}. \tag{14}$$

Setting $T_0 = 2.74$ K and $\rho_{\mathrm{present}} = 10^{-29}$ g cm^{-3} we obtain

$$\left(\frac{M}{\mathrm{M}_\odot}\right) = 1.4 \times 10^{16} \beta^{1/2}. \tag{15}$$

Hoyle et al. then showed, from (13) and (15), that a value of M a little above 10^{15} M$_\odot$ gives a consistent solution, thereby relating the microwave background temperature to the size of a typical supercluster. They also subsequently related the picture to the synthesis of light nuclei. Creation can, however, also take place on smaller scales. We shall therefore perform our calculations for MCEs of arbitrary masses.

2.2 Anisotropic MCEs

Because we are interested in the possible emission of gravity waves from an MCE, we have to depart from the above idealized spherically symmetric model. It is indeed more realistic to expect the MCEs to be anisotropic if they are eventually to become superclusters (which have generally flattened ellipsoidal shapes). Accordingly, we shall construct a toy model for simulating an anisotropically expanding mass M created in an MCE.

It is convenient to describe the expansion through a Bianchi Type I model with a line element given by

$$\mathrm{d}s^2 = c^2\,\mathrm{d}\tau^2 - X^2(\tau)\,\mathrm{d}x^2 - Y^2(\tau)\,\mathrm{d}y^2 - Z^2(\tau)\,\mathrm{d}z^2, \tag{16}$$

where (x, y, z) are the comoving coordinates and τ the proper time of a typical dust particle moving outwards. The Einstein field equations can be solved for a dust-dominated system to yield the solution

$$X(\tau) = S(\tau)\{F(\tau)\}^{2\sin\alpha}, \qquad Y(\tau) = S(\tau)\{F(\tau)\}^{2\sin[\alpha+(2\pi)/3]},$$

$$Z(\tau) = S(\tau)\{F(\tau)\}^{2\sin[\alpha+(4\pi)/3]}, \qquad F(\tau) = \frac{(GM)^{1/3}\tau^{2/3}}{S(\tau)},$$

$$\{S(\tau)\}^3 = X(\tau)\,Y(\tau)\,Z(\tau). \tag{17}$$

The anisotropy of expansion arises through the parameter α. The 'average' scale factor of expansion is given by $S(\tau)$, and is related to the mass M by

$$S^3(\tau) = \frac{9}{2} GM\tau(\tau+\Sigma), \qquad \Sigma = \text{constant}. \tag{18}$$

The mass M and the density ρ are related by

$$M = \frac{4\pi}{3}\rho S^3. \tag{19}$$

The comoving coordinates (x, y, z) for the dust particles within the MCE have values lying in the range $(-1, 1)$ so that at any time τ during expansion the physical coordinates on the surface (x_s, y_s, z_s) satisfy the ellipsoidal equation

$$\frac{x_s^2}{X^2(\tau)} + \frac{y_s^2}{Y^2(\tau)} + \frac{z_s^2}{Z^2(\tau)} = 1. \tag{20}$$

Such an expanding object has a changing quadrupole moment. The quadrupole moment tensor has principal axes in the xyz directions. A straightforward calculation gives the three diagonal elements of the quadrupole moment tensor as

$$I_{xx} = \frac{1}{5}MX^2(\tau), \qquad I_{yy} = \frac{1}{5}MY^2(\tau), \qquad I_{zz} = \frac{1}{5}MZ^2(\tau). \tag{21}$$

We therefore expect that a typical MCE of the above type will radiate gravitationally. We shall estimate its effect in the following section. For this we shall need the so-called anisotropy parameter η, which is estimated as follows.

From (17) we have, for $\tau \gg \Sigma$,

$$\frac{X(\tau)}{Y(\tau)} = \left(\frac{2}{9}\right)^{(2/3)\sin\alpha - (2/3)\sin[\alpha+(2\pi)/3]} \equiv \left(\frac{2}{9}\right)^{q_{xy}} \tag{22}$$

and similar ratios for $Y(\tau)/Z(\tau)$, $Z(\tau)/X(\tau)$, etc. For complete isotropy these ratios are all unity. In general, the exponents q_{xy}, q_{yz}, q_{zx} of (2/9) are non-zero and indicate departure from isotropy.

