Viable cosmology with a scalar field coupled to the trace of the stress tensor

M. Sami* and T. Padmanabhan†
IUCAA, Post Bag 4, Ganeshkhind, Pune 411 007, India

(Received 1 January 2003; published 24 April 2003; publisher error corrected 12 May 2003)

We study the cosmological evolution of a scalar field that couples to the trace $T=T^a_a$ of the energy momentum tensor of all the fields (including itself). In the case of a shallow exponential potential, the presence of coupling to the trace T in the field equation makes the energy density of the scalar field decrease faster, thereby hastening the commencement of radiation domination. This effect gradually diminishes at later epochs, allowing the scalar field to dominate the energy density again. We interpret this phase as the current epoch of cosmic acceleration with $\Omega_\phi=0.7$. A variant of this model can lead to accelerated expansion at the present epoch, followed by an $a(t)\propto t^{2/3}$ behavior as $t\rightarrow\infty$, making the model free from a future event horizon. The main features of the model are independent of initial conditions. However, fine-tuning of parameters is necessary for viable evolution.

DOI: 10.1103/PhysRevD.67.083509 PACS number(s): 98.80.−k, 98.80.Cq

I. INTRODUCTION

The interpretation of current observations in terms of an accelerating universe requires an exotic form of energy density with $(\rho+3p)<0$. A wide variety of scalar field models have been conjectured for this purpose including quintessence [1], K essence [2], and tachyonic scalar fields [3–5], with the last one being originally motivated by string theoretical ideas [6]. (For a recent review of issues related to the cosmological constant and dark energy, see [7].) Since all these models have a free, undetermined function $V(\phi)$, it is possible to incorporate any reasonable $a(t)$ in any of these models [4]. Thus, at a fundamental level, all these models can be objected to—quite correctly—as lacking in predictive power. The appropriate way of judging these models, therefore, will be based on the physical input that is used in constructing the model and the generic features which arise in the model.

We investigate here a model in which the coupling of the scalar field to the matter is based on a physical principle which can be motivated as follows: The most obvious generalization of the Newtonian gravity, based on $\nabla^2\phi=4\pi G\rho$ will be $\Box\phi=4\pi GT$ in which the trace $T=T^a_a$ of the energy momentum of fields acts as a source for a scalar field. This theory, however, has to be nonlinear, since the scalar field itself has a nonzero trace for the energy momentum tensor which should appear on the right hand side. Further, since the cosmological constant has a nonzero trace for the stress tensor, this scalar field will also couple to the cosmological constant and the effective cosmological constant will become dynamical. Such a model was developed and investigated more than a decade ago [8,9] as a possible solution to the cosmological constant problem. This work, however, was ambitious in the sense that it attempted to tackle the problem *without any fine-tuning* and did not succeed [8]. The current thinking in cosmology is remarkably tolerant to the fine-tuning of parameters, and virtually every scalar field model suggested in the literature has either explicit or implicit fine-tuning of parameters. In view of this, one can resurrect the above model, in which the form of the interaction arises from a clear physical requirement, and explore its consequences for cosmology. We attempt to do this in this paper and show that the resulting models have very nice properties.

II. DYNAMICS OF THE SCALAR FIELD WITH A COUPLING TO T^a_a

The Lagrangian for a system made of the gravitational field, matter sources, and the scalar field which couples to the trace of the total energy momentum tensor is derived in [8]. (This derivation is briefly summarized in Appendix A for completeness.) The complete action, which takes into account the trace coupling of the scalar field, has the form (see Appendix A)

$$S=(16\pi G)^{-1}\int R\sqrt{-g}d^4x+\frac{1}{2}\int \beta(\phi)\phi^i\phi_i\sqrt{-g}d^4x-(8\pi G)^{-1}\int \alpha(\phi)V(\phi)\sqrt{-g}d^4x+S_{\rm source}, \tag{1}$$

where the coupling to the trace is given by an arbitrary function $f(\phi)$, and

$$\alpha(\phi)=\frac{1}{1+4f(\phi)},\quad \beta(\phi)=\frac{1}{1+2f(\phi)}. \tag{2}$$

The $S_{\rm source}$ is the remaining source term (radiation, matter, etc.), with appropriate coupling to ϕ (see Appendix A). The energy momentum tensor for the field ϕ which arises from the action (1) is given by

$$T^{ik}=\alpha(\phi)V(\phi)g^{ik}+\beta(\phi)\left[\phi^i\phi^k-\frac{1}{2}g^{ik}\phi^j\phi_j\right]. \tag{3}$$

*On leave from Department of Physics, Jamia Millia, New Delhi 110025, India. Electronic address: sami@iucaa.ernet.in
†Electronic address: nabhan@iucaa.ernet.in; http://www.iucaa.ernet.in/ paddy

We shall assume that the scalar field is evolving in an isotropic and homogeneous space time and that ϕ is a function of time alone. The energy density ρ_ϕ and pressure p_ϕ obtained from T^{ik} are

$$\rho_\phi = \beta(\phi)\frac{\dot{\phi}^2}{2} + \alpha(\phi)V(\phi), \quad p_\phi = \beta(\phi)\frac{\dot{\phi}^2}{2} - \alpha(\phi)V(\phi). \tag{4}$$

The coupling to T_a^a modifies the field energy density ρ_ϕ, pressure p_ϕ, and equation of state parameter $w_\phi = (p_\phi/\rho_\phi)$.

In a universe containing dustlike matter and radiation, the action (1) leads to the following evolution equations for the field:

$$\ddot{\phi} + \frac{3\dot{a}}{a}\dot{\phi} = \dot{\phi}^2\frac{f_{,\phi}}{1+2f} + 4V(\phi)\frac{(1+2f)}{(1+4f)^2}f_{,\phi} - V_{,\phi}(\phi)\frac{(1+2f)}{(1+4f)} + \frac{\rho_m^i}{a^3}f_{,\phi}\frac{(1+2f)}{(1+f)^2}. \tag{5}$$

(The radiation, being traceless, does not couple to ϕ.) The Friedmann equation with the modified energy momentum tensor given by Eq. (3) is

$$\frac{\dot{a}^2(t)}{a^2(t)} = \frac{8\pi G}{3}\left[\frac{\dot{\phi}^2}{2(1+2f)} + \frac{V(\phi)}{1+4f} + \rho_b(a)\right], \tag{6}$$

where the background energy density due to radiation and matter is given by

$$\rho_b(a) = \frac{\rho_R^i}{a^4} + \frac{\rho_m^i}{a^3(1+f)}. \tag{7}$$

The simplest form of coupling, which we shall adopt, is the linear one

$$f(\phi) = g_c(\phi/M_p), \tag{8}$$

where $M_p = \sqrt{1/(8\pi G)}$ is the reduced Planck mass and g_c is a coupling constant. Since Eqs. (5) and (6) reduce to the standard form for $g_c = 0$, this parameter gives the strength of the forcing term. Equation (5) describes the evolution of the scalar field in the expanding universe under the influence of an external ''force,'' which depends upon the field and its kinetic energy $(1/2)\dot{\phi}^2$. The influence of this term on the dynamics of the scalar field is accumulative.

