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Cosmologies with Variable Gravitational Constant

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In 1937 Dirac presented an argument, based on the socalled large dimensionless numbers, which led him to the conclusion that the Newtonian gravitational constant G *changes with epoch. Towards the end of the last century Ernst Mach had given plausible arguments to link the property of inertia of matter to the large scale structure of the universe. Mach's principle also leads to cosmological models with a variable gravitational constant. Three cosmologies which predict a variable* G *are discussed in this paper both from theoretical and observational points of view.*

1. INTRODUCTION

The gravitational constant G appeared for the first time in physics literature through Newton's law of gravitation[(1)]

$$F = G\frac{m_1 m_2}{r^2} \tag{1}$$

F being the force of attraction between two masses m_1, m_2 separated by distance r. The success of Newtonian gravitation in explaining gravitational phenomena in our local neighborhood (the Earth and the Solar System) generates confidence that the above law is valid at least as a first approximation.

It was in this spirit that when writing down his field equations of gravitation,

$$R_{ik} - 1/2(g_{ik}R) = -\kappa T_{ik} \tag{2}$$

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Einstein[2] determined the constant κ by requiring that his law reduces to (1) in the weak-field approximation. Reference to Newtonian mechanics then gave the answer as

$$\kappa = \frac{8\pi G}{c^4} \tag{3}$$

where c = speed of light.

Gravitation, whether according to Newton or according to Einstein, is a long range force and a force most significant to cosmology, to the large scale structure of the universe. However, cosmological considerations pose deeper questions. Since the universe by definition contains everything, it also includes the physical laws which govern its structure. That is, the laws themselves are part and parcel of the universe and are not independent of it. To what extent therefore is the law of gravitation itself determined by the universe? And if the universe evolves, does the law also change its form? Evidently, the cosmological problem becomes much more difficult if, as a result of the above questions, we are forced to deal with a law of gravitation which changes with time and space.

Symmetry considerations, however, do restrict the scope for change. For example, coordinate invariance in space requires the form (1) to be preserved, allowing at the most a dependence of the 'constant' G on time. With its general covariance, relativity is much more restrictive and the tensor form of Einstein's equations demands that κ be a constant. Therefore if c is a constant G must also be a constant.

Constancy of physical laws can, of course, be justified in a universe whose large scale structure does not change. Such a cosmological model was deduced by Bondi and Gold[3] on the basis of their perfect cosmological principle. This model is known as the steady state model and its line element

$$ds^2 = c^2\,dt^2 - e^{2H_0 t}(dr^2 + r^2\,d\theta^2 + r^2 \sin^2\theta\,d\phi^2) \tag{4}$$

follows uniquely (apart from the arbitrary positive constant H_0) from the perfect cosmological principle.

However, in a speculative subject like cosmology it is wise not to put all one's eggs in one basket. Thus, cosmologists have explored other possibilities besides the steady state model mentioned above and the standard big bang models[4] which follow from Einstein's equations. It was Dirac who first raised the interesting (and provocative) possibility that in a changing universe the gravitational constant may also be changing.[5,6] Although since Dirac's original proposal several "G-varying" cosmologies have been proposed by various authors, in this article we limit ourselves to a discussion of only three distinct approaches including Dirac's original approach. We will briefly review these theories and make a comparison between them.

2. THE LARGE NUMBERS HYPOTHESIS

We begin with Dirac's approach which is based on the so-called large dimensionless numbers. Basically there are two large numbers which emerge from a mixture of the fundamental constants of cosmology and physics. We will denote them by N_1 and N_2, with

$$N_1 = \frac{m_e c^3}{e^2 H_0} \tag{5}$$

$$N_2 = \frac{e^2}{G m_e m_p} \tag{6}$$

Here m_e abd m_p are the masses of the electron and proton, respectively, $\mp e$ their electric charges and H_0 is the Hubble constant denoting the rate of expansion of the universe at the present epoch.

