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STATISTICAL TESTS OF PEAKS AND PERIODICITIES IN THE OBSERVED REDSHIFT DISTRIBUTION OF QUASI-STELLAR OBJECTS

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ABSTRACT

There have been claims from time to time that there are periodicities in the redshift distribution of quasi-stellar objects. These claims are examined from various statistical angles for the 2164 QSO redshifts available in the latest compilation (1990) by Hewitt & Burbidge. The tests include the power-spectrum analysis carried out earlier (1972) by Burbidge & O'Dell, the generalized Rayleigh test often used in gamma-ray astronomy, the Kolmogorov-Smirnov test, and the "comb-tooth" test that is especially suited to detect periodicity. The tests reveal moderate to strong evidence (in terms of the various statistical significance levels) for periodicities $\xi = 0.0565$ and 0.0127–0.0129. The confidence level of the periodicity $\xi = 0.0565$ in fact marginally increases when the redshifts are transformed to the Galactocentric frame. The same periodicity was first noticed by Burbidge in 1968, and it is remarkable that it persists to date with a QSO population that has since grown about 30 times its original size. The prima facie evidence for periodicities in $\ln(1 + z)$ is, however, found to be of no great significance, and its reality needs to be investigated further.

Subject headings: cosmology: observations — galaxies: distances and redshifts — quasars: emission lines

1. INTRODUCTION

In a recent paper Burbidge & Hewitt (1990) have drawn attention to the peak at redshift $z = 0.06$ in the distribution of the redshifts of objects similar in their optical properties to the quasi-stellar objects (QSOs). These objects are drawn from the catalog of Hewitt & Burbidge (1991), which includes galaxies with optical spectra containing strong, broad emission lines and continua which are probably of nonthermal origin. These authors considered some 700 such objects with the criterion $z < 0.02$. While highlighting the peak at $z = 0.06$, they emphasize: "However, the vast majority of the objects now known with redshifts > 0.1 are QSOs, and to see whether there is new evidence for those peaks the data on the QSO redshifts will have to be added to those given in the catalog described here." The present work may be looked upon as a continuation of the above, and it is concerned largely with the data on QSOs with redshift limit $z > 0.1$. (The details of the data are given in the following section.) In particular, we are interested in testing how the earlier claims for periodicities in the z distributions of QSOs stand up to scrutiny with the increased number of data now available.

To set the background: in the late 1960s, studies based on the redshifts of quasi-stellar and related objects had shown that the distribution of absorption-line redshifts peaked near $z = 1.95$ (Burbidge 1967), while that of emission-line redshifts at $z = 1.955$ (Burbidge & Burbidge 1967). Considering a sample of 73 strong emission-line objects, which, besides QSOs, also included radio galaxies and Seyfert galaxies that did not unambiguously show evidence of the presence of stars, Burbidge (1968) concluded that the distribution of redshifts had peaks at $z = 0.061$ and integral multiples of it. Later, Cowan (1969) found peaks with periodicities $\xi = 0.0625$ and $\xi = 0.1666$ for a sample of 178 objects.

Subsequently, Burbidge & O'Dell (1972) carried out an extensive power-spectrum analysis of 346 redshifts and confirmed the peaks at $z = 0.06$ and $z = 1.95$ with a high level of significance. In later years Karlsson (1977), Burbidge (1978), Fang et al. (1982), and Depaquit, Pecker, & Vigier (1985) argued that there is a periodicity in $\ln(1 + z)$ with $\Delta \ln(1 + z) = 0.206$ and that this feature cannot be explained away as being due to any observational selection effects related to the placing of the relevant emission lines. Further, Chu & Zhu (1989) have also argued in favor of this periodicity through their analysis of the z distribution of the Lyα forest in the absorption spectra of 11 QSOs.

Arp et al. (1990) found that this kind of periodicity is present in several cases, e.g., in the redshifts of multiple quasars around low-redshift galaxies, in the redshift distribution of low-redshift quasars as well as bright high-redshift quasars, and also in the C IV, Lyα, and Mg II absorption-line redshifts of the Sargent-Boksenberg sample of quasars (Sargent, Boksenberg, & Steidel 1988). However, Scott (1990) has claimed that a selection effect arising from an intrinsic periodicity of the same order in the emission-line spectra of quasars might lead to an apparent periodicity in the distribution of $\ln(1 + z)$. Evidently, further investigations are needed to resolve this controversy.

For our analysis we shall mainly concentrate on the z distribution rather than the $\ln(1 + z)$ distribution, and use several different statistical techniques, including the Burbidge-O'Dell–type power-spectrum analysis, the Rayleigh test used by gamma-ray astronomers, the Kolmogorov-Smirnov test, and the so called "comb-tooth template" test. We will, however, briefly discuss the $\ln(1 + z)$ case at the end.

2. THE DATA

The analyzed data are taken from the catalog prepared by Hewitt & Burbidge (1990). The total number of QSOs in their catalog is 4355, while 901 objects have been listed in their non–QSO catalog. Of the 4355 QSOs, emission-line redshifts of 4282 objects are listed. We have chosen a sample of 2164 objects out of these 4282 QSOs. For the chosen sample the redshifts range from a minimum value of 0.025 to a maximum of 4.430. The redshifts of the remaining 2118 QSOs were obtained by the "grism" technique and were not included because of a possible selection effect. Out of the 2164 objects considered in the present work, 173 QSOs have their absorption redshifts measured; 76 QSOs show absorption in the spectrum, but their absorption redshifts were not measured.