In the absence of matter, the conservation equation formally equivalent to Eq. (5) has the usual form

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = 0, \tag{9}$$

and the evolution of energy density is given by

$$\rho_\phi = \rho_{0\phi}e^{-\int 6[1-\zeta(a)]da/a} \tag{10}$$

with

$$\zeta(a) = \frac{1}{(K_e/P_e)+1},$$

where the ratio of effective kinetic to potential energy (K_e/P_e) is given by

$$\frac{K_e}{P_e} = \frac{\beta(\phi)}{\alpha(\phi)}\frac{\dot{\phi}^2}{2V(\phi)}. \tag{11}$$

Interaction modifies the kinetic as well as the potential energy through $\alpha(\phi)$ and $\beta(\phi)$; however, ρ_ϕ bears the same relation to their ratio as in the case of standard scalar field cosmology. Equation (9), of course, is not valid in the presence of matter; nevertheless, numerical work shows that the ratio (K_e/P_e) is still a good parameter which influences the dynamics. The evolution of the potential to kinetic energy ratio plays a significant role in the growth or decay of the energy density ρ_ϕ at a given epoch and will be crucial in the following discussion.

Let us next address the question of the choice of a potential of the scalar field that would lead to a viable cosmological model. The obvious restriction on the evolution is that, starting from Planck's time, the scalar field should survive until today (to account for the observed late time accelerated expansion) without interfering with the nucleosynthesis of the standard model.

Exponential potentials lead to solutions which are the attractors of evolution equations when $g_c = 0$ and provide the backdrop for the understanding of the dynamics in our case. A standard model with

$$V(\phi) = V_0 \exp\left[-\lambda\frac{\phi}{M_p}\right] \tag{12}$$

(and $g_c = 0$) is well studied in literature and can be divided into two categories: (a) the exponential potentials for which ρ_ϕ scales more slowly than the background density $\rho_b = (1/a^n)$, which translates to $\lambda < \sqrt{n}$; (b) potentials for which the scalar field energy density scales faster than ρ_b, i.e., $\lambda > \sqrt{n}$. In the first case, if the background energy density was subdominant, it would become more subdominant and radiation domination will never occur. In the second case, there exists a scaling solution which mimics the background energy density that is dominant, with

$$\Omega_\phi = \frac{\rho_\phi}{\rho_\phi + \rho_b} = \frac{n}{\lambda^2}, \quad \rho_\phi \propto \frac{1}{a^n}. \tag{13}$$

The discussion of this model with $g_c \neq 0$ (so that the coupling to the trace of the stress tensor is switched on) is given in Appendix B. It turns out that this model is not as attractive as another variant of the potential with $V(\phi) = V_0 e^{\lambda\phi/M_p}$ which we shall concentrate on next.

III. QUINTESSENCE WITH EXPONENTIAL POTENTIAL IN THE PRESENCE OF THE COUPLING TO THE TRACE T

We shall now consider a model with

$$V(\phi)=V_0\exp\left(\lambda\frac{\phi}{M_p}\right)\quad(\lambda<\sqrt{2}) \tag{14}$$

in which the field evolves toward the origin rather than away from the origin. In order to investigate the dynamics described by Eqs. (5) and (6), it would be convenient to cast these equations as a system of first order equations

$$Y_1'=Y_2, \tag{15}$$

$$\begin{aligned}Y_2'=&-3Y_2+\frac{1}{H(Y_1,Y_2)}\left[\frac{g_c}{(1+2g_cY_1)}Y_2^2\right.\\&\left.+g_c\frac{(1+2g_cY_1)}{(1+4g_cY_1)^2}\tilde{V}(Y_1)-\frac{d\tilde{V}(Y_1)}{dY_1}\frac{(1+2g_cY_1)}{(1+4g_cY_1)}\right]\\&+\frac{\rho_m^i}{a^3}g_c\frac{(1+2g_cY_1)}{(1+g_cY_1)^2},\end{aligned} \tag{16}$$

where

$$Y_1=\frac{\phi}{M_p},\quad Y_2=\frac{\dot{\phi}}{M_p^2},\quad \tilde{V}=\frac{V(Y_1)}{M_p^4}, \tag{17}$$

and the prime denotes the derivative with respect to the variable $N=\ln(a)$. The function $H(Y_1,Y_2)$ is given by

$$H(Y_1,Y_2)=\sqrt{\frac{1}{3}\left[\frac{Y_2^2}{2(1+2g_cY_1)}+\frac{\tilde{V}(Y_1)}{(1+4g_cY_1)}+\frac{\rho_b}{M_p^4}\right]}, \tag{18}$$

where $\rho_b=\rho_r^ie^{-4N}+\rho_m^ie^{-3N}/(1+g_cY_1)$. The initial conditions will be set at the Planck time with $a_i=1$. (The scale factor today would be nearly 10^{31}.) Further, for the background energy density $\rho_r=\rho_r^i/a^4$, we make the choice $\rho_r^i=M_p^4$ based on the point of view that, at the radiation dominated phase near the Planck epoch, the energy density of radiation was of the order of the Planck energy as it was the only scale available at that time. The initial values for ϕ and $\dot{\phi}$ as well as the the parameters in the potential are chosen so as to ensure a viable cosmological model. We take the initial values $Y_1=550$, $Y_2=0$ and $\lambda=0.52$, $g_c=5.9$. The value of V_0 is chosen so that initially $\rho_\phi^i/\rho_R^i\simeq10^2$, Of these choices, λ and g_c are important for obtaining a viable model; Y_1 is arbitrary except for the condition $Y_1\gg1$. The choices made for Y_2 and V_0 are merely done for the sake of convenience; different choices will lead to the same qualitative behavior. Some of these choices could be construed as fine-tuning in the model. But it is no worse than the fine-tuning in any other scalar field model.

The model described by Eq. (14) supports an eternal inflation in the absence of coupling to the trace T. The forcing term induced by the back reaction in the field evolution equation acts against the Hubble damping and hastens the evolution of the field energy density ρ_ϕ. We have plotted the ratio of kinetic to potential energy of the field in Fig. 1. It is seen that, due to coupling to the trace T, the kinetic energy acquires a large value, making the potential term unimportant and pushing the field into the kinetic regime. As a result, the field energy density crosses the background energy density and continues evolving as $(1/a^6)$ for quite some time (see Fig. 2). After the crossover, the dominant contribution to the Friedmann equation (6) comes from the background energy density ρ_b. This in turn leads to an increase in damping and consequent slowing down of the decay of ρ_ϕ, allowing the ratio (ρ_ϕ/ρ_b) to grow. The energy density gets locked to a constant value in this process and acts like a cosmological constant, thereby causing Ω_ϕ to grow. The forcing term containing $\dot{\phi}^2$ in the field evolution equation is ineffective in this regime.

