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A Quintessentially Geometric Model

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We consider string inspired cosmology on a solitary $D3$ brane moving in the background of a ring of branes located on a circle of radius R. The motion of the $D3$ brane transverse to the plane of the ring gives rise to a radion field which can be mapped to a massive non-BPS Born-Infeld type field with a cosh potential. For certain bounds of the brane tension we find an inflationary phase is possible, with the string scale relatively close to the Planck scale. The relevant perturbations and spectral indices are all well within the expected observational bounds. The evolution of the universe eventually comes to be dominated by dark energy, which we show is a late time attractor of the model. However we also find that the equation of state is time dependent, and will lead to late time Quintessence.

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I. INTRODUCTION

It was recently suggested that the rolling open string tachyon, inspired by a class of string theories, can have important cosmological implications. The decay of a non-BPS $D3$-brane filling four dimensional space time leads to a pressureless dust phase which we identify with the closed string vacuum. The rolling tachyon has an interesting equation of state whose parameter ranges from 0 to -1. It was therefore thought to be a candidate of inflation and dark matter, or a model of transient dark energy [1]. However if we rigorously stick to string theory, the effective tachyon potential contains no free parameter. A viable inflationary scenario should lead to enough number of $e-$folding, and the correct level of density perturbations. The latter requires a free parameter in the effective potential which could be tuned to give rise to an adequate amount of primordial density perturbations. One also requires an adjustable free parameter in the effective potential to account for the late time acceleration.

Recently a time dependent configuration in a string theory was investigated and was shown to have interesting cosmological application [12]. In this scenario a BPS $D3$-brane is placed in the background of several coincident, static $NS5$-branes which are extremely heavy compared to the $D3$-brane and form an infinite throat in the space time. This system is inherently non-supersymmetric because the two different kinds of branes preserve different halves of the bulk supersymmetries. As a result the $D3$ brane can be regarded as a probe of the warped background and is gravitationally attracted toward the $NS5$-branes. Furthermore there exists an exact conformal field theory description of this background where the number of five-branes determines the level of the WZW current algebra [13], which allows for exact string based calculations. Despite the fact that the string coupling diverges as we approach the fivebranes, it was shown that we can trust our effective Dirac-Born-Infeld (DBI) action to late times in the evolution provided that the energy of the probe brane is sufficiently high. In any event, as the probe $D3$-brane approaches the background branes the spatial components of the energy-momentum tensor tend to zero in exactly the same way as in the effective action description of the open string tachyon. Thus it was anticipated that the dynamics of branes in these backgrounds had remarkably similar properties to rolling tachyon solutions. This relationship was further developed by Kutasov who showed that it was possible to mimic the open string tachyon potential by considering brane motion in a specific kind of 10D geometry. In order to do this one must take the action of the BPS probe brane in the gravitational background and map it to a non-trivial scalar field solution described by the non-BPS action [2]. The new field is essentially a holographic field living on the world-volume of the brane, but encodes all the physics of the bulk background. This is known as the geometrical tachyon construction. Another particularly interesting solution considered the background branes distributed around a ring of radius R, which was analysed in ref [15, 16], and whose geometry is described

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by a coset model [14], again potentially opening the way for an exact string calculation.

It seems natural to enquire as to whether these geometrical tachyon solutions have any relevance for cosmology, since they neatly avoid the problems associated with open string tachyon inflation [4] by having a significantly different mass scale. This change in scale is due to the motion of the probe brane in a gravitationally warped background, provided by the branes in the bulk geometry. In essence, this is an alternative formulation of the simple Randall-Sundrum model [32]. More recently, flux compactification has opened up the possibility of realising these models in a purely four-dimensional string theory context [9]. The fluxes form a throat which is glued onto a compact manifold in the UV end of the geometry. The warp factor in the metric has explicit dependence on the fluxes, and so provides us with a varying energy scale. The recent approaches to brane cosmology [7] are based on the motion of $D3$-branes in these compactifications. Typically we find $\bar{D}3$-branes located at some point in the IR end of the throat, which provide a potential for a solitary probe brane, with the inflaton being the inter-brane distance. In this context we can obtain slow roll inflation, and also the so-called DBI inflation [10], which relies heavily on the red-shifting of energy scales. However flux compactification models have an unacceptably large number of vacua, characterised by the string landscape. They are also low energy models, where the string scale is significantly lower than the Planck scale and so there is no attempt to deal with the initial singularity. In addition, we require multiple throats attached to the compact manifold where the standard model is supposed to live, however there is no explanation for the decoupling of the inflaton sector. These problems need to be addressed if we are to fully understand early universe cosmology in a string theory context. The alternative approach is to consider cosmology in the full ten dimensional string theory. Although these models are plagued by their own problems there is a definite sense of where the standard model is assumed to live, and a natural realisation of inflation. Furthermore we can invoke a Brandenberger-Vafa type mechanism to explain the origin of our $D3$-brane, arising from the mutual cascade annihilation of a gas of $D9$-$\bar{D}9$-branes [37].

An alternative approach is compactify our theory on a compact manifold, where some mechanism is employed to stabilise the various moduli fields. This will naturally induce an Einstein-Hilbert term into the four dimensional action [30]. However this is a highly non-trivial problem whose precise details remain unknown. Despite being unable to embed this into String Theory, we can still learn a great deal about the physics of the model - as emphasised by recent works [18].

