Search for continuous gravitational wave signals from sources in binary systems

Sanjeev V. DHURANDHAR *
Alberto VECCHIO
Max Planck Institut für Gravitationsphysik
Albert Einstein Institut
Am Mühlenberg 1, Golm
D-14476, Germany.
* On sabbatical leave from IUCAA, Pune, India.

Abstract

We analyze the computational costs of searches for continuous monochromatic gravitational waves emitted by rotating neutron stars orbiting a companion object. As a function of the relevant orbital parameters, we address the computational load involved in targeted searches, where the position of the source is known; the results are applied to known binary radio pulsars and Sco-X1.

1. Introduction

The search for continuous wave (CW) sources – rapidly rotating neutron stars (NS) that emit quasi-monochromatic gravitational waves (GW's) – is one of the most computationally intensive tasks for the data analysis of GW detectors. Surveys of wide areas of the sky and/or large frequency and spin-down ranges are computationally bound (Brady et al., 1998); the only viable strategy is to set up hierarchical algorithms, where "coherent" and "incoherent" stages are alternated in order to maximize the signal-to-noise ratio (SNR), based on the CPU power available (Brady and Creighton, 1999; Schutz and Papa, 1999).

The algorithms investigated so far deal only with *isolated* sources. To search for a NS orbiting a binary companion has always been considered computationally intractable, as one would need to correct also for the Doppler phase shift caused by the orbital motion of the GW source around a companion object (star, back hole, or planet): the maximum time of coherent integration, before the signal power is spread over more than one frequency bin, is between ~ 200 sec (for a source like the Hulse-Taylor binary pulsar) and $\sim 10^5$ sec (for a NS with a

Jupiter-size planet in a 4-months period orbit). This would add five more search parameters. Nonetheless, there are several important reasons for start addressing this problem at this time: (i) we would like to quantify what "intractable" means, and estimate the computational costs as a function of the search parameters; (ii) the only known GW source in the high-frequency band is Sco X-1, a NS orbiting a low-mass companion (Bildsten, 1998); if our current astrophysical understanding is correct, Sco X-1 would be detectable by GEO600 (in narrow-band configuration) and LIGO, at a SNR$\simeq$ 3 in two years of full coherent integration; (iii) the continuous monitoring of all known NS's is planned, and around 50 radio pulsars are in a binary system; (iv) we are now starting the design of software codes to search for CW sources during the first science runs carried out by the detectors, but their general structure is likely to be used for several years.

The aim of this contribution is to estimate the additional processing power needed to correct for the orbital motion of a CW source orbiting a binary companion, with emphasis on targeting known NS's.

2. Signal model and data analysis

In order to disentangle the extra computational costs involved in dealing with the NS orbital parameters, we will make the following assumptions: the source location in the sky is exactly known – so that one can perfectly remove the phase Doppler shift due to the detector motion – and the signal is monochromatic, at frequency f_0. For a general blind search of NS's possibly in binary orbits, the total computational burden would be (roughly) the product of the one quoted for isolated sources, times the estimate that we present here.

The gravitational waveform is given by

$$h(t, \vec{\lambda}) = \Re\{\mathcal{A}\, e^{-i\left[2\pi f_0 t + \phi_D(t; f_0, \vec{\lambda}) + \Psi\right]}\}, \tag{1}$$

where $\mathcal{A}$ and Ψ are assumed (as usual) constant, and ϕ_D is the Doppler phase modulation induced by the orbital motion of the source around the companion; $\boldsymbol{\lambda} = (f_0, \vec{\lambda})$ it the signal parameter vector. We assume that the orbit is Keplerian, and elliptical in shape. The Doppler correction to the phase of the signal due to the orbital motion is therefore:

$$\phi_D(t; f_0, \vec{\lambda}) = -\frac{2\pi f_0 a \sin\epsilon}{c}\left[\cos\psi \cos E(t) + \sin\psi\sqrt{1-e^2}\sin E(t)\right], \tag{2}$$

where ϵ and ψ are the polar angles describing the direction to the detector, with respect to an appropriate reference frame attached to the binary system, c is the speed of light and E is the eccentric anomaly. It is related to the mean angular velocity $\omega \equiv 2\pi/P$ (where P is the orbital period) and the mean anomaly M by the Kepler equation: $E - e\sin E = \omega t + \alpha \equiv M$, where α is an initial phase, $0 \leq \alpha < 2\pi$.

The additional parameters on which one must launch a search are therefore five in the elliptical case, say $\vec{\lambda} = (a_p, \omega, \alpha, e, \psi)$, where $a_p \equiv a \sin\epsilon$, and three in the circular orbit case, say $\vec{\lambda} = (a_p, \omega, \alpha)$. As usual, f_0 is not a search parameter which requires a filter mesh.