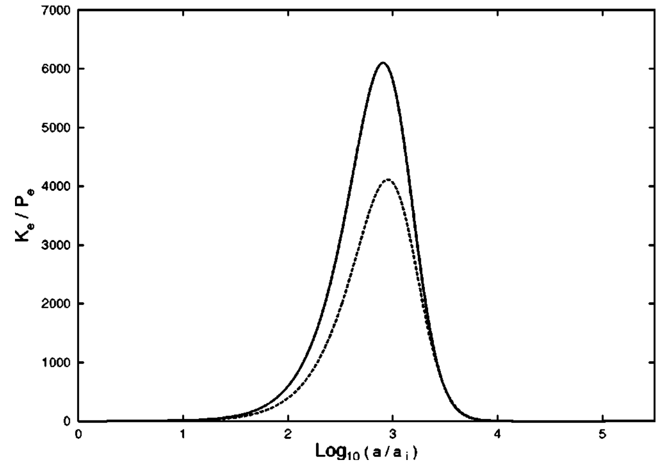

FIG. 1. The ratio of kinetic to potential energy of the scalar field is depicted for the exponential potential (14) with $\lambda=5.2$, $V_0^{1/4}\simeq2.8\times10^{-30}M_p$ with two different values for coupling (chosen for illustration): $g_c=0.04$ (dashed line) and $g_c=0.041$ (solid line). The kinetic energy (as compared to the potential energy) is seen to build up fast to a large value, forcing ρ_ϕ into the kinetic regime, which otherwise would scale slowly with the scale factor $(\rho_\phi\propto a^{-\lambda^2})$ for the model with the potential in Eq. (14).

A remarkable thing happens when the field starts rolling again at the end of the locking regime. As discussed above, the ratio of kinetic to potential energy of the field, (K_e/P_e), provides a yardstick to determine the energy scaling. At the end of the locking period, the field again starts rolling down the hill. Although the back reaction again builds up, the kinetic to potential energy ratio is now much smaller than it was in the beginning when ρ_ϕ was dominant. The ratio first increases and then fluctuates near its background value, finally heading towards zero (Fig. 3). As a result, ρ_ϕ first (nearly) tracks the ρ_b, gradually approaches it, and ultimately overtakes it. This allows the scalar field to dominate and gives rise to accelerated expansion at late times. By suitably tuning the parameters of the model, it is possible to get $\Omega_\phi=0.7$ today (Fig. 4). In Fig. 5, we have displayed the equation of state parameter w_ϕ to demonstrate the convergence of different initial conditions to the inflationary attrac-

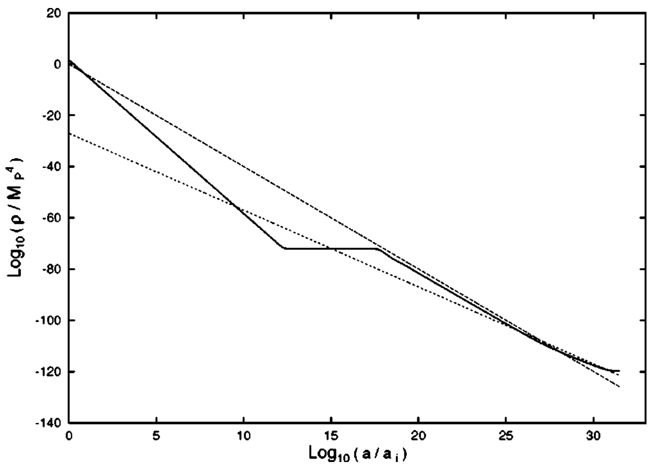

FIG. 2. The energy density is plotted against the scale factor: solid line corresponds to ρ_ϕ for $g_c = 5.9$ in case of the model described by Eq. (14) with $\lambda = 0.52$ and $V_0^{1/4} \simeq 2.8 \times 10^{-30} M_p$. The dashed and dotted lines correspond to energy densities of matter and radiation, respectively. The large value of the scalar field kinetic energy (relative to the potential energy) induced by the forcing term in the evolution equation ensures that the ρ_ϕ drops below ρ_b, after which ρ_ϕ remains approximately constant for a period of time. At the end of this phase, the scalar field tracks the background energy density before becoming dominant and driving the current accelerated expansion of the universe.

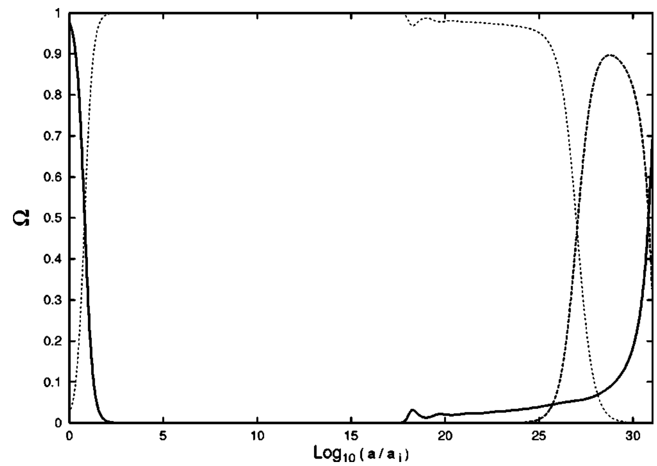

FIG. 4. Evolution of dimensionless density parameter Ω is shown as a function of the scale factor for the model described in Fig. 2 for (i) the scalar field (solid line), (ii) radiation (dotted line), and matter (dashed line). Late time behavior of the scalar field leads to the present day value of $\Omega_\phi = 0.7$ and $\Omega_m = 0.3$.

tor. Figure 6 shows that the w_ϕ mimics de Sitter–like behavior shortly after the current epoch.

The role of the $\dot{\phi}^2$ term on the right-hand side of evolution equation (5) is very important in the beginning. The size of the undershoot crucially depends upon the coupling constant g_c. A larger value of g_c leads to a deeper undershoot because the ratio K_e/P_e is sensitive to coupling initially. After the locking period this ratio is of the order of a few as background (radiation) dominates over the field at this time. During evolution, the scalar field moves from larger values toward the origin. As a result the influence of the back reaction in the Friedmann equation (6) decreases, and the field energy density gradually moves toward the background, overtakes it, and joins with the (noncoupled) inflationary attractor. [This may be contrasted with the behavior in the model described by Eq. (12), in which the scalar field evolves from the origin to larger values. In this case the contribution of the field energy density in the Friedmann equation gets gradually suppressed, allowing the background to dominate forever. In this model, ρ_ϕ tracks the background and never comes to dominate it (see Figs. 13, 14, and 15)]. This development is displayed in Fig. 7 which clearly shows (i) a deeper undershoot for larger coupling and (ii) joining of

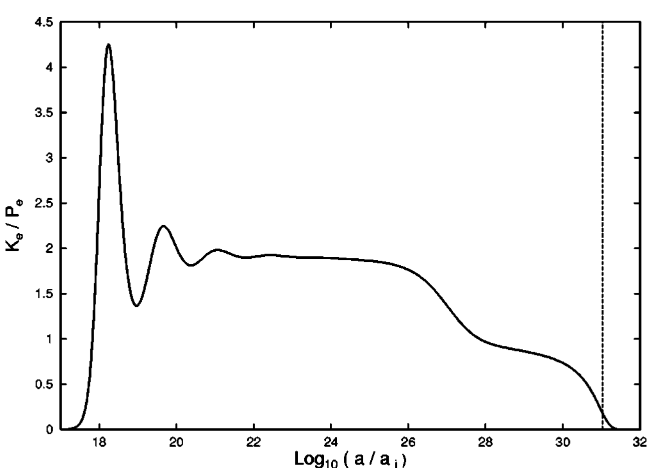

FIG. 3. Evolution of the kinetic to potential energy ratio of scalar field for the model described in Fig. 2 starting from the locking regime onward. At the end of the locking period, the ratio fluctuates about its background value and then nearly tracks it, slowly decreasing toward its attractor value. The ratio does not lock itself to its attractor value and approaches zero where it stays asymptotically. The vertical dashed line marks the current epoch.

