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On the gravitational influence of direct particle fields

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The problem of the contribution of direct particle interaction of the Fokker type to the gravitational equations is solved. It is shown that the usual procedure for obtaining the gravitational equations, of making small variations of geometry, $g_{ik}+\delta g_{ik}$ replacing g_{ik} in finite regions, with $\delta g_{ik} = 0$ on their boundaries, and of requiring that the action be stationary for such variations, can be carried through with the aid of Green functions. This procedure, due to Hilbert, serves to define the energy tensor T_{ik} associated with each of the fields. That for the C-field turns out exactly the same as we have used in the macroscopic form of the theory. That for the electromagnetic field turns out to have some new features. These are terms containing the vector potential and its derivative when world-lines are broken, although these terms vanish when there is charge conservation. The terms in the field F_{ik} are identical with the usual tensor if the field is calculated from retarded potentials. In former work no decision has been made on the form the tensor should take when the potentials are $\frac{1}{2}$ (retarded + advanced). Wheeler & Feynman showed that alternative choices are possible and that a decision cannot be made from electromagnetic considerations alone. Our analysis leads to a unique result, the Frenkel tensor.

Introduction

The total action, including the electromagnetic field and the C-field as direct particle fields is given by

$$J = \frac{1}{16\pi G}\int R\sqrt{(-g)}\,\mathrm{d}^4x - \sum_a m_a\int \mathrm{d}a - \sum_{a<b}\sum 4\pi e_a e_b \iint \bar{G}_{i_A i_B}\,\mathrm{d}a^{i_A}\,\mathrm{d}b^{i_B}$$
$$+\frac{1}{f}\sum_{a<b}\sum\iint \bar{G}_{;\,i_A i_B}\,\mathrm{d}a^{i_A}\,\mathrm{d}b^{i_B}, \tag{1}$$

where we imagine the particles ordered in a numerical sequence so as to give meaning to $a < b$ in the double sums (cf. Hoyle & Narlikar 1964). Variation of the metric tensor, $g_{ik}+\delta g_{ik}$ replacing g_{ik}, in a four-dimensional volume V with $\delta g_{ik} = 0$ on the boundary, leads to a certain variation of J. The first two terms in the action contribute

$$-\frac{1}{16\pi G}\int_V\left[R^{ik}-\tfrac{1}{2}g^{ik}R + 8\pi G\sum_a m_a\int(-\bar{g})^{-\frac{1}{2}}\,\delta^{(4)}_{(X,A)}\frac{\mathrm{d}a^{i_A}}{\mathrm{d}a}\frac{\mathrm{d}a^{k_A}}{\mathrm{d}a}\bar{g}^i_{i_A}\,\bar{g}^k_{k_A}\,\mathrm{d}a\right]\delta g_{ik}\sqrt{(-g)}\,\mathrm{d}^4x \tag{2}$$

in first order.

If only these two terms were involved, and if we require J to be stationary for all finite V subject to $\delta g_{ik} = 0$ on the boundary of V, we obtain Einstein's equations for the gravitational field of a set of massive particles $m_a, m_b, \ldots$, namely,

$$R^{ik}-\tfrac{1}{2}g^{ik}R = -8\pi G\sum_a m_a\int(-\bar{g})^{-\frac{1}{2}}\delta^{(4)}_{(X,A)}\frac{\mathrm{d}a^{i_A}}{\mathrm{d}a}\frac{\mathrm{d}a^{k_A}}{\mathrm{d}a}\,\bar{g}^i_{i_A}\,\bar{g}^k_{k_A}\,\mathrm{d}a \tag{3}$$

(the velocity of light is taken as unity throughout).

