Reprinted from "PHYSICS NEWS" Vol. 10 No. 1, March, 1979.

EINSTEIN AND COSMOLOGY

J. V. Narlikar

Tata Institute of Fundamental Research
Homi Bhabha Road, Bombay-400 005

Writing about Einstein's contributions to cosmology on the occasion of his 70th birthday, his long time colleague Leopold Infeld[1] had remarked :

Speculations about the Universe in which men live are as old as human thought and art; as old as the view of shining stars on a clear night. Yet it was the general relativity theory which, only thirty years ago, shifted cosmological problems from poetry or sepculative philosophy into physics. We can even fix the year in which modern cosmology was born. It was in 1917, *when Einstein's paper appeared in the Russian Academy of Science under the title 'Cosmological Considerations in General Relativity Theory'.*

What exactly was the role Einstein played in shaping cosmology into its modern scientific form ? How did his article[2] cited above by Infeld start a new approach to cosmology ? To what extent are Einstein's ideas of six decades back useful to the present-day cosmologist?

To answer these questions it is necessary first to look at the background of astronomy and physics at a time when Einstein began his researches into gravitation and cosmology.

On the astronomical front the observational data of relevance to cosmology was very scanty if not *nil.* Modern cosmologist concerns himself mainly with extragalactic astronomy, i.e., with the information about the universe which comes from beyond our Galaxy. In the first decade or two of the present century there was practically no extra-galactic astronomy. The nineteenth century view that our Galaxy spanned by the Milky Way contains all that there is in the universe had still not been got rid of. A number of extragalactic objects were mistakenly believed to be part of the Galaxy. Supporting astro-physical information such as stellar structure and evolution, nucleosynthesis, the various radiation phenomena (like black body, synchrotron etc.), cosmic rays, was not available. From hindsight now it seems that to have embarked on theoretical cosmology with such poor observational input was bound to be highly speculative.

The theoretical equipment in the pre-Einstein era was also highly inadequate for such a project. The physicist knew of two basic interactions, gravitation as given by Newton's law and electromagnetism as given by Maxwell's equations. Although both are long range forces, the latter was expected to be of little importance in determining the large scale structure and dynamics of the universe. This is be cause there was no observational indication of the presence of large scale electric charges or currents in the universe. Atempts had been made to construct models of a static universe within the Newtonian framework. These, however, did not succeed for a reason which is not difficult to follow. In a static universe some pressure gradient is needed to withstand the gravitational contraction induced by the attraction between the different parts of the universe. If the model of the universe is *homogeneous*, the pressure gradient becomes zero and the universe begins to contract.

Apart from this difficulty, there were other problems. With the advent of special relativity, it becames clear that Newtonian gravitation with its 'instantaneous action at a distance' was inconsistent with it. Like any other physical interaction gravitation also must propagate at a speed not greater than that of light. Not only this; the concept of inertial frame so basic to special relativity is not realizable in practice while gravitation is around. How can we define a body moving with uniform velocity under no forces if we cannot ever get rid of the gravitational force acting on it ?

Einstein appreciated these difficulties without whose resolution it would have been difficult to make progress with theoretical cosmology. The decade between 1905 and 1915 represents the time span taken by him to find an answer to these problems. His general theory of relativity arose out of his attempts to construct a theory of gravitation which does not suffer from the defects mentioned above.

General relativity takes account of the permanance (or the indestructible character) of gravitation by relating it to another permanent entity, the space-time itself. In a

remarkable stroke of genius, Einstein argued that the presence of gravitation is equivalent to the fact that the geometry of spacetime is non-Euclidean. Thus he replaced the Newtonian concept of gravitation as a force by the above geometrical interpretation. The first law of motion can be stated in the same form as before; however, 'the uniform motion in a straight line' is to be determined by applying the rules of a non-Euclidean geometry.

A simple example will illustrate the difference between the Newtonian and the Einsteinian approaches. A projectile on the surface of the Earth follows a parabolic trajectory according to Newtonian dynamics. (The air resistance, curvature of the Earth and its rotation are neglected). In this trajectory the speed of the projectile changes with its height. Its direction also changes. Thus by no stroke of imagination could we say that the projectile is moving in a straight line with uniform speed. Nor do we have to in the Newtonian framework. For, we invoke the second law of motion and argue that the acceleration of the projectile is caused by the force of Earth's gravity. In general relativity, gravitation *per se* is eliminated as a force. We argue that the geometry on the Earth's surface is no longer Euclid's. And, if we suitably change the rules of geometry we can demonstrate that according to the new rules the observed trajectory of the projectile *does* represent uniform motion in a straight line.