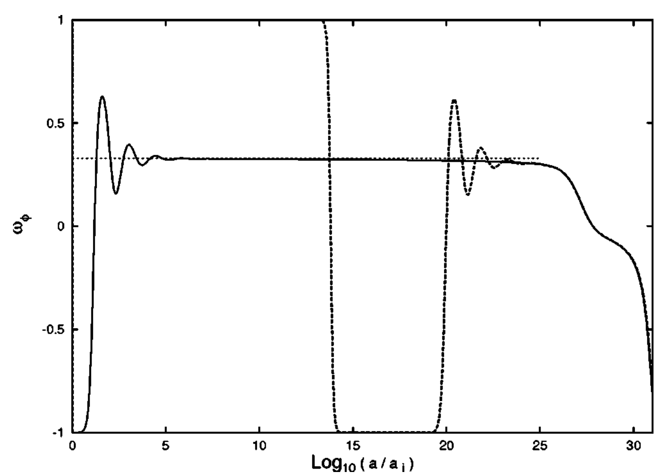

FIG. 5. The equation of state parameter of the scalar field for the exponential potential (14), showing that different initial conditions converge to the attractor solution at late times. The horizontal dashed line depicts $w_b = 1/3 = w_R$. The solid and dashed lines start with widely different initial conditions but lead to similar behavior at late times.

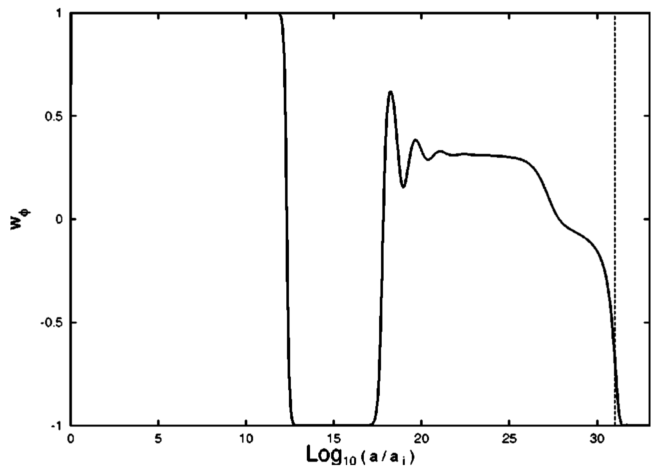

FIG. 6. Evolution of equation of state parameter of the scalar field. The coupling to the trace T brings the scalar field (which was slowly rolling down the shallow potential) into the kinetic regime. The scalar field stays in this regime for a long time before ρ_ϕ grows toward ρ_b. During this phase, w_ϕ gets locked to $w_\phi=-1$ (locking regime). After the locking period ends, ρ_ϕ tracks the background energy density slowly, approaching it from below with the equation of state $w_\phi \simeq 1/3$. At late times ρ_ϕ overtakes the background and becomes dominant to account for the current epoch of cosmic acceleration. During this time w_ϕ decreases and heads toward its inflationary attractor value $[w_\phi=(\lambda^2-3)/3)]$. However, before it could settle there, the field ϕ slowly moving toward the origin gets trapped near the barrier formed there due to coupling to the trace T. The equation of state locks to -1 permanently. The vertical dashed line marks the present epoch.

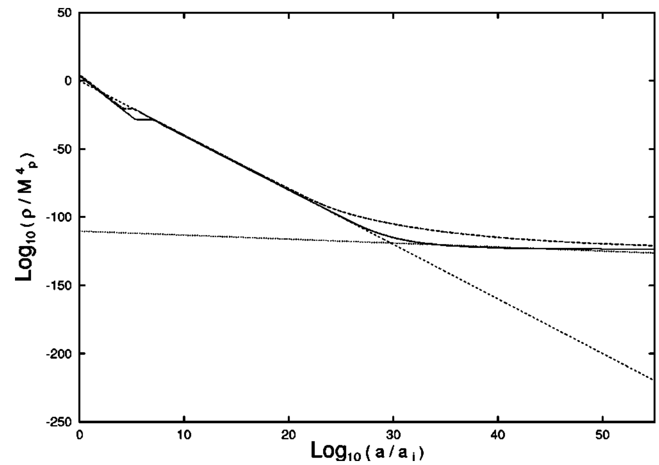

FIG. 7. The evolution of energy density in the field is shown for two values of $g_c=0.9$ (solid line) and $g_c=0.07$ (dotted line) for the model described (14) with $\lambda=0.52$. The radiation energy density is depicted by the dashed line. The second dashed line below shows the (noncoupled) inflationary behavior of ρ_ϕ ($\rho_\phi \propto a^{-\lambda^2}$). The decay of ρ_ϕ slows down earlier for smaller value of the coupling constant g_c and approaches the inflationary attractor behavior after overtaking the background (radiation) energy density. After staying with the attractor value for quite some time, ρ_ϕ becomes constant.

the inflationary track earlier for smaller values of g_c. Figure 7 also shows that the field continues to stay in the usual (noncoupled) attractor phase for quite some time, until the potential terms induced by the coupling in the field evolution equation become important. This happens at very late times and the influence of these terms will be discussed below. As for the matter terms, numerical analysis shows that they do not influence the dynamics significantly. Thus, coupling plays an important role in the intermediate regime (which is also manifest in Figs. 2, 5, and 6), thereby allowing production of the currently observed accelerated expansion with necessary tuning of g_c along with α.

Let us briefly summarize the role of the coupling to the trace T. The exponential potential under consideration is shallow and supports an eternal inflation in the standard scenario. The introduction of the coupling to the trace T hastens the decay of ρ_ϕ, pushing the scalar field into the kinetic regime. After the locking period, its role gradually diminishes, making it possible for the scalar field, at late times, to rejoin the inflationary track it was deflected from. At this stage, the scalar field is approaching the origin and we would expect the equation of state w_ϕ to settle at its attractor value. However, the equation of state becomes equal to -1 and stays there—which is evident from Fig. 6. The same effect is reflected in the behavior of ρ_ϕ shown in Fig. 7, where the field energy density becomes constant after staying at its attractor value for some time. This seems to be a general feature of the model under consideration and requires explanation.