In Figure 1*a* is given the histogram of 2164 QSOs, with redshifts lying in the range 0.025–4.430, with a bin size of 0.01. A subhistogram containing 769 QSOs corresponding to the redshift range 0.025–0.8 has been shown on a magnified scale in Figure 1*b*, from which it is easy to identify the peaks at 0.06, 0.18, 0.24, 0.30, 0.32, 0.36, 0.40, 0.47, 0.55, 0.62, etc. The peak at $z \simeq 0.06$, although present, is not a very prominent one. This is because the number of QSOs toward the lower end of the redshift range is small. For example, the number of QSOs with redshifts not exceeding 0.085 is only 22, of which nine have redshifts between 0.055 and 0.065 [i.e., an excess of slightly more than five sources around $z \simeq 0.06$ assuming a uniform distribution in the interval (0.025, 0.085)]. Although a periodicity of $\xi = 0.06$ is quite apparent from the histogram of Figure 1*b*, we have subjected the data to various other standard statistical analyses to test for the presence and significance of a periodicity. The tests are not interrelated and have the advantage of checking the claim for nonuniformity of the distribution from various independent angles. In each case the test procedure described is followed by its application to the actual data.

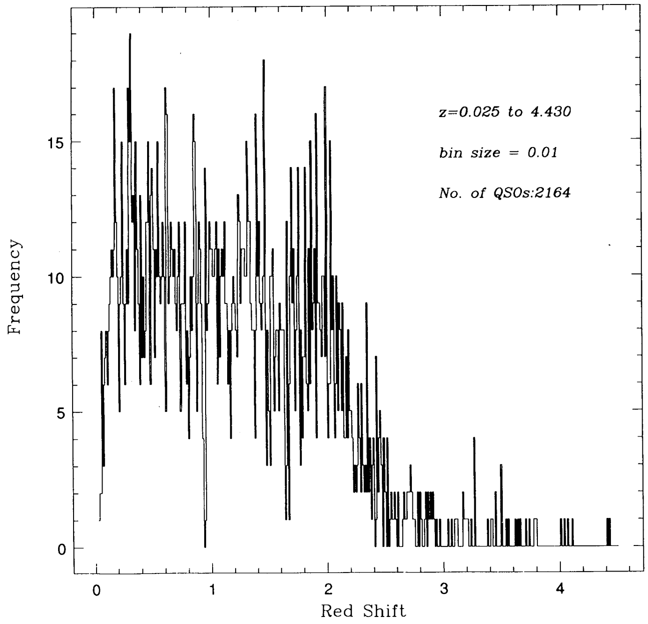

FIG. 1*a*

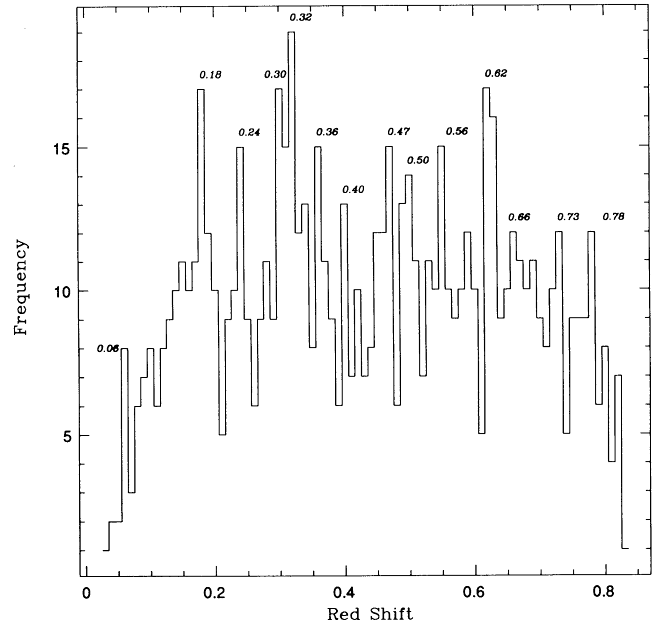

FIG. 1*b*

FIG. 1.—(*a*) Histogram of redshifts of the 2164 QSOs from the Hewitt-Burbidge catalog. (*b*) Histogram of 769 QSOs with redshifts ranging from 0.025 to 0.800.

3. THE POWER-SPECTRUM ANALYSIS

3.1. *The Test*

The power-spectrum analysis of Burbidge & O'Dell (1972) was based on the earlier work of Lake & Roeder (1972). Our method here exactly follows the approach, the salient features of which are as follows: Let $z_1, z_2, \ldots, z_N$ be the sets of redshifts in the observed distribution. For a "wavenumber" k, define the amplitude

$$F_N(k) = \sum_{n=1}^{N} \exp(-ikz_n) \tag{1}$$

and the spectral power function

$$S_N(k) = \frac{1}{N} |F_N(k)|^2 . \tag{2}$$

Then it can be shown (cf. Burbidge & O'Dell 1972) that in a sample randomly drawn from a population of uniform distribution, the statistic $S_N(k)$ is distributed with the probability density

$$p[S_N(k)]dS_N(k) = \exp[-S_N(k)]dS_N(k) . \tag{3}$$

So for K independent trials, the probability that at least one spectral power exceeds a specified value S_0 is, asymptotically,

$$P_1(S_N > S_0) = 1 - \exp[-K \exp(-S_0)] . \tag{4}$$

This formula holds for $K \gg 1$. Likewise, the probability that at least K_0 ($\ll K$) spectral powers exceed S_0 is approximated by

$$P_{K_0}(S_N > S_0) \approx \sum_{K'=K_0}^{\infty} \frac{[K \exp(-S_0)]^{K'}}{K'!} \times \exp[-K \exp(-S_0)] . \tag{5}$$

How do we determine K? In practice the analysis is performed within a given interval $k_{\min} < k < k_{\max}$ with a given "resolution." Suppose there are $(M + 1)$ discrete values of k distributed uniformly over this interval, so that the spacing between two successive values k_j and $k_{j+1}, j = 0, \ldots, M - 1$, is

$$\Delta k = \frac{1}{M}(k_{\max} - k_{\min}) . \tag{6}$$

The power-spectrum function $S_N(k)$ is, in practice, a discrete distribution $S_N(k_j)$, and if we join successive points in the $[k_j, S_N(k_j)]$-plot, we will, in general, get a jagged curve with "peaks" and "troughs." We identify $S_N(k_j)$ as a peak if it satisfies the inequalities

$$S_N(k_j) > S_N(k_{j-1}); \quad S_N(k_j) > S_N(k_{j+1}) . \tag{7}$$

The number K then is the number of such peaks in a given distribution. In the case of a uniform distribution, the maximum number of independent experiments K within a redshift range $(z_{\min}, z_{\max})$ is given by

$$K = \frac{1}{2\pi}(k_{\max} - k_{\min})(z_{\max} - z_{\min}) . \tag{8}$$

When, however, the data are not distributed uniformly, the above equation is weakened, and in practice the estimate of K is provided by the method of counting peaks described above. As Burbidge & O'Dell pointed out, in order to ignore structure on a scale greater than z in redshifts, one may limit the analysis to wavenumbers not less than $2\pi/\Delta z$. The periodicity corresponding to a value k will be denoted by $\xi = 2\pi/k$.

