

Two Astrophysical Applications of Conformal Gravity

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The conformally invariant theory of Hoyle and Narlikar is applied to two astrophysical scenarios. First it is shown how the existence of zero mass surfaces in the four-dimensional manifold in this theory can lead to black holes and white holes. Next it is shown that the so-called anomalous redshifts of QSOs and galaxies can be understood in terms of local shifts in the zero mass surfaces of this theory. Some observable consequences are discussed.

1. INTRODUCTION

The concept of conformal invariance has been discussed extensively by theoreticians although its application to physical theories has not been considered very essential. For example, Maxwell's equations are conformally invariant but general relativity is not; the Dirac equation for the neutrino is conformally invariant but that for the electron is not. The requirement of conformal invariance is perhaps not very compelling in a purely local physical theory, e.g., the field theory. However, in a global theory such as an action at a distance theory, where measurements at different well separated space-time points are involved, this concept becomes important. For example, if the action at a distance is not to be instantaneous, but with the speed of light, the light cones play an important part in such a theory. And, globally, the light cones are left invariant under a conformal transformation.

This and other arguments in favor of conformal invariance have been discussed in detail elsewhere by Hoyle and Narlikar [1, 2]. In this paper we shall consider some of the properties of a conformally invariant gravitation theory and its applications to astrophysics. This theory was first proposed by Hoyle and Narlikar [3] in 1964 in analogy to the action-at-a distance theory of electrodynamics [4-9]. The theory is Machian in nature, i.e., it requires the inertia of a particle to arise from the rest of the particles in the universe. The scalar nature of the mass and the property of conformal invariance uniquely fix the form of the basic interaction underlying the theory. In the following section the details of this theory relevant to the present work are briefly described.

The cosmological models arising in this theory and their interpretation have been given elsewhere (cf. [2] for details). We shall be concerned here mainly with two astrophysical phenomena. We shall first consider the interpretation of black holes and

white holes in this theory. We shall then consider an interpretation of the phenomenon of the so-called anomalous redshifts of quasi-stellar objects (QSOs) within the framework of conformal gravitation.

2. CONFORMAL GRAVITATION: THE BASIC FRAMEWORK

Consider first a system of particles labeled by letters a, b, c, \dots , in a Riemannian $(3 + 1)$ -dimensional manifold. Let g_{ik} denote the metric tensor defined on this manifold, and x^i ($i = 1, 2, 3, 4$) be the four coordinates. We shall assume generally that $i = 4$ is timelike, and that the signature of the metric is of the $(-, -, -, +)$ type. Denoting by a^i the coordinates of a typical point A on the world line of a , we write the relation

$$da^2 = g_{ik} da^i da^k \tag{1}$$

where da = element of proper time along the world line of a at A .

We now define the mass m_a of the particle a at A as arising from the rest of the particles in the universe

$$m_a(A) = \sum_{b \neq a} m^{(b)}(A) \tag{2}$$

where the contribution $m^{(b)}(A)$ from particle b is given by

$$m^{(b)}(A) = \lambda(A) \int \tilde{G}(A, B) \lambda(B) db. \tag{3}$$

Here λ is a coupling constant and $\tilde{G}(A, B)$ is a biscalar (cf. [10]) satisfying the conformally invariant scalar wave equation

$$\square_A \tilde{G}(A, B) + \frac{1}{6} R(A) \tilde{G}(A, B) = [-\bar{g}(A, B)]^{-1/2} \delta_4(A, B). \tag{4}$$

The \square_A denotes the covariant wave operator with respect to the coordinates of A , $R(A)$ is the scalar curvature at A , and $\delta_4(A, B)$ is the four-dimensional Dirac delta function. $\bar{g}(A, B)$ is the determinant of the parallel properator matrix $\bar{g}_{i_A i_B}$ at A and B (cf. [10]).