The anisotropy parameter η_{xy} is then defined by

$$\eta_{xy} = \left(\frac{2}{9}\right)^{(4\sin\alpha)/3} - \left(\frac{2}{9}\right)^{(4/3)\sin[\alpha+(2\pi)/3]}$$

$$= \left(\frac{2}{9}\right)^{(4/3)\sin[\alpha+(2\pi)/3]}\left[\left(\frac{2}{9}\right)^{2q_{xy}} - 1\right], \tag{23}$$

with similar expressions for η_{yz} and η_{zx}. The reason for this particular expression will become clear in the next section.

We first estimate q_{xy}, etc., from the shapes of clusters and superclusters on the assumption that after the MCE ceases to expand it retains its shape (equation 20) as galaxies form within it. We shall return to this assumption in Section 4.2. For clusters of galaxies, the mean axial ratio is ~ 0.5 (Plionis, Barrow & Frenk 1991), while for superclusters it is ~ 0.14 (Oort 1983). From (22) we estimate $q_{xy} \sim 0.46$ for clusters and ~ 1.3 for superclusters.

Since the α parameter effectively varies between $-\pi/6$ and $\pi/2$ we find from (23) that

$$\begin{aligned} &0.098 \lesssim |\eta_{xy}| \lesssim 2.04 \qquad \text{for clusters,} \\ &0.127 \lesssim |\eta_{xy}| \lesssim 2.66 \qquad \text{for superclusters.} \end{aligned} \tag{24}$$

Again, $\eta_{xy}=0$ corresponds to complete spherical symmetry. The importance of η will become clear in the next section when we discuss the actual magnitude of the gravitational wave amplitude due to an MCE.

3 GRAVITATIONAL RADIATION FROM AN MCE

3.1 Amplitude of radiation

We now proceed with the anisotropic MCE discussed above as a typical source of gravitational waves located at a cosmological distance. We first perform a calculation for flat space–time and subsequently make corrections to the results due to various causes. Typically, the gravitational wave amplitude is given by

$$h^{\mathrm{TT}}_{ik}(\boldsymbol{r},\tau)=\frac{2G}{c^4}\cdot\frac{1}{r}\left[\ddot{I}_{ik}\left(\tau-\frac{r}{c}\right)\right]^{\mathrm{TT}} \tag{25}$$

(e.g. Landau & Lifshitz 1941), where the superscript TT refers to the transverse traceless gauge. Here $\boldsymbol{r}$ is the position vector of the observer from the source and $\tau-(r/c)$ is the retarded time at which the effect from the source reaches the observer.

For simplicity, we choose the line of sight to coincide with the z-axis of the ellipsoid (20). (For a general orientation our final answer has to be multiplied by some projection factors.) In this case we have the only non-zero components of I^{TT}_{ik} as

$$I^{\mathrm{TT}}_{xx}=I_{xx}-I_{yy}=\frac{1}{5}M[X^2(\tau)-Y^2(\tau)]=-I^{\mathrm{TT}}_{yy}. \tag{26}$$

Thus we see that (22) is non-zero only if $X\neq Y$.

Substituting from (17), we obtain after a somewhat lengthy but straightforward manipulation, for $\tau\gg\Sigma$,

$$\ddot{I}^{\mathrm{TT}}_{xx}\approx\frac{2}{5}M(GM)^{2/3}\tau^{-2/3}\left[\left(\frac{2}{9}\right)^{(1/3)[1+4\sin\alpha]}-\left(\frac{2}{9}\right)^{(1/3)\{1+4\sin[\alpha+(2\pi)/3]\}}\right]$$

$$\approx\frac{2}{5}\left(\frac{2}{9}\right)^{1/3}M(GM)^{2/3}\tau^{-2/3}\eta_{xy}, \tag{27}$$

where η_{xy} is given by (23), and hence h from (25) is given by

$$h^{\mathrm{TT}}_{xx}(\boldsymbol{r},\tau)=\frac{4}{5}\left(\frac{2}{9}\right)^{1/3}(GM)^{5/3}\frac{1}{c^4r}\left(\tau-\frac{r}{c}\right)^{-2/3}\eta_{xy}. \tag{28}$$

To set the above result in astronomical perspective, we rewrite it as

$$h(\boldsymbol{r},\tau)\cong3.2\times10^{-20}\eta\left(\frac{10\ \mathrm{kpc}}{r}\right)\left(\frac{M}{10\ \mathrm{M}_\odot}\right)^{5/3}\left[\frac{\tau-(r/c)}{1\ \mathrm{s}}\right]^{-2/3}. \tag{29}$$

Henceforth we shall drop the subscript xy on η.

