

ORIGIN AND INTERPRETATION OF KILOHERTZ QPOS FROM STRANGE STARS IN X-RAY BINARY SYSTEM: THEORETICAL HYDRODYNAMICAL DESCRIPTION

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ABSTRACT

We model and interpret the Kilohertz QPOs from the hydrodynamical description of accretion disk around a rapidly rotating compact strange star. The higher QPO frequency is described by the viscous effects of accretion disk leading to shocks, while the lower one is taken to be the Keplerian motion of the accreting matter. Comparing our results with the observations for two of the fastest rotating compact stellar candidates namely, 4U 1636–53 and KS 1731–260, we find that they match to a very good approximation, thus interpreting them as strange stars.

Subject headings: accretion, accretion disks — hydrodynamics — shock waves — stars: neutron — stars: individual (4U 1636-53, KS 1731-260) — X-rays: binaries

1. INTRODUCTION

In X-ray binaries, matter is transferred from a normal star to a compact star. As the characteristic velocities near the compact object are of order $(GM/R)^{1/2} \sim 0.5c$, the (dynamical) time scale for the motion of matter through the emitting region, is short. So, the significance of millisecond X-ray variability from X-ray binaries is clear from the phenomenon that milliseconds is the natural time scale of the accretion process in the X-ray emitting regions, and hence strong X-ray variability on such time scales is certainly caused by the motion of matter in these regions. Orbital motion, stellar spin, disk and the stellar oscillations are all expected to happen on these time scales.

In recent years, kilohertz quasi-periodic oscillation (kHz QPOs) peaks, with a certain width, have been found in the power spectra of more than 20 low-mass binaries (LMXBs). In most of the sources, the kHz QPOs are found in pairs and at least one of the peaks should reflect the orbital motion of matter outside, but not too far from the compact star, in near-Keplerian orbits.

Another high frequency phenomenon, namely nearly coherent oscillations which slightly drift in frequency, was detected during some type I X-ray bursts. These so-called burst oscillations have been detected in the power spectra of some ten sources and their X-ray flux modulation is consistent with being due to the changing aspect of a drifting hot spot on the surface of the compact star. Therefore, these burst oscillations are thought to reflect the spin frequency of the star.

In a very important work, Jonker et al. (2002, hereinafter JMK) report on high frequency QPO phenomena that have been observed in the LMXBs, particularly 4U 1636–53; two kHz QPOs (Zhang et al. 1996a; Wijnands et al. 1997) as well as a sideband to the lower of the two kHz QPOs (Jonker et al. 2000) and burst oscillations with frequency $\nu_b = 581$ Hz (Zhang et al. 1996b; Strohmayer et

al. 1998). Miller (1999) presented evidence that these oscillations are in fact the second harmonic of the spin of the star ~ 290.5 Hz. However, using another data set these findings were not confirmed (Strohmayer 2001).

Since the frequency difference between the two kHz QPO peaks (the peak separation $\Delta\nu$) is nearly equal to half the burst oscillation frequency, a beat frequency model was proposed for the kHz QPOs (Strohmayer et al. 1996). In such a model, the higher frequency QPO is attributed, as in most other models, to the orbital frequency of chunks of plasma at a special radius near the compact star while the lower QPO is due to beat between the orbital frequency and the stellar spin frequency. A specific model incorporating the beat frequency mechanism, the sonic-point beat frequency model, is due to Miller et al. (1998). The model faced criticism in the work of Méndez et al. (1998) when the $\Delta\nu$ was found to be less than half the burst oscillation frequency ν_b . This could be explained in the sonic-point model (Lamb & Miller 2001) by taking into account the inward plasma velocity (see JMK for details). However, in the recent observations of JMK, the $\Delta\nu$ is significantly larger than half of ν_b causing further problems. The only change in the flow pattern previously described that could produce the observed change in $\Delta\nu$ is that in which the accretion disk changes from prograde to retrograde when the lower kHz QPO is seen to move from 750 to 800 Hz.

