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PLANE-SYMMETRIC INHOMOGENEOUS BULK VISCOUS COSMOLOGICAL MODELS WITH VARIABLE Λ

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A plane-symmetric non-static cosmological model representing a bulk viscous fluid distribution has been obtained which is inhomogeneous and anisotropic and a particular case of which is gravitationally radiative. Without assuming any *adhoc* law, we obtain a cosmological constant as a decreasing function of time. The physical and geometric features of the models are also discussed.

1. Introduction

In recent years cosmological models exhibiting plane symmetry have attracted the attention of various authors. At the present state of evolution, the universe is spherically symmetric and the matter distribution in it is isotropic and homogeneous. But in its early stages of evolution, it could have not had a smoothed out picture. Close to the big bang singularity, neither the assumption of spherically symmetry nor of isotropy can be strictly valid. So, we consider plane symmetry, which is less restrictive than spherical symmetry and provides an avenue to study inhomogeneities. Inhomogeneous cosmological cosmological models play an important role in understanding some essential features of the universe such as the formation of galaxies during the early stages of evolution and process of homogenization. The early attempts at the construction of such models have done by Tolman[1] and Bondi[2] who considered spherically symmetric models. Inhomogeneous plane-symmetric models were considered by Taub[3] and later by Tomimura,[4] Szekeres,[5] Collins and Szafron,[6] Szafron and Collins.[7] Recently, Senovilla[8] obtained a new class of exact solutions of Einstein's equation without big bang singularity, representing a cylindrically symmetric, inhomogeneous cosmological model filled with perfect fluid which is smooth and regular everywhere satisfying energy and causality conditions. Later, Ruis and Senovilla[9] have separated out a fairly large class of singularity free models through a comprehensive study of general cylin-

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drically symmetric metric with separable function of r and t as metric coefficients. Dadhich et al.[10] have established a link between the FRW model and the singularity free family by deducing the latter through a natural and simple inhomogenization and anisotropization of the former. Recently, Patel et al.[11] presented a general class of inhomogeneous cosmological models filled with non-thermalized perfect fluid by assuming that the background spacetime admits two space-like commuting killing vectors and has separable metric coefficients. Bali and Tyagi[12] obtained a plane-symmetric inhomogeneous cosmological models of perfect fluid distribution with electromagnetic field. Recently, Pradhan and Yadav[13] have investigated a plane-symmetric inhomogeneous viscous fluid cosmological models with electromagnetic field.

Models with a relic cosmological constant Λ have received considerable attention recently among researchers for various reasons (see Refs.[14−18] and references therein). Some of the recent discussions on the cosmological constant "problem" and on cosmology with a time-varying cosmological constant by Ratra and Peebles,[19] Dolgov[20−22] and Sahni and Starobinsky[23] point out that in the absence of any interaction with matter or radiation, the cosmological constant remains a "constant", however, in the presence of interactions with matter or radiation, a solution of Einstein equations and the assumed equation of covariant conservation of stress-energy with a time-varying Λ can be found. For these solutions, conservation of energy requires decrease in the energy density of the vacuum component to be compensated by a corresponding increase in the energy density of matter or radiation. Earlier researchers on this topic, are contained in Zeldovich,[24] Weinberg[15] and Carroll, Press and Turner.[25] Recent observations by Perlmutter *et al.*[26] and Riess *et al.* [27] strongly favour a significant and positive Λ. Their finding arise from the study of more than 50 type Ia supernovae with redshifts in the range $0.10 \leq z \leq 0.83$ and suggest Friedmann models with negative pressure matter such as a cosmological constant, domain walls or cosmic strings (Vilenkin, [28] Garnavich *et al.*[29] Recently, Carmeli and Kuzmenko[30] have shown that the cosmological relativity theory (Behar and Carmeli[31]) predicts the value $\Lambda = 1.934 \times 10^{-35} s^{-2}$ for the cosmological constant. This value of Λ is in excellent agreement with the measurements recently obtained by the High-Z Supernova Team and Supernova Cosmological Project (Garnavich et al;[29] Perlmutter et al.;[26] Riess et al.;[27] Schmidt et al.[32]). The main conclusion of these works is that the expansion of the universe is accelerating.

