

Unifying Brane World Inflation with Quintessence

M.Sami* and N. Dadhich†

*IUCAA, Post Bag 4, Ganeshkhind,
Pune 411 007, India.*

We review the recent attempts of unifying inflation with quintessence. It appears natural to join the two ends in the framework of brane world cosmology. The models of quintessential inflation belong to the class of *non-oscillatory* models for which the mechanism of conventional reheating does not work. Reheating through gravitational particle production is inefficient and leads to the excessive production of relic gravity waves which results in the violation of nucleosynthesis constraint. The mechanism of *instant preheating* is quite efficient and is suitable for brane world quintessential inflation. The model is shown to be free from the problem of excessive production of gravity waves. The prospects of Gauss-Bonnet brane world inflation are also briefly indicated.

PACS numbers: 98.80.Cq, 98.80.Hw, 04.50.+h

I. INTRODUCTION

Universe seems to exhibit an interesting symmetry with regard to accelerated expansion. It has gone under inflation at early epochs and is believed to be accelerating at present. The inflationary paradigm was originally introduced to address the initial value problems of the standard hot big bang model. Only later it became clear that the scenario could provide important clues for the origin of structure in the universe. The recent measurement of the Wilkinson Microwave Anisotropy Probe (WMAP) [2, 3] in the Cosmic Microwave Background (CMB) made it clear that (i) the current state of the universe is very close to a critical density and that (ii) primordial density perturbations that seeded large-scale structure in the universe are nearly scale-invariant and Gaussian, which are consistent with the inflationary paradigm. Inflation is often implemented with a single or multiple scalar-field models[4] (also see the excellent review on inflation by Shinji Tsujikawa[5]). In most of these models, the scalar field undergoes a slow-roll period allowing an accelerated expansion of the universe. After drawing the required amount of inflation, the inflaton enters the regime of quasi-periodic oscillation where it quickly oscillates and decays into particles leading to (p)reheating.

As for the current accelerating of universe, it is supported by observations of high redshift type Ia supernovae treated as standardized candles and, more indirectly, by observations of the cosmic microwave background and galaxy clustering. Within the framework of general relativity, cosmic acceleration should be sourced by an energy-momentum tensor which has a large negative pressure (dark energy)[6]. Therefore, the standard model should, in order to comply with the logical consistency and observation, be sandwiched between infla-

tion at early epochs and quintessence at late times. It is natural to ask whether one can build a model with scalar fields to join the two ends without disturbing the thermal history of universe. Attempts have been made to unify both these concepts using models with a single scalar field [7]. In these models, the scalar field exhibits the properties of tracker field. As a result it goes into hiding after the commencement of radiation domination; it emerges from the shadow only at late times to account for the observed accelerated expansion of universe. These models belong to the category of *non oscillating* models in which the standard reheating mechanism does not work. In this case, one can employ an alternative mechanism of reheating via quantum-mechanical particle production in time varying gravitational field at the end of inflation [8]. However, then the inflaton energy density should red-shift faster than that of the produced particles so that radiation domination could commence. And this requires a steep field potential, which of course, cannot support inflation in the standard FRW cosmology. This is precisely where the brane[9, 10] assisted inflation comes to the rescue.

The presence of the quadratic density term (high energy corrections) in the Friedman equation on the brane changes the expansion dynamics at early epochs [11](see Ref[12] for details on the dynamics of brane worlds) Consequently, the field experiences greater damping and rolls down its potential slower than it would during the conventional inflation. Thus, inflation in the brane world scenario can successfully occur for very steep potentials[13, 14]. The model of quintessential inflation based upon reheating via gravitational particle production is faced with difficulties associated with excessive production of gravity waves. Indeed the reheating mechanism based upon this process is extremely inefficient. The energy density of so produced radiation is typically one part in 10^{16} [14] to the scalar-field energy density at the end of inflation. As a result, these models have prolonged kinetic regime during which the amplitude of primordial gravity waves enhances and violates the nucleosynthesis constraint[15](see also [16]). Hence, it is necessary to

*On leave from: Department of Physics, Jamia Millia, New Delhi-110025; Electronic address: sami@iucaa.ernet.in

†Electronic address: nkd@iucaa.ernet.in

look for alternative mechanisms more efficient than the gravitational particle production to address the problem.

A proposal of reheating with Born-Infeld matter was made in Ref[17](see also Ref[18, 19] on the related theme). It was shown that reheating is quite efficient and the model does not require any additional fine tuning of parameters[17]. However, the model works under several assumptions which are not easy to justify.

The problems associated with reheating mechanisms discussed above can be circumvented if one invokes an alternative method of reheating, namely ‘instant preheating’ proposed by Felder, Kofman and Linde [20] (see also Ref[21] on the related theme. For other approaches to reheating in quintessential inflation see [22]). This mechanism is quite efficient and robust, and is well suited to non-oscillating models. It describes a new method of realizing quintessential inflation on the brane in which inflation is followed by ‘instant preheating’. The larger reheating temperature in this model results in a smaller amplitude of relic gravity waves which is consistent with the nucleosynthesis bounds[23]. However, the recent measurement of CMB anisotropies by WMAP places fairly strong constraints on inflationary models [24, 25]. It seems that the steep brane world inflation is on the verge of being ruled out by the observations[26]. Steep inflation in a Gauss-Bonnet braneworld may appear to be in better agreement with observations than inflation in a RS scenario [27].

II. QUINTESSENTIAL INFLATION

Quintessential inflation aims to describe a scenario in which both inflation and dark energy (quintessence) are described by the same scalar field. The unification of these concepts in a single scalar field model imposes certain constraints which were spelled out in the introduction. These concepts can be put together consistently in context with brane world inflation. Let us below list the building blocks of such a model.

- Alternative Mechanisms of Reheating
 - (i) Reheating via gravitational particle production.
 - (2) Curvaton reheating.
 - (3) Born-Infeld induced reheating.
 - (4) Instant preheating.
- Steep Inflaton Potential.
- Brane World Assisted Inflation.
- Tracher Field.
- Late Time Features in the Potential
 - (1) Potentials which become shallow at late time (such as inverse power law potentials)
 - (2) Potentials reducing to particular power law type at late times.

A. Steep Brane World Inflation

In what follows we shall work with the steep exponential potential which exhibits the aforementioned features necessary for the description of inflationary as well as post inflationary regimes. The brane world inflation with steep potentials becomes possible due to high energy corrections in the Friedmann equation. The exit from inflation also takes place naturally when the high energy corrections become unimportant.

In the 4+1 dimensional brane scenario inspired by the Randall-Sundrum [9] model, the standard Friedman equation is modified to [11]

$$H^2 = \frac{1}{3M_p^2}\rho \left(1 + \frac{\rho}{2\lambda_b}\right) + \frac{\Lambda_4}{3} + \frac{\mathcal{E}}{a^4} \quad (1)$$

where \mathcal{E} is an integration constant which transmits bulk graviton influence onto the brane and λ_b is the three dimensional brane tension which provides a relationship between the four and five-dimensional Planck masses and also relates the four-dimensional cosmological constant Λ_4 to its five-dimensional counterpart.

The four dimensional cosmological constant Λ_4 can be made to vanish by appropriately tuning the brane tension. The “dark radiation” \mathcal{E}/a^4 is expected to rapidly disappear once inflation has commenced so that we effectively get [11, 13]

$$H^2 = \frac{1}{3M_p^2}\rho \left(1 + \frac{\rho}{2\lambda_b}\right), \quad (2)$$

where $\rho \equiv \rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$, if one is dealing with a universe dominated by a single minimally coupled scalar field. The equation of motion of a scalar field propagating on the brane is

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \quad (3)$$

From (2) and (3) we find that the presence of the additional term ρ^2/λ_b increases the damping experienced by the scalar field as it rolls down its potential. This effect is reflected in the slow-roll parameters which have the form [13, 14]

$$\begin{aligned} \epsilon &= \epsilon_{\text{FRW}} \frac{1 + V/\lambda_b}{(1 + V/2\lambda_b)^2}, \\ \eta &= \eta_{\text{FRW}} (1 + V/2\lambda_b)^{-1}, \end{aligned} \quad (4)$$

where

$$\epsilon_{\text{FRW}} = \frac{M_p^2}{2} \left(\frac{V'}{V}\right)^2, \quad \eta_{\text{FRW}} = M_p^2 \left(\frac{V''}{V}\right) \quad (5)$$

are slow roll parameters in the absence of brane corrections. The influence of the brane term becomes important when $V/\lambda_b \gg 1$ and in this case we get

$$\epsilon \simeq 4\epsilon_{\text{FRW}}(V/\lambda_b)^{-1}, \quad \eta \simeq 2\eta_{\text{FRW}}(V/\lambda_b)^{-1}. \quad (6)$$

Clearly slow-roll ($\epsilon, \eta \ll 1$) is easier to achieve when $V/\lambda_b \gg 1$ and on this basis one can expect inflation to occur even for relatively steep potentials, such the exponential and the inverse power-law which we discuss below.