In contrast to the beat frequency, the sonic-point and other models, Osherovich & Titarchuk (1999) and Titarchuk & Osherovich (1999) take the inner QPO to be the Keplerian accretion disk, leading to a substantially smaller radius for the compact object that is hard to explain from any of the known neutron star models. Li et al. (1999) showed that the observed compact star 4U 1728–34 can be fitted to a strange star model that uses the realistic strange matter equation of state of Dey et al. (1998). The higher QPO in the Osherovich & Titarchuk (1999)

model is due the modification of Keplerian frequency under the influence of the Coriolis force in a rotating frame of reference.

The present letter deals with a model which takes the lower QPO to be the Keplerian motion of the accreting fluid around compact strange star, rotating very fast. It also seeks to explain the higher QPO as being due to viscous effects that lead a shock formation (Chakrabarti 1989, 1996; Molteni et al. 1996b, hereinafter MSC) and solves the hydrodynamic equations in the presence of a *pseudo-Newtonian potential* (Mukhopadhyay 2002a; Mukhopadhyay & Misra 2003, hereinafter MM) which describes the relativistic properties of accretion disk close to the compact object with an effective cutoff for an appropriate boundary condition that needs the accreting particles to go to asymptotic zero velocity. The same model applied to black holes would allow them to get to luminal velocities.

The formation of shock in accretion disk around compact object was discussed by several independent groups (e.g. Sponholz & Molteni 1994; Nabuta & Hanawa 1994; Molteni et al. 1994; Yang & Kafatos 1995; Chakrabarti 1996; Molteni et al. 1996a; MSC; Lu & Yuan 1997; Chakrabarti & Sahu 1997) either by analytical approach or numerical simulations. Recently, Mukhopadhyay (2002b) showed, even two shocks are possible to form in accretion disk around neutron star (or other compact object which has hard surface). It was shown in MSC that the shock location may oscillate in the disk and this oscillatory nature is directly related to the cooling and advective time scale of the matter. Subsequently it was shown that corresponding oscillation frequency is related to the location of the shock and the observed QPO frequency for the various black hole candidates could be explained. They (MSC) showed the theoretical calculations for QPO tally with that of observation for black hole candidate (which is of the order of Hz), say GS 339-4 and GS 1124-68. As the shock location comes closer to the compact object, the QPO frequency increases (MSC). In case of accretion flow around compact object other than black hole, the shock(s) may form even closer to the compact object with respect to the case of black hole (Mukhopadhyay 2000b). Thus the corresponding QPO frequencies are expected to be higher as of kilohertz order which could explain the origin of observed higher frequency (HF) oscillations of kilohertz QPO for compact objects.

Recently, MM prescribed a couple of potentials to describe the time varying relativistic properties of accretion disk around compact objects. They considered the Keplerian accretion disk and proposed modified gravitational force which could give the Keplerian angular frequency of the accreting matter around the compact object. As mentioned above, the lower frequency (LF) oscillation of the kilohertz QPO may arise due to the Keplerian motion of the accreting matter towards compact object, and if we know the radius of the Keplerian orbit, following MM we can calculate the LF.

In this letter we will calculate the pair of kHz QPOs for fast rotating candidates mainly for 4U 1636–53 ($\nu = 582\text{Hz}$) and KS 1731–260 ($\nu = 523.92\text{Hz}$). To calculate HF, we will more or less follow MSC but with the necessary modification using the model of MM due to the rotation of compact object (the work of MSC was confined with non-rotating black holes only). For LF, we will follow MM

where the *pseudo-Newtonian potentials and corresponding forces* are given to describe the Keplerian motion of the accreting matter. In next section, we will briefly describe the basic viscous set of accretion disk equation and formalism to calculate QPO frequencies. Then in §3, we will tabulate QPO frequencies from our theory and compare with observation and finally in §4, we will present a summary.

