

The Bending of Light by Gravity

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The P.A. Pandya Endowment Lecture (1987) of The Indian Physics Association delivered at the Indira Gandhi Centre for Atomic Research, Kalpakkam on February 25, 1988.

The guidelines for the Pandya Endowment Lecture emphasize the need for teaching the basic concepts in physics in a way that the students understand and appreciate them. Very often it happens that the same basic idea appears in different forms in different branches of physics. It is important to perceive that unity beneath diversity in order to appreciate what physics is all about.

Unfortunately, most text books present the subject in a compartmentalized fashion as a compendium of 'mechanics and gravitation', 'heat', 'light', 'sound', 'electromagnetic theory', 'atomic and nuclear physics' etc. These branches no doubt deal with different aspects of natural phenomena. But it must not be forgotten that these phenomena are manifestations of a few basic underlying principles.

To illustrate this aspect Prof. Narlikar has chosen for this lecture the topic of bending of light. Most students are familiar with the phenomenon of refraction. Yet they would probably consider the bending of light by gravity an esoteric idea. Indeed, when (in 1919) Einstein's prediction of the bending of light by gravity was observationally verified, it not only surprised the professional physicists but also sent ripples of wonder amongst the laymen. However, it is worth recalling that it was not Einstein but Newton who first contemplated this possibility.

In his famous book 'opticks' Isaac Newton wrote: "Do not bodies act upon light at a distance and by their action bend its rays; and is not this action strongest at the least distance?"

In Newton's law of gravitation, it was assumed that the phenomenon of gravitational attraction is universal in the sense that it extends to all forms of matter. Thus the law extends from the apple immortalized in Newton's folklore all the way up to the largest superclusters of galaxies in the universe and all the way down to subatomic particles. But in Newton's time the nature of light was not properly understood and as the above conjecture tells us, Newton had no definitive answer as to whether light is subject to the law of gravitation or immune to it.

It is only with the advent of the theories of relativity, special and general, that the above question was reviewed and answered satisfactorily. The special theory established the equivalence of mass and energy. So, light being a form of energy must be subject to the law of gravitation like any other mass. How the gravitational effect of a massive object on a ray of light is to be calculated was shown by the general theory of relativity.

In this lecture I will describe different aspects of this phenomenon with special emphasis on its manifestations in astronomy. Before dealing with the actual observational aspects, I will first consider the theory behind the phenomenon.

2. Refraction

It is well known to us in our daily life that light rays bend when they pass from one medium to another. Known as 'refraction' this phenomenon is responsible for the rainbow, the binoculars, the spectacles, the camera and so on. Why do light rays refract?

The underlying principle behind this behaviour of light rays had been first stated by the French mathematician Pierre de Fermat in 1650: "The ray of light travelling between any two points in space selects the track that minimises the time taken to make the journey".

Let us apply Fermat's principle to the situation illustrated in Figure 1. The surface $X'X$ separates two media I and II in which the light velocities are c_1 and c_2 respectively. With respect to the origin O on $X'X$ and the y -axis $Y'OY$ in the plane of the paper, the coordinates of points A_1 and A_2 in I and II respectively are (x_1, y_1) and $(-x_2, -y_2)$ respectively. What should be the track of light ray joining A_1 and A_2 ?

If the track intersects $X'X$ in a point C then clearly it will be straight between C and A_1 as well as between C and A_2 (since the straight line is the shortest distance between two points and hence in the same medium it will be the track of shortest time). Let $OC = \xi$. Then the time taken by the ray to go from A_1 to A_2 will be

$$T(\xi) = \sqrt{\{(x_1 - \xi)^2 + y_1^2\}}/c_1 + \sqrt{\{(x_2 + \xi)^2 + y_2^2\}}/c_2 \quad (1)$$