A specific case of interest has been to study inflation in the ring solution ref [19]. Due to the unusual nature of the harmonic function we find decoupled scalar modes, one transverse to the ring plane and the other inside the ring. The cosmology of modes inside the ring have been studied in ref [17]. In this note we will consider the situation in which the $D3$-brane moves in the transverse direction to the ring. Performing the tachyon map in this instance yields a cosh type potential implying that the resulting scalar field in the dual picture is massive. It is interesting that in this setting we do not have to worry about the continuity condition around the ring. And unlike the longitudinal motion, we have an analytic expression for the effective potential every where in the transverse directions. We study the cosmological application of the resulting scenario and show that the model leads to an ever accelerating universe. We study the autonomous form of field evolution equation in the presence of matter and radiation and show that the de-Sitter solution is a late time attractor of the model. We also demonstrate the viability of the geometrical tachyon for dark energy in the setting under consideration, arising in a natural way due to the non-linearity of the DBI action. In the next section we will introduce the string theory inspired model, and discuss how we an relate it to four dimensional cosmology. In section III we will consider the more phenomenological aspects of our model by comparing our results with experimental observation. Section IV shows how we have a natural realisation of reheating in our model, whilst section V discusses the final stage of dark energy domination. Our model predicts that the equation of state parameter will tend to $\omega \sim -1$, but on even larger timescales we expect it to increase toward zero as in models of quintessence [41]. We will conclude with some remarks and a discussion of possible future directions.

II. GEOMETRICAL SCALAR FIELD AND COUPLING TO GRAVITY.

We begin with the string frame CHS solution for k parallel, static $NS5$ branes in type IIB String Theory [20, 21]. The metric is given by:

$$ds^2 \;=\; \eta_{\mu\nu}dx^\mu dx^\nu + F(x^n)dx^m dx^m, \qquad (1)$$

where χ is the dilaton field define as $e^{2(\chi-\chi_0)} = F(x^n)$, and there exists the three form field strength of the NS B-field $H_{mnp} = -\varepsilon^q_{mnp}\partial_q\phi$. Here $F(x^n)$ is the harmonic function describing the position of branes. For a large number of branes we can consider the throat approximation, which amounts to dropping the factor of unity in the function. Inherently we are decoupling Minkowski space time from the theory, and therefore only interested in the region around the $NS5$-branes. The harmonic function is given by:

$$\begin{aligned} F \;&=\; 1 + \frac{kl_s^2\ \sinh(ky)}{2R\rho\ \sinh(ky)\ (\cosh(ky)-\cos(k\theta))} \\ &\approx\; \frac{kl_s^2\ \sinh(ky)}{2R\rho\ \sinh(ky)\ (\cosh(ky)-\cos(k\theta))}, \end{aligned} \qquad (2)$$

where ρ, θ parameterise polar coordinates in the ring plane, and the factor y is given by:

$$\cosh(y) \;=\; \frac{R^2+\rho^2}{2R\rho}. \qquad (3)$$

We put a probe $D3$ brane at the centre of $NS5$ branes, as mention in the introduction this brane will move toward the circumference due to gravitational interaction if it shifted a little from the centre keeping the brane in the plane of the ring; the cosmology in this case is described elsewhere. We consider the case where the probe brane lies in the centre of the ring but shifted a little from the plane. In this case the probe brane shows transverse motion. Note that because of the form of the DBI action, the configuration here is actually S-dual to the $D5$-brane ring solution. The only difference is the shift of $k \to 2g_sk$ in the harmonic function. The physics however are very different as we know that F-strings cannot end on the $NS5$-branes, but can end on the $D5$-branes. This implies that in the case of the $D5$-brane ring we can have additional open string tachyonic modes once the probe brane starts to resolve distances of order of the string scale. The cosmological implications for this extra field were discussed in [18].

For the brane at the center ($\rho = 0$) moving transverse to the ring ($\dot\rho = 0$), the harmonic function is given by:

$$F(\sigma) \;=\; \frac{kl_s^2}{R^2+\sigma^2}, \qquad (4)$$

and the DBI action for the probe brane can be written in the following form, in static gauge

$$S = -\tau_3\int d^4\xi\sqrt{F^{-1}-\dot\sigma^2}. \qquad (5)$$

The tachyon map in this instance arises via field redefinition. We define the following scalar field, which has dimensions of length

$$\phi(\sigma) = \int\sqrt{F}d\sigma, \qquad (6)$$

which maps the BPS action to a form commonly used in the non-BPS case [2]

$$S = -\int d^4\xi V(\phi)\sqrt{1-\dot\phi^2}, \qquad (7)$$

where $V(\phi)$ is the potential for the scalar field which describes the changing tension of the D-brane.

From the above mapping we get the solution of field as:

$$\begin{aligned}\phi(\sigma) \;&=\; \int_0^\sigma \sqrt{F(\sigma')}d\sigma' \\ &=\; \sqrt{kl_s^2}\ln\left(\frac{\sigma}{R}+\sqrt{1+\frac{\sigma^2}{R^2}}\right) \\ &=\; \sqrt{kl_s^2}\,\mathrm{arcsinh}\left(\frac{\sigma}{R}\right) \qquad (8)\\ V(\phi) \;&=\; \frac{\tau_3}{\sqrt{F}} \\ &=\; \frac{\tau_3 R}{\sqrt{kl_s^2}}\cosh\left(\frac{\phi}{\sqrt{kl_s^2}}\right). \qquad (9)\end{aligned}$$

Clearly we see that $\phi \to \pm\infty$ as $\sigma \to \pm\infty$, and that at the minimum of the potential we have $\phi = 0$ [1]. The potential of the field suggests that the mass is given by $1/kl_s^2$, corresponding to a massive scalar fluctuation. One may ask if there is a known string mode exhibiting this profile. In fact the fluctuations of a massive scalar were computed in [29] using a similar approach to the construction of the open string tachyon mode in boundary conformal field theory [1]. This field was then used in ref. [8, 31] as a candidate for the inflaton living on a $\bar{D}3$-brane in the KKLT scenario ref. [34]. The potential for the scalar is known to fourth order and was been assumed to be exponential in profile, although globally it may be hyperbolic.

