COMMENTS ON "REMARKS ON A DECRUMPLING MODEL OF THE UNIVERSE"

R. G. Vishwakarma[1]

IUCAA
Post Bag 4, Ganeshkhind
Pune 411007, India

Abstract

A recent paper by Lima and Mohazzab has been examined and it is found that their results, that the Einstein field equations are consistent only for a dust and a turning point in the dynamics of the evolution is possible, are erroneous. This is due to their ignorance of the fact that the field equations and the energy conservation equation are not independent.

[1]Email: vishwa@iucaa.ernet.in
Permanent Address: *BP 54, Patel Nagar, Mughal Sarai- 232101, Varanasi, India*

In a recent paper[1], Lima and Mohazzab have claimed that the Einstein field equations are consistent only for a dust filled universe and a turning point in the dynamics of the evolution is possible (in flat Friedmann model with a vanishing cosmological constant Λ) unless one imposes a constraint given by the time-component of the Einstein field equations! Their confusion arises due to their ignorance of the fact that the space and time components of the Einstein field equations and energy conservation equation are not mutually independent, rather they constitute a set of only two independent equations.

The authors consider the spatially flat ($k = 0$) R-W metric

$$ds^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2]. \tag{1}$$

The Einstein field equations (in the units with $8\pi G = c = 1$) then yield

$$\rho = 3\left(\frac{\dot{a}}{a}\right)^2, \tag{2}$$

$$p = -2\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2. \tag{3}$$

The conservation of energy momentum tensor ($T^{ij}{}_{;j} = 0$) implies

$$\dot{\rho} + 3(\rho + p)\dot{a}/a = 0. \tag{4}$$

By assuming that the material medium obeys the barotropic equation of state

$$p = (\gamma - 1)\rho, \tag{5}$$

one can find, from eqs. (2) and (3), an equation for the scale factor a, as

$$a\ddot{a} + \left(\frac{3\gamma - 2}{2}\right)\dot{a}^2 = 0. \tag{6}$$

One can also solve eq. (4) with the help of (5), giving

$$\rho = Ba^{-3\gamma} = \frac{p}{\gamma - 1}, \qquad B = constant > 0. \tag{7}$$

With this, eq. (3) reduces to

$$a\ddot{a} + \frac{\dot{a}^2}{2} + \frac{\gamma - 1}{2}Ba^{2-3\gamma} = 0. \tag{8}$$

The solutions of eqs. (6) and (8) can easily be obtained, respectively, as

$$\dot{a}^2 = Aa^{2-3\gamma}, \tag{9}$$

$$\dot{a}^2 = \frac{C}{a} + \frac{B}{3}a^{2-3\gamma}, \tag{10}$$

where A and C are constants of integration.

Now the authors claim that eqs. (6) and (8) supply quite different equations of motion and they are consistent only for $\gamma = 1$. They further claim that the constant of integration C appearing in (10) is arbitrary and hence a turning point (where $\dot{a} = 0$) can occur which can only be avoided by imposing eq. (2) on (8) or equivalently on (10). The reason of their misunderstanding is that they regard eqs. (2)-(4) as three independent equations and thus they argue that eq. (8) [obtained by using (3) and (4)], having solution (10), is independent from eq. (6) [obtained by using (2) and (3)], with solution (9). However, the authors do not seem to remember that the energy conservation equation (4) [which can easily be derived from the time and space components of Einstein field equations, i.e., eqs. (2) and (3)] is not an extra imposition but a natural consequence of the Einstein field equations through Bianchi identities:

$$\left(R^{ij} - \frac{1}{2}Rg^{ij}\right)_{;j} = 0 = T^{ij}{}_{;j}. \tag{11}$$

In fact eqs. (6) and (8) simply imply

$$\dot{a}^2 = \frac{B}{3}a^{2-3\gamma} \tag{12}$$

which is nothing but eqs. (2) and (7) taken together [this guarantees that eq. (2) is already present in (8)] and holds for all γ, not only for $\gamma = 1$ as the authors claim. $\gamma = 1$ simply refers to a dust solution.

Obviously there is no arbitrariness in the values of the constants A and C. By comparing (12) with (9) and (10), one can see that $A = B/3$ and $C = 0$. Hence, there is no possibility of a turning point as could happen if C could be negative. However, this might be possible in some non-trivial cases, for example, when the energy momentum conservation is not an inbuilt consequence of the Einstein field equations but rather an imposition put by hand on the system as it is in the practice when one considers both G and Λ as functions of time[2].

Acknowledgements

The author is grateful to the *Inter University Centre for Astronomy and Astrophysics* (IUCAA), India for its kind hospitality where this work was completed during a visit.

References

1. J.A.S. Lima and M. Mohazzab, Int. J. Mod. Phys. D **7**, 657, (1998).
2. Abdussattar and R.G. Vishwakarma, Class. Quant. Gravit. **14**, 945, (1997) and the references therein.

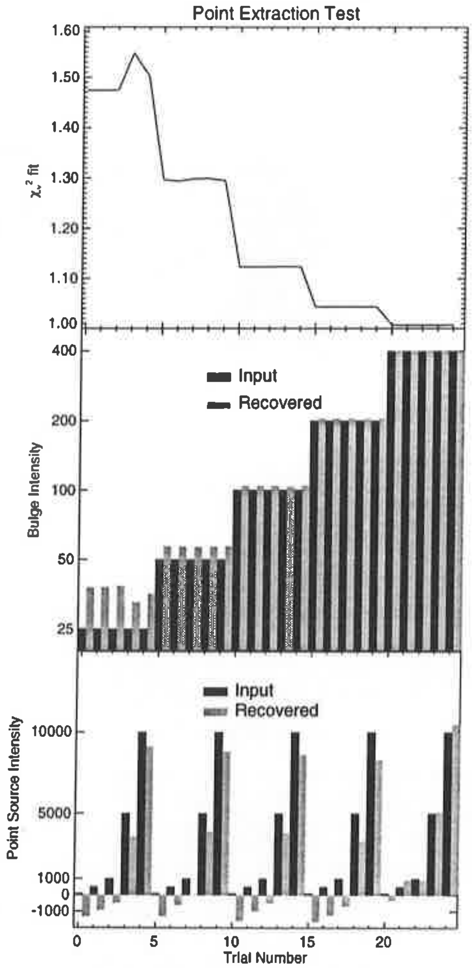

Fig. 4.— Results of point source extraction in 25 runs. The top panel shows the χ_ν^2 value obtained in 25 runs. The bulge intensity increases after every 5 runs, resulting in a steep drop in χ^2, due to improved S/N ratio. The middle panel shows 25 input values for bulge intensity and the corresponding recovered value adjacent to each other. The lowest panel shows input point source intensities for each run and the corresponding recovered value. Note that the middle panel has the Y axis plotted on a logarithmic scale. See the text for further explanation.