

Quintessential Inflation on the Brane and the Relic Gravity Wave Background

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Quintessential inflation describes a scenario in which both inflation and dark energy (quintessence) are described by the same scalar field. In conventional braneworld models of quintessential inflation gravitational particle production is used to reheat the universe. This reheating mechanism is very inefficient and results in an excessive production of gravity waves which violate nucleosynthesis constraints and invalidate the model. We describe a new method of realizing quintessential inflation on the brane in which inflation is followed by ‘instant preheating’ (Felder, Kofman & Linde 1999). The larger reheating temperature in this model results in a smaller amplitude of relic gravity waves which is consistent with nucleosynthesis bounds. The relic gravity wave background has a ‘blue’ spectrum at high frequencies and is a generic byproduct of successful quintessential inflation on the brane.

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I. INTRODUCTION

One of the most remarkable discoveries of the past decade is that the universe is accelerating. An accelerating universe is supported by observations of high redshift type Ia supernovae treated as standardized candles [1, 2] and, more indirectly, by observations of the cosmic microwave background and galaxy clustering [3, 4]. Within the framework of general relativity, cosmic acceleration should be sourced by an energy-momentum tensor which has a large negative pressure (dark energy). The simplest form of dark energy is undoubtedly the cosmological constant for which $p = -\rho = \text{constant}$. However, due to its un-evolving nature, the cosmological constant must be set to an extremely small value in order to dominate the expansion dynamics of the universe at precisely the present epoch. This gives rise (according to one’s viewpoint) either to a fine-tuning problem or to a ‘cosmic coincidence’ problem. For this reason theorists have suggested making dark energy a dynamical quantity associated with a Lagrangian and having well defined equations of motion (see [5, 6, 7, 8, 9] for reviews of dark energy). Perhaps the simplest dynamically evolving dark energy models are quintessence fields – scalar fields which couple minimally to gravity and which roll down a steep potential [10, 11]. Although quintessence models do not resolve the ‘cosmic coincidence’ conundrum they do alleviate, to some extent, the fine-tuning problem faced by the cosmological constant since they approach a common evolutionary path from a wide range of initial conditions.

Braneworld models [12, 13] add an interesting new dimension to scalar field dynamics on the brane. The pres-

ence of a quadratic density term (due to high energy corrections) in the Friedman equation on the brane fundamentally alters the expansion dynamics at early epochs by greatly increasing the Hubble parameter and hence the damping experienced by the scalar field as it rolls down its potential [14]. Consequently, inflation on the brane can be realized by very steep potentials – precisely those used to describe quintessence. The braneworld scenario therefore provides us with the opportunity to unify inflation and dark energy through a mechanism called quintessential inflation [15, 16, 17, 18, 19, 20]. Models of quintessential inflation have a single major drawback: they are usually derived from *non-oscillating* potentials for which the standard reheating mechanism does not work. Indeed, in order to ensure that the inflaton survives until today one usually invokes a method of reheating based on the quantum mechanical production of particles in the time-varying gravitational field after inflation [21, 22]. This method of reheating is very inefficient, and leads to a ‘kinetic regime’ of prolonged duration when braneworld corrections are no longer important and the scalar field rapidly drops down a steep potential, resulting in $p_\phi \simeq \rho_\phi \simeq \dot{\phi}^2/2$ and $a \propto t^{1/3}$. Gravity waves, created quantum mechanically during the kinetic regime have a ‘blue tilt’ and, for a prolonged kinetic regime, their energy density can dominate the energy density of the universe and violate nucleosynthesis constraints [18] (see also [23, 24, 25, 26, 27, 28, 29, 30]). Thus conventional braneworld models of quintessential inflation run into serious problems associated with copious graviton production which renders them unviable for an extended region in parameter space. As we shall show in this paper, this problem is easily circumvented if, instead of gravitational particle production, one invokes an alternative method of reheating, namely ‘instant preheating’ proposed by Felder, Kofman and Linde [31, 32, 33] (see also [34, 35, 36, 37]). This method results in a much higher reheat temperature and therefore

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in a much shorter duration kinetic regime. As a result the amplitude of relic gravity waves is greatly reduced and there is no longer any conflict with nucleosynthesis constraints. (For other approaches to reheating in quintessential inflation see [38, 39, 40, 41, 42]. An alternate approach to dark energy in braneworld models is provided in [43, 44].) Before we apply the instant preheating method to quintessential inflation on the brane let us briefly review scalar field dynamics in braneworld cosmology.

