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Rotating embedded black holes: Entropy and Hawking's radiation

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Abstract

In this paper, by applying Newman-Janis algorithm to a spherically symmetric 'seed' metric, we present general rotating metrics in terms of Newman-Penrose (NP) quantities involving Wang-Wu functions. From these NP quantities we present a class of rotating solutions including (i) Vaidya-Bonnor, (ii) Kerr-Newman-Vaidya, (iii) de Sitter, (iv) Kerr-Newman-Vaidya-de Sitter and (v) Kerr-Newman-monopole. The rotating Kerr-Newman-Vaidya solution represents a black hole that the Kerr-Newman black hole is embedded into the rotating Vaidya radiating universe. In the case of Kerr-Newman-Vaidya-de Sitter solution, one can describe it as the Kerr-Newman black hole is embedded into the rotating Vaidya-de Sitter universe, and similarly, Kerr-Newman-monopole. We have also discussed the physical properties by observing the energy momentum tensors of these solutions. These embedded solutions can be expressed in Kerr-Schild forms describing the extensions of Glass and Krisch superposition, which is further the extension of Xanthopoulos superposition. It is shown that, by considering the charge to be a function of radial coordinate, the Hawking's continuous radiation of black holes can be expressed in classical spacetime metrics for these embedded black holes. It is also found that the electrical radiation will continue to form 'instantaneous' charged black holes and creating embedded negative mass naked singularities describing the possible life style of radiating embedded black holes during their contineous radiation processes. The surface gravity, entropy and angular velocity, which are important parameters of a horizon, are also presented for each of the embedded black holes.

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1. INTRODUCTION

In the theory of black hole expressed in spherical polar coordinates, there is a singularity at the origin $r = 0$; whereas at infinity $r = \infty$, the metric approaches the Minkowski flat space. The event horizons of a black hole are expressed by this coordinate. Thus, the nature of black holes depends on the radial coordinate r. In an earlier paper [1] it is shown that Hawking's radiation [2] can be expressed in classical spacetime metrics, by considering the charge e to be function of the radial coordinate r of non-rotating Reissner-Nordstrom as well as rotating Kerr-Newman black holes. The variable-charge $e(r)$ with respect to the coordinate r is followed from Boulware's suggestion [3] that *the stress-energy tensor may be used to calculate the change in the mass due to the radiation*. According to Boulware's suggestion, the energy momentum tensor of a particular space can be used to calculate the change in the mass in order to incorporate the Hawking's radiation effects in classical spacetime metrics. This idea suggests to consider the stress-energy tensor of electromagnetic field of different forms or functions from those of Reissner-Nordstrom, as well as Kerr-Newman, black holes as these two black holes do not seem to have any direct Hawking's radiation effects. Thus, a variable charge in the field equations will have the different function of the energy momentum tensor of the charged black hole. *Such a variable charge e with respect to the coordinate r in Einstein's equations is referred to as an electrical radiation* (or *Hawking's electrical radiation) of the black hole*. So, for every electrical radiation we consider the charge e to be a function of r in solving the Einstein-Maxwell field equations and we have shown *mathematically* how the electrical radiation induces to produce the changes of the mass of *variable*-charged black holes. One may incorporate the idea of losing (or changing) mass at the rate as the electrical energy is radiated from the charged black hole. In fact, the change in the mass of a charged black hole takes place due to the vanishing of Ricci scalar of the electromagnetic field. Every electrical radiation $e(r)$ of the black holes leads to a reduction in its mass by some quantity. If we consider such electrical radiation taking place continuously for a long time, then a continuous reduction of the mass will take place in the black hole body, and the original mass of the black hole will evaporate completely. At that stage the complete evaporation of the mass will lead the gravity of the object depending only on the electromagnetic field, and not on the mass. We refer to such an object with zero mass as an 'instantaneous' naked singularity - a naked singularity that exists for an instant and then continues its electrical radiation to create negative mass [1]. So this naked singularity is different from the one mentioned in Steinmular *et al* [4], Tipler *et al* [5] in the sense that an 'instantaneous' naked singularity, discussed in [4,5] exists only for an instant and then disappears.

It is also noted that the time taken between two consecutive radiations is supposed to be so short that one may not physically realize how quickly radiations take place. Thus, it seems natural to expect the existence of an 'instantaneous' naked singularity with zero mass only for an instant before continuing its next radiation to create a negative mass naked singularity. This suggests that it may also be possible in the common theory of black holes that, as a black hole is invisible in nature, one may not know whether, in the universe, a particular black hole has mass or not, but electrical radiation may be detected on the black hole surface. Immediately after the complete evaporation of the mass, if one continues to radiate the remaining remnant, there may be a formation of a new mass. If one repeats the electrical radiation further, the new mass might increase gradually and then the spacetime geometry will represent the negative mass naked singularity. The classical spacetime metrics, for both stationary rotating and non-rotating, which represent the negative mass naked singularities have given in [1].

The aim of this paper is to give examples of rotating charged solutions of Einstein's field equations from the general metric for studying the Hawking's electrical radiation effects. The solutions derived here describe rotating embedded black holes i.e., the Kerr-Newman black hole is embedded into (i) the rotating Vaidya null radiating space, (ii) the rotating Vaidya-de Sitter cosmological universe and (iii) the rotating monopole space to generate the Kerr-Newman-Vaidya, the Kerr-Newman-Vaidya-de Sitter and the Kerr-Newman-monopole black holes respectively. The definitions of these embedded black holes are in agreement with the one defined by Cai et at. [6] , that *when the Scharzschild black hole is embedded into the de Sitter space, one has the Schwarzschild-de Sitter black hole*. Thus, these solutions describe new *non-stationary* rotating, Kerr-Newman-Vaidya and Kerr-Newman-Vaidya-de Sitter, and *stationary* Kerr-Newman-monopole black holes. These embedded rotating solutions can be expressed in Kerr-Schild forms to regard them as the extension of Glass-Krisch superposition [7], which is further the extension of that of Xanthopoulos [8]. These Kerr-Schild ansatze show that these embedded black holes are solutions of Einstein's field equations. It is also noted that such generation of embedded solutions in *non-rotating* cases can be seen in [9]. Here we try to extend the earlier results [1] based on Hawking's radiation of *non-embedded*, Reissner-Nordstrom as well as Kerr-Newman, black holes to these *embedded* Kerr-Newman-Vaidya, Kerr-Newman-Vaidya-de Sitter and Kerr-Newman-monopole black holes.

