

INFLATION FROM QUANTUM GRAVITY

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A model for inflation based on a quantum gravity scenario is presented. The process allows inflation of a Planck size bubble to the observed universe.

In recent years cosmologists are repeatedly invoking quantum fluctuations to "explain" the origin of the universe [1]. We discuss here one such model in which our observed universe originates from a primordial "Planck size bubble" through inflation. The final state of the universe will have zero cosmological constant and zero spatial curvature with overwhelming probability. The analysis is based on our approach to quantum gravity in which the conformal degree of freedom is quantised using the path integral approach [2]. Thus our model has *no free parameters at all* (except the Planck length), as compared to inflationary scenarios based on GUT potentials etc. The large inflationary factor arises purely from the quantum gravitational tunnelling process.

Classical Einstein equations can be derived from the action principle written in the form ($\hbar = c = 1$)

$$\mathcal{A} = (12L_p^2)^{-1} \int \sqrt{-g} d^4x (R - 2\Lambda), \quad L_p^2 = \frac{4}{3}\pi G, \quad (1)$$

where R is the scalar curvature and Λ is the so-called cosmological constant. In this form Λ includes the contributions from any gravitational Λ_0 and from vacuum expectation values of quantum fields. At this juncture, we keep L_p^2 and Λ as free parameters. The solution to Einstein's equations (obtained from $\delta\mathcal{A} = 0$) is the de Sitter metric, which may be written in the form

$$ds^2 = dt^2 - e^{2Ht} [dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)] \quad (2)$$

$$= (1 - \frac{1}{3}\Lambda\bar{r}^2) d\bar{t}^2 - (1 - \frac{1}{3}\Lambda\bar{r}^2)^{-1} d\bar{r}^2 - \bar{r}^2(d\theta^2 + \sin^2\theta d\varphi^2). \quad (3)$$

For this classical solution the scalar curvature is given by

$$R = 4\Lambda. \quad (4)$$

A classical solution normally corresponds to a quantum ground state. We have written the metric in two coordinate systems (t, r) and (\bar{t}, \bar{r}) to show the "static nature" of the ground state in (\bar{t}, \bar{r}) . Of course, for cosmological purposes one should use the comoving coordinates (t, r) .

A classical ground state need not provide a stable quantum mechanical ground state. Any local minimum will be classically stable to small perturbations. Quantum fluctuations, however, can induce a tunnelling through the potential barrier and render the local minimum unstable. We will show that this is precisely what happens to the ground state, described by eq. (2). Quantum conformal fluctuations make the de Sitter spacetime unstable, and gives a finite lifetime τ . The universe tunnels out of the de Sitter space after an inflation by the factor $(\exp H\tau)$, which we shall compute.

Such an analysis, of course, requires a workable non-perturbative model for quantum cosmology. This goal can be achieved in a limited sense by using the approach discussed in ref. [2]. Various issues of principle that arise in this approach are discussed in ref. [2] and will not be repeated here.

We consider the path integral over the class of all metrics of the form

$$g_{ik} = \Omega^2 \bar{g}_{ik}, \quad (5)$$

where \bar{g}_{ik} is the background de Sitter solution given in eq. (2). This path integral can be written as

$$K = \int \mathcal{D}\Omega \exp(i\mathcal{A}) = \int \mathcal{D}\Omega \exp\left(-i/2L_p^2 \int \sqrt{-g} d^4x (\Omega^i \Omega_i - \frac{1}{6}R\Omega^2 + \frac{1}{3}\Lambda\Omega^4)\right). \quad (6)$$

We shall define a scalar field ϕ and dimensionless cosmological constant λ by

$$\phi = L_p^{-1}\Omega, \quad \lambda = \frac{1}{3}\Lambda L_p^2, \quad (7)$$

in terms of which the kernel K becomes

$$K[\phi_2 t_2; \phi_1 t_1] = \int \mathcal{D}\phi \exp\left(-\frac{i}{2} \int \sqrt{-g} d^4x \{\phi^i \phi_i - \lambda[(2/L_p^2)\phi^2 - \phi^4]\}\right). \quad (8)$$