In order to discuss the cosmological evolution of our scalar field we need to couple our effective action to four dimensional Einstein gravity. There are several ways we can accomplish this. Firstly we can consider the Mirage Cosmology scenario [33]. This requires us to rewrite the induced metric on the $D3$-brane world-volume in a Friedmann-Robertson-Walker (FRW) form. The universe will automatically be flat, or closed if we imagine the D-brane to be spherical. The problem here is that there is no natural way to couple gravity to the brane action and therefore we must insert it by hand, however the cosmological dynamics are expected to be reliable virtually all the way to the string scale. The second option is a slight modification of the first. We imagine that the bulk is infinite in extent, and that the $D3$-brane is again coupled to gravity through some unknown mechanism. However rather than writing the induced metric in FRW form, we switch to the holographic theory. Now, the tachyon mapping in this case is only concerned with time-dependent quantities, and in particular only with the temporal component of the Minkowski metric. Therefore we choose to include a scale factor component in the spatial directions. This means that we have a cosmological coupling for the holographic scalar field, and the universe lives on the $D3$-brane world-volume. The

[1] We must bear in mind that our approximation of the harmonic function prevents us from taking the $\sigma \to \infty$ limit.

final approach would be to compactify the theory down to four dimensions. In order to do this we need to truncate the background to ensure the space is compact [9]. In our case the ring can naturally impose a cut-off in the planar direction, however we must still impose some constraint in the transverse direction to the ring plane. Our solution simplifies somewhat if we can consider the $R \to 0$ limit, or equivalently the $\sigma \gg 1$ limit, as the background will appear point like. Smoothly gluing the truncated space to a proper compact manifold will now automatically include an Einstein-Hilbert term in the effective action [30]. However, although we now have a natural coupling to gravity, the compactification itself is far from trivial as we also need to wrap two of the world-volume directions of the $NS5$-branes on a compact cycle. In order to proceed we must first uplift the full solution to M-theory [2], where we now have a ring of $M5$-branes magnetically charged under the three-form $C_{(3)}$. Compactification demands that the magnetic directions of the three-form are wrapped on toroidal cycles, which is further complicated by the ring geometry and will generally result in large corrections to the potential once reduced down to four-dimensions. So, although we have a natural gravitational coupling we may have large corrections to the theory. The complete description of this compactification is interesting, but well beyond the scope of this note and should be tackled as a future problem. However we could also assume a large volume toroidal compactification, where again all the relevant moduli have been stabilised. Provided we introduce some 'sink' for the five-brane charge, located at the some distant point in the compact space, and also only concentrate on the region close to the branes so that the harmonic function remains valid and we will have an induced gravitational coupling in the low energy theory. The corrections to the scalar potential in this region of moduli space may well be sub-leading with respect to the scalar field dynamics and thus we can treat our model as the leading order behaviour.

Recent work in this direction has been concerned with the compactification approach [18, 19], where it was assumed all the relevant moduli are fixed along the lines of the KKLT model [34] and that all corrections to the potential are sub dominant. We will tentatively assume that this will also hold in our toy model.

We can now analyse our four dimensional minimally coupled action, where we find the following solutions to the Einstein equations

$$H^2 = \frac{V(\phi)}{3M_p^2\sqrt{1-\dot{\phi}^2}} \tag{10}$$

$$\frac{\ddot{a}}{a} = \frac{V(\phi)}{3M_p^2\sqrt{1-\dot{\phi}^2}}\left(1-\frac{3\dot{\phi}^2}{2}\right). \tag{11}$$

These expressions are different to those associated with a traditional canonical scalar field. In particular we see that inflation will automatically end once $\dot{\phi}^2 \sim 2/3$ as in the tachyon cosmology models [3, 5, 11]. For completeness we write the equation of motion for the inflaton derived from the non-BPS action as follows

$$\frac{V(\phi)\ddot{\phi}}{1-\dot{\phi}^2} + 3HV(\phi)\dot{\phi} + V'(\phi) = 0, \tag{12}$$

where dots are derivatives with respect to time and primes are derivatives with respect to the field. Note that we are suppressing all delta functions in the expressions. We can now proceed with the analysis of our theory in the usual manner. It must be noted that this model corresponds to large field inflation, where the initial value of the scalar field must satisfy the following condition

$$\phi_0 \ll \sqrt{kl_s^2}\,\mathrm{arccosh}\left(\frac{\sqrt{kl_s^2}}{R}\right), \tag{13}$$

according to our truncation of the harmonic function.

Note that in what follows we will frequently switch between the field theory and the bulk geometry. The latter is more geometrical and so provides us with extra intuition about the physics of the solution, however both are equivalent - at least in this simplified model.

Using the slow-roll approximation, $H^2 \simeq V(\phi)/3M_p^2$ and $3H\dot{\phi} \simeq -V_\phi/V$, the e-folding

$$\begin{aligned} N &= \int_t^{t_f} H dt \\ &= \frac{\tau_3 R\sqrt{kl_s^2}}{M_p^2}\int_{x(\phi_f)}^{x(\phi)} \frac{\cosh^2 x}{\sinh x} dx \\ &= s\left[-\cosh(x_f) + \cosh(x) - \ln\left(\frac{\tanh(x_f/2)}{\tanh(x/2)}\right)\right]. \end{aligned} \tag{14}$$

Where we have introduced the dimensionless quantities $x = \phi/\sqrt{kl_s^2}$ and $s = \tau_3 R\sqrt{kl_s^2}/M_p^2$.