B. Exponential Potentials

The exponential potential

$$V(\phi) = V_0 e^{\alpha\phi/M_P} \quad (7)$$

with $\dot{\phi} < 0$ (equivalently $V(\phi) = V_0 e^{-\alpha\phi/M_P}$ with $\dot{\phi} > 0$) has traditionally played an important role within the inflationary framework since, in the absence of matter, it gives rise to power law inflation $a \propto t^c$, $c = 2/\alpha^2$ provided $\alpha \leq \sqrt{2}$. For $\alpha > \sqrt{2}$ the potential becomes too steep to sustain inflation and for larger values $\alpha \geq \sqrt{6}$ the field enters a kinetic regime during which field energy density $\rho_\phi \propto a^{-6}$. Thus within the standard general relativistic framework, steep potentials are not capable of sustaining inflation. However extra-dimensional effects lead to interesting new possibilities for the inflationary scenario. The increased damping of the scalar field when $V/\lambda_b \gg 1$ leads to a decrease in the value of the slow-roll parameters $\epsilon = \eta \simeq 2\alpha^2 \lambda_b/V$, so that slow-roll ($\epsilon, \eta \ll 1$) leading to inflation now becomes possible even for large values of α . The steep exponential potentials satisfies the post inflationary requirements mentioned earlier. Infact, the cosmological dynamics with steep exponential potential in presence of background (radiation/matter) admits scaling solution as the attractor of the system. The attractor is characterized by the tracking behavior of the field energy density ρ_ϕ . During the 'tracking regime', the ratio of ρ_ϕ to the background energy density ρ_B is held fixed

$$\frac{\rho_\phi}{\rho_\phi + \rho_B} = \frac{3(1+w_B)}{\alpha^2} \lesssim 0.2 \quad (8)$$

where w_B is the equation of state parameter for background ($w_B = 0, 1/3$ for matter and radiation respectively) and the inequality (8) reflects the nucleosynthesis constraint which requires $\alpha \gtrsim 5$. It is therefore clear that the field energy density in the post inflationary regime would keep tracking the background being subdominant such that it does not interfere with the thermal history of the universe.

Within the framework of the braneworld scenario, the field equations (2) and (3) can be solved *exactly* in the slow-roll limit when $\rho/\lambda_b \gg 1$. In this case

$$\frac{\dot{a}(t)}{a(t)} \simeq \frac{1}{\sqrt{6M_P^2 \lambda_b}} V(\phi), \quad (9)$$

which, when substituted in

$$3H\dot{\phi} \simeq -V'(\phi) \quad (10)$$

leads to

$$\dot{\phi}(t) = -\alpha\sqrt{2\lambda_b/3} \quad (11)$$

The expression for number of inflationary e-foldings is easy to establish

$$\begin{aligned} \mathcal{N} &= \log \frac{a(t)}{a_i} = \int_{t_i}^t H(t') dt' \\ &= \frac{V_0}{2\lambda_b \alpha^2} (e^{\alpha\phi_i} - e^{\alpha\phi(t)}) \\ &= \frac{V_i}{2\lambda_b \alpha^2} \left[1 - \exp \left\{ -\sqrt{\frac{2\lambda_b}{3M_P^2}} \alpha^2 (t - t_i) \right\} \right], \end{aligned} \quad (12)$$

where $V_i = V_0 e^{\alpha\phi_i}$. From Eq. (13) we find that the expansion factor passes through an inflection point marking the end of inflation and leading to

$$\phi_{\text{end}} = -\frac{M_P}{\alpha} \log \left(\frac{V_0}{2\lambda_b \alpha^2} \right), \quad (14)$$

$$V_{\text{end}} \equiv V_0 e^{\alpha\phi_{\text{end}}/M_P} = 2\lambda_b \alpha^2. \quad (15)$$

The COBE normalized value for the amplitude of scalar density perturbations allows to estimate V_{end} and the brane tension λ_b

$$\begin{aligned} V_{\text{end}} &\simeq \frac{3 \times 10^{-7}}{\alpha^4} \left(\frac{M_p}{\mathcal{N} + 1} \right)^4 \\ \lambda_b &\simeq \frac{1.3 \times 10^{-7}}{\alpha^6} \left(\frac{M_p}{\mathcal{N} + 1} \right)^4, \end{aligned} \quad (16)$$

We work here under the assumption that scalar density perturbations are responsible for most of the COBE signal. We shall, however, come back to the important question about the tensor perturbations later in our discussion.

The scenario of quintessential inflation we are discussing here belongs to the class of non-oscillatory models where the conventional reheating mechanism does not work. We can use the gravitational particle production to do the required. This is a democratic process which leads to the production of a variety of species quantum mechanically at the end of inflation when the space time geometry suffers a crucial change. Unlike the conventional reheating mechanism, this process does not require the introduction of extra fields. The radiation density created via this mechanism at the end of inflation is given by

$$\rho_r \sim 0.01 \times g_p H_{\text{end}}^4 \quad (17)$$

where $g_p \sim 100$ is the number of different particle species created from vacuum. Using the relation (17) and the expressions of λ_b and V_{end} obtained above, it can easily be shown that

$$\left(\frac{\rho_\phi}{\rho_r} \right)_{\text{end}} \sim 2 \times 10^{16} \left(\frac{\mathcal{N} + 1}{51} \right)^4 g_p^{-1} \quad (18)$$

This leads to a prolonged ‘kinetic regime’ during which scalar matter has the ‘stiff’ equation of state .

Using Eqs. (13) & (13) one can demonstrate that inflation proceeds at an exponential rate during early epochs which plays an important role for the generation of relic gravity waves during inflation.

III. LATE TIME EVOLUTION

As discussed above, the scalar field with exponential potential (20) leads to a viable evolution at early times. We should, however, ensure that the scalar field becomes quintessence at late times which demands a particular behavior of the scalar field potential as discussed above. Indeed, any scalar field potential which interpolates between an exponential at early epochs and the power law type potential at late times could lead to a viable cosmological evolution. The *cosine hyperbolic* potential provides one such example [29]

$$V(\phi) = V_0 [\cosh(\tilde{\alpha}\phi/M_p) - 1]^p, \quad p > 0 \quad (19)$$

which has asymptotic forms

$$V(\phi) = \frac{V_0}{2^p} e^{\alpha\phi/M_p}, \quad \tilde{\alpha}\phi/M_p \gg 1, \quad \phi > 0 \quad (20)$$

$$V(\phi) = \frac{V_0}{2^p} \left(\frac{\tilde{\alpha}\phi}{M_p} \right)^{2p} \quad |\tilde{\alpha}\phi/M_p| \ll 1 \quad (21)$$

where $\alpha = p\tilde{\alpha}$. As the *cosine hyperbolic* potential (19) exhibits power law type of behavior near the origin, field oscillations build up in the system at late times. For a particular choice of power law, the average equation of state parameter may turn negative[29, 30]

$$\langle w_\phi \rangle = \left\langle \frac{\frac{\dot{\phi}^2}{2} - V(\phi)}{\frac{\dot{\phi}^2}{2} + V(\phi)} \right\rangle = \frac{p-1}{p+1} \quad (22)$$

As a result the scalar field energy density and the scale factor have the following behavior

$$\rho_\phi \propto a^{-3(1+\langle w \rangle)}, \quad a \propto t^{\frac{2}{3}(1+\langle w \rangle)^{-1}}.$$

The average equation of state $\langle w(\phi) \rangle < -1/3$ for $p < 1/2$ allowing the scalar field to play the role of dark energy. We have numerically solved for the behaviour of this model after including a radiative term (arising from inflationary particle production discussed in the previous section) and standard cold dark matter. Our results for a particular realization of the model are shown in figures 1 & 2. We find that, due to the very large value of the scalar field kinetic energy at the commencement of the radiative regime, the scalar field density overshoots the radiation energy density. After this, the value of ρ_ϕ stabilizes and remains relatively unchanged for a considerable length

of time during which the scalar field equation of state is $w_\phi \simeq -1$. Tracking commences late into the matter dominated epoch and the universe accelerates today during rapid oscillations of the scalar field. This model provides an interesting example of ‘quintessential inflation’. However as we shall discuss next, the long duration of the kinetic regime in this model results in a large gravity wave background which comes into conflict with nucleosynthesis constraints.