II. BRANEWORLD INFLATION WITH AN EXPONENTIAL POTENTIAL

It is well known that braneworld models can give rise to successful inflation by greatly expanding the parameter space for which inflation can take place [16, 17, 18]. This is true both for the braneworld based on the Randall-Sundrum II scenario, (henceforth RS II) as well as in Gauss-Bonnet cosmology, for which the five dimensional action is (see [46, 47] and references therein)

$$S = \frac{M_5^3}{16\pi} \int d^5x \sqrt{-g} \{ \mathcal{R} - 2\Lambda_5 + \alpha_{GB} [\mathcal{R}^2 - 4\mathcal{R}_{AB}\mathcal{R}^{AB} + \mathcal{R}_{ABCD}\mathcal{R}^{ABCD}] \} + \int d^4x \sqrt{-h} (\mathcal{L}_m - \lambda), \quad (1)$$

\mathcal{R} , R refer to the Ricci scalars in the metric g_{AB} and h_{AB} ; α_{GB} has dimensions of $(length)^2$ and is the Gauss-Bonnet coupling, while λ is the brane tension. The analysis of cosmological dynamics based upon action (1), shows that there is a characteristic GB energy scale M_{GB} [48] such that,

$$\rho \gg M_{GB}^4 \Rightarrow H^2 \approx \left[\frac{\kappa_5^2}{16\alpha_{GB}} \rho \right]^{2/3} \quad (GB), \quad (2)$$

$$M_{GB}^4 \gg \rho \gg \lambda \Rightarrow H^2 \approx \frac{\kappa_4^2}{6\lambda} \rho^2 \quad (RS), \quad (3)$$

$$\rho \ll \lambda \Rightarrow H^2 \approx \frac{\kappa_4^2}{3} \rho \quad (GR). \quad (4)$$

The modified Einstein equations on the brane contain high-energy corrections as well as the projection of the Weyl tensor from the bulk on to the brane. The Friedmann equation on the RS brane is

$$H^2 = \frac{1}{3M_p^2} \rho \left(1 + \frac{\rho}{2\lambda} \right), \quad M_p = \frac{M_4}{\sqrt{8\pi}}, \quad (5)$$

(we have dropped the *dark radiation* term and the effective four dimensional cosmological constant since neither is likely to be relevant for inflation). Our discussion below is quite general, and can easily incorporate the effect of GB correction.

A homogeneous scalar field which couples minimally gravity and propagates on the brane satisfies the evolution equation

$$\ddot{\phi}(t) + 3H\dot{\phi}(t) + V_{,\phi} = 0, \quad (6)$$

and its energy density and pressure are given by

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi), \quad p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi). \quad (7)$$

We shall assume that for $|\phi|/M_p \gg 1$ the scalar-field potential can be described by a pure exponential

$$V(\phi) = V_0 e^{\alpha\phi/M_p}. \quad (8)$$

For $\alpha > \sqrt{2}$, the exponential potential (8) is too steep to sustain inflation in standard cosmology. However, as discussed in the introduction, such steep potentials can easily drive inflation in braneworld cosmology [16], [58]. In this case, the increased damping due to the quadratic term in (5) leads to slow-roll inflation on the brane at high energies, when $\rho/\lambda \gg 1$. In this limit, the slow-roll equations become [16, 18]

$$\frac{\dot{a}(t)}{a(t)} = \sqrt{\frac{1}{6M_p^2\lambda}} V(\phi), \quad 3H\dot{\phi}(t) = -V_{,\phi}, \quad (9)$$

so that

$$\dot{\phi} = -\alpha \sqrt{\frac{2\lambda}{3}}. \quad (10)$$

As a result

$$\phi(t) = \phi_i - \sqrt{\frac{2\lambda_b}{3}} \alpha (t - t_i). \quad (11)$$

The scale factor obtained from (9) passes through an inflection point marking the end of inflation, the corresponding scalar field value is [18]

$$\phi_{\text{end}} = -\frac{M_p}{\alpha} \log \left(\frac{V_0}{2\lambda\alpha^2} \right), \quad (12)$$

substitution in (8) leads to

$$V_{\text{end}} \equiv V_0 e^{\alpha\phi_{\text{end}}/M_p} = 2\lambda\alpha^2. \quad (13)$$

After inflation ends, it takes a little while for brane corrections to disappear and for the kinetic regime to commence. The passage from the end of inflation to the commencement of the kinetic regime is reflected in the following fitting formula which relates the Hubble parameter at the onset of the kinetic regime to its counterpart at the end of inflation [18]

$$H_{\text{kin}} = H_{\text{end}} F(\alpha), \quad (\alpha \gtrsim 3) \quad (14)$$

where the fitting formula $F(\alpha) = 0.085 - \frac{0.688}{\alpha^2}$ was obtained by numerically integrating the equations of motion. A brief comment is in order on the influence of brane term, B_{brane} after inflation ($H^2 \sim \rho B_{\text{brane}}$). Around $\phi = \phi_{\text{end}}$, we can approximately write B_{brane} as [18], $B_{\text{brane}} \simeq 1 + \alpha^2 e^{-\alpha(\phi_{\text{end}} - \phi)/M_p}$. Hence the

influence of brane term diminishes faster for larger values of α which manifests in the fitting formulas (14) and others which occur in text below.

