Hyperdiffusion in non-linear, large and small-scale turbulent dynamos

Kandaswamy Subramanian*
Inter University Centre for Astronomy and Astrophysics, Post bag 4, Ganeshkhind, Pune University Campus, Pune 411 007, India.
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The generation of large-scale magnetic fields is generically accompanied by the more rapid growth of small-scale fields. The growing Lorentz force due to these fields back reacts on the turbulence to saturate the mean-field and small-scale dynamos. For the mean-field dynamo, in a quasi-linear treatment of this saturation, it is generally thought that, while the alpha-effect gets renormalised and suppressed by non-linear effects, the turbulent diffusion is left unchanged. We show here that this is not true and the effect of the Lorentz forces, is also to generate additional non-linear hyperdiffusion of the mean field. A combination of such non-linear hyperdiffusion with diffusion at small scales, also arises in a similar treatment of small-scale dynamos, and is crucial to understand its saturation.

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Introduction: Large-scale magnetic fields in astrophysical bodies are thought to be generated by dynamo action involving helical turbulence and rotational shear [1]. For turbulent motions, with a large enough magnetic Reynolds number (R_m henceforth), this is also accompanied initially by the more rapid growth of small-scale fields, correlated on the tubulent eddy scales and smaller [1, 2, 3]. An important problem is to understand how the Lorentz forces due to these fields back-react on the turbulence and hence lead to mean-field and small-scale dynamo saturation.

Semi-analytic treatments of the back-reaction have typically used the quasi-linear approximation (see below) or closure schemes to derive corrections to the mean-field dynamo coefficients. It is then found that the α-effect gets "renormalised" by the addition of a term proportional to the current helicity of the small scale fields. But at the same time the mean field turbulent diffusion does not get affected, if one imposes the incompressibility condition on the velocity field (including the component induced by the Lorentz force) [4, 5]. This is perhaps somewhat intriguing, as one would have naively expected the Lorentz forces to affect all the transport coefficients. Further, if the effective "turbulent diffusion" does not get modified at all due to non-linear effects, one also wonders how the non-helical, small-scale dynamo would saturate at all? We clarify these two issues here.

Large-scale dynamo: The induction equation for the magnetic field is given by,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B}), \qquad (1)$$

where $\mathbf{B}$ is the magnetic field, $\mathbf{v}$ the velocity of the fluid, and η the ohmic resistivity. In the kinematic limit, it is usual to take $\mathbf{v} = \mathbf{v}_0 + \mathbf{v}_T$, the sum of an externally prescribed large scale velocity field $\mathbf{v}_0$ and a random field $\mathbf{v}_T$. Also $\mathbf{v}_T$ is generally assumed to be an isotropic, homogeneous, Gaussian random velocity field with zero mean, and have a short (ideally infinitesimal) correlation time τ (Markovian approximation). Splitting $\mathbf{B}$ into a mean (large-scale) magnetic field $\langle \mathbf{B} \rangle = \overline{\mathbf{B}}$ and a stochastic small-scale field $\mathbf{b} = \mathbf{B} - \overline{\mathbf{B}}$, one derives the mean-field dynamo equation [1],

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times (\mathbf{v}_0 \times \overline{\mathbf{B}} + \mathcal{E} - \eta \nabla \times \overline{\mathbf{B}}), \qquad (2)$$

Here $\mathcal{E} = \langle \mathbf{v} \times \mathbf{b} \rangle \approx \alpha_0 \overline{\mathbf{B}} - \beta_0 \nabla \times \overline{\mathbf{B}}$, is the turbulent EMF, where $\alpha_0 = -(\tau/3)\langle \mathbf{v}_T \cdot \nabla \times \mathbf{v}_T \rangle$ is the dynamo α-effect, proportional to the kinetic helicity and $\beta_0 = \tau \langle \mathbf{v}_T^2 \rangle /3$ is the turbulent magnetic diffusivity proportional to the specific kinetic energy of the turbulence. These equations predict the exponential growth of the mean magnetic field. One can also derive the equations for small-scale magnetic field correlations [1, 2, 3], which predict the exponential growth of small-scale fields on a shorter time scale. The kinematic theory then needs modification to take account of the back-reaction due to the growing Lorentz forces.