This paper is organized as follows: Section 2 deals with the application of Newman-Janis algorithm to a spherically symmetric 'seed' metric with the function M and e of two variables u, r and the presentation of general metric in terms of Newman-Penrose (NP) quantities. In section 3, by using the NP quantities of the general metric obtained in section 2, we derive a class of rotating solutions, including Kerr-Newman-Vaidya, de Sitter and Kerr-Newman-Vaidya-de Sitter, Kerr-Newman-monopole black holes. We discuss the physical properties of the solutions observing the nature of their energy momentum tensors and Weyl scalars. We also present the surface gravity, entropy and angular velocity for each of these embedded black holes as they are important parameters of a black hole. Section 4 deals with the Hawking's radiation on the variable-charged, Kerr-Newman-Vaidya, Kerr-Newman-Vaidya-de Sitter and Kerr-Newman-monopole black holes. We present various classical spacetime metrics affected by the change in the masses describing the possible life style of radiating embedded black holes in different stages during radiation process. In section 5 we conclude with a discussion of our results. The spin coefficients, the Weyl scalars and the Ricci scalars for the rotating metric discussed here are, in general, cited in an appendix for future use.

Here, as in [1] it is convenient to use the phrase '*change in the mass*' rather than '*loss of mass*' as there may be a possibility of creation of mass after the exhaustion of the original mass if one continues the same process of electrical radiation. This will be seen later in the paper. The presentation of this paper is essentially based on the Newman-Penrose (NP) spin-coefficient formalism [10]. The NP quantities are calculated through the technique developed by McIntosh and Hickman [11] in (-2) signature.

2. NEWMAN-JANIS ALGORITHM AND GENERAL METRICS

To begin with we consider a spherical symmetric 'seed' metric written in the form

$$ds^2 = e^{2\phi}\, du^2 + 2du\, dr - r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \quad (2.1)$$

where

$$e^{2\phi} = 1 - 2M(u,r)/r + e^2(u,r)/r^2$$

and the coordinates chosen are $\{x^1, x^2, x^3, x^4\}$ $=\{u, r, \theta, \phi\}$. The u-coordinate is related to the retarded time in flat space-time. So u-constant surfaces are null cones open to the future. The r-constant is null coordinate. The θ and ϕ are usual angle coordinates. The retarded time coordinates are used to evaluate the radiating (or outgoing) energy momentum tensor around the astronomical body [10]. Here M and e are the metric functions of the retarded time coordinate u and the radial coordinate r. Initially, when M, e are constant, this metric provides the non-rotating Reissner-Nordstrom solution and also when both M, e are functions of u, it becomes the non-rotating Vaidya-Bonnor solution [13].

Now we apply Newman-Janis (NJ) algorithm [14] which is a complex coordinate transformation,

$$\begin{aligned} r &= r' - i\,a\cos\theta,\ u = u' + i\,a\cos\theta, \\ \theta &= \theta',\ \phi = \phi'. \end{aligned} \quad (2.2)$$

to make the metric (2.1) rotation. This complex transformation can be done only when r' and u' are considered to be real. All the primes are being dropped for convenience of notation. The application of NJ algorithm is also employed by various authors [15,16,17,18] to different 'seed' metrics. Then the transformed metric takes the following form

$$\begin{aligned} ds^2 &= e^{2\phi}\,du^2 + 2du\,dr + 2a\sin^2\theta(1-e^{2\phi})\,du\,d\phi \\ &\quad -2a\sin^2\theta\,dr\,d\phi - R^2 d\theta^2 \\ &\quad -\{R^2 - a^2\sin^2\theta\,(e^{2\phi}-2)\}\sin^2\theta\,d\phi^2, \end{aligned} \tag{2.3}$$

where

$$e^{2\phi} = 1 - \frac{2rM(u,r,\theta)}{R^2} + \frac{e^2(u,r,\theta)}{R^2}. \tag{2.4}$$

and $R^2 = r^2 + a^2\cos^2\theta$. It is noted that after the complex coordinate transformation (2.2), M and e should be arbitrary functions of three variables u, r, θ. Then the covariant complex null tetrad vectors take the forms

$$\begin{aligned} \ell_a &= \delta_a^1 - a\sin^2\theta\,\delta_a^4, \quad (2.5)\\ n_a &= \frac{1}{2}\,H\,\delta_a^1 + \delta_a^2 - \frac{1}{2}\,H\,a\sin^2\theta\,\delta_a^4, \\ m_a &= -\frac{1}{\sqrt{2}R}\,\{-ia\sin\theta\,\delta_a^1 + R^2\,\delta_a^3 \\ &\quad +i(r^2+a^2)\sin\theta\,\delta_a^4\}. \quad (2.6) \end{aligned}$$

The null vectors ℓ_a and n_a are real, and m_a is complex null vector with the normalization condition $\ell^a\,n_a = -m^a\,\overline{m}_a = 1$ and $R = r + ia\cos\theta$. Here,

$$H(u,r,\theta) = R^{-2}\{r^2 - 2rM(u,r,\theta) + a^2 + e^2(u,r,\theta)\}. \tag{2.7}$$

The NP quantities (*i.e.* the NP spin coefficients, the Ricci and Weyl scalars cited in appendix below) have been calculated with the arbitrary metric functions M and e of three variables of the metric (2.3). From the NP spin coefficients, it is found that the rotating spherically symmetric metric (2.3) possesses, in general, a geodesic ($\kappa = \epsilon = 0$), shear free ($\sigma = 0$), expanding ($\hat{\theta} \neq 0$) and non-zero twist ($\hat{\omega}^2 \neq 0$) null vector ℓ_a [19] where

$$\hat{\theta} \equiv -\frac{1}{2}(\rho + \overline{\rho}) = \frac{r}{R^2}, \tag{2.8}$$

$$\hat{\omega}^2 \equiv -\frac{1}{4}(\rho - \overline{\rho})^2 = -\frac{a^2\cos^2\theta}{R^2\,R^2}. \tag{2.9}$$