The COBE normalized amplitude for density perturbations allows us to determine the value of the potential at the end of inflation and the value of brane tension

$$\begin{aligned} V_{end} &\simeq \frac{3 \times 10^{-7}}{\alpha^4} \left(\frac{M_p}{\mathcal{N} + 1} \right)^4 \frac{1}{(1 + A_T^2/A_S^2)^{1/4}}, \\ \lambda &\simeq \frac{1.3 \times 10^{-7}}{\alpha^6} \left(\frac{M_p}{\mathcal{N} + 1} \right)^4 \frac{1}{(1 + A_T^2/A_S^2)^{1/4}} \end{aligned} \quad (15)$$

from which one fixes the value of $\dot{\phi}$ at the end of inflation

$$\dot{\phi}_{end} = -\sqrt{\frac{V_{end}}{3}}, \quad (16)$$

here A_T and A_S are the amplitudes of tensor and scalar modes created during inflation [14, 27], and \mathcal{N} is the number of e -foldings which we take to be $\mathcal{N} \simeq 70$. At the end of inflation, in addition to the scalar field energy density, a small amount of radiation is also present $\rho_r \simeq 0.01 g_p H_{end}^4$, arising due to particles produced quantum mechanically by the changing gravitational field after inflation (g_p is the number of particle species created) [21, 22]. Using equations (5) and (15) one finds $\rho_\phi/\rho_r \sim 10^{16} g_p^{-1}$. This implies that equality between the inflaton field and radiation ($\rho_\phi \simeq \rho_r$) is reached very late when the temperature has dropped below [18]

$$T_{eq} \lesssim \sqrt{g_p} \text{GeV}, \quad (17)$$

for $\alpha \gtrsim 5$. If gravitational particle production is the sole means of generating reheating, then, prior to the radiative regime ($T > T_{eq}$) the universe will enter into an extended kinetic regime, during which $\rho_\phi \gg \rho_r$ and $p_\phi \simeq \rho_\phi$ so that $a(t) \propto t^{1/3}$.

It is well known that an inflationary universe gives rise to a stochastic background of relic gravity waves of quantum mechanical origin [23]. If the post-inflationary epoch is characterised by an equation of state $w = p/\rho$ then the spectral energy density of gravity waves produced during slow-roll inflation is [24]

$$\rho_g(k) \propto k^{2\left(\frac{w-1/3}{w+1/3}\right)}. \quad (18)$$

In our braneworld model $w \simeq 1$ during the kinetic regime, consequently the gravity wave background generated during this epoch will have a blue spectrum $\rho_g(k) \propto k$. In this case it can be shown that the energy density of relic gravity waves at the commencement of the radiative regime is given by [18]

$$\left(\frac{\rho_g}{\rho_r} \right)_{eq} = \frac{64}{3\pi} h_{GW}^2 \left(\frac{T_{kin}}{T_{eq}} \right)^2, \quad (19)$$

where h_{GW} is the dimensionless amplitude of gravity waves and COBE normalization (with $\mathcal{N} \simeq 70$) gives

$$h_{GW}^2 \simeq 1.7 \times 10^{-10}. \quad (20)$$

Substituting for T_{kin} from (14) and (15) and for T_{eq} from (17) we find that $(\rho_g/\rho_r)_{eq} \gg 1$ in this scenario. Thus braneworld inflation with a steep potential invariably results in an over-production of gravity waves which grossly violate the nucleosynthesis bound $(\rho_g/\rho_r)_{eq} \lesssim 0.2$.

Before showing how instant preheating can resolve this issue let us mention a few important formulae which will help in clarifying the issues involved. As mentioned earlier, the commencement of the kinetic regime is not instantaneous and brane effects persist for some time after inflation has ended. The temperature at the commencement of the kinetic regime T_{kin} is related to the temperature at the end of inflation by

$$T_{kin} = T_{end} \left(\frac{a_{end}}{a_{kin}} \right) = T_{end} F_1(\alpha) \quad (21)$$

where $F_1(\alpha) = (c + \frac{d}{\alpha^2})$, $c \simeq 0.142$, $d \simeq -1.057$ and $T_{end} \simeq (\rho_r^{end})^{1/4}$. The equality between the scalar field and radiation densities takes place at the temperature

$$T_{eq} = T_{end} \frac{F_2(\alpha)}{(\rho_\phi/\rho_r)_{end}^{1/2}}, \quad (22)$$

where $F_2(\alpha) = (e + \frac{f}{\alpha^2})$, $e \simeq 0.0265$, $f \simeq -0.176$. (The fitting formulas (21) and (22) were obtained in [18] by numerically integrating the equations of motion.) A useful formula relates the Hubble parameters at the commencement of the radiative and kinetic regimes

$$H_{eq} = \sqrt{2} H_{kin} \left(\frac{T_{eq}}{T_{kin}} \right)^3. \quad (23)$$

Using equations (21), (22) and (19) we obtain a useful expression which links the ratio of the scalar-field density to the radiation density at the end of inflation with the ratio between the gravity wave density and the radiation density at the start of the radiative era

$$\left(\frac{\rho_\phi}{\rho_r} \right)_{end} = \frac{3\pi}{64} \left(\frac{F_2^2(\alpha)}{h_{GW}^2 F_1^2(\alpha)} \right) \left(\frac{\rho_g}{\rho_r} \right)_{eq}. \quad (24)$$