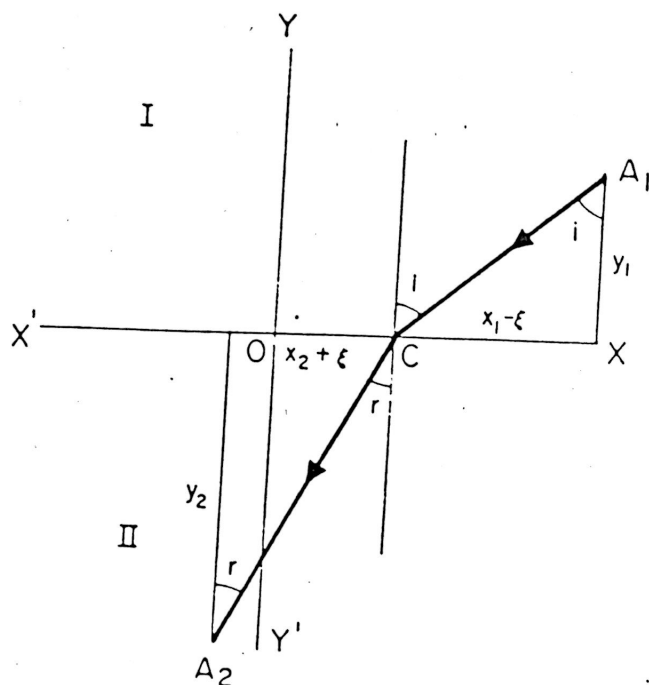


Figure 1 : The refraction of light at the interface $X'X$ between medium I and medium II.

By Fermat's principle $T(\xi)$ must be the least possible value. It is easy to see that this requirement leads to

$$0 = dT/d\xi = -(x_1 - \xi)/c_1 [\sqrt{\{(x_1 - \xi)^2 + y_1^2\}}] + (x_2 + \xi)/c_2 [\sqrt{\{(x_2 + \xi)^2 + y_2^2\}}] \quad (2)$$

From the geometry of Figure 1 we see that this is nothing but Snell's law

$$\sin i / \sin r = c_1/c_2 = \mu \quad (3)$$

where μ is the refractive index of II relative to I.

Notice that the light track between A_1 and A_2 is not the straight line joining them. If light is propagating through a medium of variable refractive index, it will follow a curved track, as happens for example when radio waves propagate through the atmosphere.

Fermat's principle is an example of principles of stationary action that govern the various branches of physics.

3. Straight Lines in Curved Spacetime

Let us apply this principle to determine the shortest route between two points on the surface of a globe. In practice the answer is found by stretching a piece of thread between the two points. It is the arc of a great circle joining the two points. We may consider it the equivalent of a straight line on the surface of a sphere. In general such a curve of shortest distance between any two points on a curved surface is called a 'geodesic'.

Since any two great circles on a spherical surface

'intersect' in two points there are no parallel geodesics on it. In other words, one of Euclid's postulates that 'given a straight line and a point outside it one can draw one and only one straight line parallel to it', fails. And so do all theorems which use that postulate. In particular, a spherical triangle will have its three angles always adding up to greater than two right angles.

In other words the geometry on the surface of a sphere will be different from Euclid's. The realization that such non Euclidean geometries can exist in their own right came to the mathematicians during the last century. It was Einstein who brought it to physics in an ingenious hypothesis that led him finally to the general theory of relativity.

According to Einstein the phenomenon of gravity is a manifestation of non-Euclidean geometry for space and time taken together.

To see this hypothesis in action consider Figure 2 describing the spacetime trajectory of a projectile thrown vertically upwards. The projectile describes a parabolic track, travelling up to a maximum height and dropping down. Had there been 'no gravity of the Earth, the projectile would have gone on along the dotted straight line: travelling with a constant speed.

According to Newtonian mechanics, the dotted track describes the first law of motion while the parabolic one is a consequence of the second law with gravity as the force. However, according to Einstein, both are instances of the first law of motion. The trajectories are geodesics computed as per rules of the spacetime geometry. The dotted straight line motion with uniform speed is a geodesic of Euclidean geometry while the