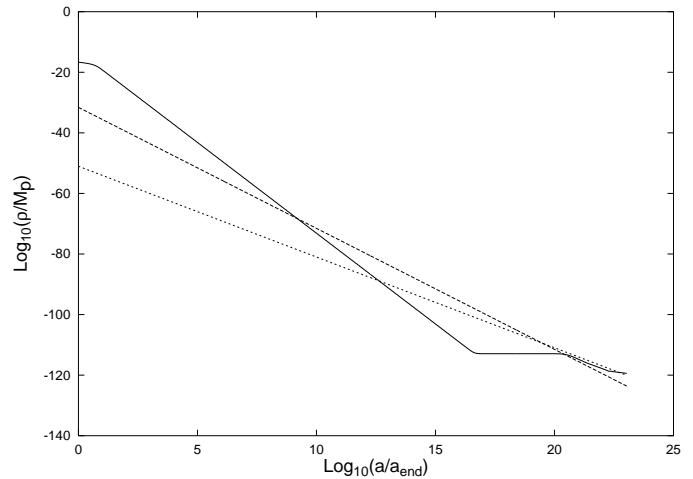


FIG. 1: The post-inflationary energy density in the scalar field (solid line) radiation (dashed line) and cold dark matter (dotted line) is shown as a function of the scale factor for the model described by (19) with $V_0 \simeq 5 \times 10^{-46} \text{ GeV}^4$, $\tilde{\alpha} = 5$ and $p = 0.2$. The enormously large value of the scalar field kinetic energy (relative to the potential) ensures that the scalar field density overshoots the background radiation value, after which ρ_ϕ remains approximately constant for a substantially long period of time. At late times the scalar field briefly tracks the background matter density before becoming dominant and driving the current accelerated expansion of the universe. From Sahni, Sami and Souradeep[15]

A. Relic Gravity Waves and Nucleosynthesis Constraint

The tensor perturbations or gravity waves get quantum mechanically generated during inflation and leave imprints on the micro-wave background.

Gravity waves in a spatially homogeneous and isotropic background geometry satisfy the minimally coupled Klein-Gordon equation $\square h_{ik} = 0$, which, after a separation of variables $h_{ij} = \phi_k(\tau)e^{-i\mathbf{k}\cdot\mathbf{x}}e_{ij}$ (e_{ij} is the polarization tensor) reduces to

$$\ddot{\phi}_k + 2\frac{\dot{a}}{a}\dot{\phi}_k + k^2\phi_k = 0 \quad (23)$$

where $\tau = \int dt/a(t)$ is the conformal time coordinate and $k = 2\pi a/\lambda$ is the comoving wavenumber. Since brane driven inflation is near-exponential we can write $a = \tau_0/\tau$

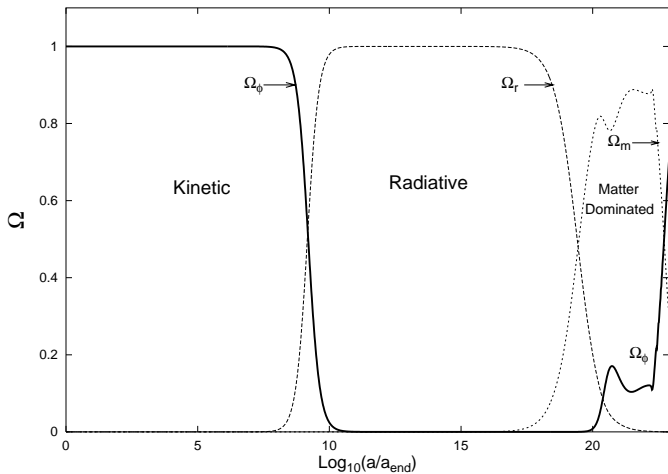


FIG. 2: The dimensionless density parameter Ω is plotted as a function of the scale factor for the model in figure 1. Late time oscillations of the scalar field ensure that the mean equation of state turns negative $\langle w_\phi \rangle \simeq -2/3$, giving rise to the current epoch of cosmic acceleration with $a(t) \propto t^2$ and present day values $\Omega_{0\phi} \simeq 0.7, \Omega_{0m} \simeq 0.3$. From Sahni, Sami and Souradeep[15]

($|\tau| < |\tau_0|$), in this case normalized positive frequency solutions of (23) corresponding to the adiabatic vacuum in the ‘in state’ are given by [15]

$$\phi_{\text{in}}^+(k, \tau) = \left(\frac{\pi\tau_0}{4}\right)^{1/2} \left(\frac{\tau}{\tau_0}\right)^{3/2} H_{3/2}^{(2)}(k\tau) F(H_{\text{in}}/\tilde{\mu}) \quad (24)$$

where $\tilde{\mu} = M_5^3/M_p^2$ and $H_{\text{in}} \equiv -1/\tau_0$ is the inflationary Hubble parameter. The term

$$F(x) = \left(\sqrt{1+x^2} - x^2 \log\left\{\frac{1}{x} + \sqrt{1 + \frac{1}{x^2}}\right\}\right)^{-1/2} \quad (25)$$

is responsible for the increased gravity wave amplitude in braneworld inflation [28]. The ‘out state’ is described by a linear superposition of positive and negative frequency solutions to (23). For power law expansion $a = (t/t_0)^p \equiv (\tau/\tau_0)^{1/2-\mu}$, we have

$$\phi_{\text{out}}(k, \tau) = \alpha\phi_{\text{out}}^{(+)}(k\tau) + \beta\phi_{\text{out}}^{(-)}(k\tau) \quad (26)$$

The energy density of relic gravity waves is given by [15]

$$\rho_g = \langle T_0^0 \rangle = \frac{1}{\pi^2 a^4} \int dk k^3 |\beta|^2, \quad (27)$$

Computing the Bogolyubov coefficient β , one can show that the spectral energy density of gravity waves produced during slow-roll inflation is [15, 23]

$$\rho_g(k) \propto k^2 \left(\frac{w-1/3}{w+1/3}\right). \quad (28)$$

where w is the equation of state parameter which characterizes the post-inflationary epoch. In braneworld model

under consideration, $w \simeq 1$ during the kinetic regime, consequently the gravity wave background generated during this epoch will have a blue spectrum $\rho_g(k) \propto k$.

We imagine that radiation through some mechanism was generated at the end of inflation with radiation density ρ_r . Then the ratio of energy in gravity waves to ρ_r at the commencement of radiative regime is given by [15]

$$\left(\frac{\rho_g}{\rho_r}\right)_{\text{eq}} = \frac{64}{3\pi} h_{GW}^2 \left(\frac{T_{kin}}{T_{eq}}\right)^2 \quad (29)$$

where h_{GW} is the dimensionless amplitude of gravity waves (from COBE normalization, $h_{GW}^2 \simeq 1.7 \times 10^{-10}$, for $\mathcal{N} \simeq 70$). We should mention that the commencement of the kinetic regime is not instantaneous and the brane effects persist for some time after inflation has ended. The temperature at the commencement of the kinetic regime T_{kin} is related to the temperature at the end of inflation as

$$T_{kin} = T_{\text{end}} \left(\frac{a_{\text{end}}}{a_{kin}}\right) = T_{\text{end}} F_1(\alpha) \quad (30)$$

where $F_1(\alpha) = (c + \frac{d}{\alpha^2})$, $c \simeq 0.142$, $d \simeq -1.057$ and $T_{\text{end}} = (\rho_r^{\text{end}})^{1/4}$. The equality between scalar field matter and radiation takes place at the temperature

$$T_{eq} = T_{\text{end}} \frac{F_2(\alpha)}{(\rho_\phi/\rho_r)_{\text{end}}^{1/2}} \quad (31)$$

with $F_2(\alpha) = (e + \frac{f}{\alpha^2})$, $e \simeq 0.0265$, $f \simeq -0.176$. The fitting formulas (30) and (31) are obtained by numerical integration of equations of motion.