The coupling to the trace T modifies the field evolution through three extra terms on the right hand side of Eq. (16). The first term contains $\dot{\phi}^2$ and is important at the early epochs and during tracking as discussed above. The second term contains the potential, which contributes in the potential dominated regimes: during the turnaround of ρ_ϕ and when the field is rolling slowly. We will show that this term is responsible for the cosmological-constant-like behavior of the scalar field at very late times, i.e., $a\to\infty$. In order to see this, we will cast Eq. (15) in the form

$$\dot{Y}_2+\frac{\dot{a}}{a}Y_2=g_c\frac{Y_2^2}{1+2g_cY_1}-\frac{d}{dY_1}V_{\mathrm{tot}}(Y_1) \tag{19}$$

where $V_{\mathrm{tot}}(Y_1)$ is the effective potential given by

$$V_{\mathrm{tot}}(Y_1)=\int\left[\lambda\frac{(1+2g_cY_1)}{(1+4g_cY_1)}-g_c\frac{(1+2g_cY_1)}{(1+4g_cY_1)^2}\right]\tilde{V}(Y_1)dY_1, \tag{20}$$

where $\tilde{V}=(V_0/M_p^4)e^{\lambda Y_1}$. (The matter term is subdominant in this limit and is hence ignored.) In the model described by Eq. (14), the scalar field is evolving from large values toward the origin and can take negative values. But when $Y_1 \to -(1/2g_c)$ the system will become singular because of the divergence of the first term in Eq. (19). However, the second term in the expression of $V_{\mathrm{tot}}(Y_1)$ does not let it happen. In fact, this term sets an infinite potential barrier at $Y_1=-(1/4g_c)$ and does not allow the field to reach the singularity. The potential which was attractive away from the origin now becomes repulsive, followed by the infinite barrier on its left (see Fig. 8). This happens when

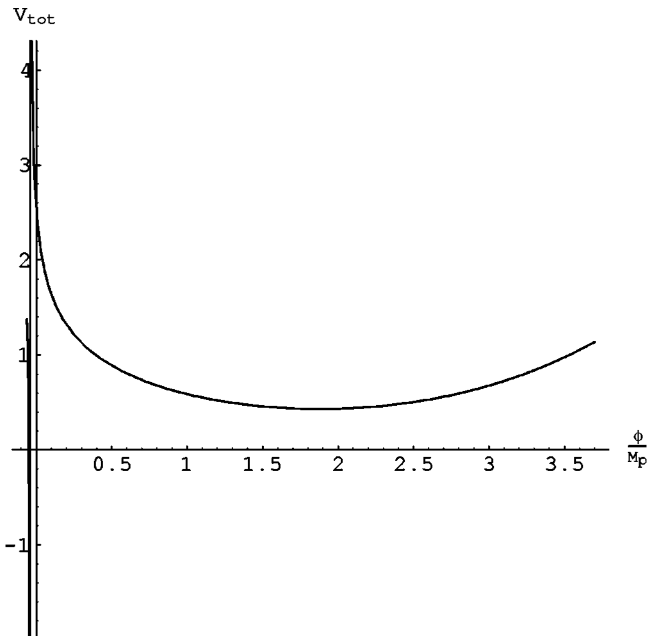

FIG. 8. The effective potential in Eq. (20) for $\lambda=0.52$, $g_c=5.9$. This effective potential becomes repulsive near the origin when $(\phi_{\rm cr}/M_p)=1.88$ followed by an infinite barrier at $(\phi^b/M_p)=-0.042$, which shields the singularity situated at $(\phi^s/M_p)=-0.084$.

$$Y_1^{\rm cr}=\frac{4g_c-\lambda}{4\lambda g_c}. \tag{21}$$

At this stage, the field ϕ is rolling slowly. It crosses the critical point and moves toward the barrier but due to insufficient kinetic energy gets reflected back by the barrier. After making a few such oscillations, the field settles at the critical point permanently and mimics the cosmological constant. For the model described by Eq. (14) with $\lambda=0.52$ and $g_c=5.9$, $Y_1^{\rm cr}\equiv\phi^{\rm cr}/M_p\simeq1.88$, which is confirmed by numerical integration (see Fig. 9).

IV. MODELS OF QUINTESSENCE FREE FROM FUTURE EVENT HORIZON PROBLEM

In the model described above, as in any generic model of quintessence, the universe will be accelerating forever, leading to a future event horizon—which could pose a problem in the string theoretical context. It is, however, possible to tackle this problem by using any potential that is exponential for large ϕ and is a power law near the origin: That is, we require $V(\phi)\to\exp(\lambda\phi/M_P)$ for large ϕ and $V(\phi)\to(\lambda\phi/M_P)^{2p}$ for $\phi\to0$. The power law behavior near the origin will lead to oscillations when the field approaches $\phi\simeq0$. The mean equation of state of the scalar field is then given by [10]

$$\langle w_\phi\rangle\simeq\left\langle\frac{\frac{\dot\phi^2}{2}-V(\phi)}{\frac{\dot\phi^2}{2}+V(\phi)}\right\rangle=\frac{p-1}{p+1}. \tag{22}$$

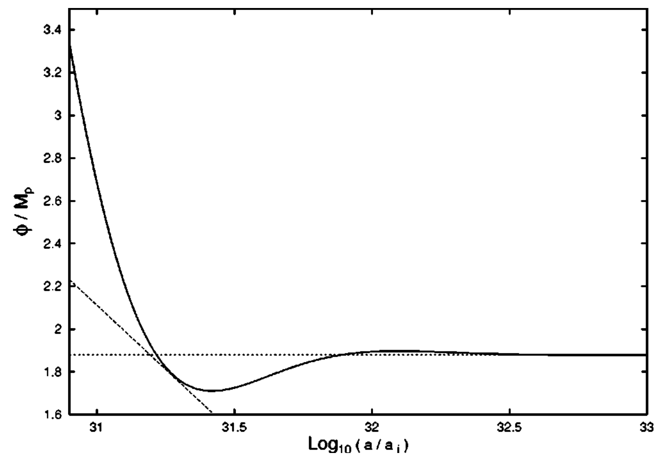

FIG. 9. Late time evolution of the scalar field (solid line). After the current epoch, the scalar field passes across the critical point, moves toward the barrier, and gets reflected in the effective potential shown in the previous figure. It makes few oscillations and settles at the critical point $(\phi_{\rm cr}/M_p)=1.88$. The dashed line depicts the inflationary attractor behavior of the scalar field in case of the exponential potential (14) with $\lambda=0.52$ $[\phi/M_p=-\lambda\ln(a/a_i)+{\rm const}]$ and the dotted line corresponds to $(\phi_{\rm cr}/M_p)=1.88$.

As a result the scalar field energy density and scale factor have the following behavior:

$$\rho_\phi\propto a^{-3(1+\langle w\rangle)},\quad a\propto t^{(2/3)(1+\langle w\rangle)^{-1}}.$$

The scalar field behaves like pressureless dust for $p=1$. By suitably adjusting the parameters in the model one could ensure that the oscillations occur in the future, i.e., well after the present epoch when Ω_ϕ has reached a value equal to 0.7. One can avoid the future event horizon in such a model, irrespective of the explicit form of $V(\phi)$ that is used, as long as it interpolates between exponential and power law behavior. One of the many possible choices of interpolation is a potential of the form $V(\phi)=V_0\sinh^{2p}(\lambda\phi/2M_p)$, for which we have evolved the field equations numerically. The results for a particular choice of parameters are shown in Figs. 10 and 11.