Further defining the new quantity: $y \equiv \cosh x$ we can write the number of e-folds as follows:

$$N = s\left[-y_f + y - \frac{1}{2}\ln\left(\frac{(y_f-1)(y+1)}{(y_f+1)(y-1)}\right)\right] \tag{15}$$

Now, the relevant slow-roll parameter is defined as $\epsilon \equiv -\dot{H}/H$ which in our solution reduces to

$$\epsilon = \frac{y^2-1}{2sy^3}. \tag{16}$$

[2] This was discussed by Ghodsi et al in [18]. We refer the interested reader there for more details.

Note that our model is explicitly non-supersymmetric, and therefore we don't need to calculate the second slow roll parameter η since we anticipate that this will be trivially satisfied if ϵ is. At the end of inflation $\epsilon = 1$, then $y_f \equiv f(s)$ is given by the root of above equation, setting $\epsilon = 1$

$$f(s) \;=\; \frac{1}{6s}\left[g(s) + \frac{1}{g(s)} + 1\right] \tag{17}$$

where $g(s) = \left(-54s^2 + 1 + 6s\sqrt{3(27s^2-1)}\right)^{1/3}$ From eqn(15) the equation for y is:

$$\ln\left(\frac{y+1}{y-1}\right) - 2y \;=\; -\frac{2N}{s} - 2f(s) - \ln\left(\frac{f(s)-1}{f(s)+1}\right) \tag{18}$$

For $s > 1$ and as $y_{\min} = 1$, ϵ always remains less than one leading to an ever accelerating universe. Thus, in this case the geometrical scalar field in the present setting is not suitable to describe inflation but can become a possible candidate of dark energy. However if τ_3 is small enough so that $s < 1$, then we will find that inflation is possible as the slow roll parameter will naturally tend toward unity. There is a critical bound $s \leq 1/(3\sqrt{3})$, which must be satisfied if we are to consider inflation in this context.

III. INFLATIONARY CONSTRAINTS.

To know the observational constraint on s we have to calculate the density perturbations. In the slow-roll approximation, the power spectrum of curvature perturbation is given by [22, 23, 24]:

$$\begin{aligned} P_S \;&=\; \frac{1}{12\pi^2 M_p^6}\left(\frac{V^2}{V_\phi}\right)^2 \\ &=\; \frac{\tau_3^2 R^2}{12\pi^2 M_p^6}\left(\frac{\cosh^2(\phi/\sqrt{kl_s^2})}{\sinh(\phi/\sqrt{kl_s^2})}\right)^2 \end{aligned} \tag{19}$$

The COBE normalisation corresponds to $P_S \simeq 2\times 10^{-9}$ for modes which crossed $N = 60$ before the end of inflation [6] which gives the following constraint:

$$k(l_s M_p)^2 \;\simeq\; \frac{10^9}{12\pi^2}\frac{s^2\cosh^4(\phi/\sqrt{kl_s^2})}{\cosh^2(\phi/\sqrt{kl_s^2}) - 1} \tag{20}$$

From the numerics using eqn(18) and eqn(19), we find that

$$k(l_s M_p)^2 \geq 3\times 10^{10} \tag{21}$$

which corresponds to $s \sim 10^{-3}$ when we impose the constraints $\tau_3 = 10^{-10} M_p^4$ and $R = 10^2/M_p$ which we regard as being typical values. The constraint on the tension in fact implies the following relationship

$$\frac{M_p}{M_s} \sim \frac{10^2}{g_s^{1/4}}, \tag{22}$$

which we need to be consistently satisfied. However, note that because of our basic assumptions about the theory we will generally obtain the bound

$$\frac{\tau_3 R}{M_p^3} \leq \frac{1}{9\times 10^5}. \tag{23}$$

If we write the tension of the brane in terms of fundamental parameters we can estimate the relationship between the String and Planck scales using the fact that we require $R > M_s^{-1}$ for the action to be valid

$$\frac{M_p}{M_s} \geq \frac{15}{g_s^{1/3}}, \tag{24}$$

where g_s is the string coupling constant. Note that this potentially constrains the String scale to be close to the Planck scale, as even if we demand weak coupling with $g_s = 0.001$ this gives us $M_p \geq 10^2 M_s$. Of course this is only a bound, and in our model we are treating this as a free parameter. In any event our typical values are consistent and thus we feel free to proceed. We should note that from a string theoretic point of view we should not take s as being a variable in this model. However our earlier analysis has shown that if we wish to consider non-eternal inflation, there exists a maximum bound on this parameter which is quite small. Thus we can make the assumption that s will always be small, with appropriate tuning of the ratio of the string and Planck scales.. In the following analysis we will always be assuming that this is satisfied so as to avoid en eternal inflation scenario. Of course, in the string theory picture we have a probe brane moving in a non-trivial background geometry, and we would expect that the RR charge on the brane will be radiated away in the form of closed string modes. This effectively means that there is an additional decay constant in the definition of the field ϕ, which we have neglected in this note. Thus what we have here is a first-order approximation to the behaviour of the solution. It remains an open question as to whether we can define a tachyon map in this instance - and how this changes the inflationary scenario described here.

At leading order in our solutions, where s is assumed to be small and making sure our effective action remains valid, we obtain

$$k(l_s M_p^2)^2 \;\simeq\; \frac{10^9}{48\pi^2}(2N+1)^2 \tag{25}$$

which corresponds to $s \sim 10^{-5}(2N+1)$ and $y \sim \frac{(2N+1)}{2s}$, when $\tau_3 = 10^{-10} M_p^4$ and $R = 10^2/M_p$. Again, more generally we would find the following upper limit on the solution

$$s \leq 10^{-3}(2N+1), \tag{26}$$

which is easily satisfied by our typical values. In fact our results remain robust when compared to the WMAPII and SDSS results combined [27]. The new data constrains $n_s = 0.98 \pm 0.02$ at the 68 confidence level, and $r < 0.24$ at the 95 confidence level.

