HAWKING PROCESS AND THE COSMIC MICROWAVE BACKGROUND IN A STEADY STATE UNIVERSE

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It is investigated whether a large number of primordial black holes can account for the observed microwave background in a steady state universe. The answer is shown to be negative.

1. Introduction. The principal observational objection to the steady state cosmology rests on its apparent inability to provide a satisfactory explanation for the observed 2.7 K background radiation. Since there is no "hot era" in this cosmology any such explanation must rely on astrophysical processes of relatively recent origin. In this case the radiation background should be traceable either directly to sources or to some mechanism which thermalizes the radiation produced by sources in other wavelengths. Although several attempts have been made in the past (see, for example, refs. [1–4]), none has been completely satisfactory. The issue therefore still remains unresolved.

Here we discuss the feasibility of producing such a microwave background from a steady state distribution of a large number of black holes radiating according to the quantum process first suggested by Hawking [5]. It is, of course, clear straightaway that the Hawking process cannot produce any significant radiation from black holes of stellar masses or from supermassive black holes usually invoked for various astrophysical processes. The radiation produced by the de Sitter horizon has too low a temperature to be of any relevance to this problem [6]. The only possibility is radiation by black holes of masses considerably lower than the solar mass $M_\odot$. Carr [7] has discussed the possibility of explaining the microwave background in a big bang universe by postulating the existence of primordial black holes (PBH) with a certain mass spectrum. It is argued there that the radiation from evaporating black holes could be thermalized before matter and radiation decouple, so that we would now see that radiation in the form of 2.7 K photons.

2. The mass spectrum. We will asssume that the steady state model is governed by the perfect cosmological principle (PCP) of Bondi and Gold [8]. Although considerable thought has been devoted towards understanding (theoretically) the concept of continuous creation of matter (cf. refs. [9–12]) we will not assume any particular model proposed for it. The PCP, however, tells us that if the line element of the cosmological space–time is given by

$$ds^2 = c^2 dt^2 - e^{2Ht}(dx^2 + dy^2 + dz^2) , \qquad (1)$$

the mean creation rate per unit volume must be

$$Q = 3H\rho , \qquad (2)$$

where ρ is the mean density of matter in the universe, H is the Hubble constant and c the speed of light.

Since we have not assumed any particular model for primary matter creation, we will not go into the question of how small-sized black holes come into existence. We will adopt an empirical approach and simply suppose that there is a continuous supply of such black holes, which we will designate as primordial black holes (to distinguish them from the usual astrophysical black holes). How the PBH's form in a steady state universe becomes a relevant question only after their feasibility for microwave background has been demonstrated.

Let $n(m)\,dm$ denote the number density of PBH's in the mass range $(m, m + dm)$ and suppose that primary creation adds $h(m)\,dm$ PBH's per unit volume per unit time. A typical black hole after creation starts radiating by the Hawking process with a mass loss rate given by

$$dm/dt = -\lambda/m^2 , \tag{3}$$

where

$$\lambda = (15\,360\,\pi)^{-1}\hbar c^4/G^2 \approx 3.968 \times 10^{24}\ \mathrm{g^3 s^{-1}} . \tag{4}$$

Thus after creation each black hole slides along the mass axis towards $m = 0$, where it ultimately evaporates. Taking into account the fact that the universe is expanding and that the PCP must hold, we get the following differential equation for $n(m)$:

$$\frac{\lambda}{m^2}\frac{dn}{dm} - \frac{2\lambda}{m^3} n - 3Hn + h(m) = 0 . \tag{5}$$

Using the boundary condition $n(\infty) = 0$, we integrate this to

$$n(m) = \frac{m^2}{\lambda} \exp(Hm^3/\lambda) \int_m^\infty h(x) \exp(-Hx^3/\lambda)\, dx . \tag{6}$$

The kernel of the differential equation (5) can be obtained by setting $h(x) = 3HN_0\delta(x - m_0)$, implying the creation of black holes of mass m_0 *only*. We then get

$$n(m) = \frac{3HN_0}{\lambda} m^2 \exp[H(m^3 - m_0^3)/\lambda] \cdot \theta(m_0 - m) . \tag{7}$$

Here $\theta(x)$ is the Heaviside function. The total number of PBH's per unit volume in existence at any time is then given by

$$N(m_0) = \int_0^{m_0} n(m)\, dm = N_0\ \{1 - \exp(-m_0^3 H/\lambda)\} . \tag{8}$$

It is convenient to define a critical mass for the PBH by

$$m_c = (\lambda/H)^{1/3} \approx 1.35 \times 10^{14}\,\mathrm{g} . \tag{9}$$

Then for $m_0 \gg m_c$, $N(m_0) \approx N_0$. We also note that in eq. (7) $n(m)$ rises rapidly with m so that any time the mass distribution is heavily concentrated at the upper mass end.

3. Background radiation from PBH's. We now consider the total contribution $S(\nu)d\nu$ to the flux at a given point in the frequency range $(\nu, \nu + d\nu)$ from the universal distribution of PBH's. Noting that a typical black hole radiates like a black body, its rate of emission of energy per unit surface area, in the frequency range $\nu, \nu + d\nu$ is given by $\mathcal{F}(\nu, T)\, d\nu$, where

$$\mathcal{F}(\nu, T) = \frac{4\pi^2\hbar}{c^2} \frac{\nu^3}{\exp(2\pi\hbar\nu/kT - 1)} , \tag{10}$$

with the temperature T related to the PBH mass m (expressed in grammes) by

$$T = \frac{\hbar}{2\pi k}\kappa = \frac{\hbar c^3}{8\pi G k m} \approx 1.2 \times 10^{26} m^{-1}\ \mathrm{K} . \tag{11}$$

Here k is the Boltzmann constant and κ the surface gravity of the PBH (taken as the Schwarzschild black hole).