The contributions of the electromagnetic field and of the C-field to the gravitational equations must evidently come from the third and fourth terms in (1). In the usual field theory formulation of the action, these contributions take the form

$$-8\pi G\left[\frac{1}{4\pi}\left(\tfrac{1}{4}g^{ik}F^{lm}F_{lm}-F^{il}F^{k}_{l}\right)-f\left(C^{i}C^{k}-\tfrac{1}{2}g^{ik}C^{l}C_{l}\right)\right] \tag{4}$$

to be added to the right-hand side of (3). The question arises as to whether the direct particle interaction theory, with action given by (1), leads to the same, or similar, terms. At first sight the situation appears unpromising. Instead of (1), the action in the usual field theory takes the form

$$\frac{1}{16\pi G}\int R\sqrt{(-g)}\,\mathrm{d}^4x-\sum_a m_a\int \mathrm{d}a-\frac{1}{16\pi}\int F_{lm}F^{lm}\sqrt{(-g)}\,\mathrm{d}^4x-\sum_a e_a\int A_i\,\mathrm{d}a^i$$
$$-\tfrac{1}{2}f\int C_lC^l\sqrt{(-g)}\,\mathrm{d}^4x+\sum_a\int C_i\,\mathrm{d}a^i, \tag{5}$$

where A_i is the electromagnetic 4-potential. When the geometry is varied, A_i, C are kept constant in (5), and (4) arises from the third and fifth terms—the fourth and sixth give no contribution. Yet, rewriting (1) in the form

$$J=\frac{1}{16\pi G}\int R\sqrt{(-g)}-\sum_a m_a\int \mathrm{d}a-\frac{1}{2}\sum_a e_a\int\sum_{b\neq a}A^{(b)}_i\mathrm{d}a^i+\frac{1}{2}\sum_a\int\sum_{b\neq a}C^{(b)}_i\,\mathrm{d}a^i, \tag{6}$$

we find that we are working with an action containing terms analogous to the fourth and sixth of (5)† but with no terms analogous to the third and fifth.

There is a difference of principle between the two cases, however. In the usual field theory the fields are taken to possess independent degrees of freedom and it is permissible to vary g_{ik} while keeping A_i, C constant. Indeed it is necessary to make an independent variation of A_i, keeping g_{ik} and C constant, in order to obtain Maxwell's equations. The situation is entirely otherwise when the fields arise directly from the particles in accordance with our definitions. Then both Maxwell's equations and the source equation for the C-field are satisfied identically. We are not permitted to make independent variations of either A_i or C. Nor are we permitted to vary g_{ik} keeping A_i, C constant. A variation of g_{ik} involves corresponding variations of A_i, C which could be calculated explicitly if necessary. We cannot therefore conclude that the third and fourth terms of (6) give zero contributions, as the corresponding terms in (5) do in the field theory case.

Yet this leaves us a long way from actually determining the contributions of these terms. When this problem was first faced it seemed rather unlikely that variations of the third and fourth terms of (1) would lead to results resembling (4). We were encouraged to proceed with the investigation by the opinion that success would give a valuable pointer against fields with independent degrees of freedom. If classical theory were the only consideration we would indeed then regard the issue as decisive. The action (6) is simpler than (5), which is a hopelessly inelegant combination of

† The extra factor $\frac{1}{2}$ appears in the last two terms of (6) because each interaction is shared by two particles. As was shown in the earlier paper (1964), this makes no difference to the equation of motion of a typical particle.

unrelated terms. It is true that (5) can be simplified by removing the C-field terms, but the price of such an omission is that the resulting gravitational theory permits the existence of singularities. In a future paper we shall attempt to develop the point of view that even (6) is implausibly inelegant.

The mathematical problem of determining the variation of the double integrals in (1) is different from the problems usually posed in the calculus of variations. One can deal with the $\int R\sqrt{(-g)}\,\mathrm{d}^4x$ term, or the single-line integrals, simply by considering the variation of the integral at a point. But to deal with the double integrals one must consider the whole of space, since the Green functions are affected, not simply by the variations δg_{ik} at a point, but by the variations everywhere. We proceed now to consider this awkward situation.

The energy momentum tensor of the C-field

We consider, first, the variation of the fourth term in the action formula (1), because it is less difficult to deal with the scalar Green function than with the vector function. The scalar function satisfies the equation

$$\frac{\partial}{\partial x^{i_X}}\left[\sqrt{(-g)}\,g^{i_X k_X}\frac{\partial \bar{G}(X,A)}{\partial x^{i_X}}\right] = -\delta^{(4)}_{(X,A)}. \tag{7}$$