From the Einstein's equations,

$$G_{ab} \equiv R_{ab} - \frac{1}{2}\,R\,g_{ab} = -K\,T_{ab}, \tag{2.10}$$

we obtain the energy momentum tensor (EMT) for the metric (2.3) as follows:

$$\begin{aligned} T_{ab} &= \mu^*\,\ell_a\,\ell_b + 2\,\rho^*\,\ell_{(a}\,n_{b)} + 2\,p\,m_{(a}\overline{m}_{b)} \\ &\quad +2\,\omega\,\ell_{(a}\,\overline{m}_{b)} + 2\,\overline{\omega}\,\ell_{(a}\,m_{b)}, \end{aligned} \tag{2.11}$$

where μ^*, ρ^*, p and ω are related to the null density, the matter density, the pressure p as well as the rotation function respectively of the rotating spherically symmetric objects and are given in terms of Ricci scalars as follows:

$$\begin{aligned} &K\,\rho^* = 2\,\phi_{11} + 6\,\Lambda, \quad K\,p = 2\,\phi_{11} - 6\,\Lambda, \\ &K\,\mu^* = 2\,\phi_{22}, \quad K\,\omega = -2\,\phi_{12}, \end{aligned} \tag{2.12}$$

where ϕ_{11}, ϕ_{12}, ϕ_{22}, Λ are the non-vanishing Ricci scalars given in appendix (A3) for the metric (2.3). Then we have

$$\begin{aligned} \mu^* &= -\frac{1}{K\,R^2\,R^2}\Big\{2r(r\,M_{,u} - ee_{,u}) \\ &\quad -\cot\theta(rM_{,\theta} - ee_{,\theta}) + a^2\sin^2\theta(rM_{,u} - ee_{,u})_{,u} \\ &\quad -(rM_{,\theta} - ee_{,\theta})_{,\theta}\Big\}, \\ \rho^* &= \frac{1}{K\,R^2\,R^2}\Big\{e^2 + 2\,r(r\,M_{,r} - e\,e_{,r})\Big\}, \quad (2.13)\\ p &= \frac{1}{K\,R^2\,R^2}\Big\{e^2 + 2\,r(r\,M_{,r} - e\,e_{,r}) \\ &\quad -R^2(2M_{,r} + r\,M_{,r\,r} - e^2_{,r} - e\,e_{,r\,r})\Big\}, \\ \omega &= -\frac{1}{\sqrt{2}\,K\,R^2\,R^2}\,\Big[i\,a\sin\theta\Big\{(R\,M_{,u} - 2\,e\,e_{,u}) \\ &\quad -(r\,M_{,r} - e\,e_{,r})_{,u}\overline{R}\Big\} \\ &\quad +\Big\{(R\,M_{,\theta} - 2\,e\,e_{,\theta}) - (r\,M_{,r} - e\,e_{,r})_{,\theta}\,\overline{R}\Big\}\Big]. \end{aligned}$$

Here the Ricci scalar $\Lambda \equiv (1/24)g^{ab}\,R_{ab}$ for the metric (2.3) is

$$\Lambda = \frac{1}{12\,R^2}\,\Big(2M_{,r} + r\,M_{,r\,r} - e^2_{,r} - ee_{,r\,r}\Big). \tag{2.14}$$

It is observed that the expression of Λ does not involve any derivative of M and e with respect to u and θ, though M and e are functions of three variables u, r, θ.

The above EMT (2.11) can be written as $T^{(\rm n)}_{ab}$ and $T^{(\rm m)}_{ab}$ to represent two fluid systems *i.e.* rotating null fluid $T^{(\rm n)}_{ab}$ and rotating matter $T^{(\rm m)}_{ab}$. Then we have

$$\begin{aligned} T^{(\rm n)}_{ab} &= \mu^*\ell_a\ell_b + \omega\ell_{(a}\overline{m}_{b)} + \overline{\omega}\ell_{(a}\,m_{b)}, \quad (2.15)\\ T^{(\rm m)}_{ab} &= 2\,(\rho^* + p)\ell_{(a}\,n_{b)} - p\,g_{ab} \end{aligned}$$

$$+\omega\,\ell_{(a}\,\overline{m}_{b)}+\overline{\omega}\,\ell_{(a}\,m_{b)}, \qquad (2.16)$$

where $\overline{\omega}$ is the complex conjugate of ω. The appearance of non-vanishing ω in $T^{(\rm n)}_{ab}$ and $T^{(\rm m)}_{ab}$ shows the rotating fluid systems in spherically symmetric spacetime geometry. When we set $\omega=0$ initially, these EMTs may be similar to those introduced by Husain [20] and Glass and Krisch [7] with the arbitrary mass $M(u,r)$ and charge $e(u,r)$ for the non-rotating objects. From the appendix (A3) we find that $\phi_{00}=0$. So the vanishing of ϕ_{00} suggests the possibility that the metric (2.3) does not possess a perfect fluid, whose energy-momentum tensor is $T_{ab}=(\rho^*+p)u_au_b-p\,g_{ab}$ with $\phi_{00}=2\phi_{11}=\phi_{22}=-K(\rho^*+p)/4$, $\Lambda=K(3p-\rho^*)/24$ with a time-like vector u^a.

3. ROTATING SOLUTIONS

In this section, from the general solutions presented in appendix we shall present a class of rotating solutions, namely (a) Vaidya-Bonnor, (b) de Sitter, (c) Kerr-Newman-Vaidya, (d) Kerr-Newman-Vaidya-de Sitter, which are to be discussed in this paper. The rotating Vaidya-Bonnor solution describes a non-stationary spherically symmetric solution of Einstein's field equations. The rotating de Sitter solution is a Petrov type D solution. The rotating Kerr-Newman-Vaidya solution represents a non-stationary black-hole, describing Kerr-Newman black hole embedded into the rotating Vaidya null radiating universe. Again the rotating Kerr-Newman-Vaidya-de Sitter solution describes the non-stationary Kerr-Newman-Vaidya black hole embedded into the rotating de Sitter cosmological universe.

(i) Rotating charged Vaidya-Bonnor solution: $M=M(u)$, $a\neq 0$, $e=e(u)$:

Once the restrictions on M and e are considered to be functions of u only, the quantities (2.13) become quite simple. In this case the energy momentum tensor (2.11) takes

$$\begin{aligned} T_{ab} &= \mu^*\,\ell_a\,\ell_b+2\,\rho^*\,\{\ell_{(a}\,n_{b)}+m_{(a}\overline{m}_{b)}\} \\ &\quad +2\,\omega\,\ell_{(a}\,\overline{m}_{b)}+2\,\overline{\omega}\,\ell_{(a}\,m_{b)}, \qquad (3.1)\end{aligned}$$

with

$$\begin{aligned}\mu^* &= -\frac{1}{K\,R^2\,R^2}\Big\{2\,r\,(r\,M_{,u}-e\,e_{,u}) \\ &\quad +a^2\sin^2\theta\,(r\,M_{,u}-e\,e_{,u})_{,u}\Big\}, \\ \rho^* &= p=\frac{e^2(u)}{K\,R^2\,R^2}, \qquad (3.2) \\ \omega &= \frac{-i\,a\sin\theta}{\sqrt{2}\,K\,R^2\,R^2}\Big\{R\,M_{,u}-2e\,e_{,u}\Big\},\end{aligned}$$

and the Weyl scalars are

$$\begin{aligned}\psi_2 &= \frac{1}{\overline{R}\,\overline{R}\,R^2}\Big(e^2-R\,M\Big), \\ \psi_3 &= \frac{-i\,a\sin\theta}{2\sqrt{2}\overline{R}\,\overline{R}\,R^2}\Big\{4\,(rM_{,u}-e\,e_{,u})+\overline{R}\,M_{,u}\Big\}, \\ \psi_4 &= \frac{a^2\sin^2\theta}{2\overline{R}\,\overline{R}\,R^2\,R^2}\Big\{R^2\,(r\,M_{,u}-e\,e_{,u})_{,u} \\ &\quad -2r\,(r\,M_{,u}-e\,e_{,u})\Big\}. \qquad (3.3)\end{aligned}$$