3.2. *Results*

As discussed in § 2, our sample consists of 2164 QSOs from the updated Hewitt-Burbidge catalog. Writing $\nu = k/2\pi = \xi^{-1}$, we take the domain of ν to be (0, 100), with $M = 10^3$, so that $k_j = 2\pi j/10$. The power spectrum $S_N(k_j)$ is computed for all k_j and plotted in Figure 2. Since we are interested in scales smaller than 0.08 and at the same time larger than 0.011, we consider only those k_j which lie between 78.5 and 570. The number of peaks in the spectral function works out to $K = 148$. It is evident from Figure 2 that there are two peaks (marked A and B in the figure) that exceed the 90% confidence level. Peak A has $S_N = 7.27$ for $k = 111.212$, corresponding to the periodicity $\xi = 0.0565$. The actual probability level is 90.21%. Peak B has $S_N = 9.05$ for $k = 486.311$, corresponding to the periodicity 0.0129, with a confidence level of 98.28%. Table 1 gives a summary of these results. The periodicity $\xi = 0.0565$ is remarkably close to the value 0.061 obtained earlier by Burbidge (1968) with a much smaller sample ($N = 73$) of strong emission-line objects. This value is also apparent from the histogram of redshifts (Figs. 1*a* and 1*b*) discussed earlier. Thus, the clustering of redshifts around multiples of 0.06 persists even though the present sample is about 30 times as large as the one considered by Burbidge in 1968. It is therefore unlikely that this result can be explained away by some kind of a selection effect.

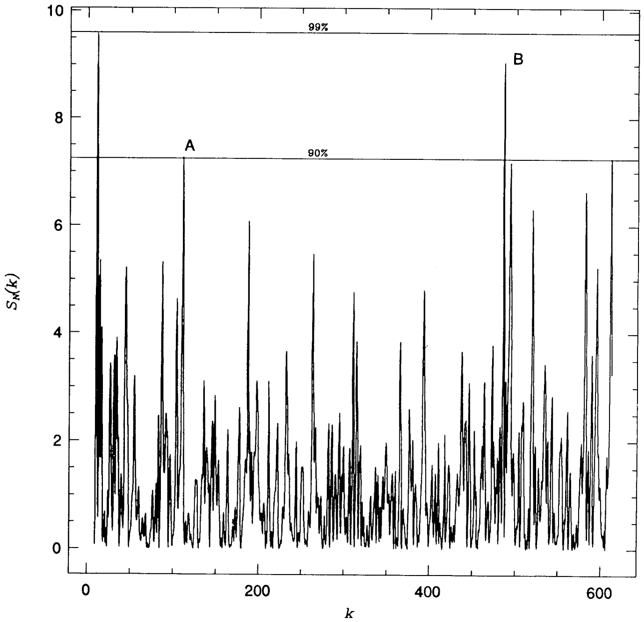

FIG. 2.—Spectral power function $S_N(k)$, plotted against the wavenumber k. Two of the peaks labeled A and B stand up above the 90% confidence level.

TABLE 1

RESULTS OF POWER SPECTRUM ANALYSIS OF REDSHIFT DISTRIBUTION

Peak	k	$S_N(k)$	$\xi = 2\pi/k$	$P_1(> S_N(k))$	Confidence Level (%)
A.....	111.212	7.26844	0.0565	0.0979	90.21
B.....	486.311	9.04624	0.0129	0.0172	98.28

Subsequent to the above work of Burbidge, a power-spectrum analysis was carried out by Burbidge & O'Dell (1972) in which there were 291 QSO redshifts (both emission and absorption redshifts were included). These authors had found a periodicity $\xi = 0.0704$ at the 96% confidence level. The present work, with 7 times as large a sample, shows that the value of ξ has come down from 0.07 to 0.06. The smaller scale (and the more significant) periodicity $\xi = 0.0129$ is difficult to detect visually from the histogram. To make it apparent, finer binning is needed, say, of redshift interval 0.003. This in turn would reduce the number per bin by a factor ~4.

4. THE GENERALIZED RAYLEIGH TEST

4.1. *The Test*

In gamma-ray astronomy one encounters a situation wherein the periodicity of a burst phenomenon is approximately known and one is interested in arriving at a better estimate from a statistical analysis of the data. The test, a generalization of the one associated with the Rayleigh test (Mardia 1972), is used for this purpose and is ideally suited to the present problem, wherein the periodicities $\xi = 0.0565$ and $\xi = 0.0129$ revealed by the power-spectrum analysis may be taken as the initial estimates. Moreover, this test takes account of the individual phases in equation (1), and provides the information that helps fine-tune the period obtained from the power-spectrum analysis. Suppose that ξ is the trial periodicity and $z_{\min}$ the minimum redshift in the distribution. The "residual phase" ϕ_n ($n = 1, \ldots, N$) is then obtained as follows. Define Rem (a, b) as the remainder of b when divided by a. Then, for the redshift z_n,

$$\phi_n(\xi) = \frac{\mathrm{Rem}\,(\xi, z_n - z_{\min})}{\xi} . \tag{9}$$

To define the test statistic, we adopt the method of Buccheri & de Jager (1989) and write

$$Z_p^2(\xi) = \frac{2}{N}\sum_{r=1}^{p}\left[\left(\sum_{n=1}^{N}\cos 2\pi r\phi_n\right)^2 + \left(\sum_{n=1}^{N}\sin 2\pi r\phi_n\right)^2\right], \tag{10}$$

where p can be a preassigned integer. For large values of N and in the absence of any real underlying periodicity in the data, the distribution of Z_p^2 is like that of χ^2 with $2p$ degrees of freedom.