The action functional for the conformal gravity is then simply

$$S = - \sum_a \frac{1}{2} \int m_a da \equiv \sum_{a < b} \iint \lambda(A) \lambda(B) \tilde{G}(A, B) da db. \tag{5}$$

From the symmetry between particles of any pair it is clearly necessary to have $\tilde{G}(A, B) = \tilde{G}(B, A)$. This therefore determines which elementary solution of (4) is to be used. In flat space we have

$$\tilde{G}(A, B) = \frac{1}{4\pi} \delta(s_{AB}^2) \tag{6}$$

where s_{AB}^2 is the invariant square of the distance between A and B . In curved space (6) is modified to a combination of $\delta(s_{AB}^2)$ and $\theta(s_{AB}^2)$, the heaviside function. The latter has been interpreted as arising from gravitational scattering [11] and its role in the action at a distance theories has been discussed by Narlikar [9].

Let us first consider the simplification $\lambda = \text{constant}$. The variation of S with respect to g_{ik} then leads to the field equations

$$\begin{aligned} & \frac{1}{6} M^{(\text{ret})} M^{(\text{adv})} \{ R_{ik} - \frac{1}{2} g_{ik} R \} \\ &= -T_{ik} + \frac{1}{2} \{ M_i^{(\text{ret})} M_k^{(\text{adv})} + M_i^{(\text{adv})} M_k^{(\text{ret})} - g_{ik} M^{l(\text{adv})} M_l^{(\text{ret})} \} \\ &+ \frac{1}{6} \{ g_{ik} \square (M^{(\text{ret})} M^{(\text{adv})}) - (M^{(\text{ret})} M^{(\text{adv})})_{;ik} \}, \end{aligned} \tag{7}$$

where

$$M^{(\text{ret})}(x) = \lambda \sum_a \int G^{(\text{ret})}(x, A) da, \quad M^{(\text{adv})}(x) = \lambda \sum_a \int G^{(\text{adv})}(x, A) da, \tag{8}$$

and

$$\tilde{G}(x, A) = \frac{1}{2} \{ G^{(\text{ret})}(x, A) + G^{(\text{adv})}(x, A) \}. \tag{9}$$

For brevity we write $M_i^{(\text{ret})} = \partial M^{(\text{ret})} / \partial x^i$, etc. The energy tensor T_{ik} is defined by

$$\begin{aligned} T_{ik}(x) = \sum_a \int & \frac{1}{2} \lambda \{ M^{(\text{ret})}(A) + M^{(\text{adv})}(A) \} \\ & \cdot \bar{g}_{iiA} \bar{g}_{kkA} \frac{da^{iA}}{da} \frac{da^{kA}}{da} [-\bar{g}(x, A)]^{-1/2} \delta_4(x, A) da. \end{aligned} \tag{10}$$

Because of (4) and (8), $M^{(\text{ret})}(x)$ and $M^{(\text{adv})}(x)$ also satisfy wave equations of the type (4); e.g.,

$$(\square + \frac{1}{6} R)_x M^{(\text{ret})}(x) = \lambda \sum_a \int [-\bar{g}(x, A)]^{-1/2} \delta_4(x, A) da. \tag{11}$$

The details of these equations have been discussed in [2]. The conformal invariance of the theory is manifested in the following way. As in general relativity there are ten unknown g_{ik} . However, there are only five instead of six independent equations in the set (7). For, because of (11) the trace as well as the divergence of (7) vanish identically. As a result, any attempt to determine the g_{ik} in a given problem will result in an arbitrariness to the extent of arbitrary coordinate transformations as well as to the extent of an arbitrary conformal factor. This latter arbitrariness means that if g_{ik} is a solution, so is $\Omega^2 g_{ik}$ where Ω is suitably well behaved (i.e., $C^{(2)}$) function satisfying

$$0 < \Omega < \infty. \tag{12}$$

Just as, with suitable response from the universe (cf. [6-8]) the time symmetric

electrodynamics can be made equivalent to the Maxwellian electrodynamics, so in this case a suitable response results in

$$M^{(\text{ret})} = M^{(\text{adv})}, \quad (13)$$

in the "local" domain of observation (see [2] for details.) Equations (7) can then be given a considerably simplified outlook by choosing a conformal frame in which

$$M^{(\text{ret})} = \text{constant} = M_0(\text{say}). \quad (14)$$

This is always possible; for the rule of transformation is

$$g_{ik} \rightarrow \Omega^2 g_{ik}, \quad M^{(\text{ret})} \rightarrow \Omega^{-1} M^{(\text{ret})}. \quad (15)$$