Using equations (30), (31) and (29) we obtain the ratio of scalar field energy density to radiation energy density at the end of inflation

$$\left(\frac{\rho_\phi}{\rho_r}\right)_{\text{end}} = \frac{3\pi}{64} \left(\frac{1}{h_{GW}^2 (F_1(\alpha)/F_2(\alpha))^2}\right) \left(\frac{\rho_g}{\rho_r}\right)_{\text{eq}} \quad (32)$$

Equation (32) is an important result which sets a limit on the ratio of scalar field energy density to radiation energy density at the end of inflation. Indeed, For the nucleosynthesis constraint to be respected, the ratio of energy density in gravity waves to radiation energy density at equality $(\rho_g/\rho_r)_{\text{eq}} \lesssim 0.2$. For a generic steep exponential potential ($\alpha \gtrsim 5$), we have

$$(\rho_\phi/\rho_r)_{\text{end}} \lesssim 10^7 \quad (33)$$

As emphasized earlier, this ratio is of the order of 10^{16} in case gravitational particle production and exceeds the nucleosynthesis constraint by nine orders of magnitudes. An interesting proposal which can circumvent this difficulty has recently been suggested by Liddle and Lopez[22]. The authors have employed a new method of reheating via curvaton to address the problems associated with gravitational particle production mechanism.

The curvaton model as shown in Ref[22] can in principal resolve the difficulties related to excessive amplitude of short-scale gravitational waves. Although this model is interesting, it operates through a very complex network of constraints dictated by the fine tuning of parameters of the model.

In the following section, we shall examine an alternative mechanism based upon Born-Infeld reheating.

IV. BORN-INFELD BRANE WORLDS

The D-branes are fundamental objects in string theory. The end points of the open string to which the gauge fields are attached are constrained to lie on the branes. As the string theory contains gravity, the D-branes are the dynamical objects. The effective D-brane action is given by the Born-Infeld action

$$S_{BI} = -\lambda_b \int d^4x \sqrt{-\det(g_{\mu\nu} + F_{\mu\nu})} \quad (34)$$

where $F_{\mu\nu}$ is the electromagnetic field tensor (Non-Abelian gauge fields could also be included in the action) and λ_b is the brane tension. The Born-Infeld action, in general, also includes Fermi fields and scalars which have been dropped here for simplicity. In the brane world scenario *a la* Randall-Sundrum one adopts the Nambu-Goto action instead of the Born-Infeld action. Shiromizu *et al* have suggested that in the true spirit of the string theory, the total action in the brane world cosmology be composed of the bulk and D-brane actions[18]

$$S = S_{bulk} + S_{BI}, \quad (35)$$

where S_{bulk} is the five dimensional Einstein-Hilbert action with the negative cosmological constant. The stress tensor appearing on the right hand side (RHS) of the Einstein equations on the brane will now be sourced by the Born-Infeld action. The modified Friedman equation on a spatially flat FRW brane acquires the form

$$H^2 = \frac{1}{3M_p^2} \rho_{BI} \left(1 + \frac{\rho_{BI}}{2\lambda_b} \right) \quad (36)$$

with ρ_{BI} given by

$$\rho_{BI} = \epsilon + \frac{\epsilon^2}{6\lambda_b} \quad (37)$$

where $E^2 = B^2 = \epsilon$ The tension λ_b is tuned so that the net cosmological constant on the brane vanishes. We have dropped the ‘dark radiation’ term in the equation (36) as it rapidly disappear once inflation sets in. Spatial averaging is assumed while computing ρ_{BI} and P_{BI} from the stress-tensor corresponding to action (34). The scaling of energy density of the Born-Infeld matter, as usual, can be established from the conservation equation Born-Infeld matter, as usual, can be established from the conservation equation

$$\dot{\rho}_{BI} + 3H(\rho_{BI} + P_{BI}) = 0 \quad (38)$$

where

$$P_{BI} = \frac{\epsilon}{3} - \frac{\epsilon^2}{6\lambda_b} \quad (39)$$

Interestingly, the pressure due to the Born-Infeld matter becomes negative in the high energy regime allowing the accelerated expansion at early times without the introduction of a scalar field. As shown in [18], the energy density ρ_{BI} scales as radiation when $\epsilon \ll 6\lambda_b$. For $\epsilon > 6\lambda_b$, the Born-Infeld matter energy density starts scaling slowly (logarithmically) with the scale factor to mimic the cosmological constant like behavior. The point is that the Born-Infeld matter is subdominant during the inflationary stage. It comes to play the important role after the end of inflation when it behaves like radiation and hence serves as an alternative to reheating mechanism.

The brane world cosmology based upon the Born-Infeld action looks promising as it is perfectly tuned with the D-brane ideology. But since the Born-Infeld action is composed of the non-linear electromagnetic field, the D-brane cosmology proposed in Ref[18] can not accommodate density perturbations at least in its present formulation. One could include a scalar field in the Born-Infeld action, say, a tachyon condensate to correct the situation. However, such a scenario faces the difficulties associated with reheating[31, 32] and formation of acoustics/kinks[33]. We shall therefore not follow this track. We shall assume that the scalar field driving the inflation (quintessence) on the brane is described by the usual four dimensional action for the scalar fields. We should remark here that the problems faced by rolling tachyon models are beautifully circumvented in the scenario based upon massive Born-Infeld scalar field on the \bar{D}_3 brane of KKLT vacua[34].

The total action that we are trying to motivate here is given by

$$S = S_{bulk} + S_{BI} + S_{4d-scalar} \quad (40)$$

where

$$S_{4d-scalar} = - \int \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi) \right) \sqrt{-g} d^4x \quad (41)$$

The energy momentum tensor for the field ϕ which arises from the action (41) is given by

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi) \right] \quad (42)$$

The scalar field propagating on the brane modifies the Friedman equation to

$$H^2 = \frac{1}{3M_p^2} \rho_{tot} \left(1 + \frac{\rho_{tot}}{2\lambda_b} \right) \quad (43)$$

where ρ_{tot} is given by

$$\rho_{tot} = \rho_\phi + \rho_{BI} \quad (44)$$

As mentioned earlier, in the scenario based upon reheating via quantum mechanical particle production during inflation, the radiation density is very small, typically one part in 10^{16} and the ratio of the field energy density to that of radiation has no free parameter to tune. This leads to long kinetic regime which results in an unacceptably large gravity background. The Born-Infeld matter which behaves like radiation (at the end of inflation) has no such problem and can be used for reheating without conflicting with the nucleosynthesis constraint. Indeed, at the end of inflation ρ_{BI} can be chosen such that $\rho_{\text{BI}}^{\text{end}} \ll 6\lambda_b$. Such an initial condition for ρ_{BI} is consistent with the nucleosynthesis constraint[17]. In that case the Born-Infeld matter energy density would scale like radiation at the end of inflation. At this epoch the scale factor will be initialized at $a_{\text{end}} = 1$. The energy density ρ_{BI} would continue scaling as $1/a^4$ below $a = a_{\text{end}}$. The scaling would slow down as ρ_{BI} reaches $6\lambda_b$ which is much smaller than V_{end} for generic steep potentials, say for $\alpha \geq 5$. Hence ρ_{BI} remains subdominant to scalar field energy density ρ_ϕ for the entire inflationary evolution. The Born-Infeld matter comes to play the important role only at the end of inflation which is in a sense similar to curvaton. But unlike curvaton, it does not contain any new parameter. The numerical results for a specific choice of parameters is shown in Fig. 3. In contrast to the ‘quintessential inflation’ based upon the gravitational particle production mechanism where the scalar field spends long time in the kinetic regime and makes deep undershoot followed by long locking period with very brief tracking, the scalar field in the present scenario tracks the background for a very long time (see figure 3). This pattern of evolution is consistent with the thermal history of the universe. We note that ‘quintessential inflation’ can also be implemented by inverse power law potentials. Unfortunately, one has to make several assumptions to make the scenario working: i) The tension of the D3 brane appearing in the Born-Infeld action is treated as constant and is identified with the brane tension in the Randall-Sundrum scenario. (ii) The fluctuations in the Born-Infeld matter are neglected. (iii) The series expansion of the Born-Infeld action is truncated beyond a certain order. In what follows, we shall examine the instant reheating mechanism discovered by Felder, Kofman and Linde and show that their mechanism is superior to other reheating mechanism mentioned above.