A rigorous way of estimating the search costs can be worked out by approaching the data analysis through a geometrical picture (Sathyaprakash and Dhurandhar, 1991; Dhurandhar and Sathyaprakash, 1994; Balasubramanian et al., 1996; Owen 1996; Brady et al., 1998): the signal is a vector in the vector space of data trains and the n-parameter family of signals traces out an n-dimensional manifold which is termed as the signal manifold. On this manifold one introduces a proper distance – and therefore a metric γ_{ij} – defined as the fractional loss of SNR – the *mismatch* μ – caused by the wrong choice of the filter parameters. The spacing of the grid of filters is decided by the fractional loss due to the imperfect match that can be tolerated. Fixing the mismatch μ, fixes the grid spacing of the filters in the parameter space $\mathcal{P}$. The number of filters N is then just:

$$N = \left[\frac{1}{2}\sqrt{\frac{n}{\mu}}\right]^n V_{\mathcal{P}}\,, \qquad V_{\mathcal{P}} = \int_{\mathcal{P}} \sqrt{\det ||\gamma_{ij}||}\, d\vec{\lambda}\,, \tag{3}$$

where $V_{\mathcal{P}}$ is the proper volume, and $n = 5\,(3)$ for eccentric (circular) orbits.

3. Computational Costs

The general expressions of $V_{\mathcal{P}}$ and N, with signal model given by Eqs. (1) and (2), are very complex. Nonetheless, in the limit of long ($T \gg P$) and short ($T \ll P$) observation times, with respect to the orbital period, one can obtain analytical closed form expressions which are actually quite simple. The expansions in these two regimes agree remarkably well with the exact expression over most of the P-range, see Figure 1. We give here some details for the circular orbit case, and just sketch the key result for eccentric orbits.

The circular orbit case is important because it provides us several insights into the problem via a comparatively easier computation; moreover, several binary radio pulsars have effectively $e = 0$ and Sco X-1 is essentially in a circular orbit; in addition, when "blind" searches will be implemented, it is likely that they will be restricted to NS's orbiting a companion in circular orbit, in order to keep the computational burden affordable. The total number of filters (for a 3% mismatch) is given by:

$$N \sim \begin{cases} 10^{17}\left(\frac{V}{10^{15}}\right) & (2\pi T/P \gg 1) \\ 10^{4}\left(\frac{V}{100}\right) & (2\pi T/P \ll 1) \end{cases}, \tag{4}$$

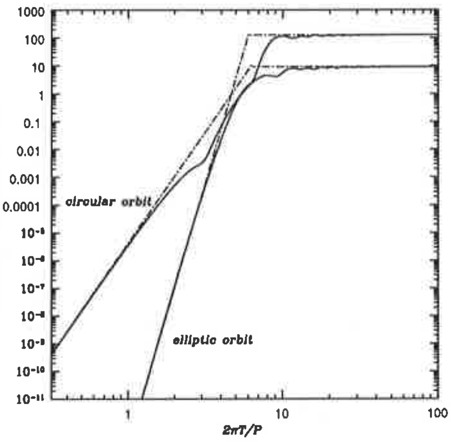

Fig. 1. The proper volume element $\sqrt{det(||\gamma_{ij}||)}$ (in arbitrary units) as a function of the time of observation T in units of the the source orbital period P. The solid and dashed lines refer to the full numerical expression and the analytical approximation, respectively (see text). The plot refers to a binary system with $a_p/(cT) = 3 \times 10^{-7}$ and $\alpha = 0$.

where the parameter volume one needs to cover is:

$$V \simeq \left\{ \begin{array}{ll} 1.5 \times 10^{15} \left(\frac{f_{\max}}{1\,\mathrm{kHz}}\right)^3 \left(\frac{a_{\max}}{5\times 10^{10}\,\mathrm{cm}}\right)^3 \left(\frac{\omega_{\max}}{6.3\times 10^{-4}\,\mathrm{s}^{-1}}\right) \left(\frac{T}{10^7\,\mathrm{s}}\right) & (2\pi T/P \gg 1) \\ 200 \left(\frac{f_{\max}}{1\,\mathrm{kHz}}\right)^3 \left(\frac{a_{\max}}{5\times 10^{10}\,\mathrm{cm}}\right)^3 \left(\frac{\omega_{\max}}{6.3\times 10^{-4}\,\mathrm{s}^{-1}}\right)^9 \left(\frac{T}{10^3\,\mathrm{s}}\right)^9 & (2\pi T/P \ll 1) \end{array} \right. ; \tag{5}$$

here we have ambitiously chosen parameter values that are appropriate for a NS/NS binary in a few hours orbit. Notice the radically different dependence on ωT in the two regimes, and the a_p^3 dependence of the computational costs. If the parameter values are known in advance with an error $\pm\delta\lambda^j$, then the number of filters reduces by a factor

$$\frac{\delta V}{V} \simeq \left\{ \begin{array}{ll} 2.4 \times 10^{-5} \prod_{j=1}^{3} \left[\left(\delta\lambda^j/\lambda^j\right)/10^{-2}\right] & (2\pi T/P \gg 1) \\ 2.2 \times 10^{-4} \prod_{j=1}^{3} \left[\left(\delta\lambda^j/\lambda^j\right)/10^{-2}\right] & (2\pi T/P \ll 1) \end{array} \right. . \tag{6}$$