Taking into account the red-shift effect, the total flux received in the frequency range $(\nu, \nu + d\nu)$ from any direction at any point P at any epoch is given by $S(\nu)d\nu$, where

$$S(\nu) = \frac{16\pi^3\hbar G^2}{c^6} \frac{c}{H} \nu^3 \times \int_{z=0}^\infty \frac{dz}{1+z} \int_{m=0}^\infty \frac{m^2 n(m)\, dm}{\exp\{(16\pi^2 G m (1+z)/c^3)\nu\} - 1} . \tag{12}$$

We now consider the implication of this formula for the microwave background.

4. Discussion. It is of course not possible to integrate eq. (12) for an arbitrary mass spectrum. It is however possible to perform the integral in the Rayleigh–Jeans limit. We then get from eq. (12), for

$$\mu \equiv (16\pi^2 G m (1+z)/c^3)\nu \ll 1 , \tag{13}$$

the following form:

$$S(\nu) = \alpha\nu^2 , \tag{14}$$

where α is a constant given by

$$\alpha = \frac{\pi G\hbar}{c^3} \frac{c}{H} \int_0^\infty m n(m)\, dm = \frac{\pi G\hbar}{c^2 H} \rho_{\mathrm{PBH}} . \tag{15}$$

Here ρ_{PBH} is the density of matter in the form of primordial black holes. Although eq. (14) does seem to reproduce the Rayleigh–Jeans spectrum, the constant α corresponding to 2.7 K demands

$$\rho_{PBH} \approx 17.2\,\mathrm{g\,cm^{-3}} . \qquad (16)$$

This is an impossibly high density, being some 31 orders of magnitude above the observed density of visible matter. Although the Bondi–Gold version of the steady steady cosmology does not mention a canonical density, it would be very difficult to argue the existence of hidden matter of this enormously high density. It may be remarked that the dynamical models of steady state cosmology [9–11] lead to densities of the order of the closure density of Friedmann models and these are ~ 29 orders of magnitude below the estimate given by eq. (16).

If instead of fitting the Rayleigh–Jeans limit we look for the peak of the function $S(\nu)$, it is expected to occur at frequencies close to that given by $\mu \approx 1$ (notwithstanding red shifts). This gives a frequency

$$\nu_{max} \approx 2.5 \times 10^{36} m_0^{-1}\,\mathrm{Hz} , \qquad (17)$$

where m_0 is the most effective contributor to the mass integral of eq. (12). From our discussions of $n(m)$, m_0 is expected to lie close to the upper end of the mass spectrum for continuous creation. Setting ν_{max} to correspond to 2.7 K black body maximum we get $m_0 \sim 10^{26}$ g. Since the luminosity of a PBH of mass m_0 (expressed in grammes) is given by $\sim 3.5 \times 10^{45} m_0^{-2}$, the amount radiated in a cosmological time scale H^{-1} by one PBH is

$$E \sim \frac{3.5 \times 10^{45}}{m_0^2} \times 6 \times 10^{17} \sim \frac{2 \times 10^{63}}{m_0^2} . \qquad (18)$$

For $m_0 \sim 10^{26}$, this gives $E \sim 2 \times 10^{11}$ erg. If there are N_0 such PBH's per unit volume, we have the energy density of radiation $\sim 2 \times 10^{11} N_0\,\mathrm{erg\,cm^{-3}}$. Equating this to the energy density $\sim 4 \times 10^{-13}$ erg $\mathrm{cm^{-3}}$ of the microwave background we get $N_0 \sim 2 \times 10^{-24}\,\mathrm{cm^{-3}}$. This gives

$$\rho_{PBH} \sim N_0 \times 10^{26}\,\mathrm{g\,cm^{-3}} \sim 2 \times 10^{2}\,\mathrm{g\,cm^{-3}} , \qquad (19)$$

thus making the case worse than that for the Rayleigh–Jeans limit. If m_0 had been fixed as being the PBH mass with temperature 2.7 K in the above calculation we would have recovered eq. (16) for ρ_{PBH}.

It might be argued that by adjusting the creation function $h(m)$ it may be possible to arrive at a mass spectrum $n(m)$ which could give the proper background. That this argument is false can be shown by the following inverse calculation. If a PBH is to be viable, its mass to luminosity ratio should not be markedly different from the observed ratio of matter to radiation density in the universe which is $2.5 \times 10^{-18}\,\mathrm{g\,erg^{-1}}$. For a PBH of mass m the ratio is $m^3/(2 \times 10^{63})\,\mathrm{g\,erg^{-1}}$. Equating the two we get $m \sim 10^{15}\mathrm{g} \sim 10\, m_c$. However, such a PBH will generate radiation mainly in the gamma range. The universe is transparent to gamma rays. There is, therefore, little chance of thermalizing this primordial radiation.

We therefore conclude that there is no way in which the Hawking process can generate the microwave background in a steady state universe.

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