V. BRANEWORLD INFLATION FOLLOWED BY INSTANT PREHEATING

Braneworld Inflation induced by the steep exponential potential (7) ends when $\phi = \phi_{\text{end}}$, see (14). Without loss of generality, we can make the inflation end at the origin by translating the field

$$V(\phi') \equiv V(\phi) = \tilde{V}_0 e^{\alpha\phi'/M_p}, \quad (45)$$

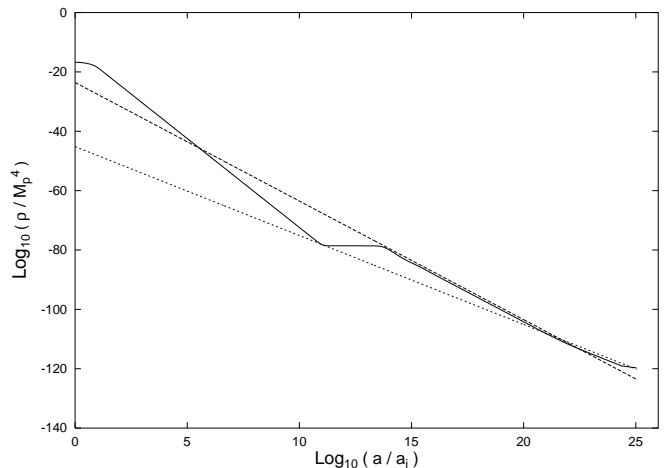


FIG. 3: The post-inflationary evolution of the scalar field energy density (solid line), radiation (dashed line) and cold dark matter (dotted line) is shown as a function of the scale factor for the quintessential inflation model described by (19) with $V_0^{1/4} \simeq 10^{-30} M_p$, $\tilde{\alpha} = 50$ and $p = 0.1$ ($\alpha = p\tilde{\alpha} = 5$). After brane effects have ended, the field energy density ρ_ϕ enters the kinetic regime and soon drops below the radiation density. After a brief interval during which $\langle w_\phi \rangle \simeq -1$, the scalar field begins to track first radiation and then matter. At very late times (present epoch) the scalar field plays the role of quintessence and makes the universe accelerate. The evolution of the energy density is shown from the end of inflation until the present epoch. From Sami, Dadhich and Shiromizu[17]

where $\tilde{V}_0 = V_0 e^{+\alpha\phi_{\text{end}}/M_p}$ and $\phi' = \phi - \phi_{\text{end}}$. In order to achieve reheating after inflation has ended we assume that the inflaton ϕ interacts with another scalar field χ which has a Yukawa-type interaction with a Fermi field ψ . The interaction Lagrangian is

$$L_{\text{int}} = -\frac{1}{2}g^2\phi'^2\chi^2 - h\bar{\psi}\psi\chi. \quad (46)$$

To avoid confusion, we drop the prime on ϕ remembering that $\phi < 0$ after inflation has ended. It should be noticed that the χ field has no bare mass, its effective mass being determined by the field ϕ and the value of the coupling constant g ($m_\chi = g|\phi|$).

The production of χ particles commences as soon as m_χ begins changing non-adiabatically [20]

$$|m_\chi| \gtrsim m_\chi^2 \quad \text{or} \quad |\dot{\phi}| \gtrsim g\phi^2. \quad (47)$$

The condition for particle production (47) is satisfied when

$$|\phi| \lesssim |\phi_{\text{prod}}| = \sqrt{\frac{|\dot{\phi}_{\text{end}}|}{g}} = \sqrt{\frac{V_{\text{end}}^{1/2}}{\sqrt{3}g}}. \quad (48)$$

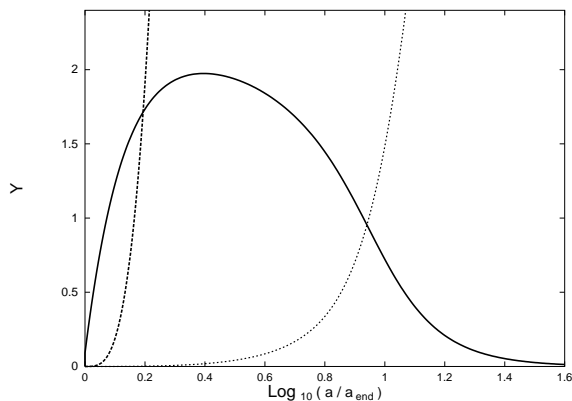


FIG. 4: The post inflationary evolution of $Y = 10^9 \times |\dot{\phi}|/M_p^2$, $g\phi^2/M_p^2$ is shown as a function of the scale factor for the model described by Eq (19) with $V_0 \simeq 10^{-30}M_p$, $\tilde{\alpha} = 50$ and $p = 0.1$. The solid curve corresponds to $\dot{\phi}/M_p^2$ whereas the dashed and dotted curves correspond to $g\phi^2/M_p^2$ for $g = 10^{-6}$ (extreme left) and $g = 10^{-9}$ (extreme right) respectively. The violation of the adiabaticity condition (necessary for particle production to take place) arises in the region bounded by the solid and dashed curves. The time duration of particle production is seen to be smaller for larger values of the coupling g . For the range of g allowed by the nucleosynthesis constraint, particle production is almost instantaneous. From Sami and Sahni[23]

From (16) we find that $\phi_{prod} \ll M_p$ for $g \gg 10^{-9}$. The production time for χ particles can be estimated to be

$$\Delta t_{prod} \sim \frac{|\phi|}{|\dot{\phi}|} \sim \frac{1}{\sqrt{gV_{end}^{1/2}}}. \quad (49)$$

The uncertainty relation provides an estimate for the momentum of χ particles created non-adiabatically: $k_{prod} \simeq (\Delta t_{prod})^{-1} \sim g^{1/2}V_{end}^{1/4}$. Proceeding as in [20] we can show that the occupation number of χ particles jumps sharply from zero to

$$n_k \simeq \exp(-\pi k^2/gV_{end}^{1/2}), \quad (50)$$

during the time interval Δt_{prod} . The χ -particle number density is estimated to be

$$n_\chi = \frac{1}{2\pi^3} \int_0^\infty k^2 n_k dk \simeq \frac{(gV_{end}^{1/2})^{3/2}}{8\pi^3}. \quad (51)$$

Quanta of the χ -field are created during the time interval Δt_{prod} that the field ϕ spends in the vicinity of $\phi = 0$. Thereafter the mass of the χ -particle begins to grow since $m_\chi = g|\phi(t)|$, and the energy density of particles of the χ -field created in this manner is given by

$$\rho_\chi = m_\chi n_\chi \left(\frac{a_{end}}{a}\right)^3 = \frac{(gV_{end}^{1/2})^{3/2}}{8\pi^3} g|\phi(t)| \left(\frac{a_{end}}{a}\right)^3. \quad (52)$$

where the $(a_{end}/a)^3$ term accounts for the cosmological dilution of the energy density with time. As shown above,

the process of χ particle-production takes place immediately after inflation has ended, provided $g \gtrsim 10^{-9}$. In what follows we will show that the χ -field can rapidly decay into fermions. It is easy to show that if the quanta of the χ -field were converted (thermalized) into radiation instantaneously, the radiation energy density would become

$$\rho_r \simeq \rho_\chi \sim \frac{(gV_{end}^{1/2})^{3/2}}{8\pi^3} g\phi_{prod} \sim 10^{-2} g^2 V_{end}. \quad (53)$$

From equation (53) follows the important result

$$\left(\frac{\rho_\phi}{\rho_r}\right)_{end} \sim \left(\frac{10}{g}\right)^2. \quad (54)$$

Comparing (54) with (33) we find that, in order for relic gravity waves to respect the nucleosynthesis constraint, we should have $g \gtrsim 4 \times 10^{-3}$. (The energy density created by instant preheating $(\rho_r/\rho_\phi) \simeq (g/10)^2$ can clearly be much larger than the energy density produced by quantum particle production, for which $(\rho_r/\rho_\phi) \simeq 10^{-16} g_p$.) The constraint $g \gtrsim 4 \times 10^{-3}$, implies that the particle production time-scale (49) is much smaller than the Hubble time since

$$\frac{1}{\Delta t_{prod} H_{end}} \gtrsim 300\alpha^2, \quad \alpha \gg 1. \quad (55)$$

Thus the effects of expansion can safely be neglected during the very short time interval in which ‘instant preheating’ takes place. We also find, from equation (48), that $|\phi_{prod}|/M_p \lesssim 10^{-3}$ implying that particle production takes place in a very narrow band around $\phi = 0$. Figure 4 demonstrates the violation of the adiabaticity condition (at the end of inflation) which is a necessary prerequisite for particle production to take place. For the range of g allowed by the nucleosynthesis constraint, the particle production turns out to be almost instantaneous.

We now briefly mention about the back-reaction of created χ -particles on the background. As shown in Ref[23], for any generic value of the coupling $g \lesssim 0.3$, the back-reaction of χ particles in the evolution equation is negligible during the time scale $\sim H_{kin}^{-1}$ (Here $\sim H_{kin}^{-1}$ characterizes the epoch the kinetic regime commences. $H_{kin} = H_{end}(0.085 - 0.688/\alpha^2)$ [15]).