4.2. *Results*

In our case we take the intervals (0.0555, 0.0575) and (0.0119, 0.0139), respectively, centered around the trial values of $\xi = 0.0565, 0.0129$, and divide each interval into 10 equal parts. Thus there are 10 trial values of ξ corresponding to each inter-

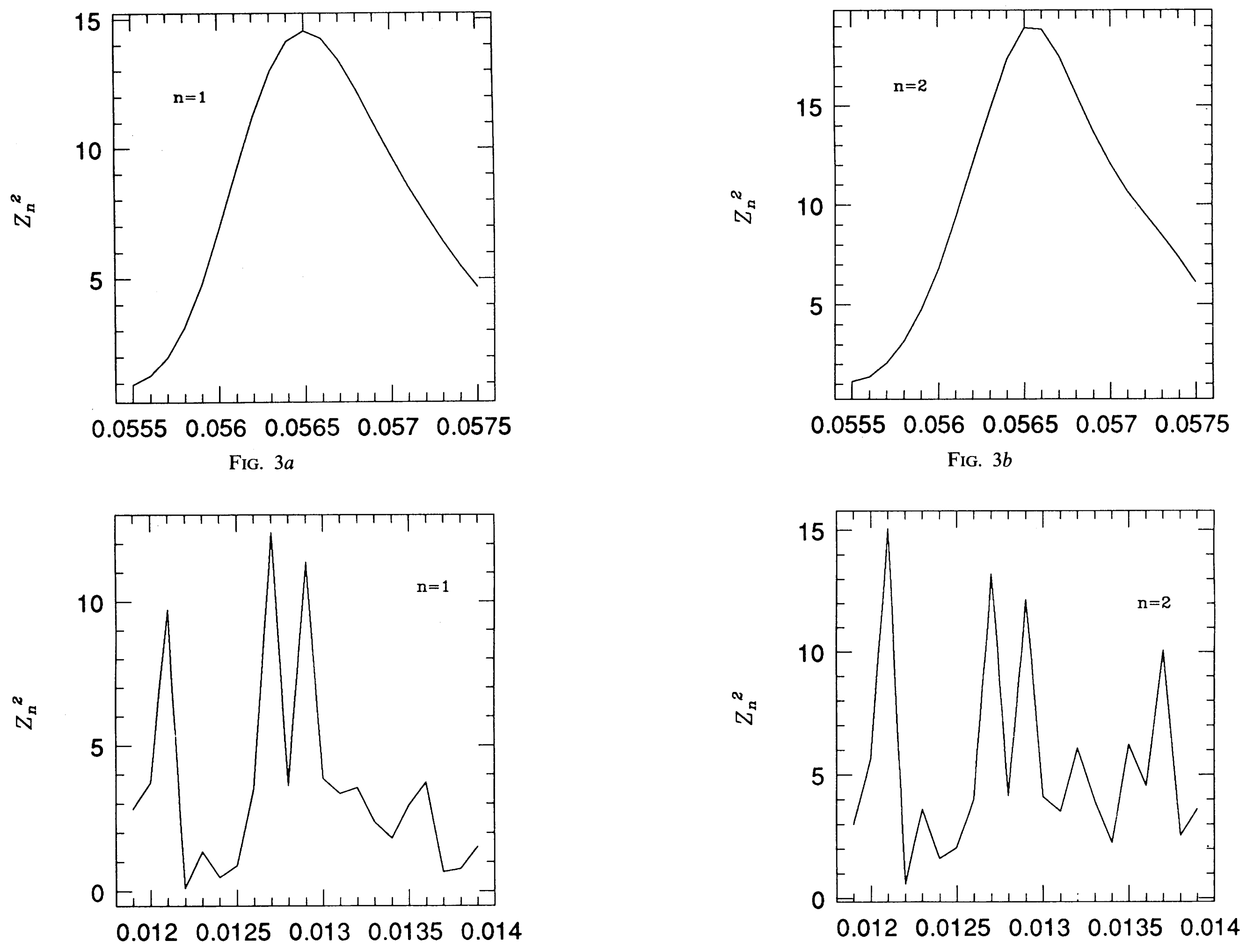

FIG. 3*a*

FIG. 3*b*

FIG. 3*c*

FIG. 3*d*

FIG. 3.—In (*a*) and (*b*) are shown the peaks of Z_n^2 at 0.0565 for $n = 1$ and $n = 2$. The corresponding tallest peaks in (*c*) and (*d*) occur at the slightly different values of 0.0127 and 0.0121, respectively, when the trial periodicities are taken in the neighborhood of 0.0129.

val, for which Z_p^2 can be calculated. For $p = 1, 2$ we see in Figure 3 the plots of Z_1^2 and Z_2^2 for the interval (0.0555, 0.0575). Both Z_1^2 and Z_2^2 take their maximum values of 14.54 and 18.94, respectively, at $\xi = 0.0565$. Had there been no periodicity, the chance of Z_1^2 exceeding 14.54 and Z_2^2 exceeding 18.94 under random fluctuations is slightly less than 0.01. Thus the periodicity of 0.0565 can be claimed at a confidence level exceeding 0.99. In the second case, Z_1^2 has its maximum value of 12.38 at $\xi = 0.0127$, which corresponds to a confidence level of 0.9797, while Z_2^2 has its maximum value of 15.05 at $\xi = 0.0121$ with a confidence level 0.95. Thus, the periodicities around 0.012 can be claimed at most with confidence of 97.97%. The behavior of Z_1^2 and Z_2^2 as a function of the trial periods lying in the interval (0.0119, 0.0139) are shown in Figures 3*c* and 3*d*, respectively.