Several ans$\ddot{a}$tz have been proposed in which the Λ term decays with time (see Refs. Gasperini,[33,34] Berman,[35] Freese *et al.*,[18] $\ddot{O}$zer and Taha,[18] Peebles and Ratra,[36] Chen and Hu,[37] Abdussattar and Viswakarma,[38] Gariel and Le Denmat,[39] Pradhan *et al.*[40]). Of the special interest is the ansatz $\Lambda \propto S^{-2}$ (where S is the scale factor of the Robertson-Walker metric) by Chen and Wu,[37] which has been considered/modified by several authors (Abdel-Rahaman,[41] Carvalho *et al.*,[18] Waga,[42] Silveira and Waga,[43] Vishwakarma.[44])

In most treatments of cosmology, cosmic fluid is considered as perfect fluid. However, bulk viscosity is expected to play an important role at certain stages of expanding universe.[45−47] It has been shown that bulk viscosity leads to inflationary like solution,[48] and acts like a negative energy field in an expanding universe.[49] Furthermore, There are several processes which are expected to give rise to viscous effects. These are the decoupling of neutrinos during the radiation era and the decoupling of radiation and matter during the recombination era. Bulk viscosity is associated with the GUT phase transition and string creation. A number of authors have discussed cosmological solutions with bulk viscosity in various context.[50−53]

Roy and Narain[54] have obtained a plane-symmetric non-static cosmological models in presence of a perfect fluid. In this paper, we will investigate a plane-symmetric inhomogeneous bulk viscous fluid cosmological models in the presence of a variable cosmological constant varying with time. In our previous paper[13] we have obtained a non-degenerate Petrov type-II solution. In the present paper we have derived a cosmological model, which has in general, as Petrov type-I solution and as a sub case it represents a gravitationally radiating Petrov type-II solution. The physical and geometric behaviour of the models will be discussed.

2. The Metric and Field Equations

We take the plane-symmetric spacetime considered by Roy and Narain[54]

$$ds^2 = dt^2 - dx^2 - B^2 dy^2 + C^2 dz^2, \tag{1}$$

where the metric potentials B and C are functions of x and t. The energy momentum tensor in the presence of bulk stress has the form

$$T_{ij} = (\rho + \bar{p}) v_i v_j - \bar{p} g_{ij} \tag{2}$$

and

$$\bar{p} = p - \xi v^i_{;i} \tag{3}$$

Here ρ, p, $\bar{p}$, and ξ are the energy density, isotropic pressure, effective pressure, bulk viscous coefficient respectively and v_i is the flow vector satisfying the relation

$$g^{ij} v_i v_j = 1 \tag{4}$$

The Einstein's field equations are

$$R_{ij} - \frac{1}{2} R g_{ij} + \Lambda g_{ij} = -8\pi T_{ij} \tag{5}$$

where Λ is the cosmological constant. Eqs. (2) and (4) for the metric (1) lead to

$$v_2 = v_3 = 0 \tag{6}$$

From Eq. (4), we have

$$v_4^2 - v_1^2 = 1 \tag{7}$$

The field Eqs. (5) for the line element (1) lead to

$$-8\pi[(\rho+\bar{p})v_1^2+\bar{p}] = \frac{B_{44}}{B}+\frac{C_{44}}{C}-\frac{B_1C_1-B_4C_4}{BC} \tag{8}$$

$$-8\pi\bar{p} = \frac{C_{44}-C_{11}}{C}-\Lambda \tag{9}$$

$$-8\pi\bar{p} = \frac{B_{44}-B_{11}}{B}-\Lambda \tag{10}$$

$$-8\pi[(\rho+\bar{p})v_4^2-\bar{p}] = \frac{B_{11}}{B}+\frac{C_{11}}{C}-\frac{B_1C_1-B_4C_4}{BC}+\Lambda \tag{11}$$

$$-8\pi[(\rho+\bar{p})v_1v_4] = \frac{B_{14}}{B}+\frac{C_{14}}{C} \tag{12}$$

The suffixes 1 and 4 at the symbols B and C denote partial differential with respect to x and t respectively.

3. Solutions of the Field Equations

From Eqs. (9) and (10), we have

$$\frac{B_{uv}}{B}=\frac{C_{uv}}{C} \tag{13}$$

where

$$u=\frac{1}{2}(x+t),\ \ v=\frac{1}{2}(x-t) \tag{14}$$

From Eqs. (8)-(13), we have

$$\left[\frac{B_{uu}+B_{vv}}{4B}+\frac{C_{uu}+C_{vv}}{4C}\right]^2-\left[\frac{B_uC_v+B_vC_u}{2BC}\right]^2$$

$$=\left[\frac{B_{uu}-B_{vv}}{4B}+\frac{C_{uu}-C_{vv}}{4C}\right]^2 \tag{15}$$

Let us assume

$$B=\alpha(u).\beta(v) \tag{16}$$

and

$$C=f(u).g(v) \tag{17}$$

From Eqs. (13), (16) and (17), we have

$$\frac{(\alpha_u/\alpha)}{(f_u/f)}=\frac{(g_v/g)}{(\beta_v/\beta)}=k_1, \tag{18}$$