Equation (24) is an important result since it allows us to set a limit to the ratio between the scalar field density and the radiation density at the end of inflation. Nucleosynthesis constraints provide an upper bound on the energy density in gravity waves at the start of the radiative era: $(\rho_g/\rho_r)_{eq} \lesssim 0.2$; substituting in (24), and assuming a steep potential ($\alpha \gtrsim 5$), we arrive at the following result

$$(\rho_\phi/\rho_r)_{end} \lesssim 10^7. \quad (25)$$

Since $(\rho_\phi/\rho_r)_{end} \sim 10^{16} g_p^{-1}$ in the case of reheating sourced by gravitational particle production [18], this mechanism leads to the nucleosynthesis constraint being violated by almost nine orders of magnitude unless $g_p \gtrsim 10^9$! (g_p is the number of particle species created

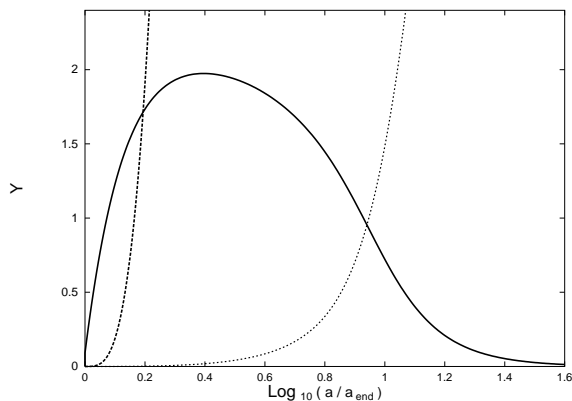


FIG. 1: The post inflationary evolution of $Y = 10^9 \times |\dot{\phi}|/M_p^2$, $g\phi^2/M_p^2$ is shown as a function of the scale factor for the model described by Eq (41) with $V_0 \simeq 10^{-30}M_p$, $\tilde{\alpha} = 50$ and $p = 0.1$. The solid curve corresponds to $\dot{\phi}/M_p^2$ whereas the dashed and dotted curves correspond to $g\phi^2/M_p^2$ for $g = 10^{-6}$ (extreme left) and $g = 10^{-9}$ (extreme right) respectively. The violation of the adiabaticity condition (necessary for particle production to take place) arises in the region bounded by the solid and dashed curves. The time duration of particle production is seen to be smaller for larger values of the coupling g . For the range of g allowed by the nucleosynthesis constraint, particle production is almost instantaneous.

using this mechanism.) Next, we shall examine the instant preheating mechanism discovered by Felder, Kofman and Linde (1999) and show that it is capable of producing $(\rho_\phi/\rho_r)_{end}$ consistent with (24) and (25).

III. BRANEWORLD INFLATION FOLLOWED BY INSTANT PREHEATING

Braneworld Inflation induced by the steep exponential potential (8) ends when $\phi = \phi_{end}$, see (12). Without loss of generality, we can make the inflation end at the origin by translating the field

$$V(\phi') \equiv V(\phi) = \tilde{V}_0 e^{\alpha\phi'/M_p}, \quad (26)$$

where $\tilde{V}_0 = V_0 e^{+\alpha\phi_{end}}/M_p$ and $\phi' = \phi - \phi_{end}$. In order to achieve reheating after inflation has ended we assume that the inflaton ϕ interacts with another scalar field χ which has a Yukawa-type interaction with a Fermi field ψ . The interaction Lagrangian is

$$L_{int} = -\frac{1}{2}g^2\phi'^2\chi^2 - h\bar{\psi}\psi\chi. \quad (27)$$

To avoid confusion, we drop the prime on ϕ remembering that $\phi < 0$ after inflation has ended. It should be noticed that the χ field has no bare mass, its effective mass being determined by the field ϕ and the value of the coupling constant g ($m_\chi = g|\phi|$).

The production of χ particles commences as soon as m_χ begins changing non-adiabatically [31, 32]

$$|m_\chi| \gtrsim m_\chi^2 \quad \text{or} \quad |\dot{\phi}| \gtrsim g\phi^2. \quad (28)$$

The condition for particle production (28) is satisfied when

$$|\phi| \lesssim |\phi_{prod}| = \sqrt{\frac{|\dot{\phi}_{end}|}{g}} = \sqrt{\frac{V_{end}^{1/2}}{\sqrt{3}g}}. \quad (29)$$

From (15) we find that $\phi_{prod} \ll M_p$ for $g \gg 10^{-9}$. The production time for χ particles can be estimated to be

$$\Delta t_{prod} \sim \frac{|\phi|}{|\dot{\phi}|} \sim \frac{1}{\sqrt{gV_{end}^{1/2}}}. \quad (30)$$

The uncertainty relation provides an estimate for the momentum of χ particles created non-adiabatically: $k_{prod} \simeq (\Delta t_{prod})^{-1} \sim g^{1/2}V_{end}^{1/4}$. Proceeding as in [32, 33] we can show that the occupation number of χ particles jumps sharply from zero to

$$n_k \simeq \exp(-\pi k^2/gV_{end}^{1/2}), \quad (31)$$

during the time interval Δt_{prod} . The χ -particle number density is estimated to be

$$n_\chi = \frac{1}{2\pi^3} \int_0^\infty k^2 n_k dk \simeq \frac{(gV_{end}^{1/2})^{3/2}}{8\pi^3}. \quad (32)$$

