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The Diffuse Gamma-ray Background in the Hoyle-Narlikar Cosmology

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Summary. In this paper we analyze the contributions of QSO's, BL-Lac's and Seyfert galaxies to the diffuse gamma-ray background within the framework of the Hoyle-Narlikar theory.

It is shown that the inconsistency reported in standard theory, namely that the evolutionary function needed to explain the gamma-ray data is very different from the one derived from the optical part of the spectrum, is no longer present.

It is also shown that the contribution of a variable gravitational "constant" to the expression for the diffuse background is the same as that of a density evolution function.

Key words: cosmology – gamma-rays

1. Introduction

Bignami et al. (1979, referred to as BFHT hereafter) have recently investigated the contributions of QSO's, BL-Lac objects and Seyfert galaxies to the diffuse, isotropic gamma-ray background. Their conclusion is that assuming reasonable average space densities and spectra for these objects, they can account for the slope and intensity of the diffuse gamma-ray spectrum *provided* that since their inception, at around the epoch $z \simeq 4$, the BL-Lac's and Seyferts have evolved practically not at all, that is

$$\varrho(z) = \varrho(z=0) = \text{constant}, \tag{1.1}$$

and the QSO's have evolved not faster than

$$\varrho(z) = \varrho(z=0)(1+z)^4, \tag{1.2}$$

where $\varrho(z)$ is used in the form of *density evolution* but could express density or luminosity evolution. This finding contrasts with the conclusions drawn from observations in other parts of the spectrum, which seem to require strong evolutionary properties. For example, in standard cosmology, it is found that the explanation of the $\log N - \log S$ relation for radio sources necessitates strong evolution of the form

$$\varrho(z) = \varrho(z=0)(1+z)^6, \tag{1.3}$$

(Schmidt, 1978 and von Hoerner, 1973) or an even stronger form

$$\varrho(z) = \varrho(z=0) \exp\{13\, t(z)\} \tag{1.4}$$

(Turner, 1979) where $t(z)$ is the "look-back time" for the epoch of redshift z. Such evolutionary scenarios would generate much more gamma-ray background than is observed.

Thus, it appears that in standard Friedmann cosmologies, the same evolutionary prescription does not seem to work over the entire electromagnetic spectrum. Of course, there is no a priori reason to expect evolution in different bands to be identical, but the discrepancy of (1.1) and (1.2) with (1.3) and (1.4) is large enough to be significant (although BFHT have suggested mitigating circumstances, not the least of which is the paucity of data), and the fact that luminosity and density evolution effects seem to cancel out entirely and arbitrarily in (1.1) is disturbing in the standard cosmologies.

It is interesting to speculate at this point what would have happened if the gamma-ray data had been available prior to the radio data in the mid-1950's instead of the other way around. The evolutionary cosmologies instead of the steady state model (which does not allow evolution) might have been on the defensive.

Speculation aside, it is the purpose of this paper to show that there now exist at least two (evolutionary) cosmologies which do not show this type of discrepancy: (1) the scale-covariant cosmology of Canuto et al. (1979), admitting a transformation $d\tau_E = \beta(t_A)dt_A$ by an unknown scalar function $\beta(t_A)$ relating the gravitational (Einstein) and electromagnetic (Atomic) metric lengths, where it has already been shown that the evolutionary prescription needed to explain the observed $\log N - \log S$ relation yields a theoretical gamma-ray background in excellent agreement with the observed one, and (2) the gravity theory of Hoyle and Narlikar (1972 hereafter, HN) which is conformally invariant under transformations $ds^2 = \Omega^2 ds^2$. In an earlier paper, Canuto and Narlikar (1979) have examined the HN model in the light of the standard observational tests, e.g. the redshift-magnitude relation, the metric and isophotal angular sizes, the radio source count and the 3 K background radiation. Since the HN cosmology passes these observational tests as well as standard cosmology, it is worth while subjecting it to the additional test of the gamma-ray background.

In the following section, we summarize the salient features of the HN cosmology, which will be used in the present work.

2. The Hoyle Narlikar Cosmology

The HN gravitation theory starts with the definition of a Machian mass function $m(X)$, which is the sum of contributions of inertia from points B to X over all world lines of typical particles b,

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with elements of proper time db, in a Riemannian manifold with metric tensor g_{ik}.

$$m(X)=\sum_b \int G(X,B)\varepsilon_B db=\int \tilde{G}(X,B)\varepsilon_B n(B)d^4b \tag{2.1}$$

The summation over the line integral has been replaced by a volume integral in the continuum approximation with $n(B)$ describing the number density of particle world lines at a typical point B. The two-point function $\tilde{G}(X,B)$ is the symmetric Green's function of the wave operator $\square+\frac{1}{6}R$, R being the scalar curvature of the manifold. This propagator conveys the contribution of inertia from B to X. The coupling constants ε_B can be positive or negative.