where k_1 is an arbitrary constant. Integrating Eq. (18), we get

$$\alpha=Mf^{k_1} \tag{19}$$

$$g=N\beta^{k_1} \tag{20}$$

where M and N are constants of integration. From Eqs. (15), (16), (17), (19) and (20), we obtain

$$\frac{(f_u/f)}{(f_{uu}/f_u)} = -\left[\frac{(k_1+1)^2\frac{\beta_{vv}}{\beta} + k_1(k_1^2-1)\frac{\beta_v^2}{\beta^2}}{k_1(k_1^2-1)\frac{\beta_{vv}}{\beta} + \{k_1^2(k_1^2-1)^2 - (k_1^2+1)^2\}\frac{\beta_v^2}{\beta^2}}\right] = k_2, \tag{21}$$

where k_2 is an arbitrary constant. Integrating Eq. (21), we get

$$f = (au+b)^{\mu} \tag{22}$$

and

$$\beta = (mv+n)^{\nu} \tag{23}$$

where a, b, m and n are arbitrary constants and

$$\mu = \frac{k_2}{k_2-1}, \quad \nu = \frac{k_1+1}{k_1^2+1} - \frac{k_2}{k_2-1} \tag{24}$$

By suitable transformation metric (1) reduces to the form

$$ds^2 = dT^2 - dX^2 - U^{2k_1\mu}V^{2\nu}dY^2 - U^{2\mu}V^{2k_1\nu}dZ^2 \tag{25}$$

where

$$U = \frac{1}{2}(X+T) \ \ and \ \ V = \frac{1}{2}(X-T) \tag{26}$$

The effective pressure and the energy density for the model (25) are given by

$$8\bar{p} = -\left[\frac{4k_1\mu\nu}{(T^2-X^2)}\right] + \Lambda \tag{27}$$

$$8\pi\rho = \left[\frac{4\mu\nu(k_1^2+k_1+1)}{(T^2-X^2)}\right] - \Lambda \tag{28}$$

For the simplification of ξ, we assume that the fluid obeys an equation of state of the form

$$p = \gamma\rho, \tag{29}$$

where $\gamma(0 \leq \gamma \leq 1)$ is a constant.

Thus, given $\xi(t)$ we can solve the cosmological parameters. In most of the investigations involving bulk viscosity is assumed to be a simple power function of the energy density[55-57]

$$\xi(t) = \xi_0\rho^k \tag{30}$$

where ξ_0 and k are constants. If $k = 1$, Eq. (30) may correspond to a radiative fluid. However, more realistic models[58] are based on n lying in the regime $0 \leq n \leq \frac{1}{2}$.

3.1. *Solutions for* $\xi = \xi_0$

In this case we assume $k = 0$ in Eq. (30). Eqs. (30) and (27) become $\xi = \xi_0$ = constant and

$$8\pi p = \frac{8\pi\xi_0}{(T^2 - X^2)}\left[1 + \frac{(k_1+1)^2}{(k_1^2+1)}\right] - \frac{4k_1\mu\nu}{(T^2 - X^2)} + \Lambda \tag{31}$$

Using Eq. (29) and eliminating $\rho(t)$ from Eqs. (28) and (31) we obtain

$$\Lambda = \frac{2\mu\nu(k_1+1)^2}{(T^2 - X^2)} - \frac{4\pi\xi_0}{(T^2 - X^2)^{\frac{1}{2}}}\left[1 + \frac{(k_1+1)^2}{(k_1^2+1)}\right] \tag{32}$$

3.2. *Solutions for* $\xi = \xi_0\rho$

In this case we assume $k = 1$ in Eq. (30). Hence, Eqs. (30) and (27) become

$$\xi = \xi_0\rho \tag{33}$$

and

$$8\pi\left[p - \frac{\xi_0\rho}{(T^2 - X^2)^{\frac{1}{2}}}\left(1 + \frac{(k_1+1)^2}{(k_1^2+1)}\right)\right] = -\frac{4k_1\mu\nu}{(T^2 - X^2)} + \Lambda \tag{34}$$