The important property shared by N_1 and N_2 is that they are dimensionless. N_1 denotes the ratio of the 'radius' of the universe (c/H_0) to the 'radius' of the electron $(e^2/m_e c^2)$ while N_2 is the ratio of the electrostatic force of attraction between the electron and the proton to their gravitational force of attraction. By eliminating the ratio e^2/m_e we arrive at a third dimensionless number

$$N_3 = N_1 N_2 = \frac{c^3}{G H_0 m_p} \tag{7}$$

Reference to various cosmological models tells us that N_3 is of the order of the number of protons in the observable universe.

The interesting feature about N_1, N_2, and N_3 which was emphasized by Dirac was their magnitude:

$$N_1 \sim N_2 \sim 10^{40}, \qquad N_3 \sim 10^{80} \tag{8}$$

Why should N_1 and N_2 be so large and yet comparable? Is the numerical coincidence a consequence of some deep physical theory as yet unknown? Suppose that such a theory exists and that it leads to a near equality of N_1 and N_2. From (5) and (6) we then get

$$G \sim \left(\frac{e^4}{m_e^2 m_p c^3}\right) H_0 \tag{9}$$

Suppose further that the quantities in large brackets, containing noncosmological information, stay fixed at all epochs. We then deduce that

$$G \propto H_0 \tag{10}$$

Dirac argued that the relationship (10) has nothing special to do with our present epoch t_0 but that it must hold at all times past and present. If the scale factor giving the expansion of the universe at any epoch t is denoted by $S(t)$, then $H(t) \propto \dot{S}/S$. For $S \propto t^n$ $(n > 0)$, we therefore get from (10), $G \propto t^{-1}$, i.e.,

$$\dot{G}/G = -1/t \tag{11}$$

For $t \sim 10^{10}$ year, the predicted fractional rate of decrease of G is one part in 10^{10} per year.

The above semiquantitative argument was put on a formal footing by Dirac through his *large numbers hypothesis* (LNH). This hypothesis states that any large number which at the present epoch t_0 has the order of magnitude $(t_0/t_e)^k$ where $t_e \equiv e^2/m_e c^3$, must be proportional to $(t/t_e)^k$ with some coefficient close to unity. The deduction (11) is thus a spcial case of LNH, for $k = -1$.

The motivation behind the LNH is to eliminate the 'coincidence' aspect of the large numbers. If G were fixed for ever, then the near equality of N_1 and N_2 would be seen only at the present epoch. At earlier epochs $N_1 \ll N_2$ while at later epochs $N_1 \gg N_2$. Unless one invokes some anthropic principle[7] to relate the epoch when $N_1 \sim N_2$ to our existence as living and intelligent observers, the result remains mysterious. The LNH, on the other hand ensures that the near equality $N_1 \sim N_2$ would hold at all epochs, past present, and future.

Before we proceed further with evolving cosmologies it is worth noting an interesting result from the steady state cosmology. This result discussed in detail by the author elsewhere[8] demonstrates that the conservation of energy at the time of creation of a charged particle does lead us to the equality $N_1 \sim N_2$ at all epochs. The argument briefly is as follows.

The net electrical neutrality of the universe requires that corresponding to N_3 protons there are also N_3 electrons. However, in a random distribution we expect an excess of $\sqrt{N_3}$ particles of one species over the other. This excess would result in a net electrical potential of magnitude

$$\phi \sim \sqrt{N_3}\,\frac{eH_0}{c} \tag{12}$$

If a new particle were to be created with charge e, the sign of its charge should be such that its electrical potential energy is *negative* and equal in magnitude to the rest mass energy of the particle created. Assuming the created particle to be the electron, we get

$$m_e c^2 \sim \sqrt{N_3}\,\frac{e^2 H_0}{c}, \qquad \text{i.e., } N_1 \sim \sqrt{N_3} \tag{13}$$

The result (7) follows from the dynamics of the steady state model as given by Hoyle and Narlikar.[9] Hence we also recover the equality $N_1 \sim N_2$.

Returning to Dirac's hypothesis, we note that the deduction (11) was based on the assumption that except G and H the other fundamental constants entering the equality $N_1 \sim N_2$ are fixed for all epochs. Now G and H are deduced from macroscopic observations while the rest of the constants, e, m_e, m_p, and $\hbar$ feature in microscopic theories. The constant c is somewhat ambiguous in its status: it is important for cosmology as well in local physics. Taking it as part of the 'micro' rather than 'macro' group since it denotes the speed of the microparticle, the photon, Dirac assumed c to be constant along with e, m_e, m_p and $\hbar$.

However, as Dirac pointed out in later papers, the constancy or otherwise of a physical quantity depends on the system of units used.[10,11] The above assumption of constancy of e, m_e, m_p, $\hbar$, and c presupposes the use of *atomic units.* The good agreement between observations of macroscopic gravitational phenomena and Einstein's theory led Dirac to postulate another set of units in which Einstein's equations hold *exactly.* In these *gravitational units,* G is constant. Obviously, to reconcile the variation of G in terms of atomic units with this conclusion, we have to assume that the ratio of the two units changes with epoch.

Dirac expressed this idea through the notion of two metrics, atomic, and gravitational, denoted respectively by ds_A and ds_E, the suffix E in the latter denoting the fact that in these units Einstein's equations are valid. The ratio

$$\beta = \frac{ds_E}{ds_A} \tag{14}$$

varies with epoch in accordance with the arguments just given.

The LNH, when applied to N_3 tells us that $N_3 \propto t^2$. That is, the number of particles in the universe increases with time. The theory therefore predicts continuous creation of new particles. Dirac distinguishes two types of creation. In *additive* creation particles are created uniformly throughout space while in *multiplicative* creation they are created preferentially where matter already exists. In the former case a typical astronomical body will not acquire any significant number of new particles while in the latter case it would do so in such a way that its particle number at epoch t is proportional to t^2.

A comparison with general relativity in the Einstein metric then leads us to the conclusion that for (11) to hold we must have

$$\beta \propto t, \qquad \text{for additive creation} \tag{15}$$

and

$$\beta \propto t^{-1} \tag{16}$$

for multiplicative creation.

Dirac cosmology therefore needs additional inputs to decide the function $\beta(t)$. Since laboratory measurements are made in atomic units, any gravitational effect involves knowing both G and β. Hence the interpretation of any experiment (or astronomical observation) leading to a measurement of $\dot{G}/G$ cannot be made without first specifying β. This has been emphasized by Canuto.[12] For example, an astronomical object is in equilibrium under its internal pressures and self gravity. If G weakens with time it is not necessary that the object should expand. For example, β might enter its equation of state in such a way that its internal pressures also weaken and the object might actually contract.

3. THE BRANS-DICKE COSMOLOGY

In 1961 C. Brans and R. H. Dicke[13] proposed another approach to cosmology based on Mach's principle. Since its inception in the 1890s, Mach's principle[14] has played a game of hide and seek with gravitation theories. Mach's original formulation of this principle has been qualitative rather than quantitative, and its subsequent interpretations by later physicists and philosophers have been many. Broadly speaking the principle implies that the inertial properties of matter are not entirely intrinsic but also depend on the large scale structure of the universe.

Both in Newton's and Einstein's theories of gravitation, inertial, and gravitational properties of matter are inextricably mixed. Thus in Newton's law (1) the same quantity m serves as the measure of inertia and as measures of how a body generates and responds to gravitational influence. In general relativity the principle of equivalence and the equality of inertial and gravitational masses express the link between inertia and gravitation. It is not surprising therefore that with its pronouncements about inertia, Mach's principle is also echoed in theories of gravitation. Indeed, in the early days Einstein himself hoped that general relativity would turn out to contain the essence of Mach's principle. This hope was not realized.

In the Brans–Dicke theory (BD) the authors' aim was to express G in terms of the large scale structure of the universe. To understand how this comes about let us first consider the problem of measuring inertia. The quantitative measure of inertia is mass and to express the magnitude of mass we need a suitable unit. How do we compare masses of particles at two different points in space-time?

A natural unit for mass emerges from the fundamental constants c, $\hbar$ and G. This is the *Planck mass* given by

$$\sqrt{\frac{\hbar c}{G}} \cong 2.16 \times 10^{-5}\, g \tag{17}$$

If m is the mass of a particle, the dimensionless number

$$\chi = m\sqrt{\frac{G}{\hbar c}} \tag{18}$$

expresses its magnitude in the units of Planck mass. If the quantity χ changes from point to point it could be due either to the change in the inertia of the particle or to the change in the magnitude of the Planck mass, or both. If, for example, through Mach's principle the inertia of a particle does change from point to point because of the change in its relationship to the rest of the universe, then an atomic physicist using a fixed mass unit and constant $\hbar$ and c would interpret a change in χ as a change in G.

The relationship betweė G and the large scale distribution of matter in the universe can be seen as follows. In standard Friedmann cosmology, the mean density of matter in the universe is given by

$$\rho = \frac{3H_0^2}{4\pi G} q_0 \tag{19}$$

where q_0 is the deceleration parameter. Writing $R = c/H_0$ as the characteristic length scale, and using Euclidean geometry, the mass contained in a cosmological sphere of radius R is given by

$$M = \frac{4\pi\rho}{3} R^3 = \frac{Rc^2}{G} q_0$$

i.e.,

$$1/G = M/q_0 Rc^2 \sim M/Rc^2 \tag{20}$$

since q_0 is believed not to be too small or large $(0.01 \lesssim q_0 \lesssim 1)$.

In the BD theory the above relation is looked upon as an outcome of individual contributions of the type m/r (from particle of mass m at distance r) to G^{-1}. Thus G is *not* a fundamental constant, but a quantity dependent on the large scale structure of the universe. Since $1/r$ type contribution comes out of the solution of a scalar wave equation, Brans and Dicke introduced a scalar field ϕ whose reciprocal behaves as G and whose sources lie in matter.

As in general relativity, the BD theory is formulated in Riemannian space-time. The field equations for the theory resemble those of general relativity with the addition of terms in the scalar field ϕ. Starting from an action principle, it is possible to show that the wave equation satisfied by ϕ is

$$\Box\phi = \frac{8\pi}{(2\omega + 3)c^4} T \tag{21}$$

where T is the trace of T^{ik} and ω is a coupling constant.

Since ϕ cannot be a constant in a nonempty universe (with the exception of the pure radiation case with $T = 0$) the BD theory predicts a variation of G with epoch. In the homogeneous isotropic cosmologies with the scale factor for expansion $S(t)$ at epoch t, we have

$$\dot{\phi}S^3 = \frac{8\pi}{(2\omega + 3)c^2} \int_0^t TS^3\, dt + C \tag{22}$$

In the simplest models with $C = 0$ we get

$$\dot{G}/G = -\frac{2}{3\omega + 4} \cdot 1/t \tag{23}$$

This may be compared with (11) for Dirac's cosmology. Solar system tests suggest that $\omega \gg 1$, so that the actual magnitude of $|\dot{G}/G|$ predicted by (23) is much smaller than that given by (11).

There are, however, other models in BD theory, with $C \neq 0$, which for large enough C predict $\dot{G}/G > 0$. Thus the interpretation of any observations cannot be made without reference to the model type.

4. THE HOYLE-NARLIKAR COSMOLOGY

In 1964, F. Hoyle and the present author[15] proposed a new theory of gravity which was inspired by their earlier work on action at a distance electrodynamics. It should be recalled that both gravity and electromagnetic theories started off as action at a distance theories. Experimental and conceptual difficulties, however, led the action at a distance concept to be replaced by the field concept. Thus theories of Newton and Coulomb gave way to those of Einstein and Maxwell.

Nevertheless subsequent work by Schwarzschild,[16] Tetrode,[17] Fokker,[18] Wheeler and Feynman,[19] and by Hogarth[20] demonstrated that classical electrodynamics in flat (and conformally flat) space-time at any rate can be successfully described by a revived action at a distance formulation. Subsequently, Hoyle and the author showed how to formulate the concept in surved space-time[21] and how to extend it to full fledged quantum electrodynamics.[22,23] We will not go into quantitative details here but highlight two conclusions that emerge from this work on electrodynamics.

The first point is a technical one and concerns the fact that for local Lorentz invariance the Newton–Coulomb notion of *instantaneous* action at a distance has to be replaced by that of action traveling at the speed of light. Technically this concept is best expressed by Green's functions of wave equations, which have support on and inside the light cones. For example, if s_{PQ}^2 is the invariant square of the distance between two points P and Q in flat Minkowski space-time, the quantity

$$\bar{G}(P, Q) = \delta(s_{PQ}^2)/4\pi \tag{24}$$

with δ, the Dirac delta function, forms the symmetric Green's function of the wave operator $\Box$. The appearance of the delta function in the action-at-a-distance integral ensures that P and Q interact only if they are connected by a light ray.

The second point is of profound significance since it demonstrates the importance of cosmology in any local problem of electrodynamics. This point arises from (24). Suppose $s_{PQ}^2 = 0$ and that in a chronologically ordered universe Q comes later than P. A signal from P traveling into the future reaches Q. The vanishing of s_{PQ}^2 also tells us that signal from Q traveling into the past reaches P. The equality of action and reaction between P and Q ensures that both future directed and past directed signals exist on equal footing. The presence of the latter, however, seems to conflict with causality.

Not quite! It was first shown by Wheeler and Feynman[19] that if the universe is a perfect absorber of all electromagnetic radiation, then causality is preserved. In fact, later work[20,21] clarified this condition in the context of the expanding universe. If only the future half of the universe (in relation to our local region) is a perfect absorber, then only future going signals exist. Similarly, if only the past half of the universe is a perfect absorber then only past going signals survive.

Clearly, to preserve causality only the future absorber must be perfect. This condition is satisfied in the steady state cosmology but not in the Friedmann (big bang) cosmologies. The latter conclusion is seen as follows. Let us take the example of the Einstein de Sitter cosmology whose line element may be written as

$$ds^2 = \Omega^2(\tau)[c^2\, d\tau^2 - dr^2 - r^2(d\theta^2 + \sin^2\theta\, s\phi^2)] \tag{25}$$

where $\Omega \propto \tau^2$. For any finite electron-proton density the opacity along the future light cone diverges (as required by the perfect future absorber condition) provided the integral

$$J = \int_{\tau_0}^{\infty} \frac{d\tau}{m_e \Omega(\tau)} \qquad \tau_0 = \text{present epoch} \tag{26}$$

diverges. For $m_e = \text{constant}$, $\Omega \propto \tau^2$, J *does not* diverge.

Thus, provided we have a model like the steady state model, the action at a distance electrodynamics can be made to work both at the classical and the quantum levels. Can a similar action-at-a-distance approach to gravity be formulated? We found that the answer to this question lies not in gravity directly but in inertia. The clue is provided by the inertial term in the standard action for a system of particles $a, b, \ldots$ of masses $m_a, m_b, \ldots$:

$$\mathscr{A} = \sum_a \int (-m_a)\, ds_a \tag{27}$$

If we take a Machian view and argue that the quantities $m_a, m_b, \ldots$ are not intrinsic to the particles $a, b, \ldots$ but arise from their interaction with other particles, then we can write for the typical particle a at world point A

$$m_a = \sum_{b \neq a} \int \lambda^2 \tilde{G}(A, B)\, ds_a \tag{28}$$

where $\tilde{G}(A, B)$ is a Green's function of some wave operator chosen suitably. λ is a coupling constant. Substituting (28) into (27) gives a manifestly symmetric structure to the inertial action:

$$\mathscr{A} = -\sum_{a<b}\sum \iint \lambda^2 \tilde{G}(A, B)\, ds_a\, ds_b \tag{29}$$

The discrete sums over particles can be replaced by volume integrals by introducing particle number density n.

The above action is defined in the Riemannian space-time. The gravitational equations follow from a variation of g_{ik} as in relativity. For details see Ref. 15.

Note that (28) allows us to contemplate *variable* particle masses. In a homogeneous but evolving universe the mass of a particle could vary with epoch. It further follows from the gravitational equations that G also could vary with epoch.

With these inputs, Hoyle and I reexamined the integral in (26) and concluded that if m_e, the electron mass varied with epoch, the integral J could be made divergent.