The action is given by

$$S=-\sum_{a\neq b}\sum \iint \varepsilon_A\varepsilon_B G(A,B)da db. \tag{2.2}$$

Writing

$$F=\tfrac{1}{2}m^2(x), \qquad \phi_{ik}=\tfrac{1}{2}g_{ik}m^1 m_1-m_{;ik} \tag{2.3}$$

the "field" equations as given by $\delta S/\delta g^{ik}=0$ are

$$R_{ik}-\frac{1}{2}g_{ik}R=-\frac{3}{F}\left({}_{(m)}T_{ik}+\phi_{ik}+\frac{1}{3}(g_{ik}F-F_{;ik})\right) \tag{2.4}$$

where

$${}_{(m)}T^{ik}(x)=\sum_a \int \varepsilon_4(X,A)(-g(a))^{-1/2}\varepsilon_A m(A)\frac{da^i}{da}\frac{da^k}{da}da. \tag{2.5}$$

The mass of particle a at world point A, $\varepsilon_A m(A)$ in the energy-momentum tensor ${}_{(m)}T^{ik}$ of a system of particles $a,b,\ldots$ may vary in space and time. [For comparison, the RHS of (2.4) in scale-covariant cosmology is $-8\pi G(\beta)T_{\mu\nu}(\beta)-2\frac{\beta_{\mu;\nu}}{\beta}+4\frac{\beta_\mu\beta_\nu}{\beta^2}+g_{\mu\nu}\left(2\frac{\beta^\lambda_{;\lambda}}{\beta}-\frac{\beta^\lambda\beta_\lambda}{\beta^2}\right)\Big]$. The definition (2.1) and the field equations (2.4) are constrained by the property of conformal invariance. Thus if $g_{ik}(X)$ and $m(X)$ describe the metric and the mass function at X in a given solution of (2.4), then for any arbitrary $C^{(2)}$ function $\Omega(x)$ in the range $0<\Omega<\infty$, the metric and the mass function can also be described by $\Omega^2 g_{ik}$ and $\Omega^{-1}m(X)$. This property leads to different gauges in the homogeneous solution of (2.4) as described below.

The simplest geometrical description is in the 'Minkowski gauge' which has the line element of the Minkowski space-time:

$$ds_M^2=c^2d\tau^2-dr^2-r^2(d\theta^2+\sin^2\theta d\phi^2). \tag{2.6}$$

In this gauge the mass of a typical particle varies as τ while the effective gravitational parameter varies as τ^{-4}.

A conformal function $\Omega=\Omega_E\propto\tau^2$ together with a coordinate transformation $t\propto\tau^3$ defines the 'Einstein gauge' of the *same* cosmological model and change the line element (2.6) to

$$ds_E^2=c^2dt^2-\alpha t^{4/3}(dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2). \tag{2.7}$$

The particle masses still vary with epoch but the gravitational parameter is constant, thus the name "Einstein gauge".

A conformal function $\Omega=\Omega_A\propto\tau$ changes the gauge from the Mikowski gauge to the 'Atomic gauge' and the line element to

$$ds_A^2=c^2dt^2-2H_0t(dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2)), \tag{2.8}$$

where t plays the role of cosmic time and H_0 is the Hubble constant. The masses of the elementary particles stay constant in this gauge but the gravitational parameter varies as $G=G_A\propto t^{-1}$. We will use the atomic gauge in the following discussion and state below the relevant results (see Canuto and Narlikar, 1979 for a full exposition).

(i) The Redshift

Denoting by $R(t)=(2H_0t)^{1/2}$ the expansion factor of the universe, the redshift z is given by the standard formula:

$$1+z=R(t_0)/R(t)=(t_0/t)^{1/2}. \tag{2.9}$$

Here t=epoch of emission and t_0 (=present epoch)=epoch of reception of the redshift light. Note that $t_0=(2H_0)^{-1}$.

(ii) The Gravitational Constant

If G_0 is the present value of the gravitational parameter and $G(t)$ its value at an epoch t of redshift z, then $G(t)=G_0(1+z)^2$.