We summarize below the chronology of the main events in any such model with a shallow potential that behaves like a power law near the origin. (i) The scalar field starts rolling slowly in the shallow potential. The coupling to the trace T almost instantaneously kills any inflationary behavior and makes the scalar field enter into the kinetic regime. The scalar field continues to stay in the kinetic regime (with $\rho_\phi\propto a^{-6}$) and soon ρ_ϕ drops below ρ_b. (ii) Once this happens, the dominant contribution to the Friedmann equation comes from the background. This, in turn, allows ρ_ϕ to turn toward ρ_b, leading to a locking regime during which the value of ρ_ϕ stabilizes and remains relatively unchanged for a considerable length of time and the scalar field behaves like a cosmological constant. (iii) The field starts rolling down the potential again, and the back reaction builds up such that the kinetic energy of the field exceeds its potential energy by a factor of a few in the beginning. However, as the field rolls toward smaller and smaller values, the role of the back reac-

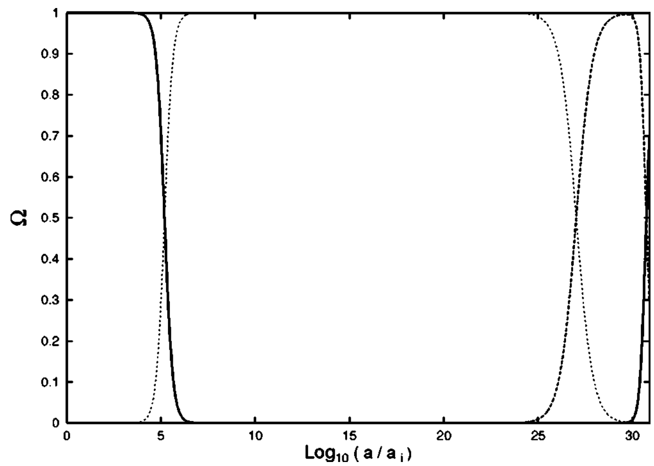

FIG. 10. The dimensionless density parameter is plotted as a function of the scale factor for (i) scalar field (solid line) with a potential $V=V_0 \sinh^{2p}(\lambda\phi/2M_p)$, radiation (dotted line), and matter (dashed line). The late time behavior of the scalar field is shown to lead to the present values of $\Omega_\phi=0.7$ and $\Omega_m=0.3$.

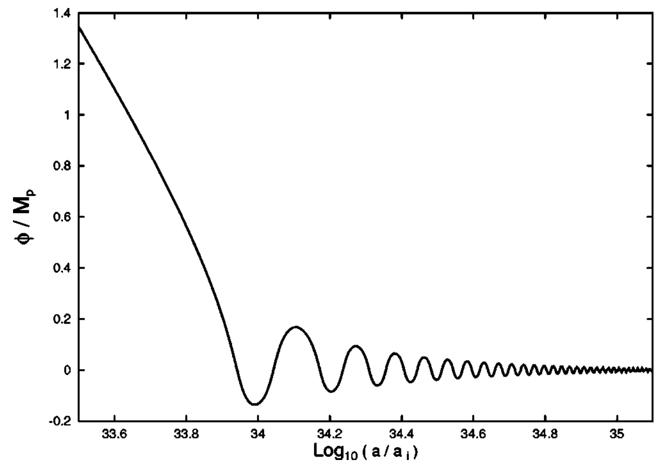

FIG. 12. Evolution of the scalar field is shown for the model described in Fig. 11 in the asymptotic future. (The current epoch is $a\approx 10^{31}$.) After driving the accelerated expansion for quite some time, the field enters into an oscillatory phase near the origin with the average equation of state $\langle w_\phi \rangle = 0$.

tion gradually diminishes and the potential energy starts catching up again with the kinetic energy. As a result ρ_ϕ slowly approaches the background energy density and finally overtakes it. The scalar field is now back on the original inflationary track. This phase is interpreted as the late time accelerated expansion of the universe, leading to the present day value of $\Omega_\phi=0.7$. (iv) After the current epoch, the universe continues in the phase of cosmic acceleration for quite some time until the scalar field approaches $\phi\simeq 0$ and starts oscillating about it, giving rise to $\langle w_\phi \rangle = 0$ and causing the scalar field to behave like cold dark matter (see Fig. 11). This is expected to happen in future, and the model is free from the event horizon problem at late times (see Fig. 12). (There are other possible solutions to this problem; see, for example, [5,11].)

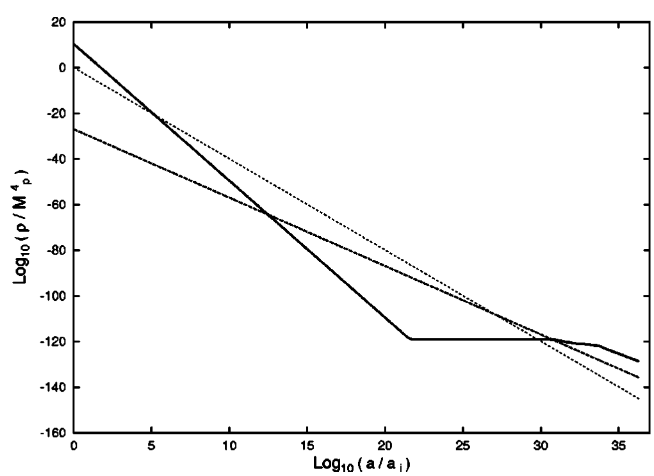

FIG. 11. Evolution of energy density is shown for scalar field (solid line), radiation (dotted), and matter (dashed) for a potential $V \propto \sinh^{2p}(\lambda\phi/2M_p)$. The energy density ρ_ϕ drops below ρ_{b} and turns around, catching up with it in the matter dominated era. The scalar field dominates at late times and drives the current cosmic acceleration. The universe continues in this phase for some time until field oscillations (near the origin $\phi=0$) build up in the system, leading to the average equation of state $\langle w_\phi \rangle = 0$. The field behaves as cold dark matter thereafter.

V. CONCLUSIONS

We have examined the dynamics of a scalar field which couples to the trace of all the fields including itself in a Friedmann-Robertson-Walker (FRW) background. The coupling to the trace T of the field modifies the energy momentum tensor and induces a forcing term in the field evolution equation. The forcing term acts against the Hubble damping, changing the evolution of field energy density significantly. In the case of a steep potential, it amounts to increasing the steepness of the potential. In the case of shallow potentials in which the field rolls slowly and mimics a cosmological constant, the coupling to the trace T plays an important role. It forces the scalar field into the kinetic regime, allowing the radiation domination to commence. We discussed a model in which the effect of the back reaction gradually diminishes, allowing the scalar field to catch up with the inflationary attractor regime at late times, thereby accounting for the observed cosmic acceleration with $\Omega_\phi=0.7$. We also presented a variant of the model in which the scalar field plays the role of quintessence at the current epoch and mimics dustlike dark matter in future, which eliminates the future event horizon. The general features of the model are shown to be independent of the initial conditions. However, fine-tuning of some of the parameters is necessary to achieve viable evolution.