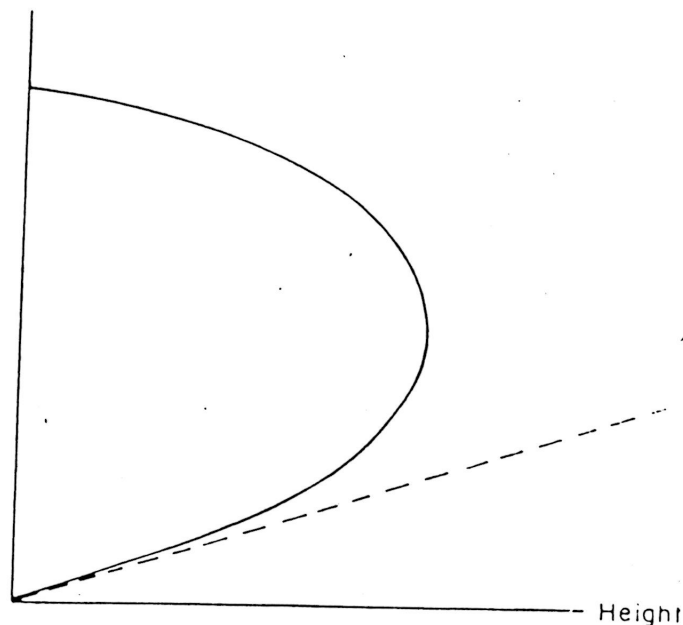


Figure 2 : The dotted straight line describes motion under no forces while the continuous line describes motion under the Earth's gravity. This is a space-time diagram.

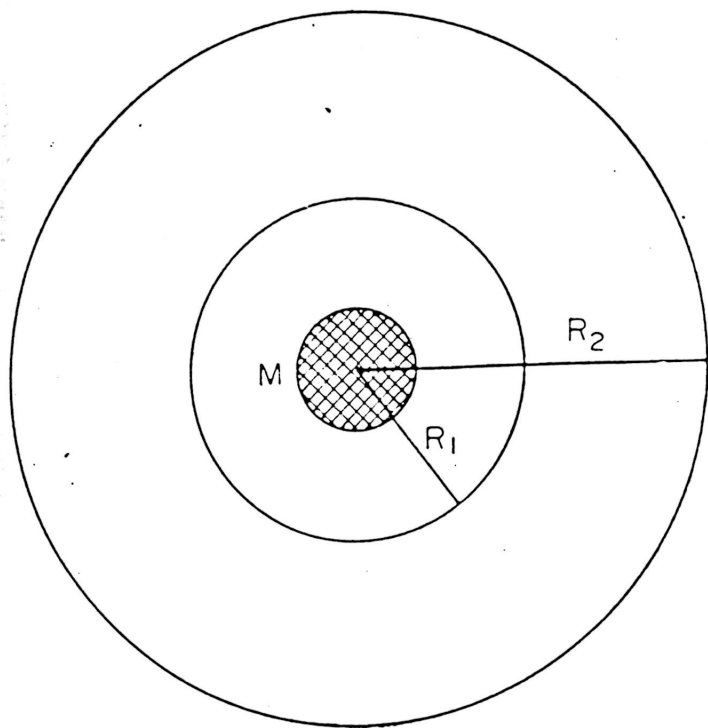


Figure 3 : Two concentric spheres around a spherical mass M shown by hatched lines.

parabolic trajectory is a geodesic of the non-Euclidean geometry caused by Earth's gravity.

Why a non-Euclidean geometry? According to general relativity, Earth's gravity implies that the spacetime measurements cannot be made as per Euclid's geometry. A new set of rules comes into effect and according to these rules the parabolic trajectory is the true geodesic for the four dimensional spacetime.

So to determine how masses move in a given gravitational environment we have to first find the geometry that obtains in that spacetime region and then calculate geodesics as per its rules of measurement.

4. The Bending of Light by the Sun

In 1916, shortly after Einstein proposed his general theory of relativity, Karl Schwarzschild solved an important problem. He found out the rules of the non-Euclidean geometry of spacetime near a spherical massive body like the Sun. The measurement rules are simply described as follows.

In Figure 3 we see two concentric spheres of 'radii' R_1 and R_2 , with the properties that

- (i) their centre coincides with that of the massive body.
- (ii) their surface areas are respectively $4 \pi R_1^2$ and $4 \pi R_2^2$.

There the similarity with Euclidean geometry ends! For, the radial measurement of distance from the centre of the first sphere to its surface is not R_1 . In fact an increase in the radial coordinate by a small amount ΔR_1 will result in an increase in the

proper distance by

$$\Delta l = \Delta R_1 / \sqrt{1 - 2GM/C^2 R_1} \quad (4)$$

where M = gravitational mass of the body. Here G = gravitational constant and c = speed of light in vacuum.

Likewise, the coordinate time T used by Schwarzschild has the property that an observer stationary of $R = R_1$ measures a proper time lapse of

$$\Delta t = \Delta \sqrt{1 - 2GM/C^2 R_1} \quad (5)$$

corresponding to a lapse ΔT of the time T

Now we may apply the Schwarzschild solution to the Sun and work out the geodesics in order to determine how planets move. The result gives the familiar Keplerian orbits with a small difference. The orbits slowly precess. The effect is very small and is barely noticeable for planet Mercury. The theory predicts that the perihelion of its orbit should advance at a rate of 43 arc sec per century. Small though this result is, it has been confirmed observationally.