Hence if $M^{(\text{ret})}$ is not constant to start with, we simply choose to make it so in a new conformal frame. The equations then reduce to the familiar Einstein equations

$$R_{ik} - \frac{1}{2} g_{ik} R = -8\pi G T_{ik}, \quad (16)$$

where

$$8\pi G = 6/M_0^2. \quad (17)$$

3. THE FRIEDMANN UNIVERSES

The Friedmann models provide the simplest cosmological solutions of Einstein's Eqs. (16). These models are usually described by the Robertson-Walker line element

$$ds^2 = dt^2 - Q^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (18)$$

where $k = 0, +1, \text{ or } -1$, and $Q(t)$ is the expansion factor satisfying a differential equation. The coordinates (r, θ, ϕ) describe the points on a typical $t = \text{constant}$ surface (which is homogeneous and isotropic). In cosmology these coordinates are identified with those of galaxies.

Since the line element (18) is conformally flat [12] we could equally well have chosen to use the flat space to describe cosmology, provided we use the above conformal theory and use a suitable conformal function. For example, in the case $k = 0$,

$$Q(t) = (3Ht/2)^{2/3} \quad (19)$$

where $H = \text{Hubble's constant at the present epoch}$. The conformal transformation to flat space is achieved by multiplying the line element (18) by $\{Q(t)\}^{-2}$. The coordinate transformation

$$\tau = (12t/H^2)^{1/3} \quad (20)$$

makes this more obvious. The new line element is

$$d\tilde{s}^2 = d\tau^2 - dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2). \tag{21}$$

The masses are given by

$$M^{(\text{ret})} = M_0[Q(t)] = (H^2\tau^2/4) M_0. \tag{22}$$

It can also be verified that the retarded contribution of all particles from $t \geq 0$ to the present epoch $t = \frac{2}{3}H^{-1}$, $\tau = 2/H$ is in fact (22). There is, however, one difficulty. The Friedmann model does not satisfy the response condition (13).

In [2] this difficulty was resolved by first noting that whereas the Robertson-Walker manifold is limited to $t \geq 0$, $\tau \geq 0$ the conformally transformed flat space can be extended to $\tau < 0$. The hypersurface $\tau = 0$ corresponds to the hypersurface of zero mass, as seen from (15). To achieve consistency we must take into account the extended space-time also. This is done in the following way.

Consider two world lines a and b (see Fig. 1) extending across $\tau = 0$ in both directions. Consider the contribution from b to a typical point A on the world line a . Suppose we make the following rule. The constant λ switches sign as the hypersurface $\tau = 0$ is crossed. That is, we assume in (3) that $\lambda(B) = \lambda > 0$ for B in the region $\tau > 0$ while $\lambda(B) = -\lambda < 0$ for B in the region $\tau < 0$. It is then easy to verify that the net contribution from the world line b to A is the retarded one if it intersects the $\tau = 0$ hypersurface at a point B_0 which lies within the past light cone

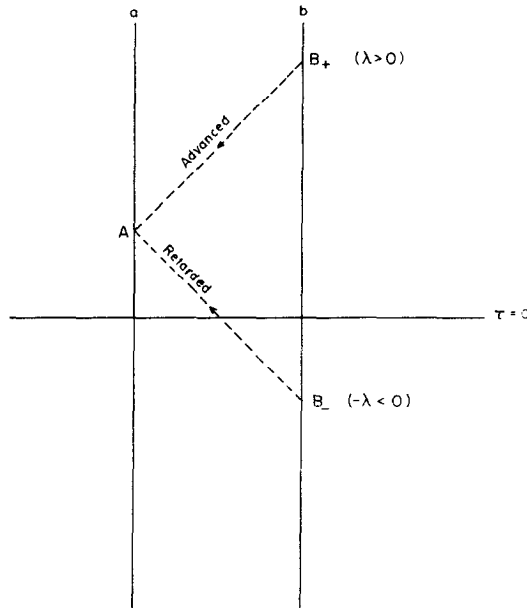


FIG. 1. The coupling constant changes from $-\lambda$ to $\lambda > 0$ as the world line b crosses $\tau = 0$. The advanced and retarded contributions at A are therefore equal and opposite.

of A . Otherwise the contribution is zero. As illustrated in Fig. 1, the positive advanced contribution from B_+ on b exactly cancels the negative retarded contribution from B_- on b in this case. Thus only those particles which cross the particle horizon of A at $\tau = 0$ really contribute.