(iii) The Apparent Bolometric Luminosity

If $L(t)$ is the absolute bolometric luminosity of a source of radiation at the radial coordinate r and an epoch t of redshift z, then its apparent bolometric luminosity at $r=0$, $t=t_0$ is given by

$$l=\frac{L(t)}{4\pi d_L^2}\frac{G(t)}{G_0}. \tag{2.10}$$

Here d_L is the standard luminosity distance:

$$d_L=rR_0(1+z). \tag{2.11}$$

3. The Gamma-ray Spectrum

To calculate the gamma-ray background at the present epoch we have to sum over the contributions of the various sources to the apparent luminosity in the gamma-ray range of frequencies *at the present epoch*. Let $B(E,z)\Delta E$ denote the total radiation generated by such sources per unit time and per unit volume in the energy range $(E,E+\Delta E)$, at the epoch t of redshift z. Then the amount of radiation received at $r=0$, $t=t_0$ per unit area per unit time and in the energy range $(E_0,E_0+\Delta E_0)$ is given by $I(E_0)\Delta E_0$ where $E_0=E(1+z)^{-1}$ and

$$I(E_0)=\int\frac{\Delta E}{\Delta E_0}\frac{G(t)}{G_0}\frac{B(E,z)}{4\pi d_L^2}dV(z). \tag{3.1}$$

In the HN cosmology, we have the volume element at the epoch t given by

$$dV(z)=4\pi R^3(t)r^2dr.$$

Hence

$$\frac{dV(z)}{4\pi d_L^2}=\frac{R^3(t)r^2dr}{R_0^2r^2(1+z)^2}=\frac{R(t)dr}{(1+z)^4}.$$

Using the formula (2.8) and the fact that radiation travels along the null trajectory

$$R^2(t)dr^2=dt^2c^2,$$

we get

$$\frac{dV(z)}{4\pi d_L^2}=\frac{c}{H}\frac{dz}{(1+z)^7}$$

and (3.1) becomes

$$I(E_0)=\frac{c}{H_0}\int_0^{z_*}\frac{\Delta E}{\Delta E_0}\frac{G(t)}{G_0}B(E,z)\frac{dz}{(1+z)^7}$$
$$=\frac{c}{H_0}\int_0^{z_*}\frac{B(E,z)dz}{(1+z)^6}\frac{G(t)}{G_0}. \quad (3.2)$$

Here we have used (2.13) and the fact that $\Delta E=(1+z)\Delta E_0$. The upper limit on integration is the epoch of redshift z_* at which the gamma-ray sources were supposed to have been turned on. This relation is to be compared with the corresponding expression in the standard cosmology:

$$I(E_0)=\frac{c}{H_0}\int_0^{z_*}\frac{B(E,z)dz}{(1+z)^5(1+2q_0z)^{1/2}}. \quad (3.3)$$

We shall now write for $B(E,z)$:

$$B(E,z)=n_0(1+z)^3 f(z)Q(E)\Delta E. \quad (3.4)$$

In the above expression n_0 is the number density of source at the present epoch so that $n_0(1+z)^3$ simply expresses the higher number density in the past due to the expansion of the universe. The function $f(z)$ which is normalized to unity at $z=0$ epitomizes our ignorance about evolution, while $Q(E)$ is the emissivity per QSO, Seyfert, or BL-Lac per unit energy interval, for which we use the same forms and parameters as BFHT in order to produce a meaningful comparison. Converting $I(E_0)$ into a function describing photons per unit area per unit time per (kev) we finally obtain from (3.2) and (3.3) respectively

$$j(E_0)=\frac{n_0c}{H_0}\int_0^{z_*}f(z)\frac{Q\{E_0(1+z)\}}{(1+z)^2}dz\frac{G(t)}{G_0} \quad (3.5)$$

and

$$j(E_0)=\frac{n_0c}{H_0}\int_0^{z_*}f_{\mathrm{BFHT}}(z)\frac{Q\{E_0(1+z)\}}{(1+z)(1+2q_0z)^{1/2}}dz. \quad (3.6)$$

To calculate $j(E_0)$ in the two cosmologies, we need to know their respective evolutionary functions $f(z)$ and $f_{\mathrm{BFHT}}(z)$. It can be seen that the expressions (3.5) and 3.6) would yield the same result if

$$f(z)=\frac{f_{\mathrm{BFHT}}(z)(1+z)}{(1+2q_0z)^{1/2}}\frac{G_0}{G(t)}=\frac{f_{\mathrm{BFHT}}(z)}{(1+z)(1+2q_0z)^{1/2}} \quad (3.7)$$

and that therefore if BFHT have found that a function $f_{\mathrm{BFHT}}\sim(1+z)^n$, with $n=0$, 4 depending on the assumed source, leads to results consistent with data, we should find that a function $f(z)\sim(1+z)^{n-3/2}$ with the same n should also be consistent with the data in HN cosmology. [We note that it is the presence of $G(t)$ in (3.7) which has reduced the exponent in $f(z)$]. This means we expect $f(z)$ to go approximately as $(1+z)^{-1.5}$ for BL-Lac's and Seyfert galaxies and as $(1+z)^{2.5}$ for QSO's. These numbers are of course dependent as are the standard cosmological (BFHT) values of n on reasonable estimates of the gamma-ray source density and the frequency distribution of spectral shapes and intensities. The need for more data cannot be over-emphasized.