2. BASIC EQUATIONS AND FORMALISM

We will follow Chakrabarti (1996) to describe the viscous set of equations for accretion disk which are given as

$$\begin{aligned} -4\pi x \Sigma v &= \dot{M}, & v \frac{dv}{dx} + \frac{1}{\rho} \frac{dP}{dx} - \frac{\lambda^2}{x^3} + F(x) &= 0, \\ v \frac{d\lambda}{dx} &= \frac{1}{\Sigma x} \frac{d}{dx} \left[x^2 \alpha \left(\frac{I_{n+1}}{I_n} P + v^2 \rho \right) h(x) \right], \\ \Sigma v T \frac{ds}{dx} &= \frac{v h(x)}{\Gamma_3 - 1} \left(\frac{dP}{dx} - \Gamma_1 \frac{p}{\rho} \frac{d\rho}{dx} \right) = Q^+ - Q^-. \end{aligned} \quad (1)$$

Here, throughout our calculations we express the radial coordinate in unit of GM/c^2 , where M is mass of the compact star, G is the gravitational constant and c is the speed of light. We also express the velocity in unit of speed of light and the angular momentum in unit of GM/c . Following Cox & Giuli (1968), we define Γ_1, Γ_3 and to define vertically integrated density, Σ and pressure, W follow Matsumoto et al. (1984). β and $h(x)$ are defined as the ratio of gas pressure to total pressure and half-thickness of the disk, $h(x) = c_s x^{1/2} F^{-1/2}$ respectively. We follow Mukhopadhyay (2002a) and MM to define the effective gravitational pseudo-Newtonian force, $F(x)$. We consider the adiabatic flow, where the speed of sound is given as $c_s^2 = \frac{\gamma P}{\rho}$, related to the temperature (T) of the system as, $c_s = \sqrt{\frac{\gamma k T}{m_p}}$. We consider the magnetic field strength in the disk to be negligible compared to the viscous effect and the heat evolved is solely due to viscosity which is defined as $Q^+ = \frac{W_{x\phi}^2}{\eta}$ where $W_{x\phi}$ and η are the viscous stress tensor and coefficient of viscosity respectively. For simplicity, the heat lost is considered proportional to the heat gained by the flow. Thus the net heat contained in the flow is chosen to be $Q^+ f$, where f is the cooling factor, which is close to 0 and 1 for cooling and heating dominated flow respectively. α is Shakura-Sunyaev (Shakura & Sunyaev 1973) viscosity parameter, s is entropy density. To get the thermodynamic properties of disk, we need to solve (1).

It is shown in MSC that when shock is formed in accretion disk, the cooling (t_{cool}) and advection (t_{adv}) time scale of matter from the shock location to the inner edge of the disk are responsible for the oscillatory behaviour of shock that is related to the QPO. When the physical condition of the disk is such that $t_{\text{cool}} \sim t_{\text{adv}}$, corresponding QPO can be found and that oscillation frequency is of the order of $1/t_{\text{adv}}$. Therefore, we can calculate

$$t_{\text{adv}} = \int_{x_s}^{x_{\text{in}}} \frac{dx}{v} \quad (2)$$

where, x_s and x_{in} are the location of shock and inner edge of the disk respectively. In our case, x_{in} is the outer radius of the compact object.

Therefore, to calculate the QPO frequency, the detailed knowledge of the thermodynamic character of the disk from shock to the surface of the compact object is very essential. Once this is known for a particular accretion flow around compact object, the corresponding QPO frequency can be calculated which is basically HF. Mukhopadhyay (2002b) already indicated that the kilohertz QPO can be calculated from the accretion disk model around slowly rotating neutron star. Here we would like to consider the rapidly rotating compact star which results the shift in shock location with respect to that of non-rotating cases and calculate the QPO frequency for different viscosity.

MM proposed a couple of pseudo-Newtonian potentials to describe the time varying properties of the Keplerian accretion disk. On the other hand, LF may arise due to the Keplerian motion of the accreting fluid. One of the potentials proposed by MM, that can describe the Keplerian angular frequency of the accretion flow is given as

$$2\pi\nu_K = \Omega_K = \frac{1}{x^{3/2}} \left[1 - \left(\frac{x_{\text{ms}}}{x} \right) + \left(\frac{x_{\text{ms}}}{x} \right)^2 \right]^{1/2}. \quad (3)$$