The line element will take the form

$$\begin{aligned} ds^2 &= [1-\{2rM(u)-e^2(u)\}R^{-2}]\,du^2+2du\,dr \\ &\quad +2aR^{-2}\{2rM(u)-e^2(u)\}\sin^2\theta\,du\,d\phi \\ &\quad -2a\sin^2\theta\,dr\,d\phi-R^2d\theta^2-\{(r^2+a^2)^2 \\ &\quad -\Delta^*a^2\sin^2\theta\}R^{-2}\sin^2\theta\,d\phi^2, \qquad (3.4)\end{aligned}$$

where $\Delta^*=r^2-2rM(u)+a^2+e^2(u)$. This solution describes a black hole when $M(u)>a^2+e^2(u)$ and has $r_\pm=M(u)^*\pm\sqrt{\{M^2(u)-a^2-e^2(u)\}}$ as the roots of the equation $\Delta^*=0$. So the rotating Vaidya-Bonnor solution has an *external event horizon* at $r=r_+$ and an *internal Cauchy horizon* at $r=r_-$. The non-stationary limit surface $g_{uu}>0$ of the rotating black hole *i.e.* $r\equiv r_e(u,\theta)=M(u)+\sqrt{\{M^2(u)-a^2\cos^2\theta-e^2(u)\}}$ does not coincide with the event horizon at r_+, thereby producing the ergosphere. The surface gravity of the event horizon at $r=r_+$ is

$$\mathcal{K}=-\frac{1}{r_+R^2}\Big[r_+\sqrt{\{f(u)^2-a^2+e^2(u)\}}+\frac{e^2(u)}{2}\Big],$$

and the entropy of the horizon is

$$\mathcal{S}=2\pi\,f(u)\Big[f(u)+\sqrt{f(u)^2-a^2-e^2(u)}\Big]-\frac{e^2}{4}.$$

The angular velocity of the horizon is given by

$$\Omega_{\rm H}=\frac{a\{2rM(u)-e^2(u)\}}{(r^2+a^2)^2}\Big|_{r=r_+}.$$

We have seen the direct involvement of the rotation parameter a in both the expressions of surface gravity and the angular velocity, showing the different structure of rotating black hole. When $a = 0$, the angular velocity will also vanish for the horizon.

From the rotating Vaidya-Bonnor metric, we can clearly recover the following solutions: (a) rotating Vaidya metric when $e(u) = 0$, (b) rotating charged Vaidya solution when $e(u)$ becomes constant, (c) the rotating Kerr-Newman solution when $M(u) = e(u)$ = constant and (d) well-known non-rotating Vaidya-Bonnor metric [13] when $a = 0$. It is also noted that when $e = a = 0$, the null density of Vaidya radiating fluid takes the form $\mu^* = -2\,M_{,u}/K\,r^2$. The non-rotating Vaidya null radiating metric is of type D in the Petrov classification of spacetime, whose one of the repeated principal null vectors, ℓ_a is a geodesic, shear free, non-rotating with non-zero expansion [21], while the rotating one is of algebraically special with a null vector ℓ_a (2.5), which is geodesic, shear free, expanding as well as non-zero twist. The rotating Vaidya-Bonnor metric (3.4) can be expressed in Kerr-Schild ansatz on the rotating Vaidya null radiating background as

$$g^{\rm VB}_{ab} = g^{\rm V}_{ab} + 2Q(u,r,\theta)\ell_a\ell_b$$

with $Q(u,r,\theta) = \{e^2(u)/2R^2\}$, indicating the existence of electromagnetic field on the rotating Vaidya spacetime geometry.

Carmeli and Kaye [22] have also obtained the rotating Vaidya metric (a) above and discussed under the name of a variable-mass Kerr solution. Herrera and Martinez [23] and Herrera *et al* [24] have discussed the physical interpretation of the solution of Carmeli and Kaye. Similarly, during the application of Newman-Janis algorithm to the *non-rotating* Vaidya-Bonnor 'seed' solution with $M(u)$ and $e(u)$, Jing and Wang [25] kept the functions $M(u)$ and $e(u)$ unchanged and studied the nature of the transformed metric with the consequent NP quantities.

(ii) Rotating solutions with $M = M(u,r)$, $a \neq 0$, $e(u,r,\theta) = 0$:

If we take M to be the function of u, r and $e(u,r,\theta) = 0$ in (2.13), the energy momentum tensor will take the form

$$\begin{aligned} T_{ab} &= \mu^*\,\ell_a\,\ell_b + 2\,\rho^*\,\ell_{(a}\,n_{b)} + 2\,p\,m_{(a}\overline{m}_{b)} \\ &\quad + 2\,\omega\,\ell_{(a}\,\overline{m}_{b)} + 2\,\overline{\omega}\,\ell_{(a}\,m_{b)}, \end{aligned} \tag{3.5}$$

with the following quantities

$$\begin{aligned} \mu^* &= -\frac{1}{K\,R^2\,\overline{R^2}}\Big\{2r^2 M_{,u} + a^2 r\sin^2\theta\,M_{,uu}\Big\}, \\ \rho^* &= \frac{2\,r^2}{K\,R^2\,\overline{R^2}}\,M_{,r}, \\ p &= -\frac{1}{K}\Big\{\frac{2\,a^2\cos^2\theta}{R^2\,\overline{R^2}}\,M_{,r} + \frac{r}{R^2}\,M_{,r\,r}\Big\}, \\ \omega &= -\frac{i\,a\sin\theta}{\sqrt{2}\,K\,R^2\,\overline{R^2}}\Big(R\,M_{,u} - r\,\overline{R}\,M_{,ur}\Big). \end{aligned} \tag{3.6}$$