We now turn to the matter of reheating which occurs through the decay of χ particles to fermions, as a consequence of the interaction term in the Lagrangian (46). The decay rate of χ particles is given by $\Gamma_{\bar{\psi}\psi} = h^2 m_\chi / 8\pi$, where $m_\chi = g|\phi|$. Clearly the decay rate is faster for larger values of $|\phi|$. For $\Gamma_{\bar{\psi}\psi} > H_{kin}$, the decay process will be completed within the time that back-reaction effects (of χ particles) remain small. Using the expression for H_{kin} this requirement translates into

$$h^2 > \frac{8\pi\alpha}{\sqrt{3}g\phi} \frac{V_{end}^{1/2}}{M_p} F(\alpha). \quad (56)$$

For reheating to be completed by $\phi/M_p \lesssim 1$, we find from equations (33) and (56) that $h \gtrsim 10^{-4} g^{-1/2} (g \gtrsim 4 \times$

10^{-3}) for $\alpha \simeq 5$. This along with the constraint imposed by the back reaction defines the allowed region in the parameter space (g, h) . We observe that there is a wide region in the parameter space for which (i) reheating is rapid and (ii) the relic gravity background in non-oscillatory braneworld models of quintessential inflation is consistent with nucleosynthesis constraints. However, this is not the complete story. One should further subject the model to the recent WMAP observations. The measurement of CMB anisotropies places fairly strong constraints on inflationary models [24, 25]. It appears that the tensor perturbations are not adequately suppressed in the models of steep brane world inflation and as a result these models are on the verge of being ruled out. As indicated by Lidsey and Nunes, inflation in a Gauss-Bonnet braneworld could appear to be in better agreement with observations than inflation in a RS II scenario [27]. In the following section, we briefly discuss the prospects of brane world inflation with the Gauss-Bonnet correction term in the bulk.

VI. GAUSS-BONNET BRANE WORLDS

Though we are trying to motivate the GB term in the bulk having a specific application in mind, the Gauss-Bonnet correction is interesting in its own right and has a deep meaning. Let us begin at the very beginning and ask for the compelling physical motivation for general relativity (GR). It is the interaction of zero mass particle with gravitation. Zero mass particle has the universal constant speed which can not change yet it must feel gravity. This can only happen if gravitational field curves space. Since space and time are already bound together by incorporation of zero mass particle in mechanics, gravitational field thus curves spacetime. In other words it can truly be described by curvature of spacetime and it thus becomes a property of spacetime - no longer an external field [35]

From the physical standpoint the new feature that GR has to incorporate is that gravitational field itself has energy and hence like any other energy it must also link to gravity. That is, field has gravitational charge and hence it is self interacting. Field energy density will go as square of first derivative of the metric and it must be included in the Einstein field equation. It is indeed included for the Riemann curvature involves the second derivative and square of the first derivative. However in the specific case of field of an isolated body, we obtain $1/r$ potential, the same as in the Newtonian case. Where has the square of $\nabla\Phi$ (Φ denotes the gravitational potential) gone? It turns out that its contribution has gone into curving the space, g_{rr} component of the metric being different from 1.

The main point is that gravitational field equation should follow from the curvature of spacetime and they should be second order quasilinear differential equations (quasilinear means the highest order of derivative must

occur linearly so that the equation admits a unique solution). Riemann curvature through the Bianchi identities leads to the Einstein equation with the Λ term. We should emphasize here that Λ enters here as naturally as the stress energy tensor. It is indeed a true new constant of the Einsteinian gravity [36]. It is a pertinent question to ask, is this the most general second order quasilinear equation one can obtain from curvature of spacetime? The answer is No. There exists a remarkable combination of square of Riemann tensor and its contractions, which when added to the action gives a second order quasilinear equation involving second and fourth power of the first derivative. This is what is the famous Gauss-Bonnet (GB) term. Thus GB term too appears naturally and should have some non-trivial meaning.

However GB term is topological in $D < 5$ and hence has no dynamics. It attains dynamics in $D > 4$. Note that gravity does not have its full dynamics in $D < 4$ and hence the minimum number of dimensions required for complete description of gravitation is 4. This self interaction of gravity arises through square of first derivative of the metric. Self interaction should however be iterative and hence higher order terms should also be included. It turns out that there exists generalization of the GB term in higher dimensions in terms of the Lovelock Lagrangian which is a polynomial in the Riemann curvature. That again yields the quasilinear second order equation with higher powers of the first derivative. Thus GB and Lovelock Lagrangian represent higher order loop corrections to the Einstein gravity.

They do however make non-trivial contribution classically only for $D > 4$ dimensions. This is rather important. If GB term had made a non-trivial contribution in 4-dimensions, it would have conflicted with the $1/r$ character of the potential because of the presence of $(\nabla\Phi)^4$ terms in the equation. The square terms (to account for contribution of gravitational field energy) were taken care of by the space curvature (g_{rr} in the metric) and now nothing more is left to accommodate the fourth (and higher) power term. However we can not tamper with the inverse square law (i.e. $1/r$ potential) which is independently required by the Gauss law of conservation of flux. That can not be defied at any cost. Thus it is not for nothing that the GB and its Lovelock generalization term makes no contribution for $D = 4$. It further carries an important message that gravitational field cannot be kept confined to 4-dimensions. It is indeed a higher dimensional interaction where the higher order iterations attain meaning and dynamics. This is the most profound message the GB term indicates. This is yet another independent and new motivation for higher dimensional gravity [36].

Self interaction is always to be evaluated iteratively. For gravity iteration is on the curvature of spacetime. It is then not surprising that GB term arises naturally from the one loop correction to classical gravity. String theory should however encompass whatever is obtained by iterative the iterative process. GB term is therefore

strongly motivated by string theoretic considerations as well. Further GB is topological in 4-D but in quantum considerations it defines new vacuum state. It is quantum mechanically non-trivial. In higher dimensions, it attains dynamics even at classical level. In the simplest case in higher dimension it should have a classical analogue of 4-D quantum case. That is what indeed happens. Space of constant curvature or equivalently conformally flat Einstein space solves the equation with GB term with redefined vacuum. This is a general result for all $D > 4$. It is interesting to see quantum in lower dimension becoming classical in higher dimension.

In the context of the brane bulk system we should therefore include GB term in the bulk and see its effects on the dynamics on the brane. The brane world gravity should thus be studied with GB term not necessarily as correction but in its own right. It is a true description of high energy gravity. However, for the purpose of following discussion, we shall treat GB as a correction term.

The Einstein-Gauss-Bonnet action for five dimensional bulk containing a 4D brane is

$$S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} \{ \mathcal{R} - 2\Lambda_5 + \alpha_{\text{GB}} [\mathcal{R}^2 - 4\mathcal{R}_{AB}\mathcal{R}^{AB} + \mathcal{R}_{ABCD}\mathcal{R}^{ABCD}] \} + \int d^4x \sqrt{-h} (\mathcal{L}_m - \lambda_b), \quad (57)$$

\mathcal{R} , R refer to the Ricci scalars in the bulk metric g_{AB} and the induced metric on the brane h_{AB} ; α_{GB} has dimensions of $(\text{length})^2$ and is the Gauss-Bonnet coupling, while λ_b is the brane tension and $\Lambda_5 (< 0)$ is the bulk cosmological constant. The constant κ_5 contains the M_5 , the 5D fundamental energy scale ($\kappa_5^2 = M_5^{-3}$). The modified Friedman equation on the (spatially flat) brane may be written as [27, 38, 39] (see also Ref[41])

$$H^2 = \frac{1}{4\alpha_{\text{GB}}} \left[(1 - 4\alpha_{\text{GB}}\mu^2) \cosh\left(\frac{2\chi}{3}\right) - 1 \right] \quad (58)$$

$$\kappa_5^2(\rho + \lambda_b) = \left[\frac{2(1 - 4\alpha_{\text{GB}}\mu^2)^3}{\alpha_{\text{GB}}} \right]^{1/2} \sinh \chi, \quad (59)$$

where χ is a dimensionless measure of the energy-density. In order to regain general relativity at low energies, the effective 4D Newton constant is defined by [39]

$$\kappa_4^2 \equiv \frac{1}{M_p^2} = \frac{\kappa_5^4 \lambda_b}{6(1 - 4\alpha_{\text{GB}}\Lambda_5/9)}. \quad (60)$$

