

Revised cosmological parameters after BICEP 2 and BOSS

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ABSTRACT: Estimation of parameters of the ‘standard’ model of cosmology have dramatically improved over past few decades due to increasingly exquisite measurements made by Cosmic Microwave Background (CMB) experiments. Recent data from Planck matches well with the minimal Λ CDM model. A likelihood analysis using Planck, WMAP and a selection of high resolution experiments (highL), tensor to scalar ratio r is found to be < 0.11 when $dn_s/d \ln k = 0$. Planck also imposes an upper bound on neutrino mass $\sum m_\nu < 0.23$ eV using Planck+WMAP+highL+BAO likelihood. However, recently results from BICEP 2 claims the detection of $r = 0.2^{+0.07}_{-0.05}$ from polarization spectra. Further, results from SDSS-III BOSS large scale galaxy survey constrains the total neutrino mass to $\sum m_\nu = 0.36 \pm 0.10$ eV. It is important to study the consequences of these new measurements on other cosmological parameters. In this paper we assess the revised constraints on cosmological parameters in light of these two measurements that are in some tension with the constraints from Planck.

The sensitive of Planck to the weak lensing effect on the CMB angular power spectrum suggests that the normalized amplitude of physical lensing power $A_L > 1$ at 2σ hinting at a potentially important internal inconsistency. Therefore, we also include a study of the constraints on A_L . Using the prior on r and $\sum m_\nu$ as measured by BICEP 2 and SDSS-III BOSS respectively, we find that the model with running spectral index ($dn_s/d \ln k \neq 0$) leads to a value of $A_L > 1$ at 3.27σ . But, the model with $dn_s/d \ln k = 0$ makes A_L consistent with 1, at 2.67σ and also shows that N_{eff} is consistent with its theoretical value of 3.046. However, the value of H_0 for this case is lower and more discrepant from value measured from high redshift supernova measurements, which can increase if the measurement on $\sum m_\nu$ decreases, without effecting N_{eff} and A_L significantly. Therefore, the analysis in this paper shows that the model with $dn_s/d \ln k = 0$ is consistent on all theoretical aspects.

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Cosmic Microwave Background (CMB) is a very powerful probe for improving our understanding of the Universe. Several CMB missions like COBE, WMAP, Planck, ACT, SPT, BICEP etc., have ushered an era of precision cosmology. High resolution CMB ESA space mission Planck [1] have measured CMB power spectra match extraordinarily well with the minimal Λ CDM model (in particular at angular scales smaller than few degrees corresponding to multipole $l > 50$). Planck has also pinpointed the allowed range of several cosmological parameters from the temperature spectra alone [2]. Further improvement on these constraints are expected in the final release that would include CMB polarization data. However, recent results from BICEP 2 [3] claimed the detection of C_l^{BB} spectra arising due to the primordial Gravitational Wave (GW) with tensor to scalar ratio (r) = $0.2_{-0.05}^{+0.07}$, which is in mild tension with Planck [2] that estimates $r < 0.11$ without running spectral index ($dn_s/d\ln k = 0$). However, this tension reduces while considered running spectral index ($dn_s/d\ln k \neq 0$) in the temperature power spectra [2].

Along with the constraint on r from BICEP 2, recently Beutler et al.[4] have imposed constraint on neutrino mass ($\sum m_\nu = 0.36 \pm 0.10$ eV) using Baryon Oscillation Spectroscopic Survey (BOSS) CMASS data release 11 in combination with several other data sets. Therefore, the constraint on neutrino mass ($\sum m_\nu < 0.23$ eV) from Planck in mild tension with the measurement of BOSS. Imposing prior constraint on $\sum m_\nu$ from BOSS result can result in significant change in the current best-fit Λ CDM parameters obtained by Planck [2].

Since the release of two prime experiments BICEP2 and BOSS, several authors, like [5, 6, 7, 8, 9, 10, 11] have placed constraints on different cosmological parameters using the available data. In this paper, we study the effects of high value of r claim by BICEP-2 [3] and high non-zero $\sum m_\nu$ claim from BOSS-CMAS [4] on other cosmological parameters, using SCoPE [12]. Since, tensor contribution to temperature