The line element will be of the form

$$\begin{aligned} ds^2 &= \{1 - 2rM(u,r)R^{-2}\}\,du^2 + 2du\,dr \\ &\quad + 4arM(u,r)R^{-2}\sin^2\theta\,du\,d\phi \\ &\quad - 2a\sin^2\theta\,dr\,d\phi - R^2 d\theta^2 - \{(r^2+a^2)^2 \\ &\quad - \Delta^* a^2\sin^2\theta\}\,R^{-2}\sin^2\theta\,d\phi^2, \end{aligned} \tag{3.7}$$

where $\Delta^* = r^2 - 2rM(u,r) + a^2$ and the Weyl scalars given in (A2) become

$$\begin{aligned} \psi_2 &= \frac{1}{\overline{R}\,\overline{R}\,R^2}\Big\{-R\,M + \frac{\overline{R}}{6}\,M_{,r}(4r + 2\,i\,a\cos\theta) \\ &\quad - \frac{r}{6}\,\overline{R}\,\overline{R}\,M_{,rr}\Big\}, \\ \psi_3 &= -\frac{i\,a\sin\theta}{2\sqrt{2}\overline{R}\,\overline{R}\,R^2}\Big\{(4\,r + \overline{R}\,)\,M_{,u} + r\,\overline{R}\,M_{,ur}\Big\}, \\ \psi_4 &= \frac{a^2\,r\sin^2\theta}{2\overline{R}\,\overline{R}\,R^2\,R^2}\Big\{R^2\,M_{,uu} - 2\,r\,M_{,u}\Big\}. \end{aligned} \tag{3.8}$$

Wang and Wu [9] have expanded the metric function $M(u,r)$ for the non-rotating solution ($a = 0$) in the power of r

$$M(u,r) = \sum_{n=-\infty}^{+\infty} q_n(u)\,r^n, \tag{3.9}$$

where $q_n(u)$ are arbitrary functions of u. They consider the above sum as an integral when the 'spectrum' index n is continuous. Here using this expression in equations (3.6) we can generate rotating metrics with $a \neq 0$ as

$$\begin{aligned} \mu^* &= -\frac{r}{K\,R^2\,\overline{R^2}}\sum_{n=-\infty}^{+\infty}\Big\{2\,q_n(u)_{,u}\,r^{n+1} \\ &\quad + a^2\sin^2\theta\,q_n(u)_{,uu}\,r^n\Big\}, \\ \rho^* &= \frac{2\,r^2}{K\,R^2\,\overline{R^2}}\sum_{n=-\infty}^{+\infty} n\,q_n(u)\,r^{n-1}, \\ p &= -\frac{1}{K R^2}\sum_{n=-\infty}^{+\infty} n q_n(u) r^{n-1} \\ &\quad \times\Big\{\frac{2a^2\cos^2\theta}{\overline{R^2}} + (n-1)\Big\}, \end{aligned} \tag{3.10}$$

$$\omega = \frac{-i\,a\sin\theta}{\sqrt{2}\,K\,R^2\,R^2}\sum_{n=-\infty}^{+\infty}(R-n\overline{R})\,q_n(u)_{,u}\,r^n.$$

Clearly these solutions (3.10) will recover non-rotating Wang-Wu solutions if one sets $a=0$. Here we find that these rotating Wang-Wu solutions include many known as well as un-known rotating solutions of Einstein's field equations with spherical symmetry as shown by Wang and Wu in non-rotating cases [9]. The functions $q_n(u)$ in (3.9) and (3.10) play a great role in generating rotating solutions. Therefore, we hereafter refer to $q_n(u)$ as Wang-Wu functions. Here a class of rotating solutions can be derived from these quantities (3.10) as follows.

(iii) Kerr-Newman-Vaidya solution

Wang and Wu [9] could combine the three *non-rotating* solutions, namely monopole, de-Sitter and charged Vaidya solution to obtain a new solution which represents a *non-rotating* monopole-de Sitter-Vaidya charged solutions. In the same way, we wish to combine the Kerr-Newman solution with the rotating Vaidya solution obtained above in (3.4) with $e(u)=0$, if the Wang-Wu functions $q_n(u)$ in (3.10) are chosen such that

$$q_n(u) = \begin{cases} m+f(u), & \text{when } n=0 \\ -e^2/2, & \text{when } n=-1 \\ 0, & \text{when } n\neq 0,-1, \end{cases} \tag{3.11}$$

where m and e are constants. [Note: This constant e is assumed to be different from the notation $e(u,r,\theta)$ which has been set to zero in subsection 3(ii), and can be seen its absence in (3.7)]. Then, the mass function takes the form

$$M(u,r) = m + f(u) - e^2/2r$$

and other quantities are

$$\rho^* = p = \frac{e^2}{K\,R^2\,R^2}, \tag{3.12}$$

$$\begin{aligned} \mu^* &= \frac{-r}{K\,R^2\,R^2}\Big\{2\,r\,f(u)_{,u} + a^2\sin^2\theta\,f(u)_{,uu}\Big\}, \\ \omega &= \frac{-i\,a\sin\theta}{\sqrt{2}\,K\,\overline{R}\,R^2}\,f(u)_{,u}, \\ \Lambda &\equiv \frac{1}{24}g^{ab}\,R_{ab} = 0, \\ \gamma &= \frac{1}{2\overline{R}\,R^2}\,\Big[\{r-m-f(u)\}\overline{R} - \Delta^*\Big], \end{aligned} \tag{3.13}$$

and ϕ_{11}, ϕ_{12}, ϕ_{22} can be obtained from equations (3.13) with (2.12). The Weyl scalars (3.8) become

$$\begin{aligned} \psi_2 &= \frac{1}{\overline{R}\,\overline{R}\,R^2}\Big[e^2 - R\,\{m+f(u)\}\Big], \\ \psi_3 &= \frac{-i\,a\sin\theta}{2\sqrt{2}\overline{R}\,\overline{R}\,R^2}\Big\{(4\,r+\overline{R})f(u)_{,u}\Big\}, \\ \psi_4 &= \frac{a^2 r\sin^2\theta}{2\overline{R}\,\overline{R}\,R^2\,R^2}\Big\{R^2 f(u)_{,uu} - 2rf(u)_{,u}\Big\}. \end{aligned} \tag{3.14}$$