When $\alpha_{\text{GB}} = 0$, we recover the RS expression. We can fine-tune the brane tension to achieve zero cosmological constant on the brane [39]:

$$\kappa_5^4 \lambda_b^2 = -4\Lambda_5 + \frac{1}{\alpha_{\text{GB}}} \left[1 - \left(1 + \frac{4}{3}\alpha_{\text{GB}}\Lambda_5 \right)^{3/2} \right]. \quad (61)$$

The modified Friedman equation (58), together with Eq. (59), shows that there is a characteristic GB energy

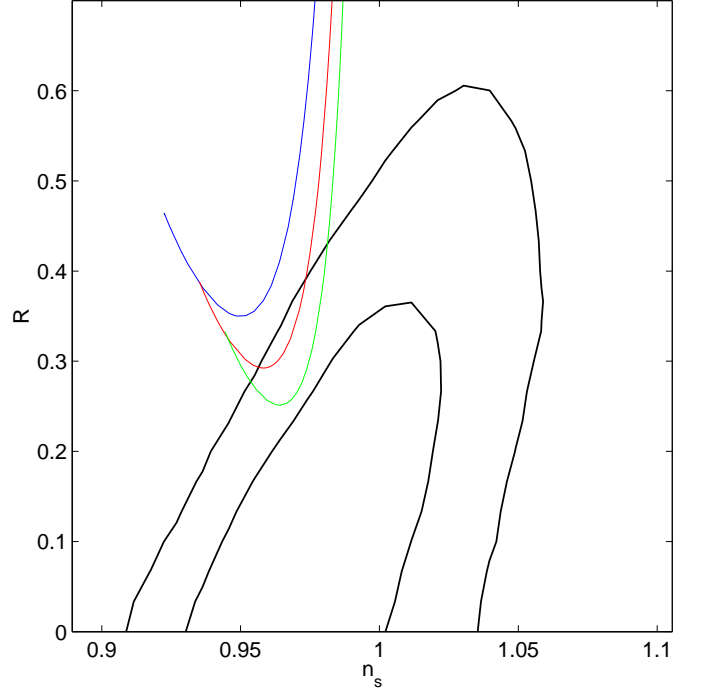


FIG. 5: Plot of R ($R \equiv 16A_7^2/A_5^2$ – according to the normalization used here[42]) versus the spectral index n_s in case of the exponential potential for $\mathcal{N} = 50, 60, 70$ (from top to bottom) along with the 1σ and 2σ observational contours. These curves exhibit a minimum in the intermediate region between GB (extreme right) and the RS (extreme left) regimes. The upper limit on n_s is dictated by the quantum gravity limit where as the lower bound is fixed by the requirement of ending inflation in the RS regime[42]. For a larger value of the number of e-folds \mathcal{N} , more points are seen to be within the 2σ bound. Clearly, steep inflation in the deep GB regime is not favored due to the large value of R inspite the spectral index being very close to 1 there. From Sami and Sahni[23]

scale M_{GB} [40] such that,

$$\rho \gg M_{\text{GB}}^4 \Rightarrow H^2 \approx \left[\frac{\kappa_5^2}{16\alpha_{\text{GB}}} \rho \right]^{2/3}, \quad (62)$$

$$M_{\text{GB}}^4 \gg \rho \gg \lambda_b \Rightarrow H^2 \approx \frac{\kappa_4^2}{6\lambda_b} \rho^2, \quad (63)$$

$$\rho \ll \lambda_b \Rightarrow H^2 \approx \frac{\kappa_4^2}{3} \rho. \quad (64)$$

It should be noted that Hubble law acquires an un-

usual form for energies higher than the GB scale. Interestingly, for an exponential potential, the modified Eq.(62) leads to exactly scale invariant spectrum for primordial density perturbations. Inflation continues below GB scale and terminates in the RS regime leading to the spectral index very close to one. This is amazing that it

happens without tuning the slope of the potential. The Gauss-Bonnet inflation has interesting consequences for steep brane world inflation (see Fig. 5 and the discussion in the next section).

VII. SUMMARY

In this paper we have reviewed the recent work on unification of inflation with quintessence in the framework of brane worlds. These models belong to the class of *non-oscillatory* models in which the underlying alternative reheating mechanism plays a crucial role. The popular reheating alternative via quantum mechanical production of particle during inflation leads to an unacceptable relic gravity wave background which violates the nucleosynthesis constraint at the commencement of radiative regime. We have mentioned other alternatives to conventional (p)reheating and have shown that 'instant preheating' discovered by Felder, Linde and Kofman is superior and best suited to brane world models of quintessential inflation. The recent measurement of

CMB anisotropies by WMAP, appears to heavily constraint these models. The steep brane world inflation seems to be excluded by observation in RS scenario[26]. As shown in Ref.[42], the inclusion of GB term in the bulk effects the constraints on the inflationary potentials and can rescue the steep exponential potential allowing it to be compatible with observations for a range of energy scales. The GB term leads to an increase of the spectral index n_s and decrease of tensor to scalar ratio of perturbations R in the intermediate region between RS and GB. As seen from Fig.5, there is an intermediate region where the steep inflation driven by exponential potential lies within 2σ contour for $\mathcal{N} = 70$. Thus, the steep inflation in a Gauss-Bonnet braneworld appears to be in agreement with observations.

VIII. ACKNOWLEDGMENTS

We thank N. Mavromatos, S. Odintsov, V. Sahni and Shinji Tsujikawa for useful comments.

-
- [1] A. Linde, *Particle Physics and Inflationary Cosmology*, Harwood, Chur (1990); A. R. Liddle and D. H. Lyth, *Cosmological inflation and large-scale structure*, Cambridge University Press (2000).
- [2] C. L. Bennett *et al.*, *Astrophys. J. Suppl.* **148**, 1 (2003) [arXiv:astro-ph/0302207].
- [3] D. N. Spergel *et al.*, *Astrophys. J. Suppl.* **148**, 175 (2003) [arXiv:astro-ph/0302209].
- [4] D. H. Lyth and A. Riotto, *Phys. Rept.* **314**, 1 (1999) [arXiv:hep-ph/9807278].
- [5] Shinji Tsujikawa, hep-ph/0304257.
- [6] V. Sahni and A. A. Starobinsky, *IJMPD* **9**, 373 (2000) [arXiv:astro-ph/9904398]; T. Padmanabhan, *Phys. Rept.* **380**, 235 (2003) [arXiv:hep-th/0212290]; B. Ratra and P. J. E. Peebles, *Phys. Rev. D* **37**, 3406 (1988); C. Wetterich, *Nucl. Phys. B* **302**, 668 (1988); J. A. Frieman, C. T. Hill, A. Stebbins and I. Waga, *Phys. Rev. Lett.* **75**, 2077 (1995) [arXiv:astro-ph/9505060]; P. G. Ferreira and M. Joyce, *Phys. Rev. Lett.* **79**, 4740 (1997) [arXiv:astro-ph/9707286]; I. Zlatev, L. M. Wang and P. J. Steinhardt, *Phys. Rev. Lett.* **82**, 896 (1999) [arXiv:astro-ph/9807002]; P. Brax and J. Martin, *Phys. Rev. D* **61**, 103502 (2000) [arXiv:astro-ph/9912046]; T. Barreiro, E. J. Copeland and N. J. Nunes, *Phys. Rev. D* **61**, 127301 (2000) [arXiv:astro-ph/9910214]; A. Albrecht and C. Skordis, *Phys. Rev. Lett.* **84**, 2076 (2000) [arXiv:astro-ph/9908085]; V. Sahni, astro-ph/0403324; D.F. Mota, C. van de Bruck, astro-ph/0401504; P. Singh, M.Sami, Naresh Dadhich, hep-th/0305110.
- [7] P. J. Peebles and A. Vilenkin, *Phys. Rev. D* **59**, 063505 (1999). E.J. Copeland, A. R. Liddle and J. E. Lidsey, *Phys. Rev. D* **64**, 023509 (2001) [astro-ph/0006421]; G. Huey and Lidsey, *Phys. Lett. B* **514**, 217(2001); A. S. Majumdar, *Phys.Rev. D* **64** (2001) 083503.
- [8] L.H. Ford, *Phys. Rev. D* **35**, 2955 (1987); B. Spokoyny, *Phys. Lett. B* **315**, 40 (1993).
- [9] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 4690 (1999); L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 3370 (1999).
- [10] S. W. Hawking, T. Hertog, and H. Reall, *Phys. Rev. D* **62** 043501 (2000); S. Nojiri, S. D. Odintsov, and S. Zerbini, *Phys. Rev. D* **62** 064006 (2000); S. Nojiri and S. Odintsov, *Phys. Lett. B* **484**, 119 (2000); L. Anchordoqui, C. Nunez, and K. Olsen, *J. High Energy Phys. Phys.* **10**, 050 (2000); S. Nojiri and S. Odintsov, *Phys. Lett. B* **494**, 135 (2000); S. Gubser, *Phys. Rev. D* **63**, 084017 (2001); T. Shiromizu and D. Ida, *Phys. Rev. D* **64**, 044015 (2001).
- [11] J.M. Cline, C. Grojean and G. Servant, *Phys. Rev. Lett.* **83** 4245 (1999); N. Kaloper, *Phys. Rev. D*, **60** 123506 (1999); T. Shiromizu, K. Maeda, and M. Sasaki, *Phys. Rev. D*, **62**, 024012 (2000); P. Binetruy, C. Deffayet, U. Ellwanger, and D. Langlois, *Phys. Lett.* **B477**, 285 (2000); E. E. Flanagan, S.-H. Tye, and I. Wasserman, *Phys. Rev. D*, **62** 044039 (2000).
- [12] Roy Maartens, "Brane-world gravity", gr-qc/0312059.
- [13] R. Maartens, D. Wands, B.A. Bassett, and I.P.C. Heard, *Phys. Rev. D*, **62**, 041301 (2000).
- [14] E.J. Copeland, A.R. Liddle, and J.E. Lidsey, astro-ph/0006421
- [15] V. Sahni, M. Sami and T. Phys. Rev. D **65**, 023518(2002).
- [16] A. A. Starobinsky, *JETP Lett.*, **30**, 682 (1979); V. Sahni, *Phys. Rev. D* **42**, 453 (1990); M. Giovannini, *Phys. Rev. D* **58**, 083504 (1998); *Phys. Rev. D* **60**, 123511 (1999); T. Kobayashi, H. Kudoh and T. Tanaka, gr-qc/0305006; T. Hiramatsu, K. Koyama and A. Taruya, hep-th/0308072; R. Easther, D. Langlois, R. Maartens and D. Wands, hep-th/0308078.
- [17] M. Sami, N. Dadhich and T. Shiromizu, *Phys.Lett. B* **568**