In the quasi-linear approximation [4, 5], this is done by assuming that the Lorentz force induces an additional non-linear velocity component $\mathbf{v}_N$, that is $\mathbf{v} = \mathbf{v}_0 + \mathbf{v}_T + \mathbf{v}_N$, with $\mathbf{v}_N$ satisfying the perturbed Euler equation

$$\rho(\partial \mathbf{v}_N / \partial t) = [\overline{\mathbf{B}} \cdot \nabla \mathbf{b} + \mathbf{b} \cdot \nabla \overline{\mathbf{B}}]/(4\pi) - \nabla p \qquad (3)$$

and $\nabla \cdot \mathbf{v}_N = 0$, where ρ is the fluid density and p the perturbed pressure including the magnetic field contribution. The turbulent EMF then becomes $\mathcal{E} = \langle \mathbf{v}_T \times \mathbf{b} \rangle + \langle \mathbf{v}_N \times \mathbf{b} \rangle$, where the quasi-linear correction to the turbulent EMF $\mathcal{E}_N = \langle \tau (\partial \mathbf{v}_N / \partial t) \times \mathbf{b} \rangle$. Here τ is again a correlation time assumed to be small enough that the time-integration (over the correlation time), can be replaced by simple multiplication. We ignore the $\langle b^3 \rangle$ contributions to $\mathcal{E}$, in the quasi-linear approximation, although these may indeed be negligible if the saturated small-scale field has a symmetric probability distribution. One expects this approximation to give a reasonable estimate of non-linear effects, when the mean field is still weak, and be also analytically tractable. Some support for the quasi-linear approximation also comes from EDQNM type closures of MHD [4].

We will calculate $\mathcal{E}_N$ in co-ordinate space representation. We can eliminate the pressure term in $\mathbf{v}_N$ using the incompressibility condition. Defining a vector $\mathbf{F} = a[\overline{\mathbf{B}}\cdot\nabla\mathbf{b} + \mathbf{b}\cdot\nabla\overline{\mathbf{B}}]$, with $a = \tau/(4\pi\rho)$, one then gets

$$\mathcal{E}_N = \langle \mathbf{F}\times\mathbf{b}\rangle - \langle[\nabla(\nabla^{-2}\nabla\cdot\mathbf{F})]\times\mathbf{b}\rangle, \tag{4}$$

where ∇^{-2} is the integral operator which is the inverse of the Laplacian, written in this way for ease of notation. We will write down this integral explicitely below, using $-1/4\pi r$ to be the Green function of the Lapalacian. We see that $\mathcal{E}_N$ has a local and non-local contributions.

To calculate these, we assume the small-scale field to be statistically isotropic and homogeneous, with a two-point correlation function $\langle b_i(\mathbf{x_1},t)b_j(\mathbf{x_2},t)\rangle = M_{ij}(r,t)$, where $\mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2$, $r = |\mathbf{r}|$ and

$$M_{ij} = M_N\left[\delta_{ij} - (\frac{r_ir_j}{r^2})\right] + M_L\left(\frac{r_ir_j}{r^2}\right) + H\epsilon_{ijf}r_f. \tag{5}$$

$M_L(r,t)$ and $M_N(r,t)$ are the longitudinal and transverse correlation functions for the magnetic field while $H(r,t)$ represents the (current) helical part of the correlations. Since $\nabla\cdot\mathbf{b} = 0$, $M_N = (1/2r)\partial(r^2M_L)/(\partial r)$. For later convenience, we also define the magnetic helicity correlation, $N(r,t)$ which is given by $H = -(N''+4N'/r)$, where a prime $'$ denotes derivative wirth respect to r. In terms of $\mathbf{b}$, we have $M_L(0,t) = \langle\mathbf{b}^2\rangle/3$, $2H(0,t) = \langle\mathbf{b}\cdot\nabla\times\mathbf{b}\rangle/3$ and $2N(0,t) = \langle\mathbf{a}\cdot\mathbf{b}\rangle/3$ (where $\mathbf{b} = \nabla\times\mathbf{a}$).