4. The Evolutionary Function

While comparing the radio source count data with the predictions of the HN cosmology, Canuto and Narlikar (op. cit.) arrived at the following conclusion. Suppose there is a luminosity evolution of the radio sources which is characterized by the relation

$$L(t)=L_0(1+z)^{2e_r}. \quad (4.1)$$

Where $L(t)$ is the luminosity at epoch t and L_0 the present luminosity; e_r is an empirically determined parameter. If α is the spectral index of a typical radio source, then it turns out that for the best fit to the data the parameter

$$r\equiv\tfrac{1}{2}(3-\alpha)+e_r \quad (4.2)$$

must be close to $\frac{1}{2}$. This gives

$$e_r=\frac{\alpha}{2}-1, \quad (4.3)$$

i.e., from (4.1)

$$L(t)=L_0(1+z)^{\alpha-2}. \quad (4.4)$$

Density evolution $(1+z)^s$ and luminosity evolution $(1+z)^{2e}$ enter into the log $N-$log S relation as $(1+z)^{3e+s}$ whereas they enter into the diffuse gamma-ray background as $(1+z)^{2e+s}$. In each case the observations separately give us only the total exponent in their frequency band. If the exponents e and s happen to be the same in all bands, we could of course solve for them individually and separate luminosity and density evolution! Returning to (4.3) and (4.4) for the moment, if density evolution instead of luminosity evolution had been postulated, it would have been found to be equivalent to

$$\varrho(z)=\varrho(z=0)(1+z)^{3e_r}=\varrho(z=0)(1+z)^{3\alpha/2-3} \quad (4.5)$$

where the exponent is small because the variability of G coupled with the luminosity function of the radio sources, of the form

$$\phi(L)\propto L^{-|\gamma|}, \quad (4.6)$$

generated a term in the integral for N, which was analogous to a density evolution in standard cosmology, of the form

$$\varrho(z)\propto(1+z)^{2(|\gamma|-1)}. \quad (4.7)$$

Thus $|\gamma|=3$ would produce a redshift dependent term $\sim(1+z)^4$. It was because of this effect that the HN cosmology was able to account for the log $N-$log S data without having to postulate density evolution. In particular, if we assume that the *same* luminosity and density evolution applies in the gamma-ray region as in the radio region, we then get

$$f(z)=(1+z)^{\alpha-2} \quad (4.8)$$

since we have assumed $f(z=0)=1$. We therefore get for (3.5)

$$j(E_0)=\frac{n_0c}{H_0}\int_0^{z_*}\frac{Q\{E_0(1+z)\}dz}{(1+z)^{4-\alpha}}\frac{G(t)}{G_0} \quad (4.9)$$

which yields an acceptable fit to the data when we take the average spectral index to be $\alpha=+0.75$ in (4.8), that is $f(z)\sim(1+z)^{-1.25}$. Considering the present accuracy of the data and the margin available in the spectral index and the parameters in $Q(E)$, this value for $2e$, the evolutionary exponent is undetectably different from the one derived from (3.7) and is equal to that found to fit the radio data. Similarly, if we had used (4.5), we would have found $s=-1.875$, thus bracketing the value -1.5 for $2e$ *or* s from the gamma-ray data. An *exact* solution, not yet warranted by the data, would be $2e=-0.75$ *and* $s=-0.75$ in both bands.

5. Conclusion

The above analysis demonstrates first of all the consistency of the Hoyle-Narlikar cosmology with observations on the one hand at the radio end and on the other at the gamma-ray end. The evolutionary estimate obtained here is lower than the equally consistent results obtained in useful cases of the scale-covariant theory (Canuto et al., 1979). To summarize, contrary to standard cosmology where the number density evolution evoked to explain the log $N - \log S$ data is claimed to be inconsistent with the observed gamma-ray background as interpreted by the gamma-ray source information now available, the HN-cosmology is able to explain the gamma-ray data with the same density evolution as derived from the radio part of the spectrum.

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