Such trajectories in a non-Euclidean geometry are called 'geodesics'. A geodesic represents a curve of stationary distance between two points in space-time. In the non-Euclidean geometry adopted by Einstein, a geodesic also represents a curve whose direction does not change as we move along it. It can be readily seen that both these attributes also characterize a straight line in the more familiar Euclidean geometry.

What type of non-Euclidean geometry did Einstein adopt? In the 19th century mathematicians like Lobatchevsky, Bolyai, Gauss, Riemann and others had shown that self consistent geometries could be obtained by altering the axioms of Euclidean geometry. Once this realization, that Euclid's geometry is not the only self consistent scheme, sank in, the mathematicians developed many different schemes of non-Euclidean geometries as exercises in abstract thought. It was Einstein, however, who first took the daring step of assuming that the *actual universe* may follow one of these non-Euclidean geometries. He selected the scheme developed by Riemann, known as Riemannian geometry. His basic equations of general relativity describe how the mathematical quantities characterizing the geometry of space-time are related to the distribution of matter and energy. There are ten, second order non-linear partial differential equations with four identities relating them. These identities describe the conservation of energy and momentum. Later Einstein, Infeld and Hoffmann were to demonstrate that this set of equations also contain the information about the motion of gravitating matter. In this respect of Einstein-description of gravitation departs from the Maxwellian electromagnetic theory. In the latter theory Maxwell's field equations have to be supplemented by the (Lorentz) equations of motion. The reasons for this remarkable behaviour of general relativity lies partly in the non-linear character of Einstein's equation and partly in the fact that the geometrical entities appearing in these equations mean more than just the gravitational potentials. They also describe the space-time structure.

In the early days of general relativity K. Schwarzschild solved Einstein's equations for spherically symmetric distribution of matter in an otherwise empty space. The so-called 'external Schwarzschild solution' describes the non-Euclidean geometry in the space outside the gravitating matter. This solution found contact with observations. By identifying the central mass with the Sun, it was possible to devise observational tests in order to decide whether the behaviour of light and of planets conforms more closely with Einstein's theory than with Newton's. The differences in the predictions of the two theories are minute and only the more recent technological developments have been able to resolve the issue in Einstein's favour.[3]

Einstein, however, was also interested in the possible applications of his theory beyond the Solar System. In fact he undertook the large scale application of his equations by constructing a model of the universe as a whole. His model of a static universe, often known as the *Einstein* Universe marks the beginning of theoretical cosmology as we know it today.

This model is the simplest possible one under the ideas prevalent in 1917. Einstein assumed that the universe is static. He also extended the Copernican doctrine to the entire universe by denying the existence of any preferred position or a preferred direction in the universe. However, to maintain this simplest picture of the universe Einstein had to pay a price. A static Copernican solution could not be obtained from his equations unless he modified them. The reason is the same as that mentioned in connection with the failure of Newtonian gravity to provide a static universe. To 'support' the universe against its self-gravity Einstein had to invent a new force of repulsion, by adding an extra term to his equations. This term, popularly known as the λ-*term* describes a force of repulsion between any two objects which varies with distance, its magnitude depending on a constant of proportionality denoted by λ. In Einstein's description the constant was expected to be very small so that this repulsive force would be insignificant on the scale of the Earth, the Solar System or even the Galaxy. The constant λ is related to the mean density of matter ρ in the universe by a simple relation:

$$\lambda = \frac{4\pi G\rho}{3}.$$

The above relation may be looked upon as describing the equality between the forces of repulsion and attraction which keeps the universe static. The geometry of space in the Einstein Universe is non-Euclidean, and can be described as that of the three dimensional surface of a four dimensional hypersphere.

In Cartesian co-ordinates x_1, x_2, x_3, x_4 such a 3-surface is given by

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = R^2$$

where R is the radius of the hypersphere. The volume of the 3-surface is

$$V = 2\pi^2 R^3,$$

R is the ' radius ' of the Einstein universe and V its volume. The universe is thus *finite* in its spacial extent. It has, however, no boundaries ! (Compare the situation in one lower dimension : the surface of a sphere has finite area but no boundaries) From Einstein's equations we get R in terms of λ or ρ :

$$R = \frac{c}{\sqrt{3\lambda}} = \frac{c}{2\sqrt{\pi G\rho}} .$$

To get an idea of the relative sizes of these quantities take for ρ the present estimate of the visible matter in the universe :

$$\rho \cong 10^{-30} \text{ g cm}^{-3} .$$

We then get

$$\lambda \cong 3 \times 10^{-37} \text{ s}^{-2}, \quad R \cong 3 \times 10^{28} \text{ cm.}$$

The first of these relations implies that the repulsive acceleration produced by the Sun at the Earth is $\sim 5 \times 10^{-24}$ cm sec^{-2}. This is $\sim 6 \times 10^{-5}$ times the gravitational acceleration of the Sun at the Earth, and thus substantiates the earlier statement that the cosmic force of repulsion is negligible in the local context.