Quanta of the χ -field are created during the time interval Δt_{prod} that the field ϕ spends in the vicinity of $\phi = 0$. Thereafter the mass of the χ -particle begins to grow since $m_\chi = g|\phi(t)|$, and the energy density of particles of the χ -field created in this manner is given by

$$\rho_\chi = m_\chi n_\chi \left(\frac{a_{end}}{a}\right)^3 = \frac{(gV_{end}^{1/2})^{3/2}}{8\pi^3} g|\phi(t)| \left(\frac{a_{end}}{a}\right)^3. \quad (33)$$

where the $(a_{end}/a)^3$ term accounts for the cosmological dilution of the energy density with time. As shown above, the process of χ particle-production takes place immediately after inflation has ended, provided $g \gtrsim 10^{-9}$. In what follows we will show that the χ -field can rapidly decay into fermions. It is easy to show that if the quanta of the χ -field were converted (thermalized) into radiation instantaneously, the radiation energy density would become

$$\rho_r \simeq \rho_\chi \sim \frac{(gV_{end}^{1/2})^{3/2}}{8\pi^3} g\phi_{prod} \sim 10^{-2}g^2V_{end}. \quad (34)$$

From equation (34) follows the important result

$$\left(\frac{\rho_\phi}{\rho_r}\right)_{end} \sim \left(\frac{10}{g}\right)^2. \quad (35)$$

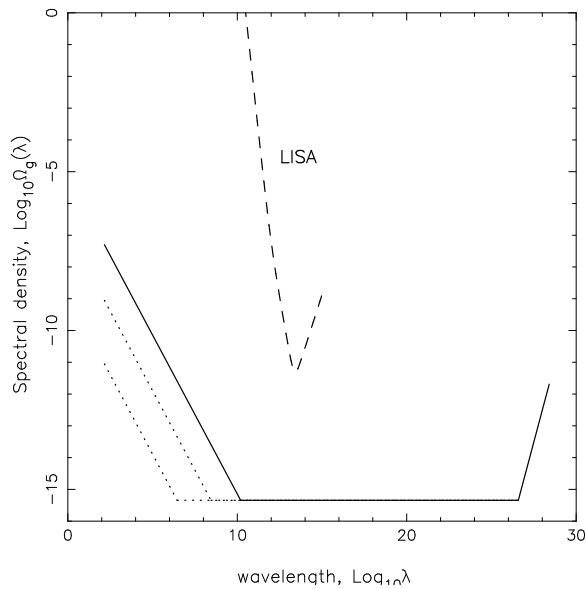


FIG. 2: The spectral energy density of the relic gravity wave background generated by quintessential inflation on the brane followed by instant preheating. The blue spectrum at short wavelengths is caused by the kinetic regime which commences soon after brane inflation ends. (The inflaton potential in (8) is assumed with $\alpha = 5$.) At these wavelengths the solid line corresponds to $g = 4 \times 10^{-3}$ and the upper (lower) dashed line corresponds to $g = 0.03$ ($g = 0.3$), where g is the coupling parameter introduced in (27). In all three cases the gravity wave background satisfies nucleosynthesis constraints. (The short-wavelength cutoff at $\lambda \simeq 10$ cm corresponds to the comoving length scale marking the commencement of the kinetic regime. The spectrum of gravity waves with $\lambda \lesssim 10$ cm will depend sensitively upon the details of the preheating process and is not shown here.) The chained line shows the expected sensitivity curve of the proposed Laser Interferometer Space Antenna (LISA). The present value of the Hubble parameter is assumed to be $H_0 = 70$ km/sec/Mpc.

Comparing (35) with (25) we find that, in order for relic gravity waves to respect the nucleosynthesis constraint, we should have $g \gtrsim 4 \times 10^{-3}$; the corresponding gravity wave background is shown in figure 2. (The energy density created by instant preheating $(\rho_r/\rho_\phi) \simeq (g/10)^2$ can clearly be much larger than the energy density produced by quantum particle production, for which $(\rho_r/\rho_\phi) \simeq 10^{-16} g_p$.)

The constraint $g \gtrsim 4 \times 10^{-3}$, implies that the particle production time-scale (30) is much smaller than the Hubble time since

$$\frac{1}{\Delta t_{prod} H_{end}} \gtrsim 300\alpha^2, \quad \alpha \gg 1. \quad (36)$$

Thus the effects of expansion can safely be neglected during the very short time interval in which ‘instant preheating’ takes place. We also find, from equation (29), that $|\phi_{prod}|/M_p \lesssim 10^{-3}$ implying that particle production takes place in a very narrow band around $\phi = 0$. Figure 1 demonstrates the violation of the adiabaticity