We shall assume that because of the maximal symmetry of the background metric, ϕ depends only on t . Then the above equation represents the path integral kernel for a one-dimensional system. If the state of the conformal factor ϕ at a time t_1 is denoted by $\psi(\phi, t_1)$ then the kernel K propagates the wavefunction ψ by the standard equation

$$\psi(\phi_2, t_2) = \int K(\phi_2 t_2; \phi_1 t_1) \psi(\phi_1 t_1) d\phi_1. \quad (9)$$

From the form of the action in eq. (8) it is clear that the conformal factor feels the presence of the potential

$$V(\phi) = \lambda[(2/L_p^2)\phi^2 - \phi^4] = (2\lambda/L_p^2)\phi^2(1 - \frac{1}{2}L_p^2\phi^2). \quad (10)$$

The behaviour of the system depends crucially on the sign of λ . In this paper we shall consider the case of $\lambda \geq 0$. We shall present later some plausible arguments as to why negative λ need not be considered. When $\lambda > 0$ the form of the potential is shown in fig. (1). The classical ground state corresponds to the local minimum at $\phi = 0$. Near $\phi = 0$, the potential may be treated as a harmonic oscillator potential with $\omega^2 \approx V''(0)$. The ground state will have $\langle \phi^2 \rangle \sim \omega^{-2}$. This ground state is separated by a potential barrier from the regions $|\phi| \rightarrow \infty$. Clearly quantum tunnelling through this potential renders the classical ground state unstable.

It must be noted that we consider the birth of the universe to be a quantum tunnelling event. Thus the "pre-birth" stage described by $\phi = 0$ will correspond to a completely degenerate metric *classically*. Quantum

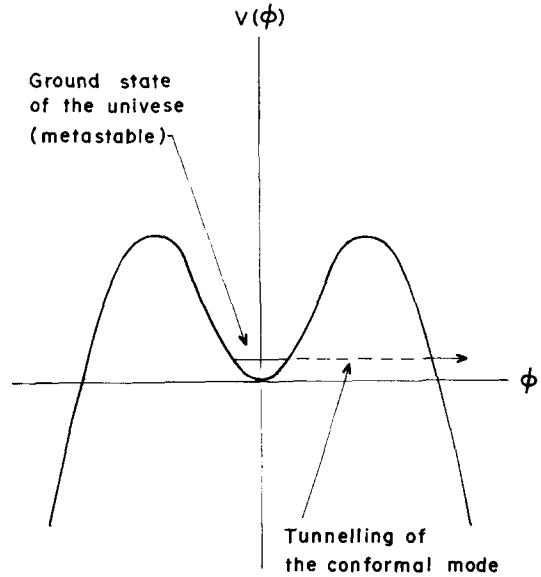


Fig. 1.

mechanically, however $\langle \phi^2 \rangle$ can be nonzero, even though $\langle \phi \rangle$ is zero. Thus a purely quantum universe can exist in this "pre-birth" stage.

This tunneling probability through the potential barrier can be computed from the WKB formula [3]

$$T \propto \exp\left(-\int_{x_1}^{x_2} [V(x) - E]^{1/2} dx\right). \quad (11)$$

We are interested in the case where $E \approx 0$ and the tunneling occurs from $\phi = 0$ (bottom of the well) to the outside $\phi^2 \geq 2/L_p^2$. We also assume that the tunneling occurs homogeneously all through the space with a spatial volume $(\frac{4}{3}\pi H^{-3})$. Then computation of the integral in eq. (11) gives the tunnelling probability per second

$$P \approx \frac{4}{3}\pi(1/L_p \lambda^{3/2}) \exp(-32\sqrt{2}\pi/3\lambda^{3/2}). \quad (12)$$

The reciprocal of P gives the lifetime of the metastable ground state τ . During the time τ , the universe inflates by a factor

$$Z = \exp(H\tau) \approx \exp[(3/4\pi)\lambda^2 \exp(\frac{32}{3}\sqrt{2}\pi \lambda^{-3/2})]. \quad (13)$$

This our main result. A glance at eq. (13) shows that Z is very large for a large range of λ . Let us assume that we require an inflation from Planck size (10^{-33} cm) to the size of the presently observed universe (10^{28}

cm). This needs $Z \gtrsim 10^{61} \sim e^{140}$. This inequality will be satisfied for all λ *except* in the range $8 \lesssim \lambda \lesssim 17$. (The minimum value of Z occurs for $\lambda \approx 11$ with $Z_{\min} \sim 10^{46}$). For example, $Z \sim \exp(10^{16})$ for the “natural choice” of $\lambda \approx 1$. If the λ term arises totally from GUTs, then $\lambda \sim 10^{-8}$ and $Z \sim \exp(10^{12})$. Thus even though λ is a free parameter in our theory, the inflation of a Planck bubble ($\sim 10^{-33}$ cm) to the observed universe ($\sim 10^{28}$ cm) can be achieved for a wide range of λ .