5. THE KOLMOGOROV-SMIRNOV TEST

5.1. *The Test*

This test compares the observed cumulative probability distribution in a sample with the predicted theoretical distribution of the parent population in a given specified range of the measured variable. In the present case, suppose that the theoretical distribution over the specified redshift range (z_1, z_2) is $P(z)$: that is, $P(z)$ is the probability of finding a redshift in the range (z_1, z). Let the observed distribution on the basis of the histogram of Figures 1*a* and 1*b* be $S(z)$. Then, for the redshifts, say n in number, found in the specified range, define

$$D_n = \mathrm{Max}\,|P(z) - S(z)|\,; \quad Z_n = \sqrt{n}D_n\,. \tag{11}$$

Then it can be shown that for large values of n, the statistic Z_n has the following probability distribution:

$$P_{\mathrm{KS}}(Z_n > \lambda) = 2\sum_{k=1}^{\infty}(-1)^{k-1}\exp(-2k^2\lambda^2)\,. \tag{12}$$

For a derivation of this result see, for example, Kendall & Stuart (1979).

The null hypothesis that the observed sample was drawn out of the parent population of the specified distribution is rejected at a given probability level like 0.05, 0.01, etc., if the above probability P turns out to be lower than the specified level.

5.2. *Results*

We have considered the application of this test in two contexts. Notice first that the redshift histogram of Figure 1*b* with bin size 0.01 shows peaks at $z = 0.18, 0.24, 0.30, \ldots$ in the range (0.025, 4.43) of redshifts. Are these peaks, consistent though they are with a periodicity $0.0565 \simeq 0.06$, statistically significant? To test this question, we first apply the Kolmogorov-Smirnov (KS) test to each of the 16 peaks individually, in the range (0.025, 0.800). That is, around each peak interval, take one interval to the left and one to the right, thus obtaining the redshift range (z_1, z_2). The null hypothesis is that the redshifts in this range are distributed uniformly, so that

$$P(z) = (z - z_1)/(z_2 - z_1) . \tag{13}$$

We compare this with

$$S(z) = \frac{\gamma(z)}{n} , \tag{14}$$

where n is the total number of redshifts in (z_1, z_2) and $\gamma(z)$ the number not exceeding z.

Table 2 lists the KS statistic Z_n and the corresponding probability level of exceeding it under the null hypothesis as applied to each of the 16 intervals. It is clear that the probability levels are high enough not to warrant rejection of the null hypothesis in each individual case. However, if we multiply the 16 probabilities, we get the low value of 4.33×10^{-3}, thus establishing a prima facie case for doubting the hypothesis of uniform distribution over the entire redshift range of the 16 peaks. A better way to see this effect is to apply the KS test over a range containing several peaks. Table 3 lists several such ranges. It is evident that in most cases the distribution is far from being uniform. For the wavelength range (0.025, 0.800), which has the peaks tested individually in Table 2, the probability of the KS statistic exceeding the observed value $Z_n = 1.775$ (for $n = 769$ redshifts) is 3.68×10^{-3}, which is comparable to that estimated by multiplying the probabilities of the individual peaks. It is probably improper to take the entire range (0.025, 4.430) for this test in view of the obvious fall of redshifts beyond $z \simeq 2$. Table 3, however shows that even for (0.025, 2.025) the KS test yields the low probability of 1.15×10^{-2}.

TABLE 2

KOLMOGOROV-SMIRNOV STATISTICS IN THE NEIGHBORHOOD OF INDIVIDUAL PEAKS IN THE REDSHIFT RANGE 0.025–0.800

z_1	z_2	Number of Sources	$\sqrt{n}D_n$	$P(>\sqrt{n}D_n)$
0.160	0.200	54	0.3402	0.9998
0.220	0.260	46	0.6709	0.7590
0.260	0.315	65	0.9359	0.3451
0.315	0.345	44	0.6633	0.7711
0.345	0.380	39	0.4850	0.9727
0.380	0.430	48	0.7044	0.7039
0.430	0.480	55	0.9304	0.3522
0.480	0.530	57	0.4477	0.9881
0.530	0.560	34	0.5259	0.9449
0.560	0.580	21	0.4255	0.9935
0.580	0.610	30	0.9129	0.3752
0.610	0.640	45	0.3727	0.9991
0.640	0.680	41	0.3904	0.9980
0.680	0.710	33	0.9922	0.2785
0.710	0.740	30	0.5477	0.9251
0.740	0.800	58	0.5477	0.9945

TABLE 3

KOLMOGOROV-SMIRNOV STATISTICS OVER REDSHIFT RANGES CONTAINING SEVERAL PEAKS

z_1	z_2	Number of Sources	$\sqrt{n}D_n$	$P(>\sqrt{n}D_n)$
0.025	0.800	769	1.7746	3.68×10^{-3}
0.025	0.525	484	2.1244	2.40×10^{-4}
0.525	1.025	481	1.2169	1.03×10^{-1}
1.025	1.525	472	0.9246	0.3597
1.525	2.025	412	1.0760	0.1972
2.025	2.525	236	3.0068	2.81×10^{-8}
2.525	4.440	87	3.1110	7.84×10^{-9}
0.025	2.025	1848	1.6062	1.15×10^{-2}
2.025	4.440	317	9.2687	4.8×10^{-75}
0.025	4.440	2164	19.488	$\ll 10^{-75}$