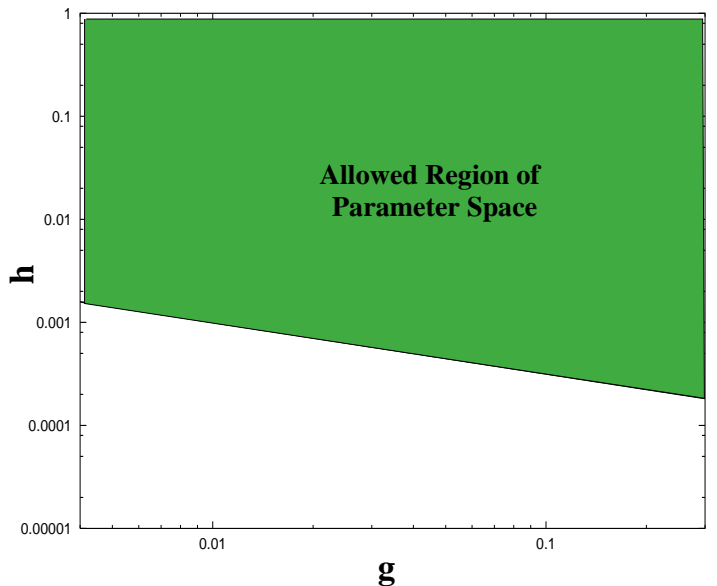


FIG. 3: Parameter space permitted by nucleosynthesis constraint and the constraint imposed by the back reaction in models of brane inflation with instant preheating. The upper limits on the parameters h is dictated by the validity of perturbation theory for the interaction described by the Lagrangian (27). Clearly there is a large region in parameter space for which the *instant preheating* mechanism can be used to construct realistic models of quintessential inflation.

condition (at the end of inflation) which is a necessary prerequisite for particle production to take place. For the range of g allowed by the nucleosynthesis constraint, the particle production turns out to be almost instantaneous.

We now briefly address the issue of back-reaction of created χ -particles on the background. As mentioned above, brane effects persist briefly after inflation has ended. $\rho_\phi \propto 1/a$ immediately after inflation, however, once the damping due to the ρ^2 term in (5) switches off, ρ_ϕ decreases faster, settling to $\rho_\phi \propto 1/a^6$ at the commencement of the kinetic regime. The transition period between the end of inflation and the commencement of the kinetic regime depends upon the value of α , as reflected in (14) (for instance $a_{kin}/a_{end} \simeq 10$, for $\alpha = 5$). The density of χ particles scales $\propto 1/a^3$ and modifies the field evolution equation

$$\ddot{\phi}(t) + 3H\dot{\phi}(t) + V_{,\phi} + g^2\phi\langle\chi^2\rangle = 0 \quad (37)$$

where $\langle\chi^2\rangle$ can be estimated according to the prescription given in Refs. [31] and [32]

$$\langle\chi^2\rangle = \frac{(gV_{end}^{1/2})^{3/2}}{8\pi^3\sqrt{3}g\phi} \left(\frac{a_{end}}{a}\right)^3. \quad (38)$$

The density of non-relativistic χ particles ρ_χ scales as $1/a^3$. In contrast, the potential term, as well as the dissipative terms in the field equation scale slower than ρ_χ for most of the evolution before the commencement of

the kinetic regime. As a result, for any generic value of the coupling $g \lesssim 0.3$, the back-reaction of χ particles in the evolution equation is negligible during the time scale $\sim H_{kin}^{-1}$. It is important to note that, although ρ_χ/ρ_ϕ is a small quantity to begin with, ρ_χ (or the decay products of the χ field) will decrease as $1/a^n(t)$, $n \leq 4$. In contrast, since the scalar field is rolling down an exponentially steep potential, its energy density decreases much faster $\rho_\phi \propto 1/a^6(t)$ during the kinetic regime. Therefore either ρ_χ or its decay products will eventually come to dominate the density of the universe.

We now turn to the matter of reheating which occurs through the decay of χ particles to fermions, as a consequence of the interaction term in the Lagrangian (27). The decay rate of χ particles is given by $\Gamma_{\bar{\psi}\psi} = h^2 m_\chi / 8\pi$, where $m_\chi = g|\phi|$. Clearly the decay rate is faster for larger values of $|\phi|$. For $\Gamma_{\bar{\psi}\psi} > H_{kin}$, the decay process will be completed within the time that back-reaction effects (of χ particles) remain small. Using (14) this requirement translates into

$$h^2 > \frac{8\pi\alpha}{\sqrt{3}g\phi} \frac{V_{end}^{1/2}}{M_p} F(\alpha). \quad (39)$$

For reheating to be completed by $\phi/M_p \lesssim 1$, we find from equations (25) and (39) that $h \gtrsim 10^{-4}g^{-1/2}$ for $\alpha \simeq 5$. In figure 3 we have shown the region $h \gtrsim 10^{-4}g^{-1/2}$, $g \gtrsim 4 \times 10^{-3}$, for which (i) reheating is rapid and (ii) the relic gravity background in non-oscillatory braneworld models of quintessential inflation is consistent with nucleosynthesis constraints. As discussed in [31] this method of reheating can give rise to super-heavy fermions with $m_\psi \sim 10^{16} - 10^{17}$ GeV, the subsequent decay of these particles completes the reheating process.