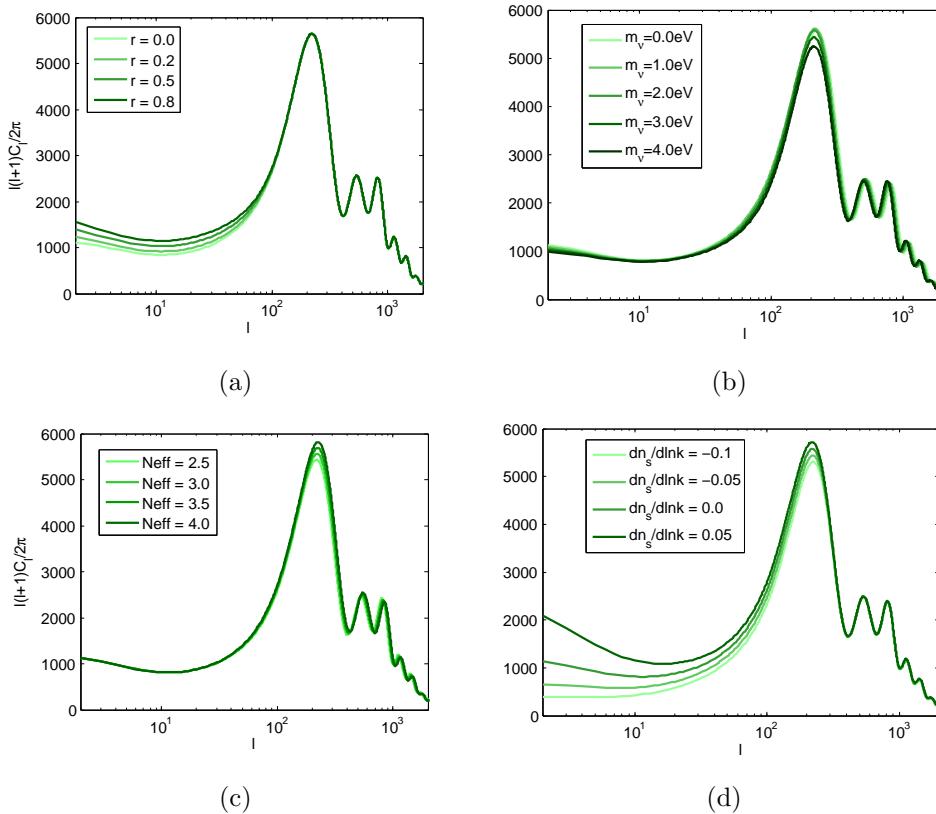


Figure 1: Temperature power spectra, $l(l+1)C_l^{TT}/2\pi$ for different sets of r , $\sum m_\nu$, N_{eff} and $dn_s/d\ln k$ values obtained using CAMB [13]. Other parameters are kept fixed at their best-fit Λ CDM values [2].

power spectra is at low multipoles $l < 100$, the detection of large tensor contribution to C_l^{TT} is revealed by the amplitude difference between the CMB power at the Sachs-Wolfe plateau and that at the acoustic peaks. Therefore, change in r leads to a significant variation of several cosmological parameters which are constrained by the plateau and acoustic peaks. On the other hand significant neutrino mass changes the relativistic matter content of the universe at early times and hence it affects the parameters like, H_0 , $\Omega_b h^2$, $\Omega_c h^2$, N_{eff} , A_L , that depend on the matter content of the universe and perturbations in them.

We make an extensive study of the effect on various parameters and search for a model that remains valid in our current theoretical regime. This improved constraint on the cosmological parameters are important for a better understanding of our cosmological models in light of the BICEP 2 and BOSS.

The paper is organized as follows, in Sec. 1 we present the estimation of cosmological parameters from WMAP-9 and Planck data using the likelihood provided by them for several parameters. Discussions and conclusions of the paper are provided in Sec. 2.

1. Cosmological Parameters Estimation

We calculate the constraints on different cosmological parameters using WMAP-9 and Planck likelihood. However, before going to the main analysis, it is important to understand the effect of different parameters like r , $\sum m_\nu$, N_{eff} and $dn_s/d\ln k$ on CMB angular power spectrum. Fig. 1 shows the effects of different parameters on C_l^{TT} . Fig. 1(a) shows the effect of r on C_l^{TT} for a range of values. All the other cosmological parameters are kept fixed at their standard values and we use $n_t = n_s - 1$. As the tensor power spectrum is dominant only at low multipoles, only lower multipoles get affected due to this variation. Plots show that r is expected to be correlated with n_s , as that also affects the low multipole power. Fig. 1(b) and Fig. 1(c) show the effect of neutrinos on C_l^{TT} . Change in either of N_{eff} or $\sum m_\nu$, affects the total matter fraction in the universe, which leads to change in the expansion history of the universe and shifts the CMB peaks towards higher or lower scales. These also affect the epoch of matter radiation equality and hence the ratio between the even and the odd peaks in the CMB power spectrum changes. In Fig. 1(b) we show the effects of variation of $\sum m_\nu$ keeping N_{eff} fixed at 3.048, whereas in Fig. 1(c) we choose the neutrinos to be massless. Finally, in Fig. 1(d) we show the effects of running spectral index $dn_s/d\ln k$ on C_l^{TT} .

We present the most of the results broadly for two different cases :- 1) for Planck+WP likelihood, where we add up the results from `commander_v4.1_lm49.clik`, `lowlike_v222.clik` and `CAMspec_v6.2TN_2013_02_26.clik` likelihood [14, 15] to perform parameter estimation. 2) for Planck+WP+Lensing likelihood, where we add the lensing likelihood along with the other three likelihoods.

We vary the Standard 6 Parameters (SP), $\{\Omega_b h^2, \Omega_m h^2, h, \tau, n_s, A_s\}$, along with the other parameters like $dn_s/d\ln k$, n_t , N_{eff} , A_L . The ranges of priors used for all these parameters are provided in Table. 1. As we are interested in finding the implications on the cosmological parameters due to the recent results from BICEP2 and SDSS-III BOSS, we use the probability density function (pdf) for r and $\sum m_\nu$, from these experiments as the prior in our MCMC analysis while estimating the parameters. We estimate parameters for the cosmological models, (a) SP, (b) SP + either of $\{dn_s/d\ln k, n_t, N_{eff}\}$ and (c) SP + $\{A_L + dn_s/d\ln k\}$ for all different sets of $\sum m_\nu$ and r mentioned below,

1. $\sum m_\nu = 0$ and $r = 0$, (Planck + WP likelihood and Planck + WP + Lensing likelihood).
2. SDSS-III BOSS prior on $\sum m_\nu$ and $r = 0$, (Planck + WP likelihood and Planck + WP + Lensing likelihood).
3. $\sum m_\nu = 0$ and BICEP 2 prior on r , (Planck + WP likelihood and Planck + WP + Lensing likelihood).