ACKNOWLEDGMENT

One of us (M.S.) is thankful to V. Sahni and T. Qureshi for useful discussions.

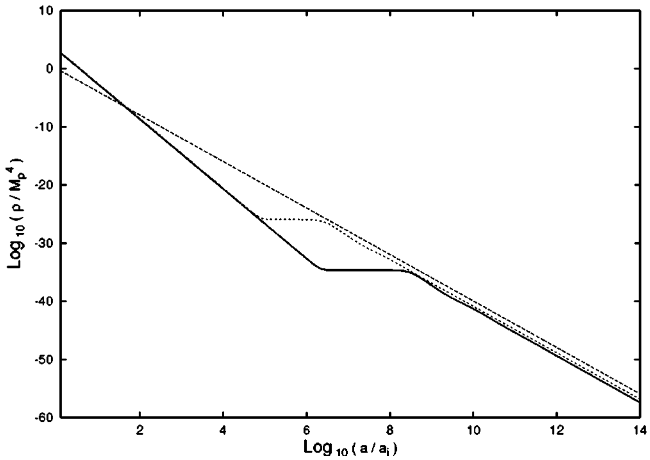

FIG. 13. The evolution of scalar field density corresponding to $g_c=0.05$ (solid line) and $g_c=0$ (dotted line) is shown for the potential (12) with $\lambda=6$. The radiation energy density is shown by the dashed line. The pattern of tracking shows that the effect of forcing terms amounts to increasing the steepness of the potential.

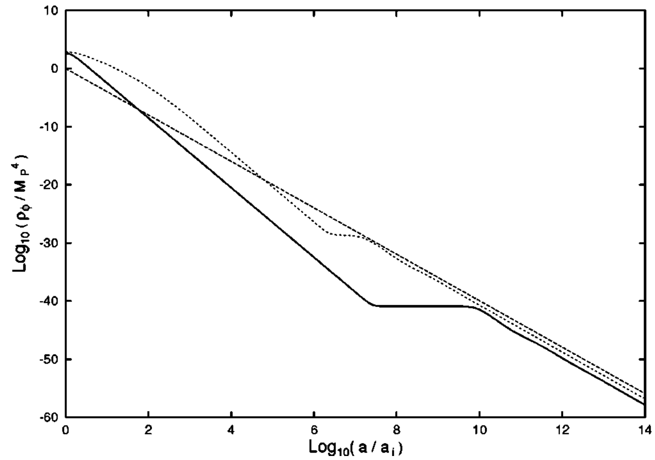

FIG. 14. The energy density of the scalar field ρ_ϕ is plotted for $g_c=1$ (solid line) and $g_c=0.1$ (dotted line) for the shallow exponential potential (12) with $\lambda=1.2$. The dashed line shows the radiation energy density. The effect of the forcing term in the evolution equations is shown to accelerate the scaling of ρ_ϕ. The stronger coupling is seen to hasten the commencement of kinetic regime. The scalar field ultimately tracks the background and gets locked up there forever.

APPENDIX A: ACTION FOR SCALAR FIELD COUPLED TO THE TRACE OF THE STRESS TENSOR

We summarize here the derivation of the self-consistent Lagrangian for a scalar field coupled to the trace of the energy momentum tensor of all matter, originally given in [8], for the sake of completeness.

Consider a system consisting of the gravitational field g_{ab}, radiation fields, and a scalar field ϕ which couples to the trace of the energy momentum tensor of all fields, including its own. The *zeroth order* action for this system is given by

$$A^{(0)}=A_{\rm grav}+A_\phi^{(0)}+A_{\rm int}^{(0)}+A_{\rm radn}, \tag{A1}$$

where

$$A_{\rm grav}=(16\pi G)^{-1}\int R\sqrt{-g}d^4x-\int \Lambda\sqrt{-g}d^4x, \tag{A2}$$

$$A_\phi^{(0)}=\frac{1}{2}\int \phi^i\phi_i\sqrt{-g}d^4x,$$

$$A_{\rm int}^{(0)}=\eta\int Tf(\phi/\phi_0)\sqrt{-g}d^4x. \tag{A3}$$

Here, we have explicitly included the cosmological constant term and η is a dimensionless number which ‘‘switches on’’ the interaction. In the zeroth order action, T represents the trace of all fields other than ϕ. Since the radiation field is traceless, the only zeroth order contribution to T comes from the Λ term, so that we have $T=4\Lambda$. The coupling to the trace is through a function f of the scalar field, and one can consider various possibilities for this function. The constant ϕ_0 converts ϕ to a dimensionless variable and is introduced for dimensional convenience.

Since the stress tensor of the scalar field has a nonzero trace, we must add to T the contribution $T_\phi=-\phi^l\phi_l$ of the scalar field. If we now add T_ϕ to T in the interaction term, $A_{\rm int}^{(0)}$ further modifies T_ϕ^{ik}. This again changes T_ϕ. Thus to arrive at the correct action an infinite iteration will have to be performed and the complete action can be obtained by summing up all the terms (see [9]). The full action can be found, more simply, by a consistency argument.

Since the effect of the iteration is to modify the expression for A_ϕ and A_Λ, we consider the following ansatz for the full action:

$$A=\frac{1}{16\pi G}\int R\sqrt{-g}d^4x-\int \alpha(\phi)\Lambda\sqrt{-g}d^4x$$
$$+\frac{1}{2}\int \beta(\phi)\phi^i\phi_i\sqrt{-g}d^4x+A_{\rm rad}. \tag{A4}$$

Here $\alpha(\phi)$ and $\beta(\phi)$ are functions of ϕ to be determined by the consistency requirement that they represent the result of the iteration of the interaction term. (Since radiation makes no contribution to T, we expect $A_{\rm rad}$ to remain unchanged.) The energy momentum tensor for ϕ and Λ is now given by

$$T^{ik}=\alpha(\phi)\Lambda g^{ik}+\beta(\phi)\left[\phi^i\phi_k-\frac{1}{2}g^{ik}\phi^\alpha\phi_\alpha\right] \tag{A5}$$

so that the total trace is $T_{\rm tot}=4\alpha(\phi)\Lambda-\beta(\phi)\phi^i\phi_i$. The functions $\alpha(\phi)$ and $\beta(\phi)$ can now be determined by the consistency requirement

$$-\int \alpha(\phi)\Lambda\sqrt{-g}d^4x+\frac{1}{2}\int \beta(\phi)\phi^i\phi_i\sqrt{-g}d^4x$$
$$=-\int \Lambda\sqrt{-g}d^4x+\frac{1}{2}\int \phi^i\phi_i\sqrt{-g}d^4x$$
$$+\eta\int T_{\rm tot}f(\phi/\phi_0)\sqrt{-g}d^4x. \tag{A6}$$

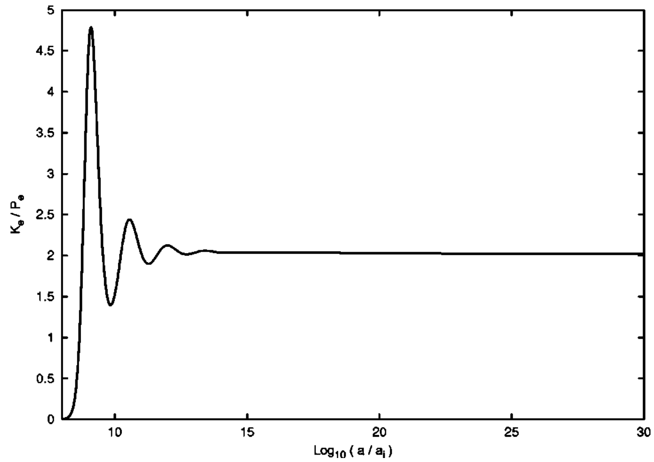

FIG. 15. Evolution of the kinetic to potential energy ratio of scalar field for the potential (12) with $\lambda=1.2$ and $g_c=1$ starting from the locking regime onward. At the end of the locking period, the ratio fluctuates about its background (radiation in this case) value ($K_e/P_e=2$) and ultimately approaches it.