This represents a rotating Kerr-Newman-Vaidya solution with the line element

$$\begin{aligned} ds^2 \;=\; & [1 - R^{-2}\{2r(m+f(u)) - e^2\}]\,du^2 \\ & +2du\,dr + 2aR^{-2}\{2r(m+f(u)) \\ & -e^2\}\sin^2\theta\,du\,d\phi - 2a\sin^2\theta\,dr\,d\phi - R^2 d\theta^2 \\ & -\{(r^2+a^2)^2 - \Delta^* a^2\sin^2\theta\}\,R^{-2}\sin^2\theta\,d\phi^2, \end{aligned} \tag{3.15}$$

where $\Delta^* = r^2 - 2r\{m+f(u)\} + a^2 + e^2$. Here m and e are the mass and the charge of Kerr-Newman solution, a is the rotational parameter per unit mass and $f(u)$ represents the mass function of rotating Vaidya null radiating fluid. The solution (3.15) will describe a black hole if $m + f(u) > a^2 + e^2$ with external event horizon at $r_+ = \{m+f(u)\} + \sqrt{[\{m+f(u)\}^2 - a^2 - e^2]}$, an internal Cauchy horizon at $r_- = \{m+f(u)\} - \sqrt{[\{m+f(u)\}^2 - a^2 - e^2]}$ and the non-stationary limit surface $r \equiv r_e(u,\theta) = \{m+f(u)\} + \sqrt{[\{m+f(u)\}^2 - a^2\cos^2\theta - e^2]}$. The surface gravity of the event horizon at $r = r_+$ is

$$\mathcal{K} = -\frac{1}{r_+R^2}\Big[r_+\sqrt{\Big\{\Big(m+f(u)\Big)^2 - (a^2+e^2)\Big\}} + \frac{e^2}{2}\Big].$$

The entropy of the horizon is given by

$$\begin{aligned} \mathcal{S} \;=\; & 2\pi\{m+f(u)\}\Big[\{m+f(u)\} \\ & +\sqrt{\{m+f(u)\}^2 - (a^2+e^2)}\,\Big] - \frac{e^2}{4}. \end{aligned}$$

The angular velocity of the horizon takes the form

$$\Omega_{\rm H} = \frac{a[2r\{m+f(u)\} - e^2(u)]}{(r^2+a^2)^2}\Big|_{r=r_+}.$$

When we set $f(u)=0$, the metric (3.15) recovers the usual Kerr-Newman black hole, and if $m=0$, then it is the 'rotating' charged Vaidya null radiating black hole (3.4).

In this rotating solution, the Vaidya null fluid is interacting with the non-null electromagnetic field whose Maxwell scalar ϕ_1 can be obtained from (3.12). Thus, we could write the total energy momentum tensor (EMT) for the rotating solution (3.15) as follows:

$$T_{ab} = T^{(\rm n)}_{ab} + T^{(\rm E)}_{ab}, \tag{3.16}$$

where the EMTs for the rotating null fluid as well as that of the electromagnetic field are given respectively as

$$T^{(\rm n)}_{ab} = \mu^* \ell_a \ell_b + 2\omega \ell_{(a}\overline{m}_{b)} + 2\overline{\omega}\ell_{(a}m_{b)} \tag{3.17}$$
$$T^{(\rm E)}_{ab} = 2\,\rho^*\,\{\ell_{(a}\,n_{b)} + m_{(a}\,\overline{m}_{b)}\}. \tag{3.18}$$

The appearance of non-vanishing ω shows the null fluid is rotating as the expression of ω (3.13) involves the rotating parameter a coupling with $\partial f(u)/\partial u$, both non-zero quantities for a rotating Vaidya null radiating universe.

This rotating Kerr-Newman-Vaidya metric (3.15) can be expressed in Kerr-Schild form on the Kerr-Newman background as

$$g^{\rm KNV}_{ab} = g^{\rm KN}_{ab} + 2Q(u, r, \theta)\ell_a\ell_b \tag{3.19}$$

where

$$Q(u, r, \theta) = -rf(u)R^{-2}, \tag{3.20}$$

and the vector ℓ_a is a geodesic, shear free, expanding as well as rotating null vector of both $g^{\rm KN}_{ab}$ as well as $g^{\rm KNV}_{ab}$ and given in (2.5) and $g^{\rm KN}_{ab}$ is the Kerr-Newman metric (3.4) with $m = e =$ constant. This null vector ℓ_a is one of the double repeated principal null vectors of the Weyl tensor of $g^{\rm KN}_{ab}$.

It appears that the rotating Kerr-Newman geometry may be regarded as joining smoothly with the rotating Vaidya geometry at its null radiative boundary, as shown by Glass and Krisch [7] in the case of Schwarzschild geometry joining to the non-rotating Vaidya space-time. The Kerr-Schild form (3.19) will recover that of Xanthopoulos [8] $g'_{ab} = g_{ab} + \ell_a\ell_b$, when $Q(u, r, \theta) \to 1/2$ and that of Glass and Krisch [7] $g'_{ab} = g^{\rm Sch}_{ab} - \{2f(u)/r\}\ell_a\ell_b$ when $e = a = 0$ for non-rotating Schwarzschild background space. Thus, one can consider the Kerr-Schild form (3.19) as the extension of those of Xanthopoulos as well as Glass and Krisch. When we set $a = 0$, this rotating Kerr-Newman-Vaidya solution (3.15) will recover to non-rotating Reissner-Nordstrom-Vaidya solution with the Kerr-Schild form $g^{\rm RNV}_{ab} = g^{\rm RN}_{ab} - \{2f(u)/r\}\ell_a\ell_b$, which is still a generalization of Xanthopoulos as well as Glass and Krisch in the charged Reissner-Nordstrom solution. It is worth mentioning that the new solution (3.15) cannot be considered as a bimetric theory as $g^{\rm KNV}_{ab} \neq \frac{1}{2}(g^{\rm KN}_{ab} + g^{\rm V}_{ab})$.

To interpret the Kerr-Newman-Vaidya solution as a black hole during the early inflationary phase of rotating Vaidya null radiating universe *i.e.*, the Kerr-Newman black hole embedded into rotating Vaidya null radiating background space, we can write the Kerr-Schild form (3.19) as

$$g^{\rm KNV}_{ab} = g^{\rm V}_{ab} + 2Q(r, \theta)\ell_a\ell_b \tag{3.21}$$

where

$$Q(r, \theta) = -(rm - e^2/2)R^{-2}. \tag{3.22}$$

Here, the constants m and e are the mass and the charge of Kerr-Newman black hole, $g^{\rm V}_{ab}$ is the rotating Vaidya null radiating black hole obtained above when $e(u)$ sets to zero in (3.4) and ℓ_a is the geodesic null vector given in (2.5) for both $g^{\rm KNV}_{ab}$ and $g^{\rm V}_{ab}$. When we set $f(u) = a = 0$, $g^{\rm V}_{ab}$ will recover the flat metric, then $g^{\rm KNV}_{ab}$ becomes the original Kerr-Schild form written in spherical symmetric flat background.