6. THE COMB-TEMPLATE TEST

6.1. *The Test*

Consider a comblike template with "teeth" at regular intervals, which is made to slide across the redshift histogram. If the "period" of the comb matches the underlying periodicity of the histogram, then we expect to see peaks arising when and only when there is a tooth for tooth matching between the two distributions. We express this idea in quantitative terms in the following way. We construct a template $c(z, z_0)$ of the form

$$\begin{aligned} c(z, z_0) &= 1 , \quad z_0 + n\xi - \frac{\omega}{2} < z < z_0 + n\xi + \frac{\omega}{2} , \\ &= 0 , \quad \text{otherwise} , \end{aligned} \tag{15}$$

where $n = 0, 1, 2, \ldots$ and $\omega < \xi$. Denote by $n(z)dz$ the number of sources in the redshift range $(z, z + dz)$, and define the correlation function

$$r(z_0) = \int_0^\infty n(z)c(z, z_0)dz . \tag{16}$$

For a discrete distribution

$$n(z) = \sum_{i=1}^{N} \delta(z - z_i) , \tag{17}$$

we get

$$r(z_0) = \sum_{i=1}^{N} c(z_i, z_0) . \tag{18}$$

If the redshift distribution is uniform, the mean value of $r(z_0)$ is

$$\rho_0 = \langle r(z_0) \rangle = \frac{N\omega}{\xi} , \tag{19}$$

and its standard deviation is

$$\sigma_0 = \left[\rho_0\left(1 - \frac{\omega}{\xi}\right)\right]^{1/2} . \tag{20}$$

For large N we expect $r(z_0)$ to be distributed normally with mean ρ_0 and the standard deviation σ_0. However, if our null hypothesis of uniform distribution is not valid and the periodicity of the template matches that of the sample, then for some z_0 we expect to see $r(z_0)$ considerably higher than ρ_0. The particular value of z_0 then also tells us where exactly to place our comb on the underlying histogram of redshifts.

6.2. *Results*

For the sample of 2164 redshifts we set $\omega = 0.01$, which is the bin size in Figure 1*a*, and try with our underlying periodicity of $\xi = 0.0565$. If the null hypothesis is correct, the mean and standard deviation should be

$$\rho_0 = 383.01 , \quad \sigma_0 = 17.75 . \tag{21}$$

The value of $r(z_0)$ is then computed for various values of z_0 and is shown in Figure 4. It peaks at $z_0 = 0.0035$, with the peak

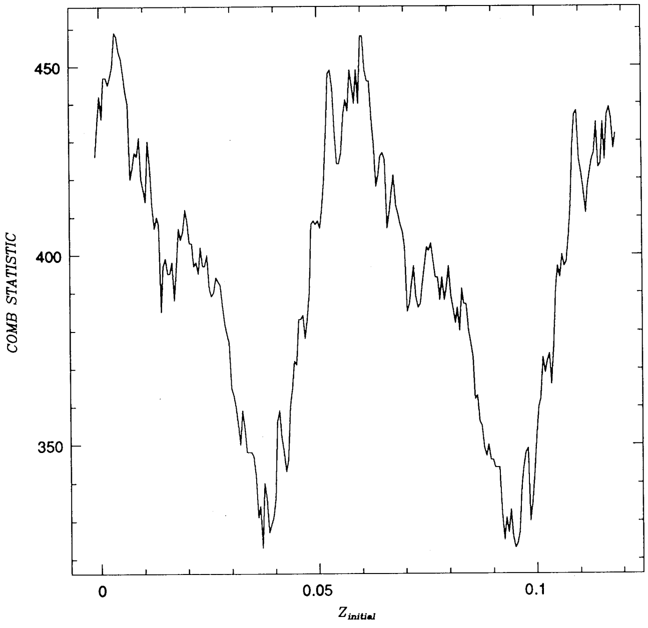

FIG. 4.—Peaks in the comb statistic clearly reveal an underlying periodicity of 0.0565.

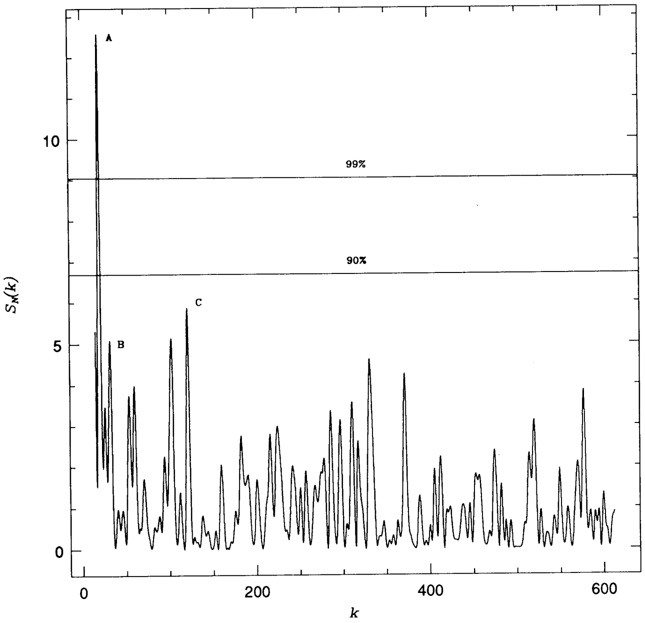

FIG. 5.—Peaks in the spectral power function of $\ln(1 + z)$ are not statistically significant.

value of 456. This is $4.11\sigma_0$ away from ρ_0, thus making the spiky effect a highly significant one. Accordingly, we conclude that the redshifts are clustered around the values

$$z_n = 0.0035 + 0.0565n \, , \tag{22}$$

giving the peak for $n = 1$ at $z_1 = 0.06$. To see whether the result is indeed sensitive to this periodicity, we repeated the calculations for $\xi = 0.0465$ and $\xi = 0.0665$. The peaks appear again, but for $\xi = 0.0465$ the peak is only $0.56\sigma_0$ away, while the peak corresponding to $\xi = 0.0665$ is $2.08\sigma_0$ away. Although the later periodicity would normally have been considered significant, our technique appears rather fine-tuned so as to reveal the more significant value of 0.0565.

7. PERIODICITY IN $\ln(1 + z)$?

We next considered the hypothesis of periodicity in $\ln(1 + z)$, especially to test the claim that a periodicity of $\Delta \ln(1 + z) = 0.206$ is present in the QSO redshift distribution (Arp et al. 1990). We employ the power-spectrum analysis of § 1 on the sample of 2164 QSOs. We used equation (1) with z_n replaced by $\ln(1 + z_n)$ to calculate $F_N(k)$. Figure 5 displays $S_N(k)$ as a function of k. Again, ν is taken to be from 0 to 100. This time the interval is divided into 4000 equal parts, so that the analysis can be carried out with a better resolution because we are dealing with $\ln(1 + z)$ instead of z. Between $k = 0$ and $k = 600$ there are 85 peaks. The three peaks which are of interest are given in Table 4. Peak A has $S_N(k) = 12.626$ for $k = 19.635$, corresponding to $\xi = 0.32$, its confidence level being 99.97%. The value $\xi = 0.32$ is somewhat large, and the effect can be treated further as we accumulate more data on high redshifts.