The spectral index of scalar perturbations is defined as [22, 23, 24]:

$$
\begin{aligned}
n_S - 1 &\equiv -4\frac{M_p^2 V_\phi^2}{V^3} + 2\frac{M_p^2 V_{\phi\phi}}{V^2} \\
&= \frac{2}{s}\left(\frac{2-y^2}{y^3}\right)
\end{aligned} \tag{27}
$$

The spectral index of tensor perturbations is defined as:

$$
\begin{aligned}
n_T &= -\frac{M_p^2 V_\phi}{V^3} \\
&= -\frac{1}{s}\left(\frac{y^2-1}{y^3}\right)
\end{aligned} \tag{28}
$$

The tensor-to-scalar ratio is:

$$
\begin{aligned}
r &\equiv 8\frac{M_p^2 V_\phi^2}{V^3} \\
&= \frac{8}{s}\left(\frac{y^2-1}{y^3}\right)
\end{aligned} \tag{29}
$$

With the limit $s \to 0$ we get

$$
n_S = 1 - \tfrac{4}{(2N+1)},\;\; n_T = -\tfrac{2}{(2N+1)},\;\; r = \tfrac{16}{(2N+1)} \tag{30}
$$

For $N = 60$, we get $n_S = 0.96694$ and $r = 0.13223$; for $N = 50$, we get $n_S = 0.96040$ and $r = 0.15842$. We know from observations that the constraint on the tensor-to-scalar ratio is $r < 0.36$ [25, 26], and so our model appears to be well within this bound.

IV. REHEATING

We see that the potential is a symmetric potential with a minima. In terms of the bulk field σ it can be written as:

$$
\begin{aligned}
V(\sigma) &= \frac{\tau_3 R}{2\sqrt{kl_s^2}}\left[\left(\frac{\sigma}{R} + \sqrt{1+\frac{\sigma^2}{R^2}}\right)\right. \\
&\quad \left. + \left(\frac{\sigma}{R} + \sqrt{1+\frac{\sigma^2}{R^2}}\right)^{-1}\right]
\end{aligned} \tag{31}
$$

Now the question is would the brane oscillate back and forth through the ring, and if so what are the necessary conditions for oscillation? In the bulk picture we would naturally anticipate oscillation with a decaying amplitude due to RR-emission. Moreover the minimum of the potential in this case is actually metastable. However this has not been verified as we need to calculate the energy emission in the coset model description [14], which we leave as future work. This will alter the dynamics of the inflaton field as discussed in the previous section.

In any event we may also expect similar behaviour once our field is coupled to gravity, with the damping being provided by the Hubble term. This is particularly important because we may find inflation occurring in the phase space region beyond $s \geq s_{\rm crit}$, once enough damping has occurred. The relevant dynamical equations are the inflaton field equation (12) and the Friedmann equation. We repeat them below for convenience.

$$
\ddot{\phi} + 3H\dot{\phi}(1-\dot{\phi}^2) + \frac{V_\phi}{V}(1-\dot{\phi}^2) = 0 \tag{32}
$$

$$
H^2 = \frac{1}{3M_p^2}\left(\frac{V}{\sqrt{1-\dot{\phi}^2}} + \rho_B\right) \tag{33}
$$

where the terms inside the curly brackets cause damping. For an easy treatment let us first consider the slow-roll approximation, then in the damping equation only the $\dot{\phi}$ term remains and all other powers of $\dot{\phi}$ can be ignored. That is to say we are considering the case near the stable point. Then $H^2 \sim \frac{\tau_3 R}{3M_p^2\sqrt{kl_s^2}} + \frac{\rho_B}{3M_p^3}$ is constant and $\frac{V_\phi}{V} \sim \frac{2}{kl_s^2}\phi$. The equation of motion is then:

$$
\ddot{\phi} + 3\dot{\phi}\sqrt{\left(\frac{\tau_3 R}{3M_p^2\sqrt{kl_s^2}} + \frac{\rho_B}{3M_p^2}\right)} + \frac{2}{kl_s^2}\phi = 0 \tag{34}
$$

for critically damped motion we need:

$$
\left(\frac{\tau_3 R}{3M_p^2\sqrt{kl_s^2}} + \frac{\rho_B}{3M_p^2}\right) = \frac{8}{9kl_s^2} \tag{35}
$$

If the RHS of (35) is greater than the LHS we will find oscillations but it is reduced by damping which depends on the size of the damping factor ($= \frac{3}{2}\sqrt{\left(\frac{\tau_3 R}{3M_p^2\sqrt{kl_s^2}} + \frac{\rho_B}{3M_p^2}\right)}$), compared to the oscillation frequency ($= \sqrt{\frac{8}{kl_s^2} - 9\left(\frac{\tau_3 R}{3M_p^2\sqrt{kl_s^2}} + \frac{\rho_B}{3M_p^2}\right)}$).