Let $\bar{G}(X,A)\to\bar{G}+\delta\bar{G}$ as a result of $g^{ik}\to g^{ik}+\delta g^{ik}$. The variation of (7) can be written in the form

$$\frac{\partial}{\partial x^{i_X}}\left[\sqrt{(-g)}\,g^{i_X k_X}\frac{\partial \delta\bar{G}}{\partial x^{k_X}}\right] = -\frac{\partial}{\partial x^{i_X}}\left[\delta(\sqrt{(-g)}\,g^{i_X k_X})\frac{\partial \bar{G}}{\partial x^{k_X}}\right]. \tag{8}$$

Evidently $\delta\bar{G}$ satisfies an equation with the same differential operator as $\bar{G}$ itself, but with a distributed source—i.e. a source which is non-zero at points X other than A—instead of the delta function singularity at A. The solution for $\delta\bar{G}$ can be written down as follows

$$\delta\bar{G}(X,A) = \int_V \frac{\partial}{\partial y^{i_y}}\left[\delta(\sqrt{(-g)}\,g^{i_y k_y})\frac{\partial \bar{G}(Y,A)}{\partial y^{k_y}}\right]\bar{G}(x,y)\,\mathrm{d}^4y, \tag{9}$$

in which the Green function $\bar{G}(X,Y)$ for a general point y^{i_y} in V has been introduced. By integrating by parts, and using $\bar{G}(X,A)=\bar{G}(A,X)$ we can write

$$\delta\bar{G}(X,A) = -\int_V \delta(\sqrt{(-g)}\,g^{i_y k_y})\,\bar{G}(X,Y)_{;i_y}\bar{G}(A,Y)_{;k_y}\,\mathrm{d}^4y. \tag{10}$$

Now take A to lie on the world-line a, and take B on world-line b in place of X. Differentiations at A, B can be moved inside the integral and we obtain

$$\delta[\bar{G}(A,B)_{;i_A i_B}] = -\int_V \delta(\sqrt{(-g)}\,g^{i_y k_y})\,\bar{G}(A,Y)_{;k_y i_A}\bar{G}(B,Y)_{;i_y i_B}\,\mathrm{d}^4y. \tag{11}$$

Hence the variation of the fourth term of (1) is given by

$$\begin{aligned} f^{-1}\delta\left[\sum_{a<b}\sum\iint \bar{G}_{;i_A i_B}\,\mathrm{d}a^{i_A}\,\mathrm{d}b^{i_B}\right] &= -f^{-1}\int_V \mathrm{d}^4y\left[\sum_{a<b}\sum\iint\delta(\sqrt{(-g)}\,g^{i_y k_y})\,\bar{G}(A,Y)_{;k_y i_A}\,\bar{G}(B,Y)_{;i_y i_B}\,\mathrm{d}a^{i_A}\,\mathrm{d}b^{i_B}\right] \\ &= -f\int_V \mathrm{d}^4y\left[\sum_{a<b}\sum\delta(\sqrt{(-g)}\,g^{i_y k_y})\,C^{(a)}{}_{;k_y}C^{(b)}{}_{;i_y}\right], \end{aligned} \tag{12}$$

in which the definition $$C^{(a)}(X) = f^{-1}\int \bar{G}(X,A);_{i_A}\,\mathrm{d}a^{i_A} \tag{13}$$

introduced in the previous paper (1964), has been used.

The energy-momentum tensor $\underset{C}{T^{ik}}$ of the C-field is defined by equating the variation of the fourth term of (1) to

$$-\frac{1}{2}\int_V \underset{C}{T^{ik}}\delta g_{ik}\sqrt{(-g)}\,\mathrm{d}^4y. \tag{14}$$

The factor $-\frac{1}{2}$ here is the same as the factor by which the variation of $-\sum\limits_a m_a\int \mathrm{d}a$ must be multiplied in order to obtain the usual dynamical energy-momentum tensor. Using (12) we obtain

$$\underset{C}{T^{ik}} = -f\sum_{a<b}\sum (C^{(a)i}C^{(b)k}+C^{(a)k}C^{(b)i}-g^{ik}C^{(a)l}C^{(b)}_l), \tag{15}$$

after inserting $\delta(\sqrt{(-g)}\,g^{pq}) = -(g^{ip}g^{kq}-\frac{1}{2}g^{ik}g^{pq})\sqrt{(-g)}\,\delta g_{ik}$.