Clearly, any prior information on the value of the source parameters greatly decreases the number of templates, and it is easy to check that a single-filter search can be performed if :

$$\frac{\delta\lambda^j}{\lambda^j} \lesssim \left\{ \begin{array}{ll} 7 \times 10^{-7} \left(\frac{V_{\max}}{10^{15}}\right)^{-1/3} & (2\pi T/P \gg 1) \\ 8 \times 10^{-3} \left(\frac{V_{\max}}{100}\right)^{-1/3} & (2\pi T/P \ll 1) \end{array} \right. , \qquad (\forall j) . \tag{7}$$

In the case of eccentric orbits, the computational burden is of course much higher, but the results show a behaviour which is very similar to the one for $e = 0$. As in the previous case, the costs increase dramatically as the observation time covers more than $\simeq 1$ rad of the source orbital phase. For $P \gg T$ one finds:

$$N^{(e)} \sim 10^{26} \left(\frac{V^{(e)}}{9 \times 10^{21}} \right) , \tag{8}$$

$$V^{(e)} \propto f_{\max}^5 \, a_{\max}^5 \, \omega_{\max} \, T \, F(e) ; \tag{9}$$

$F(e) = e^2(1 - 3e^2/8) + O(e^6)$ contains the dependency on the eccentricity, and V is normalized to the parameter values given in Eq. (5); notice that now, with a 5-dimensional parameter space, the number of filters is proportional to a_p^5.

4. Targeting known sources in binary systems

We can now apply the results of the previous section to some of the known NS in binary systems which will be targeted by laser interferometers, and estimate the processing power involved *only* in the coherent correction of the orbital motion.

Sco X-1 orbits a low-mass companion with a period $P = 0.787313(1)$ days, in an orbit which is essentially circular (here we will assume $e = 0$); the position of the NS on the orbit is known with an error $\simeq 0.1$ rad, and the projected semi-major axis is $a_p \simeq 6.3 \times 10^{10}$ cm, with $\delta a_p / a_p \simeq 5.16 \times 10^{-2}$. The uncertainties surrounding the frequency at which GW's are emitted suggest to cover a frequency band up to ≈ 600 Hz. It is easy to verify that for integration times longer than ~ 4 hours, one needs to correct for the source orbital motion. It is also evident, from the results of the previous section, that the number of filters changes dramatically when the integration time goes from ~ 6 hours to a day; in fact:

$$N_{\rm Sco} \simeq \begin{cases} 2.5 \times 10^6 \left(\frac{f_{\max}}{600\,{\rm Hz}} \right)^3 \left(\frac{T}{10^5\,{\rm sec}} \right) & (T \gtrsim 1\,{\rm day}) \\ 14 \left(\frac{f_{\max}}{600\,{\rm Hz}} \right)^3 \left(\frac{T}{5\,{\rm hour}} \right)^9 & (T \lesssim 6\,{\rm hours}) \end{cases} . \tag{10}$$

Notice also that if one allows for possible (small) departures from a perfectly circular orbit, the number of templates further increases.

We analyze now the case of radio pulsars, and assume $T = 10^7$ sec and $\mu = 0.03$. We have considered the 44 NS's with a binary companion (the total number of radio pulsars is 706) included into the catalogue by Taylor et al. (1993,1995). Seven binary radio pulsars emit at frequencies below 10 Hz, and are therefore outside the observational band; for 21 sources the parameter measurements coming from radio observations are so precise that one can simply use the quoted values of the parameters and fully correct for the orbital motion; the remaining 16 radio pulsars require a search over a limited parameter range. For these sources, all the NS's whose spin-down values yield upper-limits on the GW amplitude which are above the sensitivity curve of all proposed detectors (including LIGO III) would require at most 10 templates for $T = 10^7$ sec. Four NS's require a number of templates in the range $10^2 - 10^8$ (but are well below the LIGO III sensitivity) and eight sources, for which we do not have as yet measurements of the spin-down, would require a very substantial number of orbital filters, more than $\sim 10^{10}$.

5. Conclusions

We have presented some preliminary results regarding the additional computational costs involved in the correction of the Doppler phase shift induced in the CW signal of a NS orbiting a binary companion; we have given general expressions to compute the number of filters which are required to carry out the search as a function of the mismatch, time of integration and parameter space. A more thorough description of the issues presented here is currently in preparation, and we plan to start soon the investigation of hierarchical algorithms, that would conceivably speed up the search in a considerable way.

6. References

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