These two Kerr-Schild forms (3.19) and (3.21) certainly confirm that the metric $g^{\rm KNV}_{ab}$ is a solution of Einstein's field equations since the background rotating metrics $g^{\rm KN}_{ab}$ and $g^{\rm V}_{ab}$ are solutions of Einstein's equations. They both possess different stress-energy tensors $T^{(\rm e)}_{ab}$ and $T^{(\rm n)}_{ab}$ given in (3.18) and (3.17) respectively. Looking at the Kerr-Schild form (3.21), the Kerr-Newman-Vaidya black hole can be treated as a generalization of Kerr-Newman black hole by incorporating Visser's suggestion [26] that *Kerr-Newman black hole embedded in an axisymmetric cloud of matter would be of interest.*

(iv) Rotating de Sitter solution

Here we shall first convert the standard de Sitter cosmological universe into a rotating de Sitter space. So that the rotating Kerr-Newman-Vaidya solution can be embedded into the rotating de Sitter cosmological universe as Kerr-Newman-Vaidya-de Sitter solution. It means that one must a rotating solution in oder to embed into a rotating one, leading to a feasible solution . For this purpose, one can choose the Wang-Wu function $q_n(u)$ given in (3.9) as

$$q_n(u) = \begin{cases} \Lambda^*/6, & \text{when } n = 3 \\ 0, & \text{when } n \neq 3 \end{cases} \tag{3.23}$$

such that the mass function becomes

$$M(u,r) = \frac{\Lambda^* r^3}{6}. \tag{3.24}$$

Then the line element of the rotating de Sitter metric will take the following form

$$\begin{aligned} ds^2 &= \Big\{1 - \frac{\Lambda^* r^4}{3R^2}\Big\} du^2 + 2du\,dr - R^2 d\theta^2 \\ &\quad + \frac{2a\Lambda^* r^4}{3R^2}\sin^2\theta\, du\, d\phi - 2a\sin^2\theta\, dr\, d\phi \\ &\quad - \Big\{(r^2+a^2)^2 - \Delta^* a^2 \sin^2\theta\Big\} R^{-2}\sin^2\theta\, d\phi^2, \end{aligned} \tag{3.25}$$

where $R^2 = r^2 + a^2\cos^2\theta$ and $\Delta^* = r^2 - \Lambda^* r^4/3 + a^2$. This corresponds to the rotating de Sitter solution for $\Lambda^* > 0$, and to the anti-de Sitter one for $\Lambda^* < 0$. In general Λ^* denotes the cosmological constant of the de Sitter space. The rotating cosmological de Sitter space possesses an energy momentum tensor

$$T_{ab} = 2\rho^* \ell_{(a} n_{b)} + 2p\, m_{(a}\overline{m}_{b)}, \tag{3.26}$$

where

$$\rho^* = \frac{\Lambda^* r^4}{K R^2 R^2}, \quad p = \frac{-\Lambda^* r^2}{K R^2 R^2}(r^2 + 2a^2\cos^2\theta).$$

are related to the density and the pressure of the cosmological matter field which is, however not a perfect fluid. Then the changed NP quantities are

$$\gamma = -\frac{1}{2\overline{R}R^2}\Big\{\Big(1 - \frac{2}{3}\Lambda^* r^2\Big) r\overline{R} + \Delta^*\Big\},$$

$$\phi_{11} = -\frac{1}{2R^2R^2}\Lambda^* r^2 a^2\cos^2\theta, \tag{3.27}$$

$$\psi_2 = \frac{1}{3\overline{R}\,\overline{R}\,R^2}\Lambda^* r^2 a^2 \cos^2\theta, \tag{3.28}$$

$$\Lambda = \frac{\Lambda^* r^2}{6R^2}. \tag{3.29}$$

This means that in rotating de Sitter cosmological universe, the Λ^* is coupling with the rotational parameter a. From these NP quantities we clearly observe that the rotating de Sitter cosmological metric is a Petrov type D gravitational field $\psi_2 \neq 0$, whose one of the repeated principal null vectors, ℓ_a is geodesic, shear free, expanding as well as non-zero twist. The metric (3.25) has singularities at the values of r for which $\Delta^* = 0$ having four roots r_{++}, r_{+-}, r_{-+} and r_{--}. The singularity at $r_{++} = +\surd[(1/2\Lambda^*)\{3 + \surd(9 + 12a^2\Lambda^*)\}]$ might represent the apparent singularity for the rotating de Sitter space (3.25). When $a = 0$ for the non-rotating de Sitter space, this will reproduce the result $r_{++} = 3^{1/2}\Lambda^{*-1/2}$, discussed by Gibbon and Hawking [28]. The surface gravity of the singularity at $r = r_{++}$ can be written as

$$\mathcal{K} = -\frac{r_{++}}{12R^2}\Big\{9 - \sqrt{(9 + 12a^2\Lambda^*)}\Big\}.$$

The angular velocity for the singularity is

$$\Omega_{\rm H} = \frac{a\Lambda^* r^4}{3(r^2+a^2)^2}\Big|_{r=r_{++}}.$$

If we set the rotational parameter $a = 0$, we will recover the non-rotating de Sitter metric [28], which is a solution of the Einstein's equations for an empty space with $\Lambda \equiv (1/24)g_{ab}R^{ab} = \Lambda^*/6$ or constant curvature and $\rho^* = -p$. However, it is observed that the rotating de Sitter universe (3.25) is *non-empty* and *non-constant curvature*. It certainly describes a stationary rotating spherical symmetric solution representing Petrov type D spacetime with the Weyl scalar ψ_2 (3.28). So it is noted that to the best of the present author's knowledge, this rotating de Sitter metric has not been seen derived before.