Using T_{tot} and comparing terms in the above equation, we find that

$$\alpha(\phi)=[1+4\eta f]^{-1}, \quad \beta(\phi)=[1+2\eta f]^{-1}. \tag{A7}$$

Thus the complete action can be written as

$$A=\frac{1}{16\pi G}\int R\sqrt{-g}d^4x-\int\frac{\Lambda}{1+4\eta f}\sqrt{-g}d^4x+\frac{1}{2}\int\frac{\phi^i\phi_i}{1+2\eta f}\sqrt{-g}d^4x+A_{\mathrm{rad}}. \tag{A8}$$

(The same action would have been obtained if one used the iteration procedure.) It is obvious that the method works for any source for which the trace of the stress tensor T is proportional to the Lagrangian L. If $T=\mu L$, then the coupling replaces L by $L(1-\mu\eta f)^{-1}$. [For the kinetic energy term $L_\phi=(1/2)\phi_a\phi^a$, we have $T_\phi=-\phi^a\phi_a=(-2)L_\phi$ and for the cosmological constant term $T_\Lambda=4\Lambda=(-4)L_\Lambda$. These lead to the factors $(1+2\eta f)^{-1}$ and $(1+4\eta f)^{-1}$ in Eq. (A8).] In general, if the action is a homogeneous function of degree D in g^{ab}, $T=(2D)L$ and the coupling factor is $(1-2D\eta f)^{-1}$. For a system of dustlike particles with the action for each particle being the integral of $ds=\sqrt{g_{ab}dx^a dx^b}$, we have $D=(-1/2)$ and the coupling factor becomes $(1+\eta f)^{-1}$ (see [9]).

In the presence of a cosmological constant and sources that are conformally invariant, the action in Eq. (A8) leads to the following field equations:

$$R_{ik}-\frac{1}{2}g_{ik}R=-8\pi G\left[\beta(\phi)\left(\phi^i\phi^k-\frac{1}{2}g^{ik}\phi^\alpha\phi_\alpha\right)+\frac{\Lambda}{8\pi G}\alpha(\phi)g_{ik}+T_{ik}^{\mathrm{traceless}}\right], \tag{A9}$$

$$\Box\phi+\frac{1}{2}\frac{\beta'(\phi)}{\beta(\phi)}\phi^i\phi_i+\frac{\Lambda}{8\pi G}\frac{\alpha'(\phi)}{\beta(\phi)}=0. \tag{A10}$$

Here, $\Box$ stands for a covariant d'Lambertian, $T_{ik}^{\mathrm{traceless}}$ is the stress tensor of all fields with traceless stress tensor, and a prime denotes differentiation with respect to ϕ.

In the cosmological context, this reduces to

$$\ddot{\phi}+\frac{3\dot{a}}{a}\dot{\phi}=\eta\dot{\phi}^2\frac{f'}{1+2\eta f}+\eta\frac{\Lambda}{2\pi G}\frac{f'(1+2\eta f)}{(1+4\eta f)^2}, \tag{A11}$$

$$\frac{\dot{a}^2+k}{a^2}=\frac{8\pi G}{3}\left[\frac{1}{2}\frac{\dot{\phi}^2}{1+2\eta f}+\frac{\Lambda}{8\pi G}\frac{1}{(1+4\eta f)}+\rho_b(a)\right], \tag{A12}$$

where $\rho_b(a)=\rho_R^i/a^4$. It is obvious that the effective cosmological constant can decrease if f increases in an expanding universe. The result can be easily generalized for a scalar field with a potential by replacing Λ by $V(\phi)$ in the action. Similarly, the presence of dustlike matter adds the term $(\rho_i/a^3)(1+\eta f)^{-1}$ to the source term in Eq. (A12) and a corresponding derivative term in Eq. (A11).

APPENDIX B: COSMOLOGY WITH $V\propto\exp(-\lambda\phi/M_P)$ AND COUPLING TO T

In this case, the field evolves to large ϕ at late times, making the matter term $a^{-3}[1+\eta f]^{-1}$ subdominant to the radiation term a^{-4}. We will, therefore, ignore the matter term for simplicity. We first discuss the possibility when $\lambda>\sqrt{n}$. In this case the potential is steep leading to $\rho_\phi\propto 1/a^6$. This makes ρ_ϕ subdominant relative to ρ_b and the motion will become strongly damped, allowing the kinetic energy to decrease. This, in turn, allows ρ_ϕ to catch up with the background and stay there forever until the attractor regime is reached. The role of coupling to T will be to increase the kinetic energy and keep ρ_ϕ in the kinetic regime for longer. The ρ_ϕ will catch up again with the background, although it takes longer to reach the attractor (scaling) solution with the duration dependent upon g_c. Therefore, in this case, the role of the forcing term is equivalent to increasing the steepness of the potential given by λ (see Fig. 13). Thus nothing new or interesting happens in this case.

The case with $\lambda<\sqrt{n}$ ($n=4$ in our case) is more interesting. We shall be working with $\lambda<\sqrt{2}$, in which case the exponential potential supports a never-ending inflation. The forcing term acts against damping. Due to coupling to the trace T the kinetic energy acquires a large value, making the potential term unimportant and pushing the field into the kinetic regime. Depending upon the value of the coupling g_c, it may take a longer or shorter time to force ρ_ϕ to come into the kinetic regime, which then crosses the background energy density (see Fig. 14). The extent of the overshoot depends upon the coupling constant g_c. The evolution of ρ_ϕ in this model proceeds more or less in a similar fashion as in the case of the model described by Eq. (14) until the turnaround. At the end of the locking period, the field again starts rolling down the potential. Although the back reaction again builds up, the kinetic to potential energy ratio is now much smaller than it was in the beginning when ρ_ϕ was dominant (Fig. 15). The field does not join the inflationary attractor in the present case, because as it runs down the hill away from the origin to larger and larger values, the contribution of $V(\phi)$ and $\dot{\phi}^2/2$ to the right hand side of the Friedmann equation (6) gets suppressed systematically and ρ_ϕ remains subdominant to the background (radiation). The ratio (K_e/P_e) settles at a value of 2 $(w_\phi = 1/3)$, and the scalar field tracks the background and gets locked there forever.

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