The local contribution to $\mathcal{E}_N$ is easily evaluated,

$$\mathcal{E}_N^L \equiv \langle\mathbf{F}\times\mathbf{b}\rangle = -aM_L(0,t)(\nabla\times\overline{\mathbf{B}}) + 2aH(0,t)\overline{\mathbf{B}} \tag{6}$$

At this stage (before adding the non-local contribution) there is indeed a non-linear addition to the diffusion of the mean field (the $-aM_L(0,t)(\nabla\times\overline{\mathbf{B}})$ term). Let us now evaluate the non-local contribution. After some algebraic simplification, this is explicitely given by the integral

$$\begin{aligned}(\mathcal{E}_N^{NL})_i(\mathbf{x},t) &\equiv -(\langle[\nabla(\nabla^{-2}\nabla\cdot\mathbf{F})]\times\mathbf{b}\rangle)_i \\ &= 2\epsilon_{ijk}\int\frac{d^3r}{4\pi}\frac{r_j}{r^3}\frac{\partial M_{mk}(\mathbf{r},t)}{\partial r^l}\frac{\partial\overline{B}_l(\mathbf{y},t)}{\partial y^m}\end{aligned} \tag{7}$$

where $\mathbf{y} = \mathbf{r}+\mathbf{x}$. Note that the mean field $\overline{\mathbf{B}}$ will in general vary on scales R much larger than the correlation length l of the small-scale field. We can then use the two-scale approach to simplify the integral in Eq. (7). Specifically, assuming that $(l/R) < 1$, or that the variation of the mean field derivative in Eq. (7), over l is small, we expand $\partial\overline{B}_l(\mathbf{y},t)/\partial y^m$, in powers of $\mathbf{r}$, about $\mathbf{r} = 0$,

$$\frac{\partial\overline{B}_l}{\partial y^m} = \frac{\partial\overline{B}_l}{\partial x^m} + rn_p\frac{\partial^2\overline{B}_l}{\partial x^m\partial x^p} + \frac{r^2n_pn_q}{2}\frac{\partial^3\overline{B}_l}{\partial x^m\partial x^p\partial x^q} + \ldots \tag{8}$$

where we have defined $n_i = r_i/r$ (we will soon see why we have kept terms beyond the first term in the expansion). Simplifying the derivative $\partial M_{mk}(\mathbf{r},t)/\partial r^l$ using Eq. (5) and noting that $\epsilon_{ijk}r_jr_k = 0$, we get

$$\begin{aligned}r_j\epsilon_{ijk}\frac{\partial M_{mk}}{\partial r^l} &= r_j\epsilon_{ijk}[\frac{(M_L-M_N)}{r}n_m\delta_{kl} + M_N'n_l\delta_{mk} \\ &+ H\epsilon_{mkl} + rH'n_fn_l\epsilon_{mkf}].\end{aligned} \tag{9}$$

We substitute (8) and (9) into (7), use $\int(d\Omega/4\pi)n_in_j = \delta_{ij}/3$, and $\int(d\Omega/4\pi)n_in_jn_kn_l = [\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}]/15$ to do the angular integrals in (7), to get

$$\begin{aligned}\mathcal{E}_N^{NL} = &+aM_L(0,t)(\nabla\times\overline{\mathbf{B}}) + \frac{6a}{5}N(0,t)\nabla^2\overline{\mathbf{B}} \\ &+\frac{2a}{5}\left[\int_0^\infty dr rM_L(r,t)\right]\nabla^2(\nabla\times\overline{\mathbf{B}})\end{aligned} \tag{10}$$

The net non-linear contribution to the turbulent EMF is $\mathcal{E}_N = \mathcal{E}_N^L + \mathcal{E}_N^{NL}$, got by adding Eq. (6) and Eq. (10). We see firstly that the non-linear diffusion term proportinal to $\nabla\times\overline{\mathbf{B}}$ has the same magnitude but opposite signs in the local (Eq. (6)) and non-local (Eq. (10)) EMF's and so exactly cancels in the net $\mathcal{E}_N$. This is the often quoted result [4, 5] that the turbulent diffusion is not renormalised by non-linear additions, in the quasi-linear approximation. However this does not mean that there is no non-linear correction to the diffusion of the mean field. Whenever the first term in an expansion is exactly zero it is neccessary to go to higher order terms. This is what we have done and one finds that $\mathcal{E}_N$ has an additional hyperdiffusion $\mathcal{E}_{HD} = \eta_{HD}\nabla^2(\nabla\times\overline{\mathbf{B}})$, where

$$\eta_{HD} = \frac{2a}{5}\int_0^\infty dr\ rM_L(r,t). \tag{11}$$

Taking the curl of $\mathcal{E}_N$, the non-linear addition to the mean-field dynamo equation then becomes,

$$\nabla\times\mathcal{E}_N = [\alpha_M + h_M\nabla^2]\nabla\times\overline{\mathbf{B}} - \eta_{HD}\nabla^4\overline{\mathbf{B}} \tag{12}$$