To pass to the smooth fluid approximation, write $C = \sum\limits_a C^{(a)}$ for the total C-field. Then (15) differs from

$$-f(C^iC^k-\tfrac{1}{2}g^{ik}C^eC_e), \tag{16}$$

only in the respect that (16) contains terms of the type $C^{(a)i}C^{(a)k}$, $C^{(a)l}C^{(a)}_l g^{ik}$. The number of such terms increases linearly with the number of particles, whereas the number of terms of the type $C^{(a)i}C^{(b)k}$, $C^{(a)l}C^{(b)}_l g^{ik}$ increases as the square of the number of particles. The smooth fluid approximation consists in neglecting the former in comparison with the latter, so that (16) approximates to the more accurate result given by (15). The form (16) agrees with the usual field theory result set out in (4).

It is interesting that it is possible to attain our objective solely by symbol manipulation. An explicit calculation of the variation of the Green function is not required.

The electromagnetic energy momentum tensor

It is to be expected that the calculation of the variation of the third term of (1) will follow lines similar to the preceding section. This turns out to be the case, but the details are considerably more complicated.

Define

$$\mathscr{F}_{i_X k_X i_A} = \frac{\partial \bar{G}_{k_X i_A}}{\partial x^{i_X}} - \frac{\partial \bar{G}_{i_X i_A}}{\partial x^{k_X}}. \tag{17}$$

The electromagnetic field $F^{(a)}_{i_X k_X}$ generated by particle b can be written as

$$F^{(a)}_{i_X k_X} = 4\pi e_a\int \mathscr{F}_{i_X k_X i_A}\,\mathrm{d}a^{i_A}. \tag{18}$$

The wave equation for the vector Green function can also be expressed in terms of these new quantities:

$$\begin{aligned}&\sqrt{(-g)}[g^{i_X l_X}g^{m_X k_X}\bar{G}_{l_X i_A;\,m_X k_X}+R^{i_X m_X}\bar{G}_{m_X i_A}]\\ &= \frac{\partial}{\partial x^{k_X}}[g^{i_X l_X}g^{m_X k_X}\sqrt{(-g)}\,\mathscr{F}_{m_X l_X i_A}]+\sqrt{(-g)}\,g^{i_X l_X}\bar{G}^{k_X}{}_{i_A;\,k_X l_X}\\ &= -\delta^{(4)}_{(X,A)}\bar{g}^{i_X}{}_{i_A}.\end{aligned} \tag{19}$$

The equality of the first and third lines of (19) is, of course, just the wave equation for the vector Green function. A modest degree of manipulation is required to establish equality with the second line.

For convenience of writing we shall drop the suffix X. The variation

$$g_{ik} \to g_{ik} + \delta g_{ik}$$

requires

$$\delta\left[\frac{\partial}{\partial x^k}(g^{il}g^{mk}\sqrt{(-g)}\,\mathscr{F}_{mli_A})\right] = \frac{\partial}{\partial x^k}(g^{il}g^{mk}\sqrt{(-g)}\,\delta\,\mathscr{F}_{mli_A}) + \frac{\partial}{\partial x^k}[\delta(g^{il}g^{mk}\sqrt{(-g)})\,\mathscr{F}_{mli_A}]$$

and

$$\begin{aligned}\delta[\sqrt{(-g)}\,g^{il}\bar{G}^k{}_{i_A;\,kl}] &= \delta\left[g^{il}\sqrt{(-g)}\frac{\partial}{\partial x^l}(\bar{G}^k{}_{i_A;\,k})\right]\\ &= \delta\,(g^{il}\sqrt{(-g)})\frac{\partial}{\partial x^l}(\bar{G}^k{}_{i_A;\,k})\\ &\quad + \sqrt{(-g)}\,g^{il}\frac{\partial}{\partial x^l}\left[\delta\left(\frac{1}{\sqrt{(-g)}}\right)\frac{\partial}{\partial x^k}(\sqrt{(-g)}\,\bar{G}^k_{i_A})\right]\\ &\quad + \sqrt{(-g)}\,g^{il}\frac{\partial}{\partial x^l}\left[\frac{1}{\sqrt{(-g)}}\frac{\partial}{\partial x^k}\{\delta(\sqrt{(-g)}\,g^{km})\,\bar{G}_{mi_A}\}\right]\\ &\quad + \sqrt{(-g)}\,g^{il}\frac{\partial}{\partial x^l}\left[\frac{1}{\sqrt{(-g)}}\frac{\partial}{\partial x^k}\{\sqrt{(-g)}\,g^{km}\,\delta\bar{G}_{mi_A}\}\right].\end{aligned}$$