3.2 Fourier transform of the amplitude

Since most of the detectors, as well as the probes, of gravitational radiation are sensitive over a limited frequency band, it is useful to express (29) in frequency space. We redefine the 'zero' of time τ of the observer as the instant when the gravitational wave first hits him, so that for $\tau>0$

$$h(\boldsymbol{r},\tau)=\frac{4}{3}\left(\frac{2}{9}\right)^{1/3}\frac{(GM)^{5/3}}{c^4r}\tau^{-2/3}\eta. \tag{30}$$

The Fourier transform of $h(\boldsymbol{r},\tau)$ is given by

$$\tilde{h}(\boldsymbol{r},f)=\int_{-\infty}^{\infty}h(\boldsymbol{r},t)\exp(-2\pi\mathrm{i}f\tau)\,\mathrm{d}\tau$$

$$=\int_0^{\infty}\frac{4}{5}\left(\frac{2}{9}\right)^{1/3}\frac{(GM)^{5/3}}{c^4r}\eta\tau^{-2/3}\exp(-2\pi\mathrm{i}f\tau)\,\mathrm{d}\tau$$

$$=\frac{4}{5}\left(\frac{2}{9}\right)^{1/3}\Gamma\left(\frac{1}{3}\right)\frac{\eta\exp(-\mathrm{i}\pi/6)}{(2\pi f)^{1/3}}\frac{(GM)^{5/3}}{c^4r}. \tag{31}$$

In numerical terms this becomes

$$|\tilde{h}|=4.65\times10^{-20}\eta\left(\frac{10\ \mathrm{kpc}}{r}\right)\left(\frac{M}{10\ \mathrm{M}_\odot}\right)^{5/3}\left(\frac{f}{1\ \mathrm{Hz}}\right)^{-1/3}. \tag{32}$$

We shall next put this result in perspective with regard to observation by currently available gravitational wave detectors.

3.3 Detection by laser interferometers

For the LIGO-type detectors that are essentially broad-band detectors one uses the Wiener optimal filters $q(t)$ defined by their Fourier transform

$$\tilde{q}(f)=k\frac{\tilde{h}(f)}{S_h(f)} \tag{33}$$

(Thorne 1987; Schutz 1991), where $S_h(f)$ is the spectral density of the noise in the detector and k is some arbitrary constant. When the filter matches the signal perfectly, the cross-correlation between the outputs of the detector and the filter leads to a signal-to-noise ratio

$$\frac{S}{N}=\left[2\int_0^{\infty}\frac{|\tilde{h}(f)|^2}{S_h(f)}\mathrm{d}f\right]^{1/2}. \tag{34}$$

Assuming for simplicity that the detector is sensitive uniformly in the frequency band (f_1,f_2) and is blind outside it, we obtain

$$S_h(f)=\begin{cases}\sigma^2 & \text{for } f_1<f<f_2,\\ \infty & \text{otherwise.}\end{cases} \tag{35}$$

Here σ is a constant related to the sensitivity of the detector. So, from (34) and (31), we obtain

$$\frac{S}{N}=\left(\frac{1}{9\pi}\right)^{1/3}\frac{4\sqrt{6}}{5}\Gamma\left(\frac{1}{3}\right)\frac{\eta(GM)^{5/3}}{\sigma c^4r}(f_2^{1/3}-f_1^{1/3})^{1/2}. \tag{36}$$

To fix our ideas, we take the LIGO sensitivity which, for the characteristic frequency range between ~100 and 300 Hz, is expected to be

$$\sigma\sim10^{-24}\ \mathrm{Hz}^{-1/2} \tag{37}$$

(Abramovici et al. 1992). Using these numbers we obtain

$$\frac{S}{N} \gtrsim 16.4\eta \left(\frac{M}{10\,\mathrm{M}_\odot}\right)^{5/3} \left(\frac{100\ \mathrm{Mpc}}{r}\right) \left(\frac{\sigma}{10^{-24}\ \mathrm{Hz}^{-1/2}}\right)^{-1} \times \left[\left(\frac{f_2}{300\ \mathrm{Hz}}\right)^{1/3} - \left(\frac{f_1}{100\ \mathrm{Hz}}\right)^{1/3}\right]^{1/2}. \quad (38)$$

It is interesting that S/N is large enough for LIGO to detect an MCE of as low a mass as $10\,\mathrm{M}_\odot$ situated within a distance of ~ 100 Mpc from us, provided it is anisotropic enough to have $\eta \gtrsim 0.1$.

Are there any upper limits on the masses of MCEs detectable by gravitational waves? We must be careful here in using (28) to estimate the gravitational wave amplitude because this equation is based on the quadrupole radiation formula in which the background space–time geometry is taken to be flat, while the time coordinate τ that appears in equations (28)–(30) corresponds to the proper time in the interior of an MCE. In particular, the time coordinate external to the MCE that is relevant for the observer is a function of both the spatial and the temporal coordinates that describe the interior geometry, implying that the observed amplitude at an instant depends on the interior coordinates in a very complicated manner.

On the other hand, the expanding solution given in Section 2.2 describes an object with large-scale coherence in its dynamical motion in the τ-frame. The crucial question is to what extent this coherence would manifest itself in the gravitational waves from the object in the rest frame of an external detector (like the LIGO on Earth). An exact calculation is extremely difficult, but to obtain an insight into the problem we make use of simple physical arguments.

The net gravitational wave amplitude from an MCE, in the linear approximation, can be thought of as arising from the superposition of gravitational waves from individual fluid elements of the MCE. If f is the frequency of interest, coherent superposition of amplitudes occurs only from within a region R of size less than $\sim 0.5\lambda = 0.5c/f$. At a time τ after the birth of an MCE, the characteristic frequency f of the gravitational wave is $\sim \dot{S}/S = 2/3\tau$, so that from (18) one estimates the mass of the region R to be less than $\sim 1.3 \times 10^4 f^{-1}\,\mathrm{M}_\odot$, where f is in Hz. Thus, for frequencies in the range $\sim$ 10–100 Hz, the upper limit to the mass lies in the range $\sim$ 130–1300 $\mathrm{M}_\odot$. One can safely use the expression in (38) for events having mass less than the above upper limit. For masses above the limit, more exact and detailed calculations will be needed to compute the net observable effect. However, for an event that releases $\sim 100\,\mathrm{M}_\odot$ at a distance close to the limit of the observable Universe (~ 3000 Mpc), the signal-to-noise ratio is about 36η, well within the reach of a LIGO.

The crucial question, of course, is 'How frequent are such events?' To answer this we need to know the rate at which the MCEs occur over a given volume.

In the steady-state model, the average rate of creation of matter is given by $3H_0\rho$, where H_0 is Hubble's constant ($=100\,h_0$ km s^{-1} Mpc^{-1}) and ρ is the mean (smoothed out) density of matter. Assuming that the MCEs occur with a more or less uniform mass M per unit time, the rate of such events is

$$\mathfrak{R}(M) = \frac{3\rho H_0}{M} \cong 10^{-80} h_0 \left(\frac{\rho}{2\times 10^{-29}\ \mathrm{g\ cm^{-3}}}\right)\left(\frac{M}{10\,\mathrm{M}_\odot}\right)^{-1} \mathrm{cm^{-3}\ s^{-1}}. \quad (39)$$

The above rate implies that, within a radius of ~ 100 Mpc, one MCE of $\sim 10\,\mathrm{M}_\odot$ goes off every second. Similarly, MCEs with mass $\sim 130\,\mathrm{M}_\odot$ going off anywhere within the horizon will be seen at a rate of ~ 570 s^{-1}. On the other hand, the rate for MCEs of supercluster size occurring within ~ 3000 Mpc is still quite low, namely ~ 1 in 10^4–10^5 yr. Hoyle et al. (1993) have argued that creation events occur in phases, during which the Universe passes through a 'creative' mode. The isotropy constraints of the microwave background constrain the last epoch of such activity to redshifts of $\gtrsim 4$. Thus major supercluster-type MCEs would not be expected at smaller redshifts. In any case, destructive interference of gravitational wave amplitudes arising from points separated by a distance of $\sim 0.5\lambda$ within the MCE makes it difficult to estimate the intensity of gravitational radiation at $f \sim 100$ Hz from such high-mass events.

Reduced creation (i.e. in the form of smaller masses) may, however, be taking place sporadically even in relatively recent epochs. Thus the rate of 570 s^{-1} computed for MCEs of $\sim 130\,\mathrm{M}_\odot$ is expected to be an overestimate. For example, if the present creation rate has fallen now to $\sim 10^{-6}$ of the average value given by (39), the above rate will be reduced to 5.7×10^{-4} s^{-1}.

This calculation therefore tells us that the existence of MCEs in the stellar mass range can be easily confirmed or disproved by a detector in the LIGO class. Both the rarity of events and the lack of quantitative estimates for the gravitational wave amplitude make supercluster-type MCEs uninteresting as far as LIGOs are concerned.

3.4 A few caveats

We re-emphasize that these estimates are to be looked upon as crude 'first estimates' which need to be polished with more accurate and detailed calculations. Such calculations should include the following issues.

(a) The Bianchi Type I solution discussed by us is singular at $\tau = 0$, whereas a realistic MCE should start with a finite but very large density corresponding to a dense pocket of the *C*-field. Thus we have not taken due note of the relationship of our isolated MCE solution to the overall cosmological boundary conditions.

(b) We have ignored the matching conditions across the finite boundary of the MCE, which should tell us how to relate the time coordinate τ within the MCE to the cosmological time t. Normally, we expect the observer (like ourselves) to observe a mixture of Doppler blueshift (for the approaching surface of the MCE) and gravitational redshift of the MCE mass, in addition to the usual cosmological redshift.

(c) The exact solution is further complicated by the fact that the gravitational waves that arrive at the observer at a given time did not leave the different parts of the source at the same comoving time. (This has already been discussed in

Section 3.3.) Hence a more exact version of our calculations of $\ddot{X}$, $\ddot{Y}$, $\ddot{Z}$ etc. will have to include this non-synchronous departure effect.

(d) We have used in (22) the flat space–time formula for propagation of the gravity wave. One would expect redshift-dependent factors to appear, reducing the intensity beyond the flat space–distance effect.

We also briefly compare the MCE with the standard sources of gravitational waves in astrophysics, namely supernovae, coalescing binaries, etc. This will enable observers to distinguish between the possible sources in the event of a positive detection.

From a supernova source we expect powerful but transient bursts of gravity waves. Crude estimates (cf. e.g. Misner et al. 1973) indicate energy radiated at a rate of $\sim 3\times 10^{54}$ erg s^{-1} with a characteristic frequency of ~ 3000 Hz and lifetime of ~ 0.15 s or 300 periods. Such supernova explosions occur a few times per century per galaxy.

In the case of an MCE, since the characteristic gravitational wave frequency when the MCE is τ s old is $\sim 2/3\tau$, one expects LIGO to see a burst of radiation lasting for ~ 0.007–0.07 s, provided that the lower frequency cut-off of the detector lies in the range 10–100 Hz. An MCE of $\sim 100\ M_\odot$ would thus be observed even at a distance of ~ 3000 Mpc. Observations by the LIGO-type detectors can therefore place limits on the creation rate allowable in the form of MCEs of this order of mass.

Typical stellar binaries have longer time-scales ranging from $\sim 10^9$–10^{24} yr (Misner et al. 1973). For coalescing binaries the time-scales can be much shorter, ranging from a few years to a few seconds. For example, in the case of a neutron star binary with mass of $\sim 1.4\ M_\odot$, for each of the compact objects the characteristic gravitational wave amplitude at a frequency of ~ 300 Hz is $\sim 7\times 10^{-19}$ if the binary is 10 kpc away. For such a binary, the gravitational wave amplitude lasts for ~ 2.5 s, corresponding to frequencies larger than 100 Hz.

Energetically, therefore (and also, possibly, in terms of the event rate), both supernovae and coalescing binaries fall short of MCEs with mass less than ~ 100–$1000\ M_\odot$, so these MCEs are an interesting class of object for LIGO-type detectors.

4 THE GRAVITATIONAL WAVE BACKGROUND GENERATED BY THE MCEs

4.1 Flux and spectrum of an MCE

The energy radiated per unit time per unit area by an MCE of mass M and anisotropy parameter η is given by

$$\mathscr{F}\cong\frac{c^3}{16\pi G}\left(\frac{\partial h}{\partial\tau}\right)^2 \tag{40}$$

(cf. Thorne 1987), where h is given by (30). In frequency space, the energy crossing per unit area per unit frequency range is given by

$$\tilde{\mathscr{F}}\cong\frac{\pi c^3}{2G}f^2|\tilde{h}(f)|^2. \tag{41}$$

The net energy radiated per unit frequency range is (41) multiplied by $4\pi r^2$. This quantity becomes, after substitution from (31),

$$\mathscr{E}\cong\frac{32\pi^2}{25}\left(\frac{1}{9\pi}\right)^{2/3}\left[\Gamma\left(\frac{1}{3}\right)\right]^2\frac{\eta^2}{c^5 G}(GM)^{10/3}f^{4/3}. \tag{42}$$

In numerical terms this is

$$\mathscr{E}=1.57\times 10^{46}\eta^2\left(\frac{M}{10\ M_\odot}\right)^{10/3}\left(\frac{f}{1\ \mathrm{Hz}}\right)^{4/3}\ \mathrm{erg\ Hz^{-1}}. \tag{43}$$

Using these expressions within the framework of the steady-state model, we evaluate the spectrum of the energy density produced by all such MCEs together. We will take note of the expanding Universe in computing the dimming and frequency shift of this radiation.

4.2 The stochastic gravitational wave background

To compute the background, consider the MCEs of mass M in a shell that has inner and outer boundaries at redshifts of z and $z+\mathrm{d}z$. Denoting by $V(z)\,\mathrm{d}z$ the volume of this shell, the number of gravitons radiated in the frequency range $(f', f'+\mathrm{d}f')$ in a time $(\tau', \tau'+\mathrm{d}\tau')$ is estimated as

$$\mathrm{d}N=\frac{\mathscr{E}(f')\,\mathrm{d}f'}{hf'}\,\mathfrak{R}(M)\,V(z)\,\mathrm{d}z\,\mathrm{d}\tau'. \tag{44}$$

The corresponding frequency and time ranges at the terrestrial observer are $\mathrm{d}f=\mathrm{d}f'/(1+z)$ and $\mathrm{d}\tau=\mathrm{d}\tau'(1+z)$. Hence the flux density of gravitational radiation received from the redshift shell $(z, z+\mathrm{d}z)$ is given by

$$F(M,f,z)\,\mathrm{d}z=\frac{hf}{4\pi r^2 S_0^2}\frac{\mathrm{d}N}{\mathrm{d}f\,\mathrm{d}t}. \tag{45}$$

We first compute the background on the assumption that the MCEs are distributed uniformly in the steady-state Universe. We will subsequently discuss modifications based on our being in a bubble that extends up to a redshift of $z\sim 4$, as discussed in Section 2.1. The radial coordinate r is related to z in the steady-state model by the relation

$$rS_0=\frac{c}{H_0}z, \tag{46}$$

while $V(z)$ becomes

$$V(z)=4\pi\left(\frac{c}{H_0}\right)^3\frac{z^2}{(1+z)^3}. \tag{47}$$

Using the above expressions, we obtain

$$F(M,f,z)=\frac{c}{H_0}\frac{\mathscr{E}(f(1+z))}{(1+z)^4}\mathfrak{R}(M). \tag{48}$$

Substituting for $\mathscr{E}$ and $\mathfrak{R}$ from (42) and (39), we obtain

$$F(M,f,z)=\frac{96\pi^2}{25}\left(\frac{1}{9\pi}\right)^{2/3}\left[\Gamma\left(\frac{1}{3}\right)\right]^2\eta^2\frac{\rho(GM)^{7/3}}{c^4}f^{4/3}(1+z)^{-8/3}. \tag{49}$$

Integration of F with respect to z gives us the total stochastic gravitational wave background in the observed frequency range $(f, f+df)$. We obtain the answer as a function of M as well as f:

$$F(M,f)=\frac{288\pi^2}{125}\left(\frac{1}{9\pi}\right)^{2/3}\left[\Gamma\left(\frac{1}{3}\right)\right]^2\eta^2\frac{\rho(GM)^{7/3}}{c^4}f^{4/3}. \quad (50)$$

The corresponding energy density of the gravitational wave background is therefore

$$\mathscr{E}_{GW}=\frac{1}{c}F(M,f)\equiv\Omega_{GW}(f)\,\rho_c c^2 f^{-1}, \quad (51)$$

where ρ_c is the closure density $3H_0^2/8\pi G$.

It is now possible to set limits on $\Omega_{GW}(f)$ by analysing residuals in the pulse arrival time measurements of pulsars (Detweiler 1979; Mashhoon 1982). We make use of (51) to express Ω_{GW} as

$$\Omega_{GW}=\frac{288\pi^2}{125c^7}\left(\frac{1}{9\pi}\right)^{2/3}\left[\Gamma\left(\frac{1}{3}\right)\right]^2(GM)^{7/3}f^{7/3}\eta^2\Omega_0, \quad (52)$$

where Ω_0 is the ratio of the matter density to the closure density.

Stinebring et al. (1990) quoted an upper limit of 4×10^{-7} near $f=0.14$ yr^{-1} with 95 per cent confidence. This value may be compared with the theoretical value given by (52). Evaluation of (52) at $f=0.14$ yr^{-1} gives the inequality

$$\eta^2\left(\frac{M}{M_\odot}\right)^{7/3}<1.55\times10^{24},$$

i.e.

$$M<2.33\times10^{10}\eta^{-6/7}\,M_\odot. \quad (53)$$

These results are slightly modified if we take into account the possibility that we live in a part of space–time in which the creation process is relatively dormant. The quasi-steady-state cosmology requires that up to a redshift of $z\sim4$ we are in a Friedman bubble with the Einstein–de Sitter (flat $k=0$) model determining the expansion of the Universe, with *no* creation of matter. The relations (45)–(49) therefore apply to $z\gtrsim4$, while for smaller redshifts we need to use the Einstein–de Sitter relations. The calculation is straightforward although somewhat tedious. The resulting expression for $F(M,f)$ is that given by (50) with an additional factor $(1+z_c)^{-5/3}$, where $z_c(\approx4)$ is the redshift of the bubble boundary. The limit (53) is then changed to

$$M<7.36\times10^{10}\eta^{-6/7}\,M_\odot. \quad (54)$$

MCEs with mass $\lesssim$ 100–1000 $M_\odot$ are in obvious agreement with the above observational constraint. For high-mass events one must again be cautious. As discussed in Section 3.3, the gravitational wave amplitude formula (28), on which the above analysis is based, is valid only for MCEs of mass less than $\sim1.3\times10^4(f/1\text{ Hz})^{-1}$ $M_\odot$. For $f\approx0.14$ yr^{-1}, this translates into an upper limit of $\sim3\times10^{12}$ $M_\odot$. Strictly speaking, therefore, the constraint (54) is applicable only to MCEs of mass not exceeding typical galactic masses; and it is clear from (54) that galactic-size MCEs with η less than 0.1 are consistent with the currently available pulsar timing data.

A naive extrapolation of (54) to supercluster-size MCEs has the following implications. For MCEs with $M\cong10^{15}$ $M_\odot$, (54) will force η to values $\lesssim10^{-6}$. That is, such MCEs should expand very isotropically. This, of course, is not consistent with the relatively large values of η given by (24). One has to argue, then, that the observed large values of η in the superclusters are not characteristic of the expansion anisotropies from which they formed. In that case, the observed flattening has to be a later phenomenon related to the astrophysical processes of condensation of galaxies and clusters in the superclusters. A second line of defence involves the statement that the supercluster-size MCEs are only a small fraction of the overall creation process, which involves for the most part smaller masses. If, however, a more careful calculation shows that very massive MCEs do not act as coherent sources of gravitational waves (as discussed earlier), then the expression (54) cannot be used to rule out supercluster-type MCEs.

5 CONCLUSIONS

These calculations show that gravitational waves can provide sharply focused tests of the mini-creation event hypothesis. In terms of its magnitude, an anisotropic MCE is the biggest source of gravitational waves, far exceeding in its power and event rate the more conventional sources such as supernova explosions and binary star systems.

We have shown that a LIGO-type laser interferometric detector should frequently detect mini-creation events generating mass less than ~1000 $M_\odot$. If matter is being created through mini-bangs at the overall rate required by the quasi-steady-state cosmology proposed by Hoyle et al. (1993), then the non-detection of any event would imply either that the creation process is occurring through larger and more infrequent MCEs or that the MCEs are highly isotropic, failing which the hypothesis of continuous creation is untenable.

The calculation of the stochastic background of gravitational waves created by the MCEs is also capable of being tested through the timing measurements of millisecond pulsars. We find that up to galactic-size MCEs are consistent with the presently available pulsar residual analysis. Extrapolation of these calculations to supercluster-type MCEs seems to indicate that such MCEs should be highly isotropic in order to meet the pulsar timing constraint.

Our conclusions are based on an approximate calculation which suffices for low-mass events. A more exact (and technically much more difficult) calculation is, however, necessary for MCEs of supercluster size.

Finally, this discussion demonstrates that the concept of cosmological creation of matter through mini-bangs hypothesized by Arp et al. (1990) is capable of being tested observationally with the existing technology.

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