Here $\alpha_M = a\langle\mathbf{b}\cdot\nabla\times\mathbf{b}\rangle/3$ is the standard non-linear correction to the alpha-effect [4, 5], and $h_M = a\langle\mathbf{a}\cdot\mathbf{b}\rangle/5$ is an additional higher order non-linear helical correction derived here.

One can check that the hyperdiffusion coefficient η_{HD} is positive definite, by writing (11) in Fourier space. The longitudinal magnetic correlation is given in Fourier space by $M_L(r,t) = 2\int dkE_M(k,t)(j_1(kr)/kr)$, where $E_M(k,t)$ is the magnetic power spectrum, and $j_1(x)$ the spherical Bessel function. Using this relation one gets $\eta_{HD} = (4a/15)\int dk(E_m(k,t)/k^2)$, which is clearly positive definite. The magnitude of $\eta_{HD}\sim(2a/5)M_L(0,t)l^2$. So the importance of hyperdiffusion, relative to the turbulent diffusion is given by $(\eta_{HD}/R^4)/(\beta_0/R^2)\sim(2/5)(\mathbf{b}^2/4\pi\rho\mathbf{v}^2)(l/R)^2$. So for equipartition small scale fields, hyperdiffusion is only important in mean-field evolution, for moderate scale spearations $l/R < 1$. It could play an important role for example, in the "self-cleaning" evolution seen in simulations of Brandenburg [7], by causing a non-linear cascade of power from large-scale fields to nearby (in scale) smaller and smaller-scale fields. In case $l/R \ll 1$, the usual alpha-suppression [6], arising from helicity conservation (and consequent growth of α_M of the right sign to cancel α_0), is expected to lead to mean-field dynamo saturation, rather than hyperdiffusion. In both situations, since l/R is smaller than unity, non-linear corrections of higher order than hyperdiffusion are expected to be smaller by further factors of (l/R). Analogous effects are expected to be crucial for small-scale dynamo saturation, to which we now turn.

The non-helical small-scale dynamo: It is well known that small-scale magnetic fields can grow under the action of a stochastic velocity field $\mathbf{v}_T$, *even if the flow is non-helical*, provided R_m is greater than a critical value of order 100. The kinematic problem is well studied in the literature [2, 3]. We consider now how such a non-helical, small-scale dynamo saturates. For this one can neglect the subdominat effect of $\mathbf{v}_0$ and also the $\overline{\mathbf{B}}$ coupling to $\mathbf{b}$ (since $\mathbf{b}$ is expected to grow much faster than $\overline{\mathbf{B}}$). To model the effects of non-linearity, and in analogy to the above quasi-linear treatment, we assume that Lorentz forces due to the growing small-scale field induces as additional non-linear velocity component $\mathbf{v}_N$, satisfying an equation analogous to Eq. (3). That is now $\mathbf{v} = \mathbf{v}_T + \mathbf{v}_N$, with $\mathbf{v}_N = a[\mathbf{b}\cdot\nabla\mathbf{b} - \nabla p]$, and $\nabla\cdot\mathbf{v}_N = 0$. Here once again p includes the perturbed magnetic pressure. Using the incompressibility condition, one can again write $\mathbf{v}_N = \mathbf{v}^L + \mathbf{v}^{NL}$, as the sum of a "local" term $\mathbf{v}^L = a\mathbf{b}\cdot\nabla\mathbf{b} \equiv \mathbf{f}$, and the non-local "pressure" term $\mathbf{v}^{NL} = -\nabla(\nabla^{-2}\nabla\cdot\mathbf{f})$.