4. SDSS-III BOSS prior on $\sum m_\nu$ and BICEP 2 prior on r , (Planck + WP likelihood and Planck + WP + Lensing likelihood).

Table 1: Prior range for the Cosmological parameters used in this paper. The r and $\sum m_\nu$ we use the probability distribution function (pdf) from BICEP2 and BOSS experiment (non flat prior).

Parameters	Prior range	Defination
$\Omega_b h^2$	0.015-0.03	Physical baryon density.
$\Omega_m h^2$	0.05-0.25	Physical matter density.
h	0.55-1.2	Hubble parameter.
τ	0.01-0.2	Reionization optical depth.
n_s	0.8-1.2	Scalar spectral index.
A_s	0.5-2.5	Amplitude of temperature fluctuations.
$r_{0.002}$	pdf from BICEP 2 [3]	Ratio of tensor primordial power to curvature power at $k_0 = 0.002 \text{ Mpc}^{-1}$.
$dn_s/d \ln k$	-2.0 - 2.0	Running of the scalar spectral index.
n_t	-0.25 - 0.25	Dark energy equation of state.
N_{eff}	1.0-5.0	Effective number of neutrino-like relativistic degrees of freedom.
$\sum m_\nu$	pdf from BOSS [4]	The sum of neutrino masses .
A_L	0.5- 3.5	Amplitude of the lensing power relative to the physical value.

1.1 Standard 6 parameters with $\sum m_\nu$ and r

First we discuss the statistics of standard 6 parameters $\{\Omega_b h^2, \Omega_c h^2, h, \tau, n_s, A_s\}$ due to the change in r and $\sum m_\nu$. From the plots shown in Fig. 1, it is evident that if we change the value of r or $\sum m_\nu$ then there can be significant effects on the cosmological parameters. We run SCoPE [12] with standard 6 cosmological parameters, with and without varying r and $\sum m_\nu$. The results from our runs are shown in Fig. 2, Fig. 3 and Fig. 4. At this point it should be noted that during our MCMC analysis instead of using flat prior on r and $\sum m_\nu$, we have used the one dimensional likelihood, from BICEP 2 and BOSS experiment respectively, as the prior for r and $\sum m_\nu$. The two dimensional contour plots of r and $\sum m_\nu$ with the standard model parameters with WP + Planck likelihood and WP + Planck + lensing likelihood are shown in blue and red color respectively in Fig. 2. It can be seen that $\sum m_\nu$ is strongly correlated with h with the correlation coefficient ~ -0.74 . As $\sum m_\nu$ increases the value of h decreases, which is expected as the matter content of the universe is getting changed and leads

to change in the epoch of matter-radiation equality of the universe. $\sum m_\nu$ also has small effect on the baryon density and the dark matter content of the universe. The effect of r on h shows slight positive correlation as shown in Fig. 2. We can also notice that on using only WP + Planck likelihood, the average value of $\sum m_\nu$ is 0.349 eV whereas if lensing is included the average value increases and becomes 0.42 eV. Though, as variance is high, both the values are within 1σ of each other. The value of r is same in both the cases and is approximately 0.16.