The method discussed here can also be extended to long lived quanta of the χ field with bare mass \tilde{m}_χ . In this case it can be shown that the probability of creation of such particles will be suppressed by the factor (see also [31])

$$\exp\left[-\frac{\pi\tilde{m}_\chi^2}{g|\phi|}\right] = \exp\left[-1.7 \cdot 10^7 \left(\frac{\pi\alpha^2}{g}\right) \left(\frac{\tilde{m}_\chi}{M_p}\right)^2\right], \quad (40)$$

where we have used (10) and (15). From (40) we find that the creation of long lived quasi-stable particles of mass $\tilde{m}_\chi \simeq 2 \times 10^{15}$ GeV is strongly suppressed by the factor 8×10^{-17} if $\alpha = 5$ and $g \sim 1$. Since the density of quasi-stable particles decreases as $1/a^3(t)$, this mechanism allows us to create a small number of super-heavy WIMPs after braneworld inflation has ended and whose role becomes important at late times, when they can be candidates for dark matter or, if they decay, be capable of producing cosmic rays more energetic than the GZK limit [49] (see [31, 50, 51] for related extensive discussions of this issue).

The analysis presented here is also likely to qualitatively hold for a class of inverse square potentials. We conclude that instant preheating in the context of brane

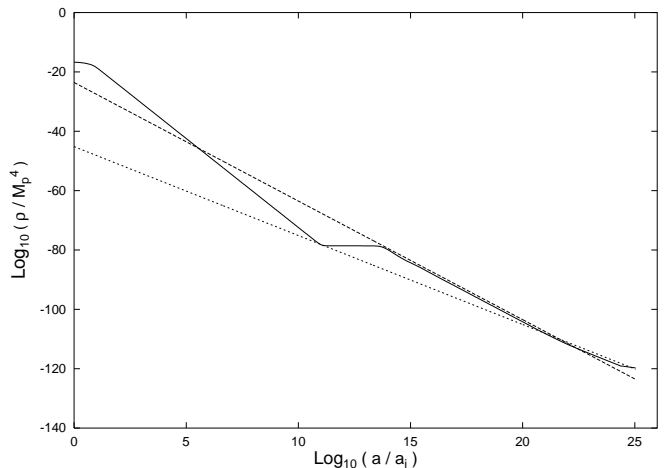


FIG. 4: The post-inflationary evolution of the scalar field energy density (solid line), radiation (dashed line) and cold dark matter (dotted line) is shown as a function of the scale factor for the quintessential inflation model described by (41) with $V_0^{1/4} \simeq 10^{-30}M_p$, $\tilde{\alpha} = 50$ and $p = 0.1$ ($\alpha = p\tilde{\alpha} = 5$). After brane effects have ended, the field energy density ρ_ϕ enters the kinetic regime and soon drops below the radiation density. After a brief interval during which $\langle w_\phi \rangle \simeq -1$, the scalar field begins to track first radiation and then matter. At very late times (present epoch) the scalar field plays the role of quintessence and makes the universe accelerate. The evolution of the energy density is shown from the end of inflation until the present epoch.

inflation is very efficient and is capable of producing successful models of non-oscillatory quintessential inflation on the brane.

IV. LATE-TIME BEHAVIOUR

It is well known that, during the post-brane epoch, a scaling solution is an attractor for the exponential potential (see [52] and references therein). After the attractor is reached, the field energy density tracks the background in such a way that ρ_ϕ/ρ_r remains constant. Compliance with nucleosynthesis constraints requires $\Omega_\phi \lesssim 0.2$ during the radiative regime, which implies $\alpha \gtrsim 5$. However, for a purely exponential potential ρ_ϕ remains subdominant forever and the scalar field never becomes quintessence. Quintessential inflation can be achieved for a potential which interpolates between an exponential and a flat or rapidly oscillating form. An example of the latter is provided by [53]

$$V(\phi) = V_0 [\cosh(\tilde{\alpha}\phi/M_p) - 1]^p, \quad 0 < p < 1/2 \quad (41)$$

which has asymptotic forms

$$V(\phi) = \frac{V_0}{2^p} e^{\tilde{\alpha} p \phi / M_p}, \quad \tilde{\alpha} |\phi| / M_p \gg 1, \quad (42)$$

and

$$V(\phi) = \frac{V_0}{2^p} \left(\frac{\tilde{\alpha} \phi}{M_p} \right)^{2p} \quad \tilde{\alpha} |\phi| / M_p \ll 1, \quad (43)$$

where $\tilde{\alpha} p = \alpha$. The power law behaviour of (41) near the origin leads to oscillations of ϕ when it approaches small values. Rewriting ϕ in terms of ϕ' as, $\phi = \phi' + \phi_{end}$, reproduces the correct asymptotes (26) for large values of ϕ' as well as the power-law behavior (43) for $\phi' \rightarrow -\phi_{end}$. The time-averaged equation of state of the scalar field during oscillations is given by [53]

$$\langle w_\phi \rangle = \frac{p-1}{p+1}, \quad (44)$$

as a result, the scalar-field energy density and the scale factor display the following behavior

$$\rho_\phi \propto a^{-3(1+\langle w_\phi \rangle)}, \quad a \propto t^{\frac{2}{3}(1+\langle w_\phi \rangle)^{-1}}.$$

Clearly, the scalar field behaves like dark energy for $p < 1/2$. By adjusting parameters in this model one can ensure that scalar field oscillations occur at late times so that Ω_ϕ reaches 0.7 today. We have evolved the field equations numerically and our results (for a particular parameter choice) are shown in figure 4.