From the definition of Ω_B setting it to 0.3, we get $\rho_B = \frac{3\tau_3 R}{7\sqrt{kl_s^2}}$, then from eqn(35) we obtain

$$
\begin{aligned}
s &> \frac{168}{90} \quad \text{Over damped} \\
&= \frac{168}{90} \quad \text{Critically damped} \\
&< \frac{168}{90} \quad \text{Oscillatory with a decaying amplitude.}
\end{aligned} \tag{36}
$$

Recall from the previous section that for us to have non-eternal inflation there is a maximal bound for s, and so only the last solution can be considered physical. From the constraint we get $\sqrt{kl_s^2} \sim 10^5 M_p^{-1}$, $\tau_3 \sim 10^{-10} M_p^4$ and $R \sim 10^2 M_p^{-1}$. Hence it is oscillatory near the critical point. The energy of the decaying scalar field is used in expansion and particle production. If the rate of expansion of universe is much less than the decaying rate of the amplitude of the field then most of the energy released by the scalar field goes to reheating. The explicit solution of eqn (34) is:

$$\phi(t) = \phi_0 e^{\left[-\frac{3}{2}t\sqrt{\frac{10\tau_3 R}{21 M_p^2\sqrt{kl_s^2}}}\right]} e^{\left[\pm i t\sqrt{\frac{2}{kl_s^2} - \frac{15\tau_3 R}{14 M_p^2\sqrt{kl_s^2}}}\right]} \tag{37}$$

The ratio of rate of field decay to the rate of expansion of universe is is defined to be:

$$\Theta \equiv \left|\frac{\dot{\phi}}{H\phi}\right| \tag{38}$$

For this case we find:

$$\Theta = \sqrt{\frac{21 M_p^2}{5\tau_3 R\sqrt{kl_s^2}}} \tag{39}$$

The above quantity can be made to be less than one by adjusting the various parameters.
Using eqn(25) we obtain:

$$\Theta \sim \sqrt{\frac{21 \times 10^5}{5(2N+1)}} \tag{40}$$

which allows us to write the parameter as a function of the number of e-foldings, provided we can trust our small s expansion. We know that reheating ends when $\Theta = 1$, thus the minimal number of e-foldings we require for this to be satisfied is

$$N_{\rm end} \sim 10^5. \tag{41}$$

Clearly this is a large number of e-foldings, and this should motivate us to do a more thorough analysis. For now it would appear that unless there is a large amount of fine tuning, reheating would not end in this scenario. The difficulty is that we cannot use the WKB approximation in this case due to rapid fluctuations in the variation of the potential. Moreover, the analysis will be incomplete without specifying the exact form the gravitational coupling - as there will be corrections to the effective action arising from any compactification. For these reasons we will postpone the analysis and return to it in a later publication.

V. DARK ENERGY

What are the implications of our model for dark energy[3]? It is well known that the non-linear form of the DBI action admits an unusual equation of state, which is of the form

$$\begin{aligned} \omega &= \frac{P}{\rho} \\ &= \dot{\phi}^2 - 1 \end{aligned} \tag{42}$$

where P and ρ are the pressure and energy densities respectively. In tachyon models the field is moving relativistically near the vacuum and the equation of state will tend to $\omega \sim 0$, which is problematic for reheating. However our model has significantly different late time behaviour because our scalar field will oscillate about the minimum of its potential, eventually coming to a halt at the minimum. Therefore we expect the equation of state to become $\omega \sim -1$, corresponding to the vacuum energy of the universe. This motivates us to analyse our system as a potential candidate for dark matter. One problem, however, is that the reheating phase doesn't seem to have a natural termination point. Rather, reheating of the universe continues whilst the brane oscillates around the minimum of the potential, and then terminates in what appears to be a dark energy dominated phase. From the perspective of model building this is obviously a difficult problem. For now let us assume that there is some ad hoc mechanism which ends inflation, and look at the evolution of the system in this dark matter dominated phase. The corresponding evolution equations of interest are:

$$\frac{\ddot{\phi}}{1-\dot{\phi}^2} + 3H\dot{\phi} + \frac{V_\phi}{V} = 0 \tag{43}$$

$$\dot{H} + \frac{V(\phi)\dot{\phi}^2}{2M_p^2\sqrt{1-\dot{\phi}^2}} + \frac{\gamma\rho_B}{2M_p^2} = 0 \tag{44}$$

where we have included contribution from a barotropic fluid in the second equation. Defining the following dimensionless quantities:

$$\begin{aligned} Y_1 &= \frac{\phi}{\sqrt{kl_s^2}} \\ Y_2 &= \dot{\phi}, \end{aligned} \tag{45}$$

and using eqn(43) and eqn(45) we get the autonomous equations:

$$Y_1' = \frac{1}{\sqrt{kl_s^2}H} Y_2 \tag{46}$$

$$Y_2' = -\left(1 - Y_2^2\right)\left(3Y_2 + \frac{1}{H}\frac{dY_3}{dY_1}\right) \tag{47}$$

Where we have switched to using the number of e-folds as the time parameter, and now primes denote derivatives with respect to N. The final expressions we require can be read off as

$$Y_3 = \ln\left(\frac{V(\phi)}{3M_p^2}\right)$$

[3] See [42] for an excellent review.

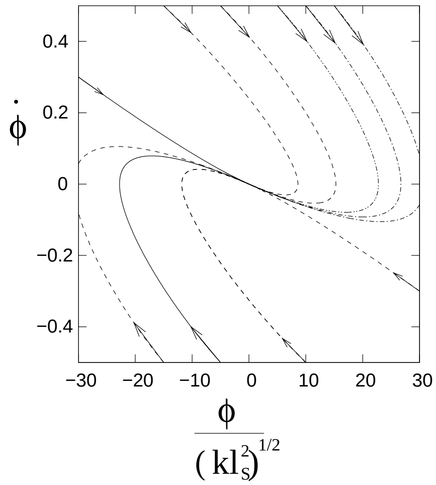

FIG. 1: Plot of the phase space solution with a variety of initial conditions. Here we see the presence of global attractor at $(\phi = 0, \dot{\phi} = 0)$