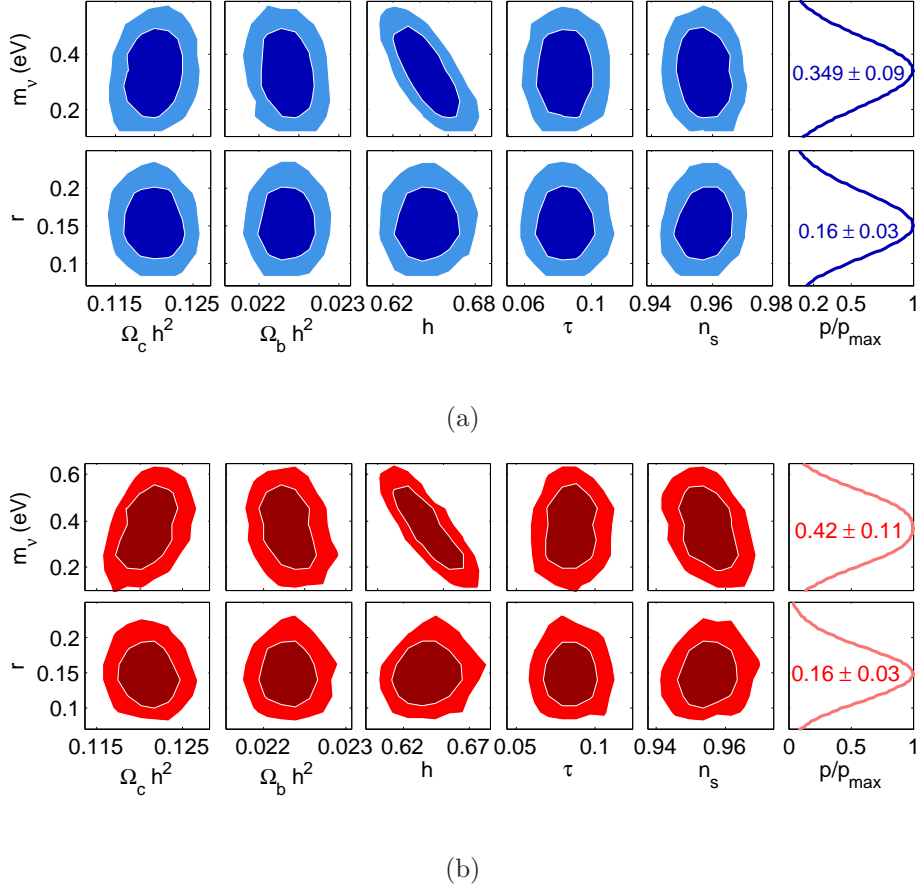


Figure 2: The two dimensional likelihood contours for the SP with $r_{0.002}$ and $\sum m_\nu$. (a): Planck+WP likelihood. (b): Planck+WP+Lensing likelihood. The one dimensional marginal probability distribution for r and $\sum m_\nu$ are shown in the last column.

Fig. 3 and Fig. 4 shows the one dimensional probability distribution for the standard cosmological parameters from Planck+WP likelihood and Planck+WP + lensing likelihood respectively. Including both m_ν and r in the estimation leads to lower value of Hubble constant ($h = 0.644$) as shown in Fig. 3 and Fig. 4, which is in tension with the results obtained from calibrated SNe magnitude-redshift relation by Riess et al. [16]. This tension increases further on taking into account of the lensing likelihood as shown in Fig. 4.

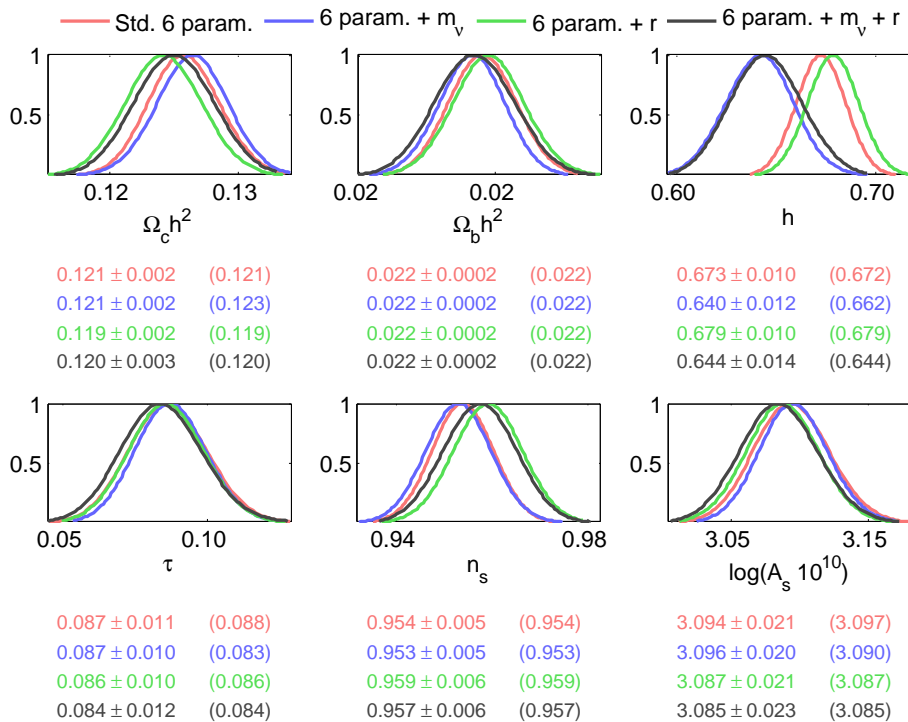


Figure 3: One dimensional marginal probability distribution of the standard model parameters when analyzed with Planck+WP likelihood. The results are shown for four different cases. Red : Only the standard model parameters (SP) are varied. The neutrinos are considered to be massless. Blue : we vary $\sum m_\nu$ along with other SP. The number of massive neutrino species here are considered to be $N_{\text{eff}} = 3.046$. For both these models we considered only the scalar power spectrum. Green : analysis with $r_{0.002}$ and SP. Neutrinos are considered to be massless and $n_t = n_s - 1$. Black : we vary both $\sum m_\nu$ and $r_{0.002}$. The average and the standard deviations for the parameters are given in the plot itself. The best fit values are quoted in the brackets.

1.2 Standard 6 parameters + $dn_s/d \ln k$ or n_t .

Simplest inflationary models predict $dn_s/d \ln k$ are related to the higher order of inflationary slow roll parameters [17]. By constructing a limit on $dn_s/d \ln k$, we can rule out several inflationary models. Planck [2] has put constraint on $dn_s/d \ln k$ with $r < 0.26$ as, $dn_s/d \ln k = -0.022 \pm 0.010$ (68%; Planck + WP + highL)

We obtain the constraints on $dn_s/d \ln k$ and n_s in Fig. 5. On considering no tensor spectrum, i.e. $r = 0$, $dn_s/d \ln k$ is well consistent with zero, which also matches Planck results. However, the constraint on $dn_s/d \ln k$ with the prior on r as measured by BICEP 2 [3] shows 2.76σ deviation from zero. The effect of $\sum m_\nu$ over $dn_s/d \ln k$ is negligible. The case with both $\sum m_\nu$ and r , shows that $dn_s/d \ln k$ is consistent with zero at 2.24σ .