This extended viewpoint has several implications. First the space-time singularity at $t = 0$ is transformed away in the flat space-time obtained by the conformal transformation. The price paid for this is the vanishing of mass at $\tau = 0$. This is, however, a lesser evil. A physicist can probably handle the properties of particles of vanishing restmass in a flat space; but he cannot talk of any meaningful physics in a space-time which itself is singular. The question is "Do all singularities of general relativity arise from the insistence on the part of the physicist that particle masses are constant?" This question is being investigated at present and preliminary work by Kembhavi [13] suggests that the answer may be in the affirmative. Kembhavi finds that a wide range of cosmological singularities in general relativity arise from the choice of the conformal frame leading to (14) and (16). Investigations by Hoyle [6, 7] suggest a wide range of interesting possibilities which link the other half of the universe ($\tau < 0$) to the astrophysics of our half ($\tau > 0$). For this reason the extended space-time appears to offer a better framework to the cosmologist trying to understand the present state of the universe, than the more familiar big bang cosmology.

4. BLACK HOLES AND WHITE HOLES

The cosmological case considered above may be looked upon as a special case of a more general situation which arises if coupling constants are allowed to switch signs. With positive and negative contributions of the type (3), the appearance of zero mass hypersurfaces will be quite common. And it is not necessary that the hypersurfaces are open as in the cosmological case (which is a highly symmetrical solution).

In Fig. 2 is shown a typical closed zero mass hypersurface Σ in a space-time manifold. Consider a world line of a intersecting Σ at two points A_1 and A_2 . Let P_+ (P_-) and Q_- (Q_+) be two points on a close to A_1 and A_2 , respectively, and lying inside (outside) Σ . For P_+ sufficiently close to A_1 , i.e., for the length A_1P_+ small compared to P_1 , a linear dimension characteristic of the radius of curvature of Σ at A_1 , the hypersurface Σ would appear to an observer at P_+ like a surface stretching out to infinity. For, the information available to P_+ about Σ is confined to the stretch of Σ within the past light cone of P_+ . The experience of P would therefore be broadly similar to that of an observer in a Friedmann model. Conversely, our own cosmological experience has a characteristic dimension H^{-1} . Hence for

$$\rho \gg H^{-1} \quad (23)$$

the Friedmann model represents an approximation to a closed zero mass hypersurface of "radius" ρ .

The observer freely and radially falling into a Schwarzschild black hole has a

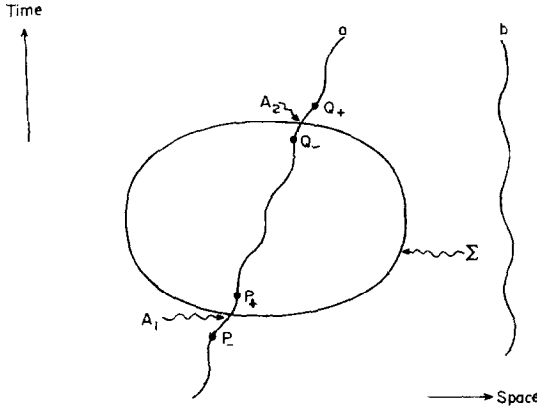


FIG. 2. The worldline a crosses a closed zero mass hypersurface Σ twice. There is a black hole followed by white hole at each of the points of intersection A_1 and A_2 .

trajectory similar to that of a fundamental observer (i.e., a galaxy) in a contracting Friedmann model. In general relativity this observer falls into a singularity, and that marks an abrupt end to his future. It has been conjectured (cf. [16]) that such an observer emerges into another universe in the form of an ejection from a white hole. In the framework of general relativity it is not possible to see how such a connection could arise.