Using Eq. (29) and eliminating $\rho(t)$ from Eqs. (28) and (34) we obtain

$$\left[1 + \gamma - \frac{\xi_0}{(T^2 - X^2)^{\frac{1}{2}}}\left(1 + \frac{(k_1+1)^2}{(k_1^2+1)}\right)\right]\Lambda =$$

$$\frac{4\mu\nu}{(T^2 - X^2)}\left[k_1 + (k_1^2 + k_1 + 1)\left(\gamma - \frac{\xi_0}{(T^2 - X^2)^{\frac{1}{2}}}\{1 + \frac{(k_1+1)^2}{(k_1^2+1)}\}\right)\right] \tag{35}$$

We have observed from Eqs. (32) and (35) that the cosmological constant is a decreasing function of time and it approaches a small value as time progresses (i.e., the present epoch). Thus, with our approach, we obtain a physically relevant decay law for the cosmological constant unlike other investigators where *adhoc* laws were used to arrive at a mathematical expression for the decaying vacuum energy.

4. Some Physical and Geometrical Features of the Models

We shall now give the expressions for kinematical quantities and components of conformal curvature tensor. With regard to the kinematical properties of the velocity vector v_i in the metric (25), a straight forward calculation leads to the following expressions for the expansion θ and shear tensor σ_{ij} of the fluid.

$$\theta = \frac{1}{(T^2 - X^2)^{\frac{1}{2}}}\left[1 + \frac{(k_1+1)^2}{(k_1^2+1)}\right] \tag{36}$$

$$\sigma_{11} = \frac{2k_1}{3(k_1^2+1)}\left[\frac{T^2}{(T^2 - X^2)^{\frac{3}{2}}}\right] \tag{37}$$

$$\sigma_{22} = \frac{U^{2k_1\mu}V^{2\nu}}{(T^2 - X^2)^{\frac{1}{2}}}\left[\frac{2k_1^2 - k_1 - 1}{3(k_1^2+1)} - \mu(k_1 - 1)\right] \tag{38}$$

$$\sigma_{33} = \frac{U^{2\mu}V^{2k_1\nu}}{(T^2 - X^2)^{\frac{1}{2}}}\left[\frac{2 - k_1(k_1+1)}{3(k_1^2+1)} - \mu(k_1 - 1)\right] \tag{39}$$

$$\sigma_{44} = -\left[\frac{(k_1 - 1)^2}{3(k_1^2+1)}\right]\left[\frac{X^2}{(T^2 - X^2)^{\frac{3}{2}}}\right] \tag{40}$$

$$\sigma_{14} = \left[\frac{(k_1 - 1)^2}{3(k_1^2+1)}\right]\left[\frac{TX}{(T^2 - X^2)^{\frac{3}{2}}}\right] \tag{41}$$

where all other components, the rotational tensor and the acceleration vanish. Hence the models are expanding, non-rotating, shearing and geodetic in general. The non-vanishing components of the flow vector are

$$v_1 = -\frac{X}{(T^2 - X^2)^{\frac{1}{2}}} \tag{42}$$

$$v_4 = \frac{T}{(T^2 - X^2)^{\frac{1}{2}}} \tag{43}$$

It is observed that the region of the spacetime in which this is valid, is $T^2 - X^2 > 0$. The reality conditions $p > 0$ and $\rho > 0$ imply that

$$\left[\frac{4k_1\mu\nu}{(T^2 - X^2)}\right] < \Lambda < \left[\frac{2\mu\nu(k_1+1)^2}{(T^2 - X^2)}\right] \tag{44}$$

The non-vanishing components of the conformal curvature tensor are

$$C_{(1212)} = \frac{1}{8}\left[\frac{\mu(k_1 - 1)\{\mu(k_1+1) - 1\}}{U^2} - \frac{\nu(k_1 - 1)\{\nu(k_1+1) - 1\}}{V^2}\right] - \frac{1}{12}\left[\frac{\mu\nu(k_1 - 1)^2}{UV}\right] \tag{45}$$

$$C_{(1313)} = \frac{1}{8}\left[\frac{\nu(k_1 - 1)\{\nu(k_1+1) - 1\}}{V^2} - \frac{\mu(k_1 - 1)\{\mu(k_1+1) - 1\}}{U^2}\right] - \frac{1}{12}\left[\frac{\mu\nu(k_1 - 1)^2}{UV}\right] \tag{46}$$

$$C_{(2323)} = \frac{1}{6}\left[\frac{\mu\nu(k_1 - 1)^2}{UV}\right] \tag{47}$$

$$C_{(2424)} = \frac{1}{8}\left[\frac{\mu(k_1 - 1)\{\mu(k_1+1) - 1\}}{U^2} - \frac{\nu(k_1 - 1)\{\nu(k_1+1) - 1\}}{V^2}\right] + \frac{1}{12}\left[\frac{\mu\nu(k_1 - 1)^2}{UV}\right] \tag{48}$$