A plausibility argument [4] for favouring very small values of λ can be given if one is prepared to make the following assumptions: (i) The cosmological term is dynamical and not a free parameter in the theory. (Part of the Λ that arises from Higgs fields is definitely dynamical, but it is not clear whether Λ_0 may be treated as dynamical). (ii) Probability amplitudes for configurations can be defined by a “suitable” analytic continuation of the path integral to the euclidian regime. As we shall discuss below, this assumption, though very fashionable nowadays, is highly nontrivial.

Based on these assumptions, one may conjecture that the probability amplitude for the primordial ground state to have a cosmological constant Λ is given by

$$P(\Lambda) \propto \exp\left(\frac{1}{6}(\Lambda/L_p^2) \int dt_E d^3x\right). \tag{14}$$

Here the euclidian continuation is made in the “standard way” – used in the conventional field theory – by substituting $t = -it_E$. When Λ is positive, the euclidian four-volume is finite and is $\sim \Lambda^{-2}$; when Λ is negative the euclidian volume is infinite. Thus we get,

$$P(\Lambda) = 0, \quad \Lambda < 0, \\ = (\text{const}) \times \exp(1/6L_p^2\Lambda), \quad \Lambda > 0. \tag{15}$$

If one accepts this argument, then values of $\lambda > 1/18$ (corresponding to $6L_p^2\lambda > 1$) are extremely improbable. For $\lambda < 1/18$, the inflation factor $Z \sim \exp(10^{720})!$

The argument based on the euclidian action, however, suffers from a well-known defect: If the $t = -it_E$ substitution is made in eq. (7) the conformal action will not have a lower bound. (This is one of the reasons why we prefer to work out the WKB tunneling amplitude rather than an euclidian instanton tunneling.) Under the $t = -it_E$ substitution eq. (7) becomes

$$K = \int \mathcal{D}\phi \exp\left(\frac{1}{2} \int dt_E d^3x \sqrt{g} [(\partial\phi)^2 + (2\lambda/L_p^2)\phi^2(1 - \frac{1}{2}L_p^2\phi^2)]\right). \tag{16}$$

Some further (ad hoc) prescription is required to make sense out of this integral.

Any attempt to cure the “wrong sign” by changing to complex ϕ by putting $\phi = i\eta$ will reverse the relative sign between the ϕ^2 and ϕ^4 terms in the potential. It is likely that we are solving a different problem altogether. In using the argument for zero value of Λ , the above ambiguity in the euclidian path integral must be kept in mind.

The physics of the universe during (and just after) the tunneling is not well understood. It is likely that a large amount of particle creation will take place at this point due to the excitation of all vacuum modes. Later when gravitational effects are negligible, the universe will be described in a conformally flat form:

$$ds^2 = \Omega^2(t)(dt^2 - dx^2 - dy^2 - dz^2). \tag{17}$$

(Notice that the huge inflation would have effectively reduced the universe to a $k = 0$ model). In the classical limit, Ω will satisfy the equation

$$\ddot{\Omega} = 0, \quad \Omega(t) \propto t. \tag{18}$$

This is equivalent to a model with the metric

$$ds^2 = d\tau^2 - S^2(\tau)(dx^2 + dy^2 + dz^2), \tag{19}$$

with $S(\tau) \propto \tau^{1/2}$. This is the standard radiation dominated model.

That the universe could have originated by a vacuum fluctuation is far from being a new idea. However, we believe that our model has the following attractive features compared to many other scenarios: (i) The inflation is produced by purely gravitational interactions. No external field, or potential, is put in by hand. (ii) The theory allows for an inflation from the Planck length to the observed scale of the universe. Though Z depends on λ , we have shown that Z is sufficiently large for a wide choice of λ . Even if one does not believe at all in the argument given above favouring low values of λ , the result in eq. (13) still remains valid. As and when some reliable dynamical process which favours zero value for λ is found, the present work can be placed in a proper setting.

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