In the conformal theory this connection could be easily established through a zero mass hypersurface. In Fig. 2, the point P_+ corresponds to the emergence from the singularity whereas the point P_- , which is one the world line of a and is chronologically before A_1 corresponds to the collapse phase. In the same way the point Q_- corresponds to collapse and the point Q_+ to emergence from a singularity.

Thus a closed zero mass hypersurface represents *two* combinations of collapse and reemergence. An external observer like b (see Fig. 2) may witness only the stages P_- and Q_- and interpret them as black hole formation and a white hole eruption. As the two points A_1 and A_2 need not have the same space coordinates the white hole at A_2 may appear unconnected with a black hole at A_1 . Nevertheless in principle it is possible to assert a connection on the basis of the wider framework provided by the conformal theory.

5. ANOMALOUS REDSHIFTS

In recent years considerable data has accumulated on the so-called anomalous redshifts (see, for example an excellent review by Arp [17]) of galaxies and the quasi-stellar objects (QSOs). The word “anomalous” implies a departure from the conventional cosmological explanation of the redshift. According to this explanation, two objects A and B close to each other should be seen to have the same redshifts. Anomaly arises when their redshifts turn out to be different.

At present the evidence for such anomalies is considered controversial. How does one know that A and B are close to each other? One way to show this is to demonstrate a material connection between A and B . Another way is to argue on statistical grounds that the chance for A and B to lie in such close directions (although they may be well separated along the line of sight) is very small. An example of the former type is the connection extending from a galaxy NGC 5296 to a quasar. Arp has argued that the quasar as well as this galaxy have been ejected from a large galaxy NGC 5297 of NGC 5296 is a companion. There is another compact peculiar galaxy close by with a redshift 0.086 whereas NGC 5296 has a redshift ~ 0.008 . A striking example (cf. [18]) of the latter type of redshift anomaly is that of a pair of QSOs identified with radio sources 4C - 1150 (a) and (b) which are separated by only $\sim 4.5''$ of arc but have redshifts 1.9 and 0.44. On a random hypothesis the chance of this happening is estimated at less than 0.001.

Although such examples are not considered to have established the case for anomalous redshifts, the increasing number of such cases over the last few years makes it worthwhile to think of theoretical explanations. The Doppler effect would require very large velocities whereas the gravitational redshift explanation demands a highly compact nature of the high redshift objects. Both these possibilities appear to be ruled out by the present assessment of the data [17]. For this reason it is tempting to look for a new explanation. An added advantage of considering a new explanation at this stage is to provide a predictive scenario which can be checked with future observations.

An explanation of this type is provided by a slight modification of the cosmological picture described earlier. Consider first the Friedmann model $k = 0$ and its flat space version discussed in Section 3. Let $t = t_0$, $\tau = \tau_0$ denote the present epoch for an observer located at $r = 0$. In flat space he will see a galaxy located at $r = r_1$ at $t = t_1$, $\tau = \tau_1$, where

$$\tau_1 = \tau_0 - r_1. \quad (24)$$

If $m_e(\tau)$ is the time-varying electron mass, then by (22) we see that the redshift of the galaxy is given by

$$z = \frac{m_e(\tau_0)}{m_e(\tau_0 - r_1)} - 1 = \frac{\tau_0^2}{(\tau_0 - r_1)^2} - 1. \quad (25)$$

When converted to the t -coordinate (25) will be seen to be equivalent to the familiar formula in the Einstein-de Sitter cosmology. We now consider the modification which will generate the anomalous redshifts.

On each world-line identify a special point which determines the switchover of the coupling constant. Let $a = a_0$ denote the proper time of this special point A_0 on the world line of a typical particle a . Then consider the form of the action

$$S = \sum_{a < b} \sum \lambda^2 \iint \epsilon(a - b_0) \epsilon(b - a_0) \tilde{G}(A, B) da db \quad (26)$$

where $\lambda > 0$ and

$$\begin{aligned} \epsilon(x) &= +1 & (x > 0), \\ &= -1 & (x < 0), \\ &= \theta(x) - \theta(-x). \end{aligned} \tag{27}$$

The cosmological case discussed in Section 3 is a special case of this where a_0, b_0, \dots all correspond to the epoch $\tau = 0$. (26) envisages the more general situation where the switchover in the coupling constant does not occur simultaneously for all particles. In Fig. 3 are shown the consequences of this type of coupling for a pair (a, b) of particles.