Here x_{ms} is the radius of marginally stable orbit, for Kerr geometry that was given by Bardeen (1973) as

$$\begin{aligned} x_{\text{ms}} &= 3 + Z_2 \mp [(3 - Z_1)(3 + Z_1 + 2Z_2)]^{1/2} \\ Z_1 &= 1 + (1 - J^2)^{1/3} [(1 + J)^{1/3} + (1 - J)^{1/3}], \\ Z_2 &= (3J^2 + Z_1^2)^{1/2}, \end{aligned}$$

where the ‘-’ (‘+’) sign is for the co-rotating (counter-rotating) flow and J (which varies from 0 – 1) is the specific angular momentum of compact object. In our present work, we propose ν_K as the lower kilohertz frequency (LF) that varies with the angular frequency of the compact object. If we know the angular frequency of the compact object and the Keplerian radius of the corresponding accretion disk, LF can be calculated for that particular candidate.

3. QPO FREQUENCIES

The observed range of frequency of the QPOs for different candidates are quite large. However, there exists a relation between the LF and the HF which relates them to the intrinsic characteristic of the central compact object like the mass-radius relation, spin, etc. Also it has been seen that for a particular candidate, there is a variability in the frequency range for different times of observation, however keeping the co-relation between the HF and the LF constant. We encash upon these to our present interpretation.

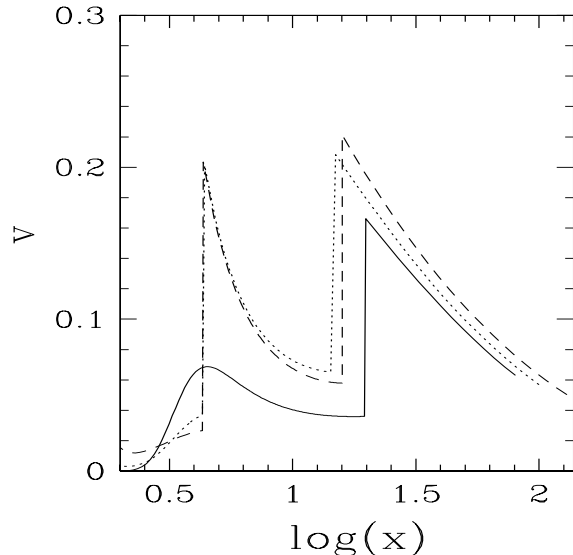


Fig. 1: Variation of accretion speed in unit of light speed as a function of radial coordinate around 4U 1636-53. Solid, dotted and dashed curves are respectively for (i) $\alpha = 0, f = 0$, (ii) $\alpha = 0.02, f = 0.2$, (iii) $\alpha = 0.05, f = 0.5$. Other parameters are $J = 0.2877, \lambda_c = 3, \dot{M} = 1, \beta = 0.03$.

Our present consideration will mainly be focussed upon the QPOs of 4U 1636-53 and KS 1731-260 which are among the fast rotating compact stars having angular frequency $\sim 582\text{Hz}$ ($J = 0.2877$) and $\sim 524\text{Hz}$ ($J = 0.2585$) respectively. Both these stars have displayed long lasting superbursts. Long lasting superbursts find a natural explanation in the diquark surface pairing after these pairs are broken (Sinha et al. 2002). Thus, it is justified to invoke the realistic strange star model for these stars and our inputs, namely the star mass and radius is taken from this model of Dey et al. (1998). We estimate the LF from the stellar frequency and the Keplerian radius of the disk. As HF is directly related to the accreting matter speed, in Fig. 1, we show the variation of accretion speed, for various viscosities of the accretion disk, around 4U 1636-53. As the angular frequency of KS 1731-260 is close to 4U 1636-53, profile of matter speed around KS 1731-260 does not differ significantly from that around 4U 1636-53 and is not shown. Obviously, we consider those cases where shock does form in the accretion disk which is responsible for HF. In case of inviscid flow, there is only a single shock formation in the accretion disk, while for the viscous flow, shock is formed twice, the shock locations being listed in Table-1. Now following (2) and (3) we can calculate HF and LF for various physical parameters of the disk and compact objects. The inner boundary condition of the model decelerates the speed of the infalling matter close to the stellar surface, and hence loses its energy. This in turn reduces the temperature of the falling matter to a considerable amount and hence the contribution of the energy transfer to the surface burst phenomena (Kuulkers 2002) is negligible.