$$C_{(3434)} = \frac{1}{8}\left[\frac{\nu(k_1 - 1)\{\nu(k_1+1) - 1\}}{V^2} - \frac{\mu(k_1 - 1)\{\mu(k_1+1) - 1\}}{U^2}\right]$$

$$+\frac{1}{12}\left[\frac{\mu\nu(k_1-1)^2}{UV}\right] \tag{49}$$

$$C_{(1414)} = -\frac{1}{6}\left[\frac{\mu\nu(k_1-1)^2}{UV}\right] \tag{50}$$

$$C_{(1224)} = -\frac{1}{8}\left[\frac{\mu(k_1-1)\{\mu(k_1+1)-1\}}{U^2} - \frac{\nu(k_1-1)\{\nu(k_1-1)-1\}}{V^2}\right] \tag{51}$$

$$C_{(1334)} = \frac{1}{8}\left[\frac{\mu(k_1-1)\{\mu(k_1+1)-1\}}{U^2} + \frac{\nu(k_1-1)\{\nu(k_1+1)-1\}}{V^2}\right] \tag{52}$$

From Eqs. (45) - (52) it is observed that if $k_1 = 1$ the spacetime is conformally flat. The spacetime is, in general, of Petrov-type I. However if $k_1 \neq 1$ and either $\mu = \frac{1}{k_1+1}$ or $\nu = \frac{1}{k_1+1}$ then the spacetime will be of Petrov-type II. For this case the reality conditions $p > 0$ and $\rho > 0$ imply that

$$\left[\frac{8k_1^2}{(k_1+1)^2(k_1^2+1)(T^2-X^2)}\right] < \Lambda < \left[\frac{4k_1}{(k_1^2+1)(T^2-X^2)}\right] \tag{53}$$

For $\mu = \frac{1}{k_1+1}$, we have $\nu = \frac{2k_1}{(k_1+1)(k_1^2+1)}$ and in this case the geometry of the spacetime (25) takes the form

$$ds^2 = dT^2 - dX^2 - U^{\frac{2k_1}{(k_1+1)}} V^{\frac{2k_1}{(k_1+1)(k_1^2+1)}} dY^2 - U^{\frac{2}{(k_1+1)}} V^{\frac{4k_1^2}{(k_1+1)(k_1^2+1)}} dZ^2 \tag{54}$$

The physical components C_{hijk} take the form

$$\mathbf{C_{AB}} = \begin{pmatrix} -2\xi & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi-\eta & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi+\eta & 0 & \eta & 0 \\ 0 & 0 & 0 & 2\xi & 0 & 0 \\ 0 & 0 & \eta & 0 & -\xi+\eta & 0 \\ 0 & \eta & 0 & 0 & 0 & -\xi-\eta \end{pmatrix} \tag{55}$$

where

$$\xi = -\frac{1}{3}\left[\frac{(k_1-1)^2}{(k_1+1)^2(k_1^2+1)}\right]\frac{1}{UV} \tag{56}$$

and

$$\eta = \frac{1}{4}\left[\frac{k_1^3(k_1-1)}{(k_1+1)(k_1^2+1)^2}\right]\frac{1}{V^2} \tag{57}$$

From Eq. (55) we conclude that the model exhibits gravitational radiation. For large value of X it gives a type-two null spacetime representing an outgoing radiation field, although it will not satisfy the reality conditions at $X = \infty$. The radiative term in C_{AB} is the quantity $\eta = \frac{1}{4}\left[\frac{k_1^3(k_1-1)}{(k_1+1)(k_1^2+1)^2}\right]\frac{1}{V^2}$.

5. Discussion and Concluding Remarks

We have obtained a new class of inhomogeneous plane-symmetric cosmological models with a bulk viscous fluid as the source of matter. Generally, the models are expanding, shearing, non-rotating and Petrov-type I non-degenerate in which the flow vector is geodetic. Under certain conditions the models will be of Petrov-type II. It is well known fact that the free gravitational field affects the flow of the fluid by inducing the shear in the flow line.[63] It is therefore interesting to study the cosmological models with given Petrov-types. The cosmological constant in all models given in Sec. 3.1 and 3.2 are decreasing function of time. The cosmological consequences of a decaying cosmological term has been discussed lucidly[35],[59−62] with an *adhoc* assumption of a decay law. However, in our models, we recover such a law without assuming any *a priori* law for Λ. Thus our models are more general than those studied earlier.

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