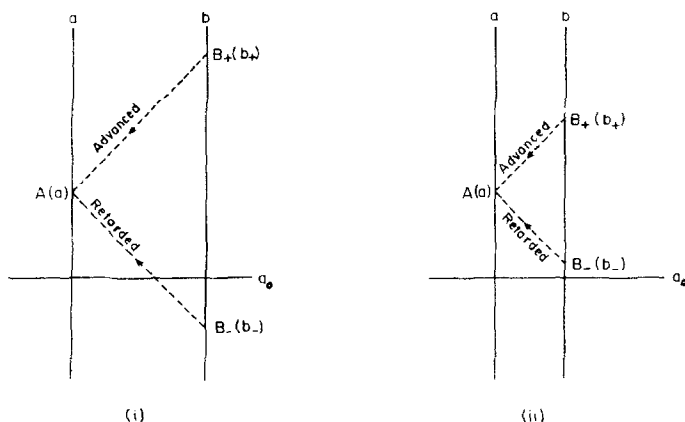


FIG. 3. In (i) the world line b does not contribute any inertia at A . In (ii) the advanced and retarded contributions are equal.

We define with $\lambda = 1$,

$$m^{(b)}(A) = \int \epsilon(a - b_0) \epsilon(b - a_0) \tilde{G}(A, B) db. \tag{28}$$

Suppose in flat Minkowski space $a_0 > 0, b_0 = 0$. (The latter case corresponds to $\tau = 0$.) If the proper time a of A is positive we have $\epsilon(a - b_0) = 1$. We then get

$$\begin{aligned} m^{(b)}(A) &= \int \epsilon(a - b_0) \epsilon(b - a_0) \tilde{G}(A, B) db, \\ &= \int \epsilon(b - a_0) \frac{1}{2} \{G^{(\text{ret})}(A, B) + G^{(\text{adv})}(A, B)\} db. \end{aligned} \tag{29}$$

Let the light cones from A intersect the worldline of b in $B_+, b = b_+$, (future) and $B_-, b = b_-$ (past). In Fig. 3i we have the situation $b_- < a_0, b_+ > a_0$. Since $\epsilon(b_+ - a_0) = 1$ and $\epsilon(b_- - a_0) = -1$, there are equal and opposite advanced and retarded contributions to $m^{(b)}(A)$ the net effect being zero. In Fig. 3ii we have $b_+ > a_0,$

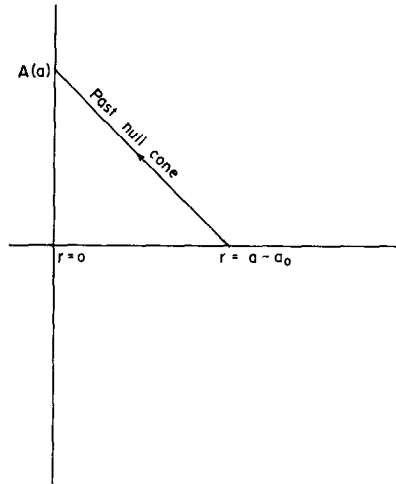


FIG. 4. The net contribution to the inertia at A is from particles with $r \leq a - a_0$.

$b_- > a_0$ and both contributions are equal, leading effectively to the *full* retarded contribution at A .

Consider now the case where in flat Minkowski space we have the particle a which has $a_0 > 0$ and all other particles b with $b_0 = 0$. The above argument then shows that the effective contribution to the mass of a at a typical epoch $a > a_0$ is limited to those source particles whose world-lines intersect the hypersurface $\Sigma(a_0)$ corresponding to the epoch $a = a_0$ inside the intercept of the past light cone from a with $\Sigma(a_0)$ (see Fig. 4). Taking the r -coordinate of the particle a to be $r = 0$, the r coordinate of contributing particles satisfies the restriction

$$r \leq (a - a_0). \tag{30}$$

Since each particle contributes an inertia $\propto r^{-1}$, the net mass of the particle a at epoch a is given by

$$m_a \propto (a - a_0)^2. \tag{31}$$

$a = a_0$ corresponds to a zero mass epoch. Similarly, for a typical other particle of b -type

$$m_b \propto b^2 \tag{32}$$

with the same constant of proportionality as in (31).