Theoretically, for simplest inflationary model, n_t , i.e. tensor spectral index is

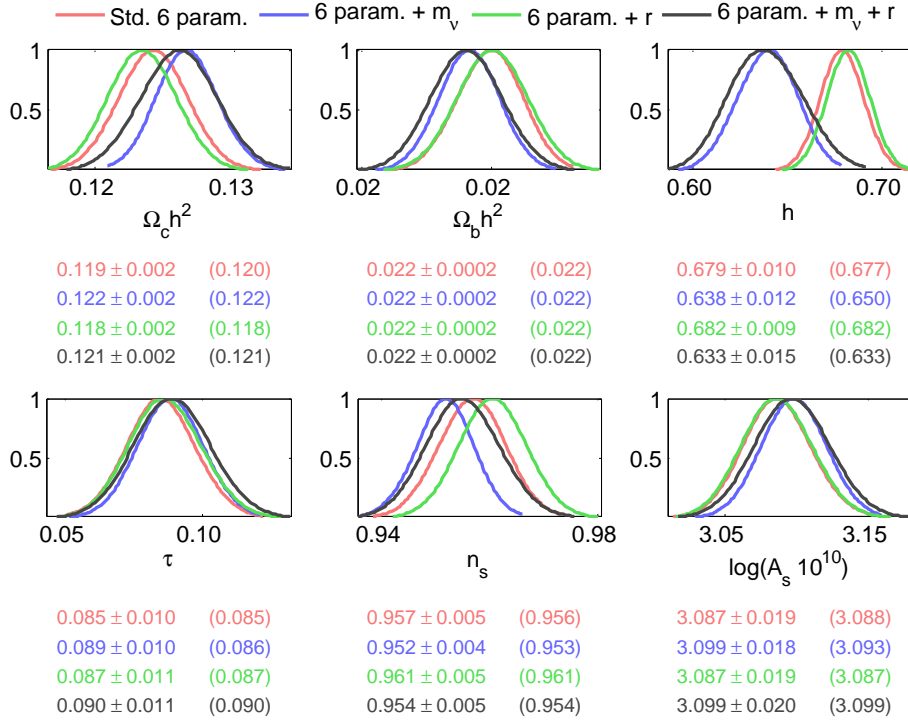


Figure 4: One dimensional marginal probability distribution of the standard model parameters when analyzed with Planck+WP+lensing likelihood. The results are shown for four different cases. Red : Only the standard model parameters (SP) are varied. The neutrinos are considered to be massless here. Blue : we vary $\sum m_\nu$ along with other SP. The number of massive neutrino species here are considered to be $N_{\text{eff}} = 3.046$. For both these models we considered only the scalar power spectrum. Green : analysis with $r_{0.002}$ and SP. Neutrinos are considered to be massless and $n_t = n_s - 1$. Black : we vary both $\sum m_\nu$ and $r_{0.002}$. The average and the standard deviations for the parameters are given in the plot itself. The best fit values are quoted in the brackets.

related to the scalar spectral index as $n_t = n_s - 1$, with the scalar to tensor ratio by $r = -8n_t$. We use the prior on r from BICEP 2 to get the constraint on n_t . Fig. 6 shows that n_t is poorly constrained from Planck + WP data and it is consistent with zero. So, only with Planck and WP we cannot impose any constraint on n_t using the value of r .

Table 2: Constraints on n_s and $dn_s/d \ln k$ for different set of models.

	SP + $\frac{dn_s}{d \ln k}$	SP + $\frac{dn_s}{d \ln k} + \sum m_\nu$	SP + $\frac{dn_s}{d \ln k} + r$	SP + $\frac{dn_s}{d \ln k} + \sum m_\nu + r$
n_s	0.953 ± 0.006	0.952 ± 0.006	0.958 ± 0.006	0.9563 ± 0.0060
$\frac{dn_s}{d \ln k}$	-0.0105 ± 0.0069	-0.0083 ± 0.0072	-0.0207 ± 0.0075	-0.0166 ± 0.0074

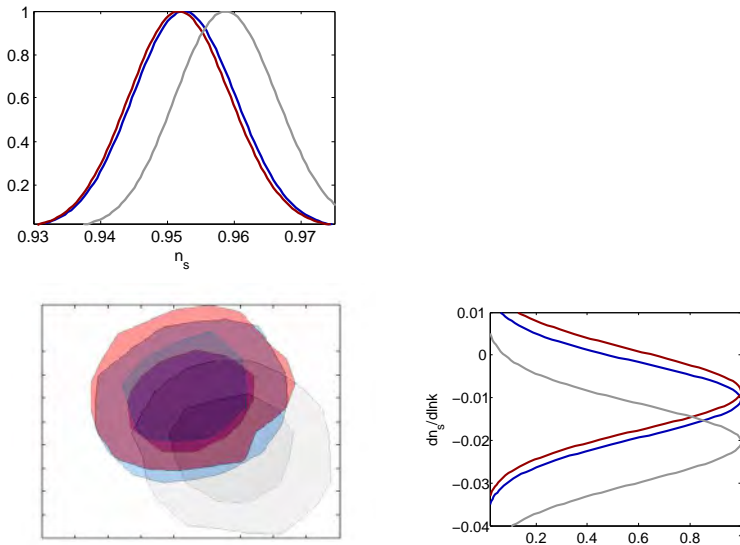


Figure 5: Variation of $dn_s/d \ln k$ and n_s are shown for following cases. Blue : For $SP+dn_s/d \ln k$, Red : For $SP+dn_s/d \ln k+\sum m_\nu$, Gray : for $SP+dn_s/d \ln k+r_{0.002}$. Introducing tensor part leads to $dn_s/d \ln k$ away from the 0 at $\sim 2.76\sigma$.