Hence the variation of (19) can be written in the form

$$\begin{aligned}&\frac{\partial}{\partial x^k}[g^{il}g^{mk}\sqrt{(-g)}\,\delta\,\mathscr{F}_{mli_A}] + \sqrt{(-g)}\,g^{il}\frac{\partial}{\partial x^l}\left[\frac{1}{\sqrt{(-g)}}\frac{\partial}{\partial x^k}(\sqrt{(-g)}\,g^{km}\,\delta\,\bar{G}_{mi_A})\right]\\ &= -\frac{\partial}{\partial x^k}[\delta(g^{il}g^{mk}\sqrt{(-g)})\,\mathscr{F}_{mli_A}] - \delta(\sqrt{(-g)}\,g^{il})\frac{\partial}{\partial x^l}(\bar{G}^k{}_{i_A;\,k})\\ &\quad - \sqrt{(-g)}\,g^{il}\frac{\partial}{\partial x^l}\left[\delta\left(\frac{1}{\sqrt{(-g)}}\right)\frac{\partial}{\partial x^k}(\sqrt{(-g)}\,\bar{G}^k{}_{i_A})\right]\\ &\quad - \sqrt{(-g)}\,g^{il}\frac{\partial}{\partial x^l}\left[\frac{1}{\sqrt{(-g)}}\frac{\partial}{\partial x^k}\{\delta(\sqrt{(-g)}\,g^{km})\,\bar{G}_{mi_A}\}\right].\end{aligned} \tag{20}$$

The differential operator acting on $\delta\bar{G}_{i_X i_A}$ in (20) is the same as that which acts on $\bar{G}_{i_X i_A}$ in (19). As in the scalar case the source on the right-hand side of (20) is nonzero at points X other than the source point A of the δ-function in (19), and just as this could be dealt with by using the original scalar Green function so we can deal with the present situation by using the vector Green function. Using B instead of X and letting Y be a variable point in V, the volume in which we are making the change of geometry, we have

$$\begin{aligned}\delta\bar{G}_{i_B i_A} &= \int_V \bar{G}_{i_B i}\frac{\partial}{\partial y^k}[\delta(g^{il}g^{mk}\sqrt{(-g)})\,\mathscr{F}_{mli_A}]\,\mathrm{d}^4y\\ &\quad + \int_V \bar{G}_{i_B i}\,\delta\,(\sqrt{(-g)}g^{il})\frac{\partial}{\partial y^l}(\bar{G}^k{}_{i_A;\,k})\,\mathrm{d}^4y\\ &\quad + \int_V \bar{G}_{i_B i}\sqrt{(-g)}\,g^{il}\frac{\partial}{\partial y^l}\left[\delta\left(\frac{1}{\sqrt{(-g)}}\right)\frac{\partial}{\partial y^k}(\sqrt{(-g)}\,\bar{G}^k{}_{i_A})\right]\mathrm{d}^4y\\ &\quad + \int_V \bar{G}_{i_B i}\sqrt{(-g)}\,g^{il}\frac{\partial}{\partial y^l}\left[\frac{1}{\sqrt{(-g)}}\frac{\partial}{\partial y^k}\{\delta(\sqrt{(-g)}\,g^{km})\,\bar{G}_{mi_A}\}\right]\mathrm{d}^4y,\end{aligned} \tag{21}$$

in which the subscript Y has been omitted from suffixes at Y.

To simplify (21) we make use of $\delta g_{ik} = 0$ on the boundary of V, transforming the first and third integrals once by the divergence theorem and the last integral twice. We get

$$\delta\bar{G}_{i_A\, i_B} = \delta\bar{G}_{i_B\, i_A} = -\frac{1}{2}\int_V \delta(g^{il}g^{mk}\sqrt{(-g)})\,\mathscr{F}_{mli_A}\,\mathscr{F}_{iki_B}\,\mathrm{d}^4y$$

$$+\int_V \delta(\sqrt{(-g)}\,g^{il})\left\{\bar{G}_{i_B\,i}\frac{\partial}{\partial y^l}(\bar{G}^k{}_{i_A;k}) + \bar{G}_{ii_A}\frac{\partial}{\partial y^l}(\bar{G}^k{}_{i_B;k})\right\}\mathrm{d}^4y$$

$$+\int_V \delta(\sqrt{(-g)})\,\bar{G}^l{}_{i_A;l}\,\bar{G}^k{}_{i_B;k}\,\mathrm{d}^4y. \tag{22}$$