In the same table, it is shown that the values of ‘ α ’ when varied from 0 to 0.05 reproduce an admissible value of the QPO frequencies. At this point, it is necessary to mention that the mass-radius relation of the central compact source also plays a vital role in the determination of the

Table-1

source	α	f	r_s (km)	r_k (km)	J	M (M_\odot)	R (km)	ν_h Hz	ν_K Hz	observed	
										HF(Hz)	LF(Hz)
4U 1636–53	0.05	0.5	15.37	18	0.2877	1.18	7.114	1030.8	719.7	1030	700
	0.02	0.2	14.27	19	0.2877	1.32	7.23	1019.2	705.2		
	0	0	13	17	0.2877	0.991	6.828	1005.2	715.2		
KS 1731–260	0.05	0.5	15.37	15.2	0.2585	1.106	7.013	1155.4	907.6	1159	898
	0.02	0.2	14.27	16	0.2585	1.23	7.16	1174	892.6		
	0	0	13	14.5	0.2585	0.893	6.64	1151.9	863.1		

frequencies. Compact objects with hard surface are generally considered as neutron stars, but in our study we find that the mass-radius relation obtained from the neutron star equations of state (EOS) are not satisfactory enough to reproduce the matching QPOs. They need to be more compact and hence we had to look for an alternative to the conventional neutron star EOS. As already hinted, recent developments in the theory of strange stars and their EOS gave us the alternative choice. We choose the EOS (SS1 as in Li et al. 1999) for compact strange star adapted from the model of Dey et al. (1998). In Table-1, we enlist the values of QPO frequencies for both the candidates 4U 1636–53 and KS 1731–260 for different cases of variation of the parameters. The corresponding chosen mass and radius of the star are all in the ‘allowed range’ as per the model of Dey et al. (1998). The ν_h s are calculated from fluid dynamical model and compared with observed HF. Also, ν_K s are calculated theoretically and compared with observed LF. It is very exciting to see that our calculated ν_h and ν_K are very close to the observed HF and LF respectively. The results not only reflect the success of the model, but also reinforces the claim of the existence of strange stars.

Considering the stellar spin frequency is half of the assumed values (Titarchuk 2003) the change of ν_K is negligible ($\ll 1\%$) (eq. [3]). In this situation that matter will adjust itself in the disk at the cost of changing other physical parameters, namely, angular momentum (λ), sonic points etc. (Mukhopadhyay 2003) to keep unchanged the shock location as well as ν_h , at least for $|J| \leq 0.3$. Similarly, with the presence of moderate magnetic field the shock can be formed at a same location for a different choices of λ , α , s etc.

4. SUMMARY

It is beyond doubt that the origin of the kilohertz QPOs is the accretion disk centering around a compact object. We model here, the source of origin of the kilohertz QPOs in the light of fluid dynamical calculation of the inflowing matter in the accretion disk. It is perhaps for the first time such a theoretical calculation is presented which matches with the results of the observation to a very satisfactory limit. Although we have used here two of the fastest rotating candidates, 4U 1636–53 and KS 1731–260, to compare our results and also identify them as strange stars rather than conventional neutron stars, a detailed study of all the other candidates in this model is necessary. The choice of the two above mentioned stars was also motivated by the fact that they have recently displayed very long lasting superbursts (Sinha et al. 2002).

Out of the QPO pair, higher one is directly related to the accretion flow around the compact object. If shock is formed in the accretion disk and the advective time-scale is of the same order as the cooling time-scale in the disk from the shock location to the stellar surface, we immediately expect higher kHz QPO. In earlier works, theoretical calculations based on this idea, had been carried out for the black hole candidates GS 339–4 and GS 1124–68 (MSC). In the present letter, similar procedure is applied for other stellar candidates and with a modification for inclusion of rotation of the compact objects. The results ended up in the discovery of those candidates as strange stars. Lower QPO frequency arises, when we consider the Keplerian motion of the accretion fluid. The most important aspect of our work is that we have not made use of any ‘toy-model’ to describe the QPOs, but they arrived naturally from the hydrodynamical calculations.

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