1.3 Standard 6 parameters + N_{eff}

In the standard model of particle physics there are 3 types of neutrinos corresponding to the three families of leptons. However, there are corrections to the N_{eff} due to non instantaneous decoupling and QED effects. This theoretically leads to $N_{\text{eff}} = 3.046$. However, any other non-interacting relativistic species also effect the CMB power spectra in the same manner as neutrinos. Signature of any such relativistic species can therefore be found by estimating N_{eff} from the CMB measurements.

In Fig. 7 we plot the one dimensional likelihood of N_{eff} for four different cases, (a) N_{eff} with standard 6 parameter case with $\sum m_\nu = 0$ and $r = 0$. (b) N_{eff} with standard 6 parameter and BICEP 2 prior on r and $\sum m_\nu = 0$. (c) N_{eff} with standard 6 parameter case with prior on $\sum m_\nu$ from BOSS experiment and $r = 0$. (d) Non-zero value for both $\sum m_\nu$ and r . All these cases are studied by including both the lensed and unlensed likelihood.

As shown in the Fig. 7, if the massive neutrinos are considered then the N_{eff} decreases, whereas if tensor modes are considered with the massless neutrinos then the N_{eff} of the neutrinos increases. The values of the effective neutrinos for different cases are shown below,

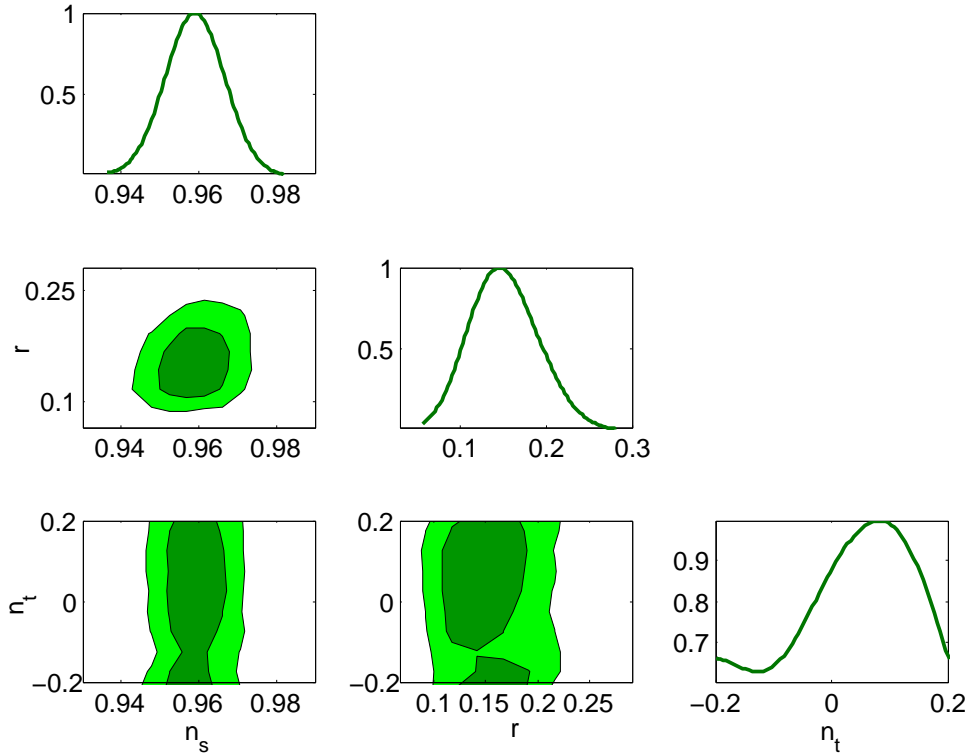


Figure 6: Constraints on r , n_s and n_t . We use the probability distribution of r obtained from BICEP [3] as the prior on r for making this estimation. The constraint on n_t is poor from Planck+WP data.

$$N_{\text{eff}} = \begin{cases} 3.0416 \pm 0.2834 & SP + N_{\text{eff}} + \sum m_\nu (\text{Planck} + WP) \\ 3.0244 \pm 0.29 & SP + N_{\text{eff}} + \sum m_\nu (\text{Planck} + WP + Lensed) \\ 3.2646 \pm 0.28 & SP + N_{\text{eff}} (\text{Planck} + WP) \\ 3.1948 \pm 0.288 & SP + N_{\text{eff}} (\text{Planck} + WP + Lensed) \\ 3.5635 \pm 0.32 & SP + N_{\text{eff}} + r_{0.002} (\text{Planck} + WP) \\ 3.5015 \pm 0.28 & SP + N_{\text{eff}} + r_{0.002} (\text{Planck} + WP + Lensed) \\ 3.41 \pm 0.32 & SP + N_{\text{eff}} + r_{0.002} + \sum m_\nu (\text{Planck} + WP) \\ 3.2923 \pm 0.34 & SP + N_{\text{eff}} + r_{0.002} + \sum m_\nu (\text{Planck} + WP + Lensed) \end{cases}$$

In Fig. 8, we plot the two dimensional likelihood of N_{eff} with all the other parameters for massless neutrino case, which shows that N_{eff} is positively correlated with almost all other parameters. Fig. 9 shows the two dimensional likelihood with $\sum m_\nu \neq 0$ and $r = 0$.