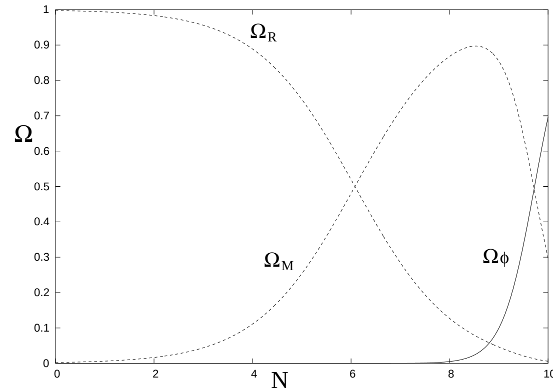

FIG. 2: Illustration of the various behaviour for Ω_i. Here we have taken $\rho_m^0 = 4.58 \times 10^6$, $\rho_R^0 = 10^{10}$ and $V_0 = 10^{-6}$. The dark line is for Ω_R, dotted line is for Ω_ϕ and light line is for Ω_m

$$H^2 = \frac{e^{Y_3}}{\sqrt{1-Y_2^2}} + \frac{\rho_B}{3M_p^2}. \tag{48}$$

Simple analysis shows us that critical point is at $Y_1 = 0$ and $Y_2 = 0$ which is a global attractor. This agrees with our physical intuition since it implies the probe brane will slow down, eventually coming to rest at the origin of the transverse space. In terms of our critical ratios we find

$$\Omega_\phi = \frac{e^{Y_3}}{e^{Y_3} + \frac{\rho_B}{3M_p^2}\sqrt{1-Y_2^2}} \tag{49}$$

$$\Omega_B = \frac{\rho_B}{\frac{3M_p^2 e^{Y_3}}{\sqrt{1-Y_2^2}} + \rho_B} \tag{50}$$

Note that they are constrained by $\Omega_\phi + \Omega_B = 1$. We also have $\Omega_B = \Omega_M + \Omega_R$, where M and R denote matter and radiation respectively, whilst ϕ is associated with our scalar field.
From the plots fig(2) we see that the Ω_ϕ goes to 0.7 and Ω_M goes to 0.3 and Ω_R goes to 0 in the presence epoch. We see that at late times, the field settles at the potential minimum leading to de-Sitter solution with energy scale $V_0 = \tau_3 R/\sqrt{kl_s^2}$. Using the numerical data from the preceding sections we can write this an upper bound on the energy density as follows

$$V_0 \leq 10^{-12} M_p^4. \tag{51}$$

Although this is several orders of magnitude higher than the observed value, we note that this value is heavily dependent on the scales in the theory, and with appropriate tuning could be substantially smaller. Since there exists no realistic scaling solution (which could mimic matter/radiation), the model also requires the fine tuning of the initial value of the scalar field. The field should remain sub dominant for most of the cosmic evolution and become comparable to the background at late times. It would then evolve to dominate the background energy density ultimately settling down in the de-Sitter phase.

However, recall from the bulk picture that the point $\sigma = 0, \rho = 0$ will be gravitationally unstable and the probe brane will eventually be attracted toward the ring. In terms of our cosmological theory we see that this de-Sitter point will actually be only quasi-stable and that a tachyonic field will eventually condense forcing the vacuum energy down toward zero. This suggests that the vacuum energy will not be constant, but will slowly varying. Furthermore our equation of state should be modified to incorporate the dynamics of this additional field. It is trivial to see that the inflationary phase will terminate and give way to a dark energy phase where

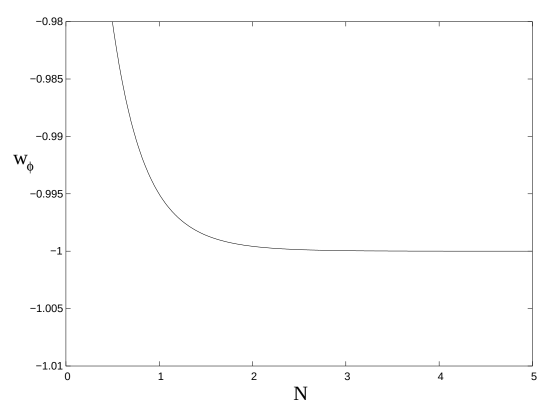

FIG. 3: Evolution of the equation of state parameter with the number of e-folds. Note that ω rapidly approaches -1 as expected.

$\omega \sim -1$. Once the tachyon field starts to roll, ω will increase toward zero from below giving rise to a phase of quintessence [41]. Eventually we will begin to probe the strong coupling regime and our effective action will break down.

let us return to the bulk picture to understand this in more detail. We introduce a complex field $\xi = \rho + i\sigma$ which can actually be globally defined in the target space. The harmonic function factorises in this coordinate system into holomorphic and anti-holomorphic parts $F(\xi, \bar{\xi}) = f(\xi)f(\bar{\xi})$. Thus the tachyon map will also split accordingly

$$\partial_t \phi = f(\xi)\partial_t \xi, \qquad \partial_t \bar{\phi} = f(\bar{\xi})\partial_t \bar{\xi}. \tag{52}$$

These expressions are exactly solvable provided we continue them into the complex plane. If we now reconstruct the potential for these fields in terms of our holographic theory we obtain the general solution

$$V(\phi, \bar{\phi}) = \frac{R\tau_3}{\sqrt{kl_s^2}} \left[\cos\left(\frac{\phi}{\sqrt{kl_s^2}} \right) \cos\left(\frac{\bar{\phi}}{kl_s^2} \right) \right]^{1/2}. \tag{53}$$

Clearly when ϕ is real we recover our *cosine* potential, whilst if it is purely imaginary we recover the *cosh* solution. These correspond to motion inside the ring and motion transverse to the ring respectively. The tachyonic instability forces the field from the false vacuum state toward the true ground state. Therefore we expect the dark energy potential to be

$$V(\phi, \bar{\phi}) \sim \frac{R\tau_3}{\sqrt{kl_s^2}} \cos\left(\frac{\phi}{\sqrt{kl_s^2}} \right), \tag{54}$$

and so the true minimum will occur when $V \sim 0$ at $\phi = \pm\pi\sqrt{kl_s^2}/2$ corresponding to the location of the ring in the bulk picture. The cosmological dynamics in this particular phase are well described by [17, 19], where it was shown to be possible for the true vacuum to be non-zero, provided the trajectory of the probe brane is sufficiently fine tuned.