According to relativity, light travels in tracks called 'null geodesics'. For a null geodesic the net proper length is zero. In our example of Schwarzschild's solution, for a light ray moving radially

$$(\Delta l)^2 - C^2 (\Delta t)^2 = 0 \quad (6)$$

For non-radial tracks also we can calculate null geodesics using the variational principle. The generalization of the statement 'light travels in a straight line' is to be found in these null geodesics.

Notice that a similar generalization was imposed in Figure 1 by Fermat's principle. There the track A_1CA_2 is not straight: but it minimizes the propagation time for light from A_1 to A_2 . The Euclidean straight line connecting A_1 to A_2 is not the right answer!

Likewise, in Figure 4 we see two lines from a star K to the observer O . The dotted line is the Euclidean straight line while the 'curved' but continuous line is the null geodesic track determined by Schwarzschild's geometry for the sun S . Had the Sun not been there the former track would be the right one. With the Sun in the way it is the second track that is correct. So the observer sees the star not at K but at K' , in the apparent direction from which the ray of light approaches O .

It was Eddington who in 1919 pioneered an observational check of this result. If a star happens to be behind the Sun at the time of a total solar eclipse, it should be possible to observe the shift in

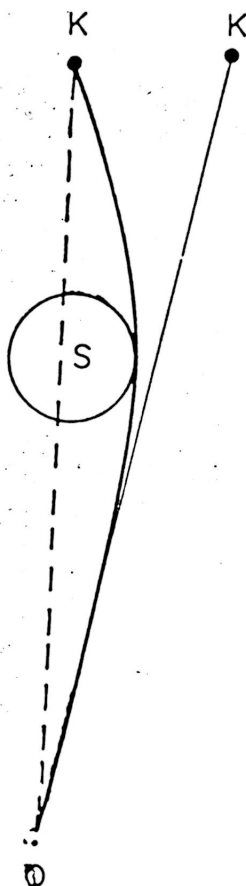


Figure 4 : The bending of light from the star K by the gravity of the Sun S produces a virtual image at K'.

its apparent position (from K to K'). Such a shift was observed and was, within the error bars, in agreement with the predicted value of

$$\theta = 4 \text{ GM/C}^2\text{R} \sim 1.75 \text{ arc Sec.} \quad (7)$$

Here R = radius of the Sun (For details of this result see Ref.1.)

The error bars in the 1919 experiment were, however, larger than estimated! In fact it turns out that the refractive bending in the Sun's atmosphere is appreciable for light rays and it is hard to estimate its contribution to the net bending. To minimize it, similar experiments must be made with longer waves, e.g. radio or microwaves.

These experiments were performed in 1975 and the predicted bending was confirmed within 5% accuracy. (See Ref.2)

It is worth recalling here Newton's conjecture of gravitational bending of light. Newton also believed in a corpuscular theory of light. If we assume that light is made of particles attracted by the Sun's gravity, then the continuous line in Figure 4 may be viewed as a hyperbolic orbit of light particles whose asymptotic speed is c. A purely Newtonian calculation then gives a bending angle that is exactly half that predicted by general relativity in formula (7).

Thus we can now safely rule out this Newtonian alternative, although Newton's conjecture was qualitatively correct.

5. Gravitational Lenses in Space

The above example is the gravitational analogue of an optical lens that forms virtual images of real objects. For, in Figure 4, K' is the virtual image of K.

It is clear that gravitational lensing may exist on larger scales where light has to pass close to or through massive bodies. In such cases the astronomer is subject to illusions: what he observes is not what actually exists out there. Let me illustrate the situation with two specific scenarios.

- (i) Superluminal separation: Figure 5 describes a situation found in about 10-20 quasars by very long baseline interferometry. Two blobs A' and B' are seen to separate at superluminal speed. For example, in the quasar 3C273, A' may be stationary in the nuclear region of a quasar while B' is seen to move away with speed $\geq 10c$. Is this observation real? If so special relativity is disproved!