1.4 Standard 6 parameters + $A_L + dn_s/d \ln k$

In this section we show the constraints on the A_L , i.e. the lensing power amplitude relative to the physical value [2]. Theoretically, A_L should be consistent with 1.

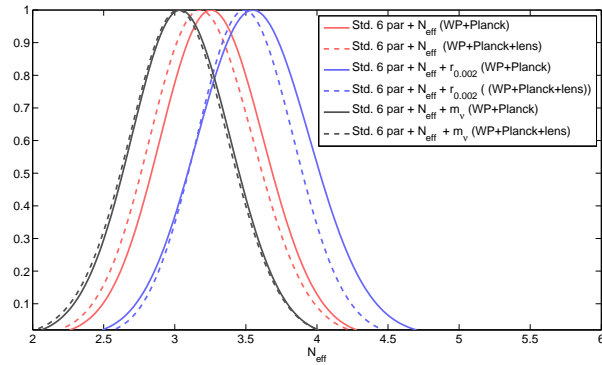


Figure 7: Distribution of N_{eff} for Planck+WP and Planck+WP+lensing likelihood.

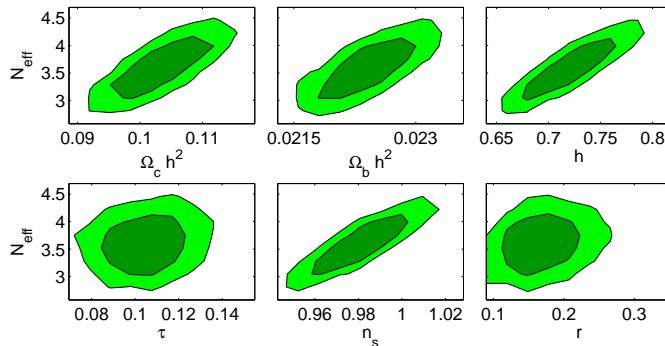


Figure 8: Two dimensional likelihood contours of N_{eff} for Planck+WP likelihood with standard model parameters and r . Neutrinos are considered to be massless for this case.

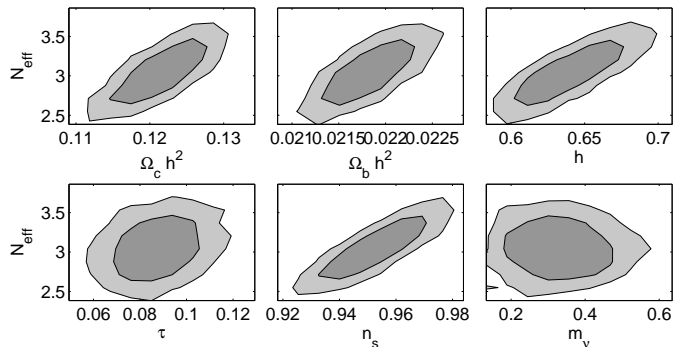


Figure 9: Two dimensional likelihood contours of N_{eff} for Planck+WP likelihood with standard model parameters and $\sum m_\nu$. We have considered $r = 0$ for this case.

However, Planck found that A_L is inconsistent with its theoretical value at 2σ level. Here we consider the effect on A_L due to the recent measurements of $\sum m_\nu$ and r . We

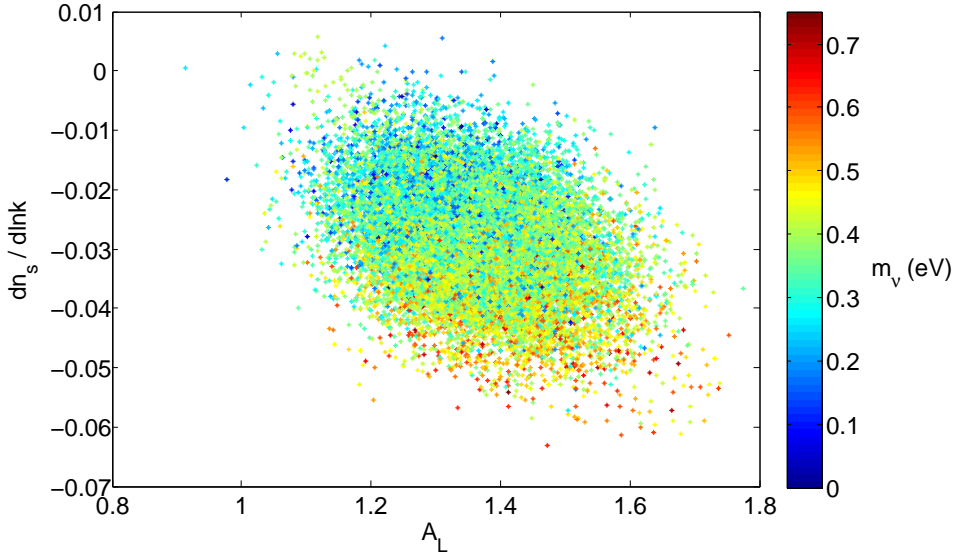


Figure 10: We plot A_L vs $dn_s/d\ln k$ in the scattered diagram and colour coded it with $\sum m_\nu$. A_L is positively correlated with $\sum m_\nu$ and is negatively correlated with $dn_s/d\ln k$.

vary A_L , $dn_s/d\ln k$, $\sum m_\nu$ and r along with the other standard model parameters. The results are shown in Fig. 10 and Fig. 11.