- (2003) 118-126[hep-th/0304187].
- [18] T. Shiromizu, T. Torii and T. Uesugi, hep-th/0302223;
- [19] S. Onda , T. Shiromizu , K. Koyama and S. Hayakawa, hep-th/0311262; K. Takahashi , K. Ichikawa, hep-th/0310142; T. Uesugi , T. Shiromizu T. Torii and K. Takahashi, hep-th/0310059; E. Elizalde , J. E. Lidsey , S. Nojiri and S. D. Odintsov, Phys.Lett.B **574**,1,2003 [hep-th/0307177]; T. Shiromizu , K. Koyama , S. Onda and T. Torii, Phys.Rev.D **68**, 063506 (2003) [hep-th/0305253]; M. Novello, S. E. Perez Bergliaffa, J. Salim, astro-ph/0312093.
- [20] G. Felder, L. Kofman, A.D. Linde, Phys.Rev. D **59** 123523 (1999) [hep-ph/9812289]; G. Felder, L. Kofman, A.D. Linde, Phys.Rev. D **60** 103505 (1999) [hep-ph/9903350]; L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. D**56** 3258 (1997); L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. Lett. **73**, 3195 (1994).
- [21] Y. Shtanov, J. Traschen and R. Brandenberger, Phys. Rev.D **51**, 5438 (1995); A. H. Campos, H. C. Reis and R. Rosenfeld, Phys. Lett. **B** 575, 151(2003); A. H. Campos, J. M. F. Maia and R. Rosenfeld, astro-ph/0402413; H. Tashiro, T. Chiba and M. Sasaki, gr-qc/0307068.
- [22] B. Feng and M. Li, Phys.Lett., **B** 564, 169 (2003)[hep-ph/0212213]; R. Liddle and L. Arturo Urena-Lopez, astro-ph/0302054.
- [23] M. Sami and V. Sahni, hep-th/0402086 (to appear in PRD).
- [24] H.V. Peiris, et al., Astrophys.J.Suppl. **148**, 213 (2003) [astro-ph/0302225].
- [25] M. Tegmark, et al., astro-ph/0310723.
- [26] S. Tsujikawa and A.R. Liddle, astro-ph/0312162.
- [27] J.E. Lidsey and N.J. Nunes, Phys. Rev. D, **67**, 103510 (2003).
- [28] D. Langlois, R. Maartens and D. Wands, Phys. Lett. B **489**, 259 (2000).
- [29] V. Sahni and L. Wang, Phys. Rev. **D62**, 103517 (2000).
- [30] M. S. Turner, Phys. Rev. **D28** ,1243 (1983); T. Damour and V. F. Mukhanov , Phys. Rev. Lett. **80**, 3440 (1998); A. R. Liddle and A. Mazumdar, Phys. Rev. **D58**,083508(1998); V. Cardenas and G. Palma ,Phys. Rev. **D61** ,027301(1999); J. Lee et al, Phys. Rev. **D61** ,027302 (1999); M. Sami, Grav. Cosmol. **8**, 309(2003); M. Sami and T. Padmanabhan, Phys. Rev. **D67**, 083509(2003)[hep-th/0212317]; Stephen D.H. Hsu, Phys. Lett. **B567** (2003) 9-11[astro-ph/0305096].
- [31] Lev Kofman and Andrei Linde, JHEP **0207**, (2002)004[hep-th/0205121].
- [32] M. Sami, Pravabati Chingangbam and Tabish Qureshi, Phys. Rev., **D66**, (2002) 043530; M. Sami, Pravabati Chingangbam and Tabish Qureshi, Pramana **62** (2004)765[hep-th/0301140]; M. Sami, hep-th/0205146.
- [33] G. Felder, Lev Kofman and Alexei Starobinsky, JHEP **0209** (2002) 026[hep-th/0208019] ; E. J. Copeland, P. M. Saffin and D. A. Steer, Phys. Rev. D **68**, 065013(2003).
- [34] M. R. Garousi, M. Sami and S. Tsujikawa, hep-th/0402075.
- [35] N. Dadhich, "Subtle is the Gravity", gr-qc/0102009.
- [36] N. Dadhich,"Universalization as physical guiding principle", gr-qc/0311028
- [37] B. Zwiebach, in *Anomalies, Geometry and Topology* - proceedings of the symposium, Argonne, Illinois, 1985, edited by W. A. Bar; D. Lorentz-Petzold, *Prog. Theor. Phys.* **78**, 969 (1987);*Mod. Phys. Lett. A* **3**, 827 (1988); A. B. Henriques, *Nucl. Phys.* **277 B**, 621 (1986); Q. Shafi and C. Wetterich, *Phys. Lett.* **129 B**, 387 (1983); *ibid* **152 B**, 51 (1984); H. Ishihara, *Phys. Lett.* **179 B**, 217 (1986); B. C. Paul and S. Mukherjee, *Phys. Rev. D* **42**, 2595 (1990); I. P. Neupane, hep-th/0108194; I. P. Neupane, hep-th/0106100.
- [38] C.Charmousis and J-F. Dufaux, CQG **19**, 4671(2002)[hep-th/0202107].
- [39] Kei-ichi Maeda, Takashi Torii, hep-th/0309152.
- [40] J.-F. Dufaux, J. Lidsey, R. Maartens, M. Sami, hep-th/0404161.
- [41] Shin'ichi Nojiri, Sergei D. Odintsov, hep-th/0006232; B. Abdesselam and N. Mohammadi, *Phys. Rev. D* **65**, 084018 (2002); J. E. Lidsey, Shin'ichi Nojiri, S. D. Odintsov, hep-th/0202198; S.C. Devis, hep-th/0208205; E. Gravanis and S. Willison, hep-th/0209076; S. Nojiri, S.Odintsov and S. Ogushi, Phys. Rev. D **65** (2002) 023521; J. P. Gregory, A. Padilla, hep-th/0304250; H. M. Lee, hep-th/0010193 (2000); J. E. Kim and H. M. Lee, gr-qc/0306116 (2003); B. C. Paul and M. Sami, Phys. Rev. D **70** (2004)027301[hep-th/0312081]; N. Deruelle and C. Germani, gr-qc/0306116 (2003); Nathalie Deruelle and John Madore, gr-qc/0305004; M. Sami, N. Savchenko and A. Toporensky, hep-th/0408140.
- [42] Shinji Tsujikawa, M. Sami and Roy Maartens, astro-ph/0406078.