If, however, what the observer O sees at A', B' are not the real positions of these blobs but apparent ones arising say, from the bending of light coming from the real positions A, B, it is possible

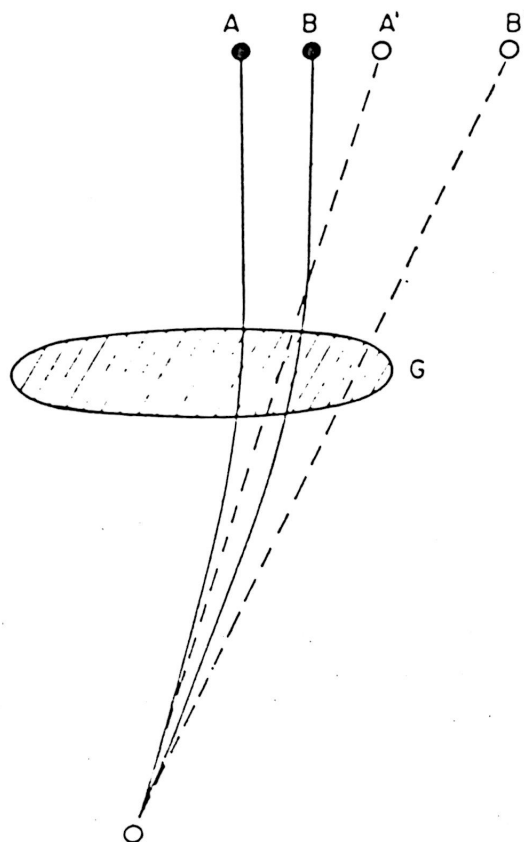


Figure 5 : The gravitational bending of radio waves from A, B by the galaxy G produces virtual images at A', B'. Note that the radio waves go through G rather than around it as in Figures 4 and 6.

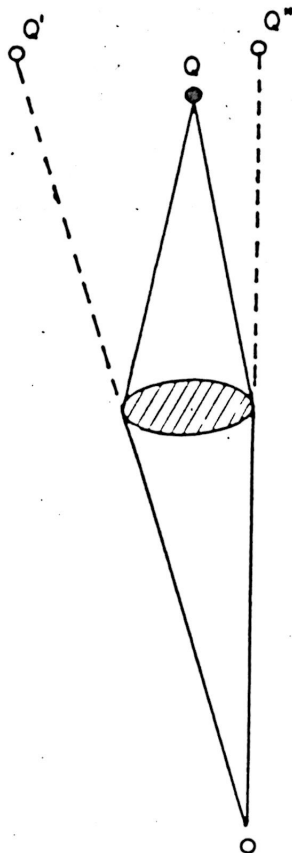


Figure 6: Two virtual images of Q are produced at Q' , Q'' by two sets of rays bent by the galaxy G as these rays reach the observer O.

to save relativity! For, an intervening galaxy may serve as the bending agent and under suitable conditions the magnification $A'B'/AB$ achieved by this gravity lens may be quite large. Thus apparent superlumina motion could be due to magnification produced by a galactic gravitational lens while the true motion may indeed be subluminal. Chitre and I had suggested this explanation in 1978 (see Ref.3).

- (ii) Twin quasars: In 1979 another manifestation of this phenomenon was suggested (see Ref.4) when the twin quasars 0957 + 561A,B were discovered. Although differing in intensity the two quasars had

identical spectra and shapes. Could these be two different images separated by only about 6 arc second produced by a gravity lens whereas the source was in fact unique?

Figure 6 describes how this could happen. There could be two null geodesics from quasar Q to observer O going round a galaxy G so that O sees two virtual images Q' , Q'' , instead of the real one at Q. In the case of 0957 + 561A,B a galaxy along with a cluster of galaxies together are believed to act as a complex gravitational lens.

Optical simulators of gravity lenses have now been designed. These bend light rays in a way similar to the bending effected by a massive spherical object. These simulators serve a useful purpose in demonstrating how the lensing phenomena might occur in the cosmos.

6. Conclusion

To summarize, today Isaac Newton would have been happy not only to have found the answer to his conjecture but also to see the law of gravitation appearing as a truly universal law, a law that affects not only matter but also radiation.

As we have seen in this lecture the 'bending of light' by gravity is in many ways similar to the phenomenon of refraction. Just as the latter allows us to construct lenses that generates illusions through virtual images so might the massive objects in the cosmic environment lead to illusions through gravitational lenses.

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Special Issue on "Physics Instrumentation in India"

The publication Directorate of CSIR proposes to publish a special issue of Indian Journal of Pure and Applied Physics on "Physics Instrumentation in India" early in 1989. Persons desirous of contributing to this

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