In Fig. 10 we show the scattered plot between A_L and $dn_s/d\ln k$ and color coded with $\sum m_\nu$, which shows for $A_L \approx 1$ is consistent for the $r = 0.2$ with lower value of $\sum m_\nu$ and without running. It shows A_L is negatively correlated with $dn_s/d\ln k$, whereas with $\sum m_\nu$, A_L shows a positive correlation. The one dimensional marginal probability distribution for different cases are shown in Fig. 11. It can be seen that if $dn_s/d\ln k = 0$ and $\sum m_\nu = 0$ then A_L is consistent with the physical value $A_L = 1$ at $\sim 2\sigma$ with the prior on r as mentioned by BICEP 2 [3]. However if we vary $dn_s/d\ln k$ and $\sum m_\nu$ then the average value of A_L shifts towards higher value. For the case with $SP + r + \sum m_\nu + dn_s/dk + A_L$ is inconsistent with 1 at 3.3σ . This shows that varying $dn_s/d\ln k$ with the prior on $\sum m_\nu$ and r leads to an inconsistent lensing amplitude. Whereas, model without $dn_s/d\ln k$ is inconsistent at 2.7σ . So, the model without running with the given $\sum m_\nu$ and r is slightly preferred.

2. Discussions and Conclusions

We study the effects of the new measurements of r and $\sum m_\nu$ claimed by BICEP 2 and BOSS respectively, impose on other cosmological parameters. We evaluate the models $dn_s/d\ln k$ and N_{eff} along with the other SP $\{\Omega_b h^2, \Omega_m h^2, h, \tau, n_s, A_s\}$ with the prior on r and $\sum m_\nu$ as obtained by BICEP 2 [3] and BOSS [4]. Our results show that $dn_s/d\ln k = 0$ at 2.24σ (Sec. 1.2) and N_{eff} is consistent with the theoretical value of 3.046 within 1σ (Sec. 1.3). This implies that the simplest inflationary

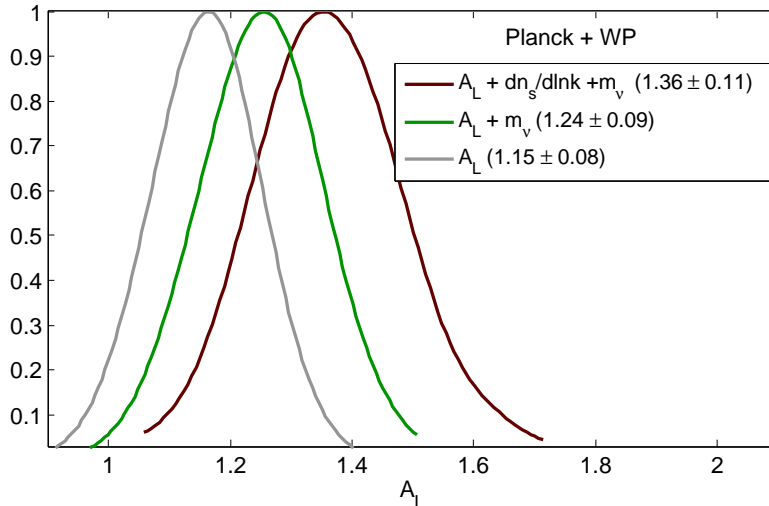


Figure 11: The brown curve is for a case where we vary r , $\sum m_\nu$, A_L and $dn_s/d\ln k$ along with the other standard model parameters. We use the priors from BICEP 2 and BOSS experiments on r and $\sum m_\nu$ respectively. In the green curve we fix the value of $dn_s/d\ln k = 0$ and in the grey curve we fix both $dn_s/d\ln k = 0$ and $\sum m_\nu = 0$. The mean value and the standard deviations of A_L are shown inside the bracket in the legend.

model and with the known relativistic species in the universe, explains the observed temperature spectra of CMB.

Lensing amplitude A_L plays an important role in estimating the value of both N_{eff} and $dn_s/d\ln k$. The model with $\sum m_\nu$, r and $dn_s/d\ln k = 0$ makes A_L is consistent with 1, at 2.66σ . But if we allow $dn_s/d\ln k$ to vary, then we get $A_L > 1$ at 3.27σ level. Therefore, the models with running spectral index, is not physical.

Hence, we can conclude that the standard cosmological model with $SP + \sum m_\nu + r$, $N_{\text{eff}} = 3.046$ and $A_L = 1$, with running of the spectral index $dn_s/d\ln k = 0$ is fully consistent with the data. Though, it provides $H_0 = 63.6 \pm 1.5 \text{ kms}^{-1} \text{ Mpc}^{-1}$, which disagrees from supernova measurements ($H_0 = 73.8 \pm 2.4 \text{ kms}^{-1} \text{ Mpc}^{-1}$) from HST [16] by 4.2σ . Improvement of measurements of H_0 and $\sum m_\nu$ from different future experiments can resolve this discrepancy. Also, the constraints on r can be improved with the full sky measurement of polarization data from Planck, which is supposed to be released in near future. All these together can lead to more precise measurement of the cosmological models.

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