We turn now to our objective, to calculate the variation of the fourth term of (1). Thus

$$-\delta\sum_{a<b}\sum 4\pi e_a e_b\iint \bar{G}_{i_A\,i_B}\,\mathrm{d}a^{i_A}\,\mathrm{d}b^{i_B}$$

$$= \frac{1}{8\pi}\sum_{a<b}\sum\int_V \delta(g^{il}g^{mk}\sqrt{(-g)})\,F^{(a)}_{ml}F^{(b)}_{ik}\,\mathrm{d}^4y$$

$$-\frac{1}{4\pi}\sum_{a<b}\sum\int_V \delta(\sqrt{(-g)}\,g^{il})\,[A^{(b)}_i(A^{(a)k}{}_{;k})_{;l} + A^{(a)}_i(A^{(b)k}{}_{;k})_{;l}]\,\mathrm{d}^4y$$

$$-\frac{1}{4\pi}\sum_{a<b}\sum\int_V \delta(\sqrt{(-g)})A^{(a)l}{}_{;l}\,A^{(b)k}{}_{;k}\,\mathrm{d}^4y$$

$$= -\frac{1}{2}\int_V \underset{em}{T^{ik}}\delta g_{ik}\sqrt{(-g)}\,\mathrm{d}^4y, \tag{23}$$

where the last line defines the electromagnetic energy-momentum tensor $\underset{em}{T^{ik}}$. After inserting the expanded forms of $\delta(g^{il}g^{mk}\sqrt{(-g)})$, $\delta(g^{il}\sqrt{(-g)})$, $\delta(\sqrt{(-g)})$, we have

$$4\pi\underset{em}{T^{ik}} = \sum_{a<b}\sum[\tfrac{1}{2}g^{ik}F^{(a)lm}F^{(b)}_{lm} - F^{(a)il}F^{(b)k}{}_l - F^{(b)il}F^{(a)k}{}_l]$$

$$+\sum_{a<b}\sum[g^{ik}\{A^{(a)m}A^{(b)l}{}_{;lm} + A^{(b)m}A^{(a)l}{}_{;lm} + A^{(a)l}{}_{;l}A^{(b)m}{}_{;m}\}]$$

$$-2\sum_{a<b}\sum[g^{mk}\{A^{(a)i}A^{(b)l}{}_{;lm} + A^{(b)i}A^{(a)l}{}_{;lm}\}]. \tag{24}$$

The second and third terms are zero if the gauge condition $A^{(a)l}{}_{;l} = 0$ is satisfied. It has already been seen that this condition must be satisfied unless world-line a is broken. Even in this case the second and third terms in (24) vanish if world-lines are broken in pairs so as to conserve charge.

The first term in (24) agrees with the usual macroscopic tensor when we again approximate by neglecting terms of the type $F^{(a)il}F^{(a)k}{}_l$, $F^{(a)lm}F^{(a)}{}_{lm}\,g^{ik}$. If we write $F_{ik} = \sum_a F^{(a)}_{ik}$, to this approximation the first term of (24) is just

$$\tfrac{1}{4}g^{ik}F^{lm}F_{lm} - F^{il}F^k{}_l, \tag{25}$$

as in the usual theory.

Wheeler & Feynman (1949), in their treatment of the Fokker action in flat space-time, discuss two distinct expressions for $\underset{em}{T^{ik}}$, the Frenkel tensor and the canonical tensor. They remark:

'From the standpoint of pure electrodynamics it is not possible to choose between the two tensors. The difference is of course significant for the general theory of relativity, where energy has associated with it a gravitational mass. So far we have not attempted to discriminate between the two possibilities by way of this higher standard.'

The first term of (24) is the Frenkel tensor. We believe therefore that it is the Frenkel and not the canonical tensor that is consistent with the general theory of relativity as derived from an action principle of the form (1).

References

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