(v) Kerr-Newman-Vaidya-de Sitter solution

Now we shall embed the Kerr-Newman-Vaidya solution (3.15) into the rotating de Sitter solution (3.25) by choosing the Wang-Wu function as

$$q_n(u) = \begin{cases} m + f(u), & \text{when } n = 0 \\ -e^2/2, & \text{when } n = -1 \\ \Lambda^*/6, & \text{when } n = 3 \\ 0, & \text{when } n \neq 0, -1, 3, \end{cases} \tag{3.30}$$

where m and e are constants and $f(u)$ is related with the mass of rotating Vaidya solution (3.15). Thus, we have the mass function

$$M(u,r) = m + f(u) - \frac{e^2}{2r} + \frac{\Lambda^* r^3}{6}$$

and other quantities are obtained from (3.6) and (2.13) as

$$\rho^* = \frac{1}{K R^2 R^2}\Big(e^2 + \Lambda^* r^4\Big), \tag{3.31}$$

$$
\begin{aligned}
p &= \frac{1}{K\,R^2\,\overline{R}^2}\Big\{e^2 - \Lambda^* r^2(r^2 + 2a^2\cos\theta)\Big\},\\
\mu^* &= \frac{-r}{K\,R^2\,\overline{R}^2}\Big\{2\,r\,f(u)_{,u} + a^2\sin^2\theta\,f(u)_{,uu}\Big\},\\
\omega &= \frac{-i\,a\sin\theta}{\sqrt{2}\,K\,\overline{R}\,R^2}\,f(u)_{,u},\\
\Lambda &\equiv \frac{1}{24}g^{ab}R_{ab} = \frac{\Lambda^* r^2}{6\,R^2},\\
\gamma &= \frac{1}{2\overline{R}\,R^2}\Big[\Big\{r - m - f(u) - \frac{2\Lambda^* r^3}{3}\Big\}\overline{R} - \Delta^*\Big],
\end{aligned}
\tag{3.32}
$$

and ϕ_{11}, ϕ_{12}, ϕ_{22} can be obtained from equations (3.32) with (2.12). The Weyl scalars take the following form

$$
\begin{aligned}
\psi_2 =&\ \frac{1}{\overline{R}\,\overline{R}\,R^2}\Big[e^2 - R\{m + f(u)\}\\
&+\frac{\Lambda^* r^2}{3}a^2\cos^2\theta\Big],\\
\psi_3 =&\ \frac{-i\,a\sin\theta}{2\sqrt{2}\overline{R}\,\overline{R}\,R^2}\Big\{(4\,r + \overline{R})f(u)_{,u}\Big\},\\
\psi_4 =&\ \frac{a^2 r\sin^2\theta}{2\overline{R}\,\overline{R}\,R^2\,R^2}\Big\{R^2 f(u)_{,uu} - 2r f(u)_{,u}\Big\}.
\end{aligned}
\tag{3.33}
$$

That is, the rotating Kerr-Newman-Vaidya-de Sitter solution will take the line element as follows

$$
\begin{aligned}
ds^2 \;=\;& \Big[1 - R^{-2}\Big\{2r\Big(m + f(u)\Big) + \frac{\Lambda^* r^4}{3} - e^2\Big\}\Big]du^2\\
&+2du\,dr + 2aR^{-2}\Big\{2r\Big(m + f(u)\Big) + \frac{\Lambda^* r^4}{3}\\
&-e^2\Big\}\sin^2\theta\,du\,d\phi - 2a\sin^2\theta\,dr\,d\phi - R^2 d\theta^2\\
&-\Big\{(r^2 + a^2)^2 - \Delta^* a^2\sin^2\theta\Big\}R^{-2}\sin^2\theta\,d\phi^2,
\end{aligned}
\tag{3.34}
$$

where $\Delta^* = r^2 - 2r\{m + f(u)\} - \Lambda^* r^4/3 + a^2 + e^2$. Here m and e are the mass and the charge of Kerr-Newman solution, a is the non-zero rotation parameter and $f(u)$ represents the mass function of rotating Vaidya null radiating fluid. The metric (3.34) will describe a cosmological black holes with the horizons at the values of r for which $\Delta^* = 0$ having four roots r_{++}, r_{+-}, r_{-+} and r_{--} given in appendix (A5) and (A6). The first three values will describe respectively the event horizon, the Cauchy horizon and the cosmological horizon. The surface gravity of the horizon at $r = r_{++}$ is

$$
\mathcal{K} = -\Big[\frac{1}{rR^2}\Big\{r\Big(r - m - f(u) - \frac{\Lambda^* r^3}{6}\Big) + \frac{e^2}{2}\Big\}\Big]_{r=r_{++}}
$$

and the entropy of the horizon is obtained as

$$
\mathcal{S} = \pi\Big\{r^2 + a^2\Big\}_{r=r_{++}}.
$$

The angular velocity of the horizon is found as

$$
\Omega_{\rm H} = \frac{a[2r\{m + f(u)\} + (\Lambda^* r^4/3) - e^2(u)]}{(r^2 + a^2)^2}\Big|_{r=r_{++}}.
$$

In this rotating solution (3.34), the Vaidya null fluid is interacting with the non-null electromagnetic field on the de Sitter cosmological space. Thus, the total energy momentum tensor (EMT) for the rotating solution (3.34) takes the following form:

$$
T_{ab} = T^{(n)}_{ab} + T^{(\rm E)}_{ab} + T^{(\rm C)}_{ab}, \tag{3.35}
$$

where the EMTs for the rotating null fluid, the electromagnetic field and cosmological matter field are given respectively

$$
\begin{aligned}
T^{(\rm n)}_{ab} &= \mu^*\,\ell_a\,\ell_b + 2\,\omega\,\ell_{(a}\,\overline{m}_{b)} + 2\,\overline{\omega}\,\ell_{(a}\,m_{b)},\\
T^{(\rm E)}_{ab} &= 4\,\rho^{*(\rm E)}\{\ell_{(a}\,n_{b)} + m_{(a}\overline{m}_{b)}\},\\
T^{(\rm C)}_{ab} &= 2\{\rho^{*(\rm C)}\,\ell_{(a}\,n_{b)} + 2\,p^{(\rm C)}\,m_{(a}\overline{m}_{b)}\},
\end{aligned}
\tag{3.36}
$$

where μ^* and ω are given in (3.32) and

$$
\begin{aligned}
\rho^{*(\rm E)} &= p^{(\rm E)} = \frac{e^2}{K\,R^2\,\overline{R}^2},\quad \rho^{*(\rm C)} = \frac{\Lambda^* r^4}{K\,R^2\,\overline{R}^2},\\
p^{(\rm C)} &= -\frac{\Lambda^* r^2}{K\,R^2\,\overline{R}^2}\Big(r^2 + 2a^2\cos^2\theta\Big).
\end{aligned}
\tag{3.37}
$$

Now, for future use we shall, without loss of generality, have a decomposition of the Ricci scalar Λ, given in (3.32), as

$$
\Lambda = \Lambda^{(\rm E)} + \Lambda^{(\rm C)} \tag{3.38}
$$

where $\Lambda^{(\rm E)}$ is the *zero* Ricci scalar for the electromagnetic field and $\Lambda^{(\rm C)}$ is the *non-zero* cosmological Ricci scalar with $\Lambda^{(\rm C)} = (\Lambda^* r^2/6R^2)$. The appearance of ω shows that the Vaidya null fluid is rotating as the expