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# The $C$ -field as a direct particle field

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It has been known for some years that a  $C$ -field, generated by a certain source equation, leads to interesting changes in the cosmological solutions of Einstein's equations. The steady-state cosmology appears as an asymptotic case. The source equation has so far only been given in the macroscopic case of a smooth fluid. In the present paper we derive the source equation in terms of discrete particles. The method adopted is similar to that we have recently given for the generalization to Riemannian space of the Fokker action principle in the electromagnetic theory. In the latter, a 4-vector is defined in terms of the world lines of particles. The definition is such that the four-dimensional *curl* of the vector satisfies Maxwell's equations, which are therefore identities. Similarly,  $C$  is a scalar defined in terms of the world-lines of particles, and the source equation used formerly then follows as an identity.

## INTRODUCTION

The motive for introducing the  $C$ -field is by no means solely cosmological. The existence of a singularity as a possible development in the theory of relativity is worrying, since it implies a breakdown of the principle of equivalence—corresponding singularities do not exist in electromagnetic theory, so a co-moving observer falling into a singularity *can* distinguish between falling in a gravitational field and being pulled by a 'rope'—i.e. being subject to an electromagnetic force. We have recently shown (Hoyle & Narlikar 1964*a*) that the presence of the  $C$ -field in the gravitational equations, satisfying the source equation we are to examine in the present paper, prevents singularities from occurring.

A singularity also implies a contradiction in what we assume about the nature of matter. Transformation of one baryon into another apart, we usually assume that the world-lines of particles are complete lines. From an arbitrary point on a world-line we can pass to infinity in either direction along the line. This property is violated at a singularity. Instead of a complete line we have a broken line. And if the whole Universe had a singular origin, the line has two ends; it becomes a segment.

Our point of view is that if world-lines can be broken then a mathematical expression of the fact must be given. The procedure we shall adopt is to require that the action of a particle be stationary, not only with respect to a slight deviation of any finite portion of the trajectory (i.e. a portion with fixed ends) but also with respect to small variations of the position of a broken end. This procedure also makes good sense in that the total action associated with a segment of a line is finite, whereas the action associated with the usual infinite line is itself infinite and the meaning of a variation of an infinite quantity is obscure.

In order to give expression to the required property of the action, we shall find it necessary to introduce a coupling between the ends of broken lines. This implies

that the ends generate a 'field', the *C*-field. The *C*-field therefore has its source at a set of discrete points, the ends of broken lines. It is natural that the field be a scalar because the source points do not themselves contain any structural information. In the electromagnetic case it is natural that the field is determined by a 4-vector because all points on the world-lines are source points and at each point there is information associated with the tangent vector.

INTERPARTICLE ACTION IN RIEMANNIAN SPACE-TIME

It will be useful to give a brief resumé of the generalization to Riemannian space-time of the Fokker action principle (cf. Hoyle & Narlikar 1964*b*). The action can be written in terms of the symmetric two-point Green functions given by DeWitt & Brehme (1960). The following notation will be adopted.

The world-lines of particles will be labelled *a, b, ...*. A typical point on world-line *a* will be denoted by *A*. The indices of various tensors at *A* will carry suffix *A, i<sub>A</sub>, k<sub>A</sub>*, etc. The co-ordinates of *A* are *a<sup>iA</sup>* and proper time at *A* will be denoted by *a*, with

$$da^2 = g_{i_A k_A} da^{i_A} da^{k_A}. \tag{1}$$

The charge and mass associated with *a* will be denoted by *e<sub>a</sub>, m<sub>a</sub>*.

The scalar and vector Green functions  $\bar{G}(A, B)$ ,  $\bar{G}_{i_A i_B}$  satisfy the wave equations

$$g^{i_A k_A} \bar{G}_{; i_A k_A} = -(-\bar{g})^{-\frac{1}{2}} \delta_{(A, B)}^{(4)}, \tag{2}$$

$$g^{i_A k_A} \bar{G}_{i_A m_B; i_A k_A} + R_{i_A}^{k_A} \bar{G}_{k_A m_B} = -(-\bar{g})^{-\frac{1}{2}} \bar{g}_{i_A m_B} \delta_{(A, B)}^{(4)}, \tag{3}$$

where  $\bar{g}_{i_A m_B}$  is the parallel propagator (Synge 1960) and  $\bar{g} = \det(\bar{g}_{i_A m_B})$ .  $\delta_{(A, B)}^{(4)}$  is given by

$$\delta_{(A, B)}^{(4)} = \delta(a^1 - b^1) \delta(a^2 - b^2) \delta(a^3 - b^3) \delta(a^4 - b^4). \tag{4}$$

The explicit forms for these Green functions are given by DeWitt & Brehme as follows:

$$\bar{G} = (1/8\pi) [\Delta^{\frac{1}{2}} \delta(\sigma_{AB}) - v \theta(\sigma_{AB})], \tag{5}$$

$$\bar{G}_{i_A i_B} = (1/8\pi) [\Delta^{\frac{1}{2}} \delta(\sigma_{AB}) \bar{g}_{i_A i_B} - v_{i_A i_B} \theta(\sigma_{AB})], \tag{6}$$

where

$$\sigma_{AB} = \frac{1}{2} \left[ \int_{\Gamma_{AB}} ds \right]^2, \tag{7}$$

$\Gamma_{AB}$  being the geodesic joining *A, B* (assumed close enough for the geodesic to be unique), and

$$\Delta = \det(\sigma_{; i_A i_B}) / \bar{g}. \tag{8}$$

The functions *v, v<sub>i<sub>A</sub> i<sub>B</sub></sub>* can be determined in terms of the metric tensor by a solution in series. In flat space-time, *v = 0, v<sub>i<sub>A</sub> i<sub>B</sub></sub> = 0, Δ = 1*. In general, however, information propagated from a source point *A* by these Green functions is not confined to the cone through *A* defined by null geodesics. The information spreads into the interior of the cone and Huygens's principle is not satisfied.

The two Green functions are connected by

$$\bar{G}_{; i_A} = -\bar{G}^{k_X}_{i_A; k_X}. \tag{9}$$

The vector potential at point *X* due to world-line *a* is defined by

$$A_{i_X}^{(a)} = 4\pi e_a \int \bar{G}_{i_X i_A} da^{i_A}, \tag{10}$$

which satisfies the gauge condition

$$A^{(a) i_X; i_X} = 0, \quad (11)$$

when the world-line  $a$  is open, but not when it is broken. The electromagnetic field associated with  $a$  is defined by

$$F_{i_X k_X}^{(a)} = A_{k_X; i_X}^{(a)} - A_{i_X; k_X}^{(a)}, \quad (12)$$

and satisfies all Maxwell's equations *identically*.

The action associated with  $a$  is

$$J^{(a)} = -m_a \int da - 4\pi e_a \sum_{b \neq a} e_b \iint \bar{G}_{i_A i_B} d\alpha^{i_A} db^{i_B}. \quad (13)$$

Variation of  $J^{(a)}$  in the usual way leads to the equations of motion of  $a$ —i.e. equations for world-line  $a$ . These are the Lorentz equations but without self-force.

#### THE DEFINITION OF THE $C$ -FIELD

We now take the world-line  $a$  to be a segment with ends at the points  $A_1, A_2$  and with  $A_2$  at the later time (in the sense in which we elect to evaluate all line integrals). The contribution of the world-line  $a$  to the total  $C$ -field at the point  $X$  is defined by

$$C^{(a)}(X) = f^{-1}[\bar{G}(X, A_2) - \bar{G}(X, A_1)]. \quad (14)$$

It would be possible, and indeed more logical, to define  $C^{(a)}$  with  $f = 1$ , just as it would be possible to define  $A_{i_X}^{(a)}$  with  $4\pi |e_a| = 1$ . Our reason for introducing the constant  $f$  in (14) is to permit a comparison with formulae given in previous papers for the macroscopic case. In a future paper, we intend to review the relationship between the coupling constants of different fields and we shall then simplify formulae such as (10), (14), leaving as few adjustable constants in the theory as we can.

The definitions (10), (14) are more similar than they would appear at first sight. This can be seen by rewriting (14) as

$$C^{(a)}(X) = \frac{1}{f} \int_{A_1}^{A_2} \bar{G}_{; i_A} d\alpha^{i_A}. \quad (15)$$

The scalar Green function  $\bar{G}$  plays a role in the definition of the  $C$ -field that is formally very similar to the part played by the vector Green function in the case of the electromagnetic field. This explains why in the macroscopic case there have always been formal similarities between the two fields. The formal similarities will become even more marked when we come to define the action of a particle. However, it must be emphasized that  $C^{(a)}$  depends only on  $A_1, A_2$ , and not on points of  $a$  between  $A_1, A_2$ .

There is an actual connexion between the two fields, worth noting before we discuss the implications of (14). This arises because of (9) which relates the scalar and vector Green functions. Hoyle & Narlikar (1964*b*) showed that

$$A^{(a) i_X; i_X} = -4\pi e_a \int \bar{G}_{; i_A} d\alpha^{i_A}, \quad (16)$$

and the right-hand side is not zero when the world-line  $a$  has ends  $A_1, A_2$ ; in fact

$$A^{(a) i_X; i_X} = -4\pi e_a f C^{(a)}(X). \quad (17)$$

It is important that after summation over all particles it is possible to have

$$\sum_a e_a C^{(a)} \approx 0, \quad \text{but} \quad \Sigma C^{(a)} \neq 0. \quad (18)$$

Charge compensation can permit the gauge condition  $A^{iX};_{iX} = 0$  even though  $C \neq 0$ . This will indeed be the case if the world-lines are correlated in such a way that charge is always conserved. However, we do not wish to introduce any such *ad hoc* postulate at the present stage.

We now show that

$$fC^{(a) iX};_{iX} = J^{(a) iX};_{iX}, \quad (19)$$

where

$$J^{(a) iX} = \int (-\bar{g})^{-\frac{1}{2}} \delta_{(A, X)}^{(A)} \bar{g}^{iX}_{i_A} d\alpha^{i_A}. \quad (20)$$

Before giving the proof of this result it is interesting to note that  $\sum_a m_a J^{(a) iX}$  becomes the mass-current when we pass to a smooth fluid approximation. Hence if we define  $C = \sum_a m_a C^{(a)}$  for the total  $C$ -field the quantity  $fC^{iX};_{iX}$  is just the divergence of the mass-current, and this is the result we have always used in the macroscopic theory. If  $C^{(a)}$  had been defined in (14) with a mass-factor  $m_a$  the same result would have followed but with the total  $C$ -field defined by  $C = \sum_a C^{(a)}$ . The effect in either case is to weight the contributions of different particles according to their masses. However, we do not wish at this stage to commit ourselves to a definite decision on how weights should be assigned. A discussion we shall give in a future paper raises doubt as to whether mass is indeed the correct quantity to describe the contribution of a particle. A general procedure could readily be followed by introducing a weight factor  $w_a$ , either into the definition of  $C^{(a)}$  or into that of  $C$ . But since we do not intend in this paper to specify any  $w_a$  we shall omit weight factors from our formulae—they can easily be inserted at any stage of an argument. The following discussion therefore refers to a situation in which all particles have equal weight.

We proceed now to prove (19), making use of (9) and of the wave equation satisfied by the vector Green function. Thus

$$\begin{aligned} fC^{(a) iX};_{iX} &= \int \bar{G};_{i_A}{}^{iX};_{iX} d\alpha^{i_A} = - \int \bar{G}^{kX}_{i_A};_{kX}{}^{iX};_{iX} d\alpha^{i_A} \\ &= - \int \{ \bar{G}^{kX}_{i_A};{}^{iX}_{kX}{}^{iX};_{iX} + (R^{iX}_{mX} \bar{G}^{mX}_{i_A});_{iX} \} d\alpha^{i_A}, \end{aligned}$$

where the commutation of the suffixes  $i_X, k_X$  introduces the term involving the Riemann-Christoffel tensor. Commuting the  $k_X, i_X$  suffixes so as to be able to use the wave equation for the Green function, we have

$$\begin{aligned} fC^{(a) iX};_{iX} &= - \int \{ \bar{G}^{kX}_{i_A};{}^{iX}_{i_X k_X} + R^{kX}_{mX}{}^{iX}_{k_X} \bar{G}^{mX}_{i_A};{}^{iX} \\ &\quad + R^{iX}_{mX}{}^{kX}_{i_X} \bar{G}^{kX}_{i_A};{}^{mX} + (R^{iX}_{mX} \bar{G}^{mX}_{i_A});_{iX} \} d\alpha^{i_A} \\ &= - \left[ \int (\bar{G}^{kX}_{i_A};{}^{iX}_{i_X} + R_{mX}{}^{kX} \bar{G}^{mX}_{i_A}) d\alpha^{i_A} \right];_{kX} \\ &= J^{(a) iX};_{iX}. \end{aligned}$$

## THE EQUATIONS OF MOTION OF A PARTICLE

We now define the action associated with  $a$  as

$$J^{(a)} = -m_a \int da - 4\pi e_a \sum_{b+a} e_b \iint \bar{G}_{i_A i_B} da^{i_A} db^{i_B} + f^{-1} \sum_{b+a} \iint \bar{G}_{;i_A i_B} da^{i_A} db^{i_B}. \quad (21)$$

The analogy between the electromagnetic field and the  $C$ -field is now very clear. Using the definitions (10), (15) the action can be written in the form

$$J^{(a)} = -m_a \int da - \sum_{b+a} \int (e_a A_i^{(b)} - C_{;i}^{(b)}) da^i, \quad (22)$$

in which the suffix  $A$  has been dropped since we are now concerned only with points on  $a$ . We now vary the whole of  $a$  including the end points  $A_1, A_2$ . The usual reductions lead to

$$\begin{aligned} \delta J^{(a)} = & - \left[ \left\{ m_a g_{ik} \frac{da^k}{da} + e_a \sum_{b+a} A_i^{(b)} - \sum_{b+a} C_{;i}^{(b)} \right\} \delta a^i \right]_{A_1}^{A_2} \\ & + \int \left\{ g_{ik} \left( m_a \frac{d^2 a^k}{da^2} + m_a \Gamma_{im}^k \frac{da^l}{da} \frac{da^m}{da} \right) - e_a \sum_{b+a} F_{il}^{(b)} \frac{da^l}{da} \right\} \delta a^i da. \end{aligned} \quad (23)$$

For  $\delta J^{(a)}$  to vanish for arbitrary  $\delta a^i$  we therefore require the line  $a$  to satisfy

$$\frac{d^2 a^k}{da^2} + \Gamma_{em}^k \frac{da^e}{da} \frac{da^m}{da} = \frac{e_a}{m_a} \sum_{b+a} F^{(b)kl} \frac{da^l}{da}, \quad (24)$$

and the ends  $A_1, A_2$  to be such that

$$m_a \frac{da^k}{da} + e_a \sum_{b+a} A^{(b)k} - \sum_{b+a} C^{(b)k} = 0, \quad (25)$$

at  $A_1, A_2$ .

Equations (24) are just the usual Lorentz equations of motion, without self-force. We see therefore that the  $C$ -field has no effect on the equations of motion of a particle. The contrary has been claimed by a number of authors. The error in these arguments arose from an invalid constraint, as was already pointed many years ago by one of us (Hoyle 1949). The smooth-fluid approximation is used, without noticing that in general the assumption of single-stream motion demands the presence of electromagnetic forces. Hence it is no surprise that departures from geodesic motion were found. The error lay in attributing these departures to the  $C$ -field instead of to the electromagnetic field. The above results make this point clearer than it has been hitherto.

In addition to equations (24) we also obtain (25). World-lines can end only at points at which this condition is satisfied. The interpretation is clear from the first two terms—the first is the usual dynamical contribution to the momentum-energy 4-vector, the second is the usual electromagnetic contribution. Evidently (25) is simply a requirement that energy and momentum be conserved at the ends of a world-line. This is achieved through the  $C$ -field—i.e. through recoil on the ends of other world-lines. There is no question of energy and momentum not being conserved. This is also made clearer by the present treatment, although a condition

equivalent to (25) has always emerged in the smooth-fluid applications to cosmology. The contrary has sometimes been maintained, largely through unfamiliarity with the concept of conservation in an open system—simply that the divergence of the total energy tensor be zero, as it always has been.

#### CONCLUSIONS

The *C*-field can be defined through a scalar Green function, the procedure being analogous to the description of the electromagnetic field through the vector Green function. Our former source-equation in the macroscopic theory emerges as an identity.

The *C*-field contribution to the action of a particle is also analogous to the electromagnetic contribution. There is no effect on the equations of motion which remain the Lorentz equations. There is, however, an effect at the ends of a world-line, where the *C*-field permits energy and momentum to be conserved, through recoil effects on other particles.

Finally, we emphasize again that by introducing the *C*-field the singularities present in the usual theory of relativity seem to be avoided. And, as the macroscopic treatment has shown, the universe itself need possess no singularity, even when the line element has the Robertson–Walker form, i.e. a synchronous time form.

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