Peaks B and C have $S_N(k)$ of 5.0997 and 5.8662, corresponding to values of 0.197 and 0.052 with confidence levels of 59.55% and 78.59%, respectively. Thus, no statistical significance can be attached to these periodicities. The period 0.197 does come close to the previously claimed value 0.206, but then, in our analysis, it is not a significant one.

In looking for periodicities in $\ln(1 + z)$, one has to exercise some caution, however. The proponents of noncosmological redshifts have frequently argued that the observed redshift z is a composite one, being made up of a cosmological part z_c and a noncosmological part z_{nc}, in the following way:

$$1 + z = (1 + z_c)(1 + z_{\text{nc}}) \, . \tag{23}$$

It is expected that any "signal" of periodicity in $\ln(1 + z)$ should come from the noncosmological component. This, however, can be masked by a scatter in $\ln(1 + z_c)$ provided that z_c is large. For example, $\ln(1 + z_c) = 0.2$ corresponds to $z_c = 0.22$. Thus a scatter of z_c values over this range suppresses the real effect. To discover whether the effect is present, therefore, G. Burbidge (1990, private communication) has argued that one needs first to study samples of high-redshift QSOs found near low-redshift galaxies. In such cases z_c is small, on the assumption that the proximity implies near-equality of cosmological redshifts (Burbidge et al. 1990). It is significant that the positive effect found by Arp et al. (1990) seems to be for such QSOs.

Nevertheless, in view of the criticism of Scott (1990), we feel that the claim of periodicity in $\ln(1 + z)$ of $\simeq 0.2$ needs to be looked at more carefully than we have done here. We hope to carry out such an analysis in the future.

TABLE 4

PERIODICITY IN $\ln(1 + z)$: RESULTS OF POWER-SPECTRUM ANALYSIS

Peak	k	$S_N(k)$	$\xi = 2\pi/k$	$P_1(> S_N(k))$	Confidence Level (%)
A.....	19.635	12.6256	0.32	2.70×10^{-4}	99.97
B.....	31.887	5.0997	0.197	0.4045	59.55
C.....	120.592	5.8662	0.052	0.2141	78.59

8. THE EFFECT OF TRANSFORMING TO THE GALACTOCENTRIC FRAME

Another check on the reality of the periodicity may be made as follows. If the effect is real, then it should be sharpened up if all the redshifts are measured with respect to the Galactocentric frame. Thus we need to "correct" the redshifts of QSOs for the Earth's motion relative to the center of the Galaxy.

Let z_{obs} and z_{GC} be the observed and the corrected cosmological redshifts, respectively, of a QSO. Now, an observer O, whose position is coincident with that of the terrestrial observer, but who is at rest with respect to the Galactic center, measures the wavelength to be

$$\lambda = \lambda_0(1 + z_{\text{GC}}) . \tag{24}$$

However, the wavelength λ is Doppler-shifted to λ' because of the terrestrial observer's motion $\boldsymbol{v}$ with respect to the observer O, so that

$$\frac{\lambda'}{\lambda} = 1 - \frac{\boldsymbol{v} \cdot \hat{\boldsymbol{n}}}{c} , \tag{25}$$

where $\hat{\boldsymbol{n}}$ is the unit vector from the Earth to the QSO. Therefore, the observed redshift as measured by the terrestrial observer is given by

$$1 + z_{\text{obs}} = \frac{\lambda'}{\lambda_0} = \frac{\lambda'}{\lambda}\frac{\lambda}{\lambda_0} = \left(1 - \frac{\boldsymbol{v} \cdot \hat{\boldsymbol{n}}}{c}\right)(1 + z_{\text{GC}}) ,$$

i.e.,

$$1 + z_{\text{GC}} = \frac{1 + z_{\text{obs}}}{1 - \boldsymbol{v} \cdot \hat{\boldsymbol{n}}/c} . \tag{26}$$

The circular speed of the "local standard of rest" (LSR) with respect to the Galactic center is 220 ± 19 km s^{-1} (Fich, Blitz, & Stark 1989), while the solar system moves with respect to the LSR with a speed of 20 km s^{-1} toward $\alpha = 18^{\text{h}}$ and $\delta = 30°$ (Kerr & Lynden-Bell 1986). For the solar system motion with respect to the Galactic center, the above data imply a speed of $|\boldsymbol{v}| = 235.8 \pm 19.0$ km s^{-1} toward $\alpha = 20^{\text{h}}.87^{+0.02}_{-0.03}$ and $\delta = 47°.6^{+0.03}_{-0.07}$. Using the central value of $\boldsymbol{v}$, redshift reduction to the Galactocentric frame was carried out for all the QSOs under investigation.

Again, since we are interested in scales smaller than 0.08 and larger than 0.011, we consider only those k_j which lie between 78.5 and 570. Figure 6 illustrates the spectral power as a function of k. The number of peaks in the spectral power function comes out as $K = 139$. It is evident from the figure that there are three peaks (marked A, B, and C) that exceed 90% or are very close to that confidence level. The peak A has $S_N = 7.23$ for $k = 111.212$, corresponding to the previously observed periodicity $\xi = 0.0565$. The actual probability level is 90.42%, marginally higher than the previous case. Peaks B and C have, respectively, $S_N = 10.23$ and $S_N = 7.03$ for $k = 486.319$ and 519.611, corresponding to the periodicities of 0.0129 and 0.0121 with confidence levels of 99.5% and 88.42%, respectively. The periodicities $\xi = 0.0565$ and $\xi = 0.0129$ which were obtained earlier without taking into consideration the peculiar motion of the solar system remain unchanged even when the redshift values are corrected. It should be emphasised that with the reduction to the Galactocentric frame, the statistical significance of $\xi = 0.0129$ crosses the 99.0% level. These results lend a greater confidence in the reality of the effect. The power-spectrum analysis was carried out also for several different values of (α, δ) and speeds within the above error bars without altering the above conclusion.