We finally comment on the instability for the field fluctuations for potential with a minimum [28]. In a flat FRW background each Fourier mode of ϕ satisfies the following equation

$$\frac{\delta\ddot{\phi}_{\tilde{k}}}{1-\dot{\phi}^2} + \left[3H + \frac{2\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2} \right] \delta\dot{\phi}_{\tilde{k}} + \Big[\frac{\tilde{k}^2}{a^2} + (\ln V)_{,\phi,\phi} \Big] \delta\phi_{\tilde{k}} = 0 \tag{55}$$

Where $\tilde{k}$ is the comoving wavenumber. We now compute the second derivatives of the potential and obtain

$$(\ln V)_{\phi,\phi} = \frac{1}{kl_s^2} \left(1 - \tanh\left[\frac{\phi}{\sqrt{kl_s^2}} \right] \right). \tag{56}$$

Here we see that $(\ln V)_{\phi,\phi}$ is never divergent for any value of ϕ, and is always non-negative i.e that $(\ln V)_{\phi,\phi} \in [0, 1]$. Thus we do not have any instability associated with the perturbation $\delta\phi_k$ with our potential (9). This is to be contrasted with the result obtained for the open string tachyon. which has rapid fluctuations and instabilities associated with its evolution.

VI. CONCLUSION

In this note we have examined the time dependant configuration of a single $D3$ brane in the background of $NS5$ branes distributed on a ring of radius R, taking the near horizon approximation. We then studied the cosmological implications of the effective potential which arises due to the transverse motion of $D3$ with respect to the plane of the ring. The model appears to describe an inflationary phase giving way to a natural reheating mechanism, and then a further phase of dark energy driven expansion. Although we cannot accurately predict the scale of the energy density at this point, we do obtain an upper bound. In this case the dark energy phase is a late time attractor of our model, and we predict that the vacuum energy will eventually decay to zero - although on extremely large time-scales [4]. In fact our results will be dramatically improved by keeping the full structure of the harmonic function, because at large distances the potential is even flatter yielding even more e-foldings of inflation. Due to the absence of scaling solutions in our field theory, we need to tune the initial value of the scalar field such that it can become relevant only at late times. With these described fine tunings, the geometrical field is a potential candidate for dark energy. The model is free from tachyon instabilities, and the field perturbations behave in a similar manner to those of the canonical scalar field.

Of the model we have several potential problems. Firstly our assumption about the coupling of the DBI to four-dimensional gravity, although as we have pointed out this can be resolved by a full string theory compactification. However there will generally be large corrections, potentially destroying the simplicity of the solution. Secondly the trajectory of the brane in the bulk space is particularly special. In the most generic case we would anticipate a general spiralling trajectory toward the ring. In this case there would be no simple decoupling of the modes and we would need to consider the full form of the potential. This amounts to a certain amount of fine tuning of the initial conditions. Another problem

[4] However we must be careful since the DBI action will not be valid once it coalesces with the $NS5$-branes so we must assume that it passes between the branes. This requires fine tuning of the initial trajectory which is not realistic. This problem may be resolved by switching to the description of the model in terms of Little String Theory [35].

is that we have not turned on any standard model fields which would be expected to couple to the inflaton on the world-volume. However the inclusion of U(1) gauge fields on the brane will act to reduce the velocity of the field by a factor of $\sqrt{1-E^2}$, where E is our dimensionless electric field. More importantly however is that we have neglected the induced two-form field strength, which can have important applications in cosmology as seen in the recent paper [40]. Despite these problems, we know there is a coset model describing the background which opens the way for exact string theory calculations. Furthermore the relationship between the two energy scales in the theory means it is possible to talk about long-standing problems such as the Transplanckian issue [38]. One further problem is the termination of reheating in this model. We have emphasised that this is indeed difficult to tackle in this model due to its analytic simplicity. One may hope that a careful analysis of the tachyon mapping will lead to more realistic behaviour for the inflaton field, and thus a possible exit from reheating. In fact this may also be possible by considering more general trajectories of the probe brane in the bulk picture. We hope to return to this issue in a future publication.

One thing that emerges though is the relationship between a dark energy dominated phase and the 'fast rolling' DBI action [10]. Although our proposal is far from rigorous, it does capture the majority of the same physics as in the flux compactification scenario. We know that D-branes moving in non-trivial backgrounds have sub-luminal velocities as measured by observers in the far UV of the geometry, due to the gravitational redshifting. In fact the branes are decelerating and for late times will have negligible velocities. This in turn implies that the equation of state parameter will tend to $\omega \sim -1$ at late times. A concrete example where this could be examined is in the case of the warped deformed conifold [39]. The RR flux will wrap the S^3 in the IR end of the geometry, and we can imagine a solitary $D3$-brane probing this part of the conifold after an inflationary phase. To an observer in the compact space the brane will slow down as it reaches the origin of the S^3 yielding a dark matter dominated phase [36].

However our model opens up the possibility that non-trivial background configurations may have important implications for brane cosmology, as we have seen how to combine inflation, reheating and dark energy in a single model. Furthermore this is not subject to the same landscape problems as the flux compactification models, and we can try and tackle higher energy issues in a clear formalism [9]. Although we acknowledge the simplicity of our solution we hope that this will encourage more research in this direction.

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