The second relation emphasizes the aspect of cosmology which Einstein found so appealing. Here is a case which shows how the large scale geometrical structure of the universe is determined by the distribution of matter in it. Matter introduces curvature of space-time. In the Einstein Universe there is so much matter that the space closes upon itself—it is finite in volume. Einstein felt (and many physicists today feel the same) that for a proper discussion of cosmology including boundary conditions, the relationship between the local and the distant, as well as for Mach's principle[4] it is necessary to have a finite closed universe.

However, there were two developments which shook Einstein's faith that this model of the universe is unique and correct. That it is not unique was demonstrated by de Sitter[5] soon after Einstein obtained his model: de Sitter showed that a non-static empty models of the universe also results from the modified equations of Einstein. The more serious blow to correctness came a decade later from the observations of Edwin Hubble and his colleagues. Hubble's work pioneered *observational* cosmology just as Einstein's work pioneered *theoretical* cosmology.

Hubble discovered the phenomenon of *nebular redshift*.[6] By the late 1920s the astronomers had established the fact that the universe extends well beyond the confines of our Galaxy. Many of the nebulae, once considered to be part of the Milky Way System turned out to lie at immensely greater distances outside the Galaxy. Hubble's observations related to these nebulae. He found that there was a significant increase in the wavelength of light received from these objects. Thus if the expected wavelength was λ the observed weavelength was $\lambda (1 + z)$ with $z > 0$. This fractional increase in wavelength is called the ' redshift '. Hubble found that there was a simple relation between the redshift and the distance D of the typical nebula, which he expressed in the form

$$cz = H D,$$

where H is a constant, now know as *Hubble's constant*. In a simple Doppler-shift interpretation cz represents the speed of recession of the nebula from us. The above relation (known as *Hubble's law*) tells us that *the universe is not static* but has a systematic large scale motion of its main observed constituents, the galaxies. Hubble's law therefore falsified Einstein's own assumption of a static universe.

Nevertheless, Hubble's observations turned out to be a triumph rather than a disaster for general relativity. In 1922, some seven years *before* Hubble's law, A. Friedmann[7] had obtained non-static solutions of Einstein's equations *without* the λ term. These equations admit solutions consistent with the Copernican principle mentioned earlier in which the universe *as a whole* expands. Thus the distance between any two galaxies increases systematically with time. Hence viewed from any galaxy, the others would exhibit the phenomenon of redshift. observed by Hubble.

Einstein himself realized the significance of the *expanding universe*. In 1931, he published a paper[8] with de Sitter in which they worked out the simplest solution of general relativistic equations *without the* λ *term*. This model, known as the *Einstein de sitter model* predicts that the scale of distances S between any two galxies increases with time according to the rule

$$S \propto t^{2/3}$$

where t is the cosmic *epoch* of observation. At this epoch Hubble's constant is given by

$$H(t) = \frac{2}{3t} .$$

Thus 2/3H gives us the ' age ' of the universe—the time taken by the universe from the state of zero scale factor ($S = 0$) to the present state. The state of zero S is taken to represent the ' origin ' of the universe. The interpretation is that the universe exploded into existence at $t = o$, a property shared by all Friedmann models, including the Einstein de Sitter model. This is the reason for calling these models ' big bang ' models.

In the Einstein de Sitter model the density of matter is given by

$$\rho = \frac{3H^2}{8\pi G} \equiv \rho_c \text{ (say).}$$

This density plays a limiting role in deciding whether the universe is open or closed. If the actual density exceeds this value the space is *closed* in the sense we had discussed earlier in the context of the Einstein Universe. If the density does not exceed this value the universe is *open*. In the open universe the geometry of the space (t = constant) is either Euclidean as in the Einstein—de Sitter model or hyperbolic. The volume is infinite in either case.