The stochastic Eq. (1) can now be converted into the evolution equation for M_L. The detailed derivation of this equation with a different non-linear velocity component (modelled as an ambipolar type drift) is given in [3]. The major difference here is the form of the non-linear term, which we evaluate explicitely below. We get

$$\frac{\partial M_L}{\partial t} = \frac{2}{r^4}\frac{\partial}{\partial r}(r^4\kappa\frac{\partial M_L}{\partial r}) + GM_L + K \qquad (13)$$

where we have defined $\kappa = \eta + T_L(0) - T_L(r)$, $G = -2(T_L'' + 4T_L'/r)$ and $T_L(r)$ is the longitudinal correlation function of $\tau\mathbf{v}_T$ defined analogous to M_L (see [3]). The diffusion κ includes microscopic diffusion (η), a scale-dependent turbulent diffusion ($T_L(0) - T_L(r)$) and the $G(r)$ term allows for the rapid dynamo generation of magnetic fluctuations by velocity shear [1, 2, 3].

The non-linear contribution is $K = (r_i r_j/r^2)K_{ij}$ where

$$\begin{aligned} K_{ij} = \quad & R^{(y)}_{jpq}(\langle[v^L_p(\mathbf{y}) + v^{NL}_p(\mathbf{y})]b_i(\mathbf{x})b_q(\mathbf{y})\rangle) \\ & + R^{(x)}_{ipq}(\langle[v^L_p(\mathbf{x}) + v^{NL}_p(\mathbf{x})]b_q(\mathbf{x})b_j(\mathbf{y})\rangle). \quad (14) \end{aligned}$$

Here the operator $R^{(x)}_{ipq} = \epsilon_{ilm}\epsilon_{mpq}(\partial/\partial x^l)$ and $R^{(y)}_{ipq} = \epsilon_{ilm}\epsilon_{mpq}(\partial/\partial y^l)$. In order to evaluate K, we need to deal with fourth moments of the fluctuating field $\mathbf{b}$. As in [3], we use a Gaussian closure to write these fourth moments in terms of the second order moments. The local contribution to K_{ij}, involving only the $\mathbf{v}^L$ terms in Eq. (14), can then be simply evaluated to give

$$K^L_{ij} = 2aM_L(0,t)\nabla^2 M_{ij}. \qquad (15)$$

The non-local contribution K^{NL}_{ij}, involving the pressure term, $\mathbf{v}^{NL}$, is again expressible as an integral, after using the Green function of ∇^2. The x and y-derivative terms in Eq. (14) give equal contributions and we get,

$$K^{NL}_{ij} = R^{(y)}_{jpq}\int 4a\frac{d^3u}{4\pi}\frac{u_p}{u^3}\frac{\partial M_{mq}(\mathbf{u},t)}{\partial u^l}\frac{\partial M_{li}(\mathbf{X},t)}{\partial X^m} \qquad (16)$$

where $\mathbf{X} = \mathbf{u} + \mathbf{y} - \mathbf{x} = \mathbf{u} + \mathbf{R} = \mathbf{u} - \mathbf{r}$.

Since $K_{ij} = K^L_{ij} + K^{NL}_{ij}$, one gets an integro-differential equation for the evolution of M_L, which is not analytically tractable. One can however make analytic headway in two limits $r = |\mathbf{x}-\mathbf{y}| \gg l$, and $r \ll l$, where $l(t)$ is now the length scale over which $M_L(r,t)$ is peaked. (For example, during kinematic evolution, $l = r_d \sim L/R_m^{1/2}$, where L is the velocity correlation length [2, 3]). For $r \gg l$, the integral (16) can then be evaluated by taking the limit $r = |\mathbf{x}-\mathbf{y}| \gg u$, and again expanding $\partial M_{li}(\mathbf{X},t)/\partial X^m$, in powers of $\mathbf{u}$, about $\mathbf{u} = 0$. So

$$\begin{aligned} \frac{\partial M_{li}(\mathbf{X})}{\partial X^m} = \quad & \frac{\partial M_{li}(\mathbf{R})}{\partial R^m} + un_s\frac{\partial^2 M_{li}}{\partial R^m \partial R^s} \\ & + \frac{u^2 n_s n_t}{2}\frac{\partial^3 M_{li}}{\partial R^m\partial R^s\partial R^t} + \ldots \quad (17) \end{aligned}$$

where we have defined now $n_i = u_i/u$. Substituting the above expansion into Eq. (16), using again Eq. (5) to simplify the derivative $\partial M_{mq}(\mathbf{u},t)/\partial u^l$, and evaluating the angular integrals with the help of various moments of n_i defined above, we get

$$K^{NL}_{ij}(r,t) = -2aM_L(0,t)\nabla^2 M_{ij} - 2\eta_{HD}\nabla^4 M_{ij}. \quad (18)$$