Other forms of the potential which will give rise to quintessential inflation can be found in [8].

V. CONCLUSIONS

In this paper we have successfully applied the instant preheating mechanism discussed in [31, 32] to braneworld inflation. Braneworld cosmology has the attractive property of allowing inflation to occur even for steep potentials thus greatly expanding the class of potentials which give rise to inflation. We have demonstrated that instant preheating works remarkably well for non-oscillating potentials such as an exponential. In such models the instant preheating mechanism results in a much higher energy density for radiation than the mechanism based on gravitational particle production [21, 22].

One of the main results of this paper is that a single scalar field on the brane can successfully describe inflation at early epochs and quintessence at late times. Braneworld models of quintessential inflation followed by instant preheating do not over-produce gravity waves and are therefore entirely consistent with the supernova data on the one hand and nucleosynthesis constraints on the other. Although the amplitude of the gravity wave background in quintessential inflation is below the projected sensitivity of LISA, the ‘blue spectrum’ of gravity waves

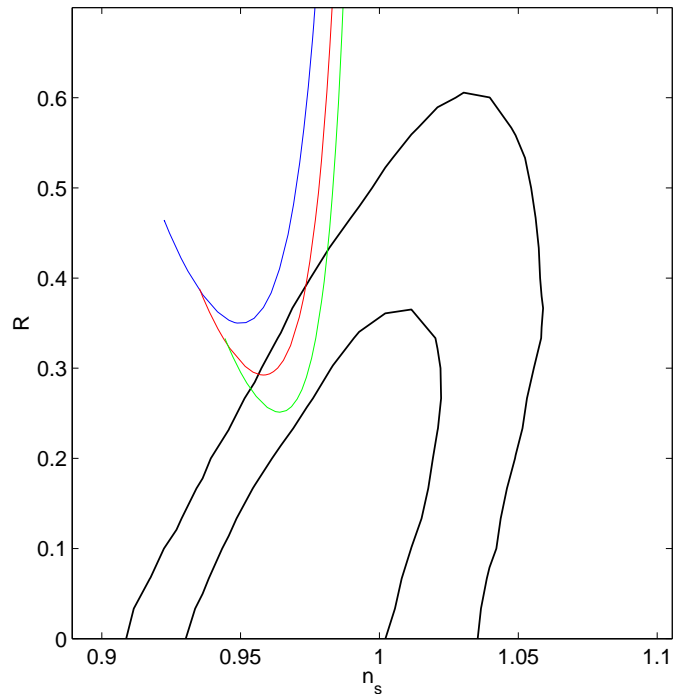


FIG. 5: Plot of $R \equiv 16A_T^2/A_S^2$ versus the spectral index n_s for inflation driven by the exponential potential (8) in a Gauss-Bonnet (GB) braneworld for $\mathcal{N} = 50, 60, 70$ e-foldings (from top to bottom). The 1σ and 2σ observational contours which follow from the WMAP+SDSS results are shown for comparison [57]. The $R(n_s)$ curves exhibit a minimum in the intermediate energy region between the GB (extreme right) and RS (extreme left) regimes. It is clear that for a larger value of \mathcal{N} (the theoretical curves are outside the 2σ contour bond for $\mathcal{N} = 50$, however the points begin to move inside the 2σ bound as \mathcal{N} becomes larger than 55) more points in this region lie within the 2σ observational bound illustrating the fact that steep inflation in the GB model is in agreement with observations (see [57] for more details).

with wavelengths $\lesssim 10^{10}$ cm, makes it likely that they could lie within the sensitivity range of future space-based gravity wave detectors (see for example [54]).

We would like to end by mentioning that the main features of instant preheating in braneworld models are sufficiently robust and are likely to carry over to other inflationary models in which enhanced damping is an important feature of early scalar field dynamics. In this paper we have laid stress on the ‘heavy damping’ experienced by a scalar field during the RS phase in order to construct a successful scenario of quintessential-inflation on the brane. It should be emphasised however that the RS phase also occurs in braneworld models which have a Gauss-Bonnet (GB) term in the bulk; see equations (1) - (4). Indeed it is to this class of models to which one must turn for a scenario, which not only

gives rise to quintessential-inflation, but also satisfies all other observational constraints – particularly those on the spectral index and the scalar to tensor ratio placed by WMAP+SDSS [4, 55]. It is well known that steep inflation in the RS scenario comes into conflict with observations [56] while GB brane world can rescue these models [57]. Indeed, it appears that steep inflation in GB models which commences at an intermediate energy scale between RS and GB regimes – can be easily reconciled with observations [57]. Figure 5 summarizes this result by showing the values of the scalar to tensor ratio R and the scalar spectral index n_S in the GB inflationary model ; for details the reader is referred to [57]. It is interesting that the $R(n_S)$ curve shows a minimum for steep inflation which begins at an intermediate energy scale in GB inflation and which agrees with observations. (The upper right branch of the curves corresponds to steep inflation

which commenced deep in the GB regime; the left branch corresponds to the RS regime.) We conclude that a successful scenario of quintessential inflation on the Gauss-Bonnet braneworld has been constructed which agrees with CMB+LSS observations and also generates an interesting blue spectrum for gravity waves on small scales.

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