This is an example of cosmology making a scientific prediction. In principle both the Hubble constant and the density of matter in the universe are measurable and so it is possible to decide observationally whether

$$\rho > \rho_c \text{ or } \rho \leqslant \rho_c ,$$

and hence whether the universe is closed or open. This possibility was opened up in Einstein's lifetime but has not been successfully tested to this day. The failure comes from the various uncertainties which surround observational cosmology even today.[9]

An interesting situation also surrounds the λ-term. As mentioned earlier, Einstein introduced the λ-term to obtain a static solution and then abandoned it when he found that a static universe was no longer consistent with observations. Other contemporary cosmologists however did not all agree with this point of view. Notably, Eddington[10] and Lemaitre[11] pointed out that the Einstein Universe was unstable and could at best be looked upon as a quasi-static phase in the evolution of a big bang universe. Their model starts existence as a big bang model, then attains the quasi static phase which can be extended for arbitrarily long-period by adjusting the λ-term and finally expands again. Asymptotically it behaves like the de Sitter universe. The quasi static phase, it was argued, would be useful in promoting the formation of galaxies. The question, not settled even today, is whether the λ-term is really necessary from a cosmological point of view. A recent survey of observational cosmology by Gunn and Tinsley[12] suggests that the λ-term appears to be needed.

Einstein's cosmological interests also extended to fundamental issues such as the origin of inertia. He was initially impressed by the persuasive arguments of Mach that inertia of a body might be related to the large scale structure of the universe. He had hoped that his theory of gravitation would turn out to incorporate Mach's principle. In this endeaver he was unsuccessful. It became clear during the course of his lifetime that general relativity does not incorporate Mach's ideas. Einstein himself became disillusioned with Mach's ideas, largely because he suspected that they required the concept of action at a distance. He writes in his autobiographical notes :[13]

..... *Mach conjectures that in a truly rational theory inertia would have to depend upon the interaction of the masses, precisely as was true for Newton's other forces, a conception which for a long time I considered as in principle the correct one. It presupposes implicitly, however, that the basic theory should be of the general type of Newton's mechanics : masses and their interactions as the original concepts. The attempt at such a solution does not fit into a consistent field theory, as will be immediately recognized* .....

After Einstein's death there have been attempts to reconcile Mach's principle with a gravitation theory[14, 15]. These attempts go beyond the general theory of relativity although they were all inspired by it. Such attempts are examples of the burst of activity on several fronts in cosmology in the last two decades. This activity is not limited to fundamental issues only. It has extended to the application of astrophysical concepts to such cosmological problems as the formation of galaxies, the origin of elements, the nature of background radiation etc. These aspects of 'physical cosmology' are the results of the growing confidence felt by cosmogists that the basic laws of physics can be applied to the universe in the large.

On the observational front also, the remarkable strides have been made, thanks to the developments in radioastronomy, electronics, and space technology. Indeed, from the day when Einstein wrote his first paper of cosmology with negligible data, the situation is so transformed today that the theoretical cosmologist finds it difficult to take proper account of all the data that is pouring in steadily. In spite of this remarkable transformation it is correct to assert that the theoretician of today is still being inspired by the approach adopted by Einstein six decades ago.

References

1. Infeld, L. In **Albert Einstein : Philosopher-Scientist,** ed. P.A. Schlipp, Library of Living Philosophers Inc., Illinois, USA, p. 475 (1949).
2. Einstein, A. Preuss. Akad. Wiss. Berlin Sitzber., 142 (1917).
3. Narlikar, J.V. **General Relativity and Cosmology,** Macmillan (1978) : for a review of the experimental situation.
4. Mach, E. The Science of Mechanics, Watts and Co. London (1893)
5. De Sitter, W. M.N.R.A. S. **78**, 3 (1917).
6. Hubble, E. Proc. Nat. Acad. Sci. (USA) **15**, 169 (1929).
7. Friedmann, A., Z. Phys. **10**, 377 (1922).
8. Einstein, A. and De Sitter, W., Proc. Nat Acad. Sci. (USA) **18**, 213 (1932).
9. Narlikar, J.V., Physics News, **7**, 4, 1 (1976).
10. Eddington, A.S. M.N.R.A.S. **90**, 668.
11. Lemaitre', G.C. M.N.R.A.S. **92**, 500 (1932).
12. Gunn, J.E. and Tinsley, B.M. Nature, **257**, 454 (1976).
13. Einstein, A. in Albert Einstein ; Philosopher-Scientist, ed. P.A. Schlipp, Library of Living Philosophers Inc., Illionois, U.S.A., p. 29 (1949).
14. Brans,C. and Dicke, R.H., Phys. Rev. **124**, 925 (1961).
15. Mayle, F. and Narlikar, J.V., **Action at a Distance in Physics and Cosmology,** Freeman (1974).