We see that, for $r \gg l$, K^{NL}_{ij} again has a diffusion term which exactly cancels the corresponding term in K^L_{ij}, leaving behind pure non-linear hyperdiffusion, that is $K_{ij} = K^L_{ij} + K^{NL}_{ij} = -2\eta_{HD}\nabla^4 M_{ij}(r,t)$. (Also no odd derivative terms appear in the absence of helicity). So

$$K(r,t) = -\frac{2\eta_{HD}}{r^4}\frac{\partial}{\partial r}\left[r^4\frac{\partial}{\partial r}\left(\frac{1}{r^4}\frac{\partial}{\partial r}(r^4\frac{\partial M_L}{\partial r})\right)\right]. \quad (19)$$

Now consider the other limit $r \ll l$. In this limit, one can assume $r \ll u$ in Eq. (16), and expand around $r = 0$. In fact since the first term (with $r = 0$) neither vanishes, nor cancels K^L_{ij} exactly (see below), we need to only keep this term. Putting $r = 0$ in Eq. (16) for K^{NL}_{ij}, straightforward but tedious algebra gives,

$K^{NL}_{ij} = 8\delta_{ij}\int(du/u)\ (M'_L)^2$. Again adding the local and non-local terms we get

$$K(r,t) = 2aM_L(0,t)\frac{1}{r^4}\frac{\partial}{\partial r}(r^4\frac{\partial M_L}{\partial r}) + 8\int_0^\infty \frac{du}{u}(M'_L)^2 \quad (20)$$

Note that now for $r \ll l$, K is still in the form of a non-linear diffusion, albeit with a partial cancellation due to the addition of the positive definite K^{NL} contribution. One can check for specific forms of M_L, that the above $K(r,t)$ does indeed lead to non-linear dissipation. For example, for a model $M_L(r,t) = M_L(0,t)\exp(-r^2/l^2)$, strongly peaked about l, we get $K(0,t) = -24aM_L^2(0,t)/l^2$ and even for $r \sim l/2$, K is still negative $\sim -6aM_L^2(0,t)/l^2$. Of course as one goes to larger $r \sim l$, one has to keep higher order terms in the expansion around $r = 0$.

From Eq. (19) and (20) we see that the back reaction on the non-helical small-scale dynamo, due to the growing Lorentz force, can be characterised in this model problem, as non-linear diffusion for small $r \ll l$ (yet partially compensated by a constant), transiting to non-linear hyperdiffusion for $r \gg l$. And the damping of M_L in both regimes have damping coefficients which are themselves proportional to the magnetic energy density, or $M_L(0,t)$. This means that as the small-scale field grows and $M_L(0,t)$ increases, the non-linear damping would increase leading eventually to a saturated state. The properties of the saturated state requires detailed numerical solution, which we hope to return to elsewhere. But taking a clue from our earlier work [3], where one had purely additional non-linear diffusion, the stationary state could have a correlation function, which corresponds to a "ropy" small scale field, and with peak magnetic fields of order the equipartion value. The way this is altered due to hyperdiffusion at larger r, albeit where M_L is subdominant, will be interesting to examine.

Conclusion: We have examined here consequences of one popular model of non-linear back reaction on the dynamo. For the mean-field dynamo, it has been thought that turbulent diffusion is not renormalised at all by the Lorentz forces. We have clarified that this is valid only at the lowest order, and at a higher order (to which one must go if the lower order term is exactly zero), one gets additional non-linear hyperdiffusion of the mean field [8]. Such hyperdiffusion may not be crucial for the mean field dynamo saturation, for $l/R \ll 1$. But it could have interesting consequences for how the field eventually "self-cleans" (or orders) itself during saturation. Further, when a similar model is applied to discuss the saturation of the non-helical small-scale dynamo, one obtains an intriguing combination of non-linear diffusion and hyperdiffusion, which governs how the small-scale fields reach a saturated state, due to the back-reaction of the Lorentz force. It remains to solve the above equations numerically and also elucidate the conditions when other possible models [3, 11] for dynamo saturation are applicable.

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* Electronic address: kandu@iucaa.ernet.in

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