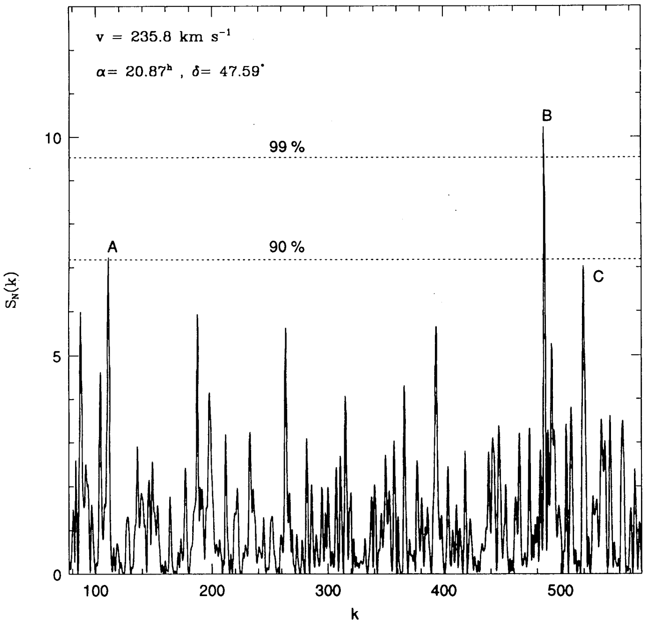

FIG. 6.—Power spectrum of corrected redshifts. Peak A corresponds to a periodicity of 0.0565, as in Fig. 2. The probability of confidence is, however, marginally higher. Peak B ($\xi = 0.0129$) stands out with a very high probability of confidence.

In a similar work, Tifft & Cocke (1984) had concluded that for the solar-motion velocity vector $(v_\phi, v_r, v_z) \approx (231, -35, 1)$ km s^{-1}, the redshift periodicities of late-type galaxies are highly significant, (here v_ϕ and v_r are the circular and radial velocities, respectively, while v_z is the component of the velocity in the direction perpendicular to the Galactic disk). The Tifft-Cocke velocity vector differs from the one used in our analysis by an amount

$$(\delta v_\phi, \delta v_r, \delta v_z) \approx -(4.44, 45.03, 6.81) \text{ km s}^{-1} . \tag{27}$$

It is interesting to estimate the change in the spectral power due to such changes in the velocity $\boldsymbol{v}$. This is briefly illustrated below (a detailed account will appear elsewhere).

Suppose that M redshifts out of N have a periodicity ξ; it is easy to show that the spectral power at $k = 2\pi/\xi$ is $\sim M^2/N$. For example, with ξ, N, and $S_N(2\pi/\xi)$ as 0.0565, 2164, and 7.23, respectively, M turns out to be ~ 125. A change from $\boldsymbol{v}$ to $\boldsymbol{v} + \delta\boldsymbol{v}$ will lead to $z_i \to z_i + \delta z_i$, $i = 1, 2, \ldots, N$, where

$$\delta z_i = \frac{\delta\boldsymbol{v} \cdot \hat{\boldsymbol{n}}_i}{c - \boldsymbol{v} \cdot \hat{\boldsymbol{n}}_i}(1 + z_i) , \tag{28}$$

where $\hat{\boldsymbol{n}}_i$ is the unit vector along the direction of the ith source. The spectral power at $k = 2\pi/\xi$ changes to

$$S'_N \sim \frac{1}{N}\left|\sum_{n=1}^{M} \exp\left(-\frac{2\pi i \delta z_n}{\xi}\right)\right|^2 . \tag{29}$$

The phases $\theta_i = 2\pi\delta z_i/\xi$ will, in general, be distributed randomly between $\theta_{\min}$ and $\theta_{\max}$, so that

$$S'_N \sim \frac{1}{N}\left[\frac{\sin(M\,\Delta\theta/2)}{\sin(\Delta\theta/2)}\right]^2 , \tag{30}$$

with $\Delta\theta = (\theta_{\max} - \theta_{\min})/M$.

Using the values given in equation (27) for $\delta \boldsymbol{v}$, we find that when $\xi = 0.0565$, we obtain $\Delta\theta \approx 2.11 \times 10^{-4}$ and $S'_N \approx 7.17$. Thus, the decrease in the spectral power is only 0.83%. Similarly, when $\xi = 0.0129$, the spectral power at $k = 2\pi/\xi$ falls by only 1.3%.

This clearly demonstrates the stability of the two periodicities, $\xi = 0.0565$ and $\xi = 0.0129$, against uncertainties in the solar system velocity $\boldsymbol{v}$.

9. CONCLUSION

To summarize, our four different statistical tests confirm an underlying spiky nature of the redshift distribution of Figures 1*a* and 1*b*. There is reasonable evidence to support the claim of periodicity of $\xi = 0.0565$ and also perhaps of a periodicity in the range 0.0127–0.0129. The former periodicity, which survives with marginally greater confidence level when the redshifts are transformed to the Galactocentric frame, is close to that observed by Burbidge (1968) and is somewhat less than that obtained by Burbidge & O'Dell (1972). It is interesting to note what the latter authors had concluded in their 1972 paper: "The $\xi = 0.0705$ ... appears not to represent a true periodicity, but rather seems to be effected by chance constructive interference of several peaks, none of which are individually significant." One feels in retrospect that they were being rather overcautious, since periodicity of the same order persists even in a much larger sample.

We do not wish to draw any deeper conclusion from these results, beyond stating the fact that the peaks and periodicities have remained for more than two decades despite a thirty-fold augmentation of the data. Any theory of redshifts, whether intrinsic or cosmological, will find in these results a stiff challenge.

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