In the explicit solution obtained by us[24] the background space-time is Minkowskian so that $\Omega(\tau) = 1$ in (25). The coupling constant λ depends on τ in such a way that $\lambda\tau =$ constant. The actual epoch dependences of the various quantities are given below:

$$\lambda \propto \tau^{-1}, \qquad n \propto \tau, \qquad G \propto \tau^{-4}, \qquad m_e, \quad m_p \propto \tau \tag{30}$$

Substitution of the various constants of proportionality into (30) shows that

$$\lambda^2(\tau^3 n)^{1/2} \sim 0(1) \tag{31}$$

which is nothing but a restatement of the connection between the large dimensionless numbers described in section 1.

The cosmology described above will be referred to as HN cosmology. We first note that with $m_e \propto \tau$, the absorption integral J does diverge. However, if we wish to restate the HN cosmology in terms of Dirac's atomic units we have to make a conformal transformation[25] of the background metric. Using a conformal function $\Omega \propto \tau$ and a coordinate transformation $t \propto \tau^2$, we get the metric of the space time in atomic units as

$$ds_A^2 = c^2\, dt^2 - 2H_0 t[dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)] \tag{32}$$

In these units m_e and m_p are constant but G varies as t^{-1}, so that at the present epoch

$$\dot{G}/G = -2H_0 \tag{33}$$

5. DISCUSSION

We have here three examples of cosmologies which predict a gravitational constant that varies with epoch. The order of magnitude predicted for the present value of $|\dot{G}/G|$ is $\sim H_0$, the value of the Hubble constant at this epoch (although in the BD theory the predicted magnitude of $\dot{G}/G$ may be small for large ω and its sign could be even positive for some models). This result, in retrospect is hardly surprising since all three theories bring cosmological considerations into the determination of G.

The detailed equations (which we have avoided in this article) of the three theories show a similarity of structure. In all three cases the Riemannian space-time is used and the Einstein tensor

$$R_{ik} - 1/2(g_{ik}R)$$

of relativity appears. In all cases three is an additional scalar field, or its equivalent, which distinguishes the theory from general relativity. The tensor through which this scalar quantity enters the gravitational equations also has a formal structure that is broadly the same for all three theories. This similarity of structure arises because a conformal transformation is involved at some stage or other of the development of each theory.

The similarities end here. The motivation of each theory which leads it to a variable G is different. In Dirac's cosmology it is the equality of two large numbers that leads to an epoch dependent G. In the BD cosmology Machian considerations lead to the desired result while in the HN cosmology the guiding considerations came from action at a distance.

Gravity has stood apart from other interactions both in the unification program and in attempts at quantization. This isolation leads one to suspect whether G is in fact a fundamental constant, or even a quantity of purely local significance. The three approaches described above suggest the opposite, that cosmological considerations cannot be brushed aside in the determination of the value of G.

There are observational methods for checking the hypothesis that G varies with time. A direct laboratory measurement is still not possible but the progress of low temperature technology suggests that one day it will be feasible to measure the change in G over several years. The use of atomic clocks has enabled astronomers to decide, in principle whether there is any significant systematic change arising from G variation in the orbits of the Earth and the Moon.(26) Direct measurement by laser ranging can tell us in principle how the size of lunar orbit changes with time. Such measurements can be used to set an upper limit to $|\dot{G}/G|$. The changes in the structure of the Earth and in the solar terrestrial relationship, the early evolution of life on the Earth are indirect checks on G variation coming from geophysics.(27,28) Astrophysics also provides indirect checks through the effect of $\dot{G} \neq 0$ on stellar evoluation. How fast stars evolve can in turn be related to changes in the luminosity of a galaxy. Such results can be tested by the observations of luminosity evolution in galaxies.(25)

Coming to cosmology proper, the evolution in the spectrum of primordial background radiation could be relected in a nonPlanckian spectrum of the presently observed microwave background.(29) Since G variation affects the rate of expansion of the universe, the very early universe calculations and the applications of grand unified theories to cosmology are affected by this hypothesis.

Indeed, in present day physics it would be hard to find another hypothesis which, besides being provocative, has effects over such a wide spectrum of physics.

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