We now envisage a small perturbation in the homogeneous and isotropic universe where the bulk of matter is made up of particles of type b but there are some which are of type a , in the above sense. The dynamics of the universe will therefore remain largely unaffected but objects made of a -type matter will exhibit the anomalous redshifts. This is illustrated with the help of Fig. 5.

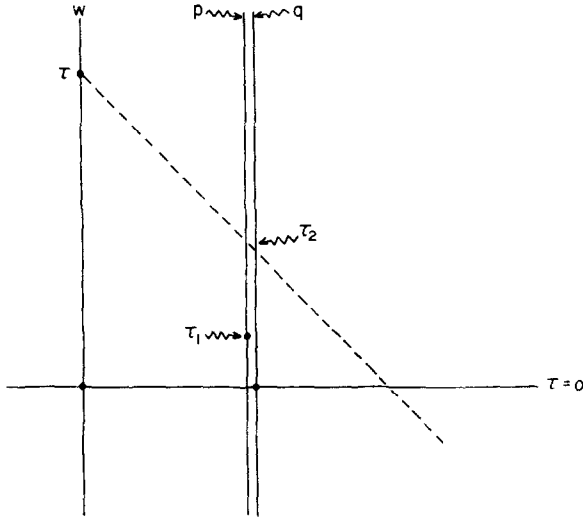


FIG. 5. The galaxies p and q are near each other and are observed by w at epoch τ . The special points on the world lines of w and q (shown by filled circles) occur at $\tau = 0$ while the special point of p occurs at τ_1 . The matter in p shows anomalous redshift.

Here $r = 0$ corresponds to the worldline of a typical b -type observer w , looking at the universe at the epoch τ . p and q are the world lines of two contiguous objects, p being of a -type and q of b -type. Thus the special epoch in the history of p is characterized by $\tau = \tau_1 > 0$. The past light cone of the observer intersects the world lines of p and q at effectively the same epoch $\tau_2 > \tau_1$, say. Then the electron mass measured by w at τ is given by

$$m_e(w, \tau) = \mu\tau^2, \quad \mu = \text{constant}. \tag{33}$$

Similarly, we have

$$m_e(p, \tau_2) = \mu(\tau_2 - \tau_1)^2, \quad m_e(q, \tau_2) = \mu\tau_2^2. \tag{34}$$

Therefore the redshifts of p and q as measured by w are respectively,

$$z_p = \frac{\tau^2}{(\tau_2 - \tau_1)^2} - 1 \tag{35}$$

and

$$z_q = (\tau^2/\tau_2^2) - 1. \tag{36}$$

There is therefore an excess redshift in p by the amount

$$z_p - z_q = \tau^2 \left\{ \frac{\tau_1(2\tau_2 - \tau_1)}{\tau_2^2(\tau_2 - \tau_1)^2} \right\}. \tag{37}$$

This is the anomalous redshift.

The epoch $\tau = \tau_1$ may in some ways be identified with the creation of p , if we decide to use the framework of general relativity. From the analogy with cosmology, p will appear to be younger than q if we decide to use the cosmic time coordinate t . Hence in any pair of associated objects showing anomalous redshifts, *the object showing the higher redshift will be younger*. This agrees with the conclusions drawn by Arp [17] from his empirical analysis of the data. In his scenario the higher redshift objects (e.g., QSOs or compact companion galaxies) are thrown out of the lower redshift object in the association in the form of new matter. It should also be noted that the redshift anomaly arises without recourse to large radial velocities or a high degree of compactness.

As more precise data on anomalous redshifts become available in the future the formulas (35)–(37) can be applied in a quantitative manner to check the predictions of this theory. In the cases where galaxies or stars are involved stellar evolution will provide a check on the ages. Thus it may be possible to verify the conclusion of this theory that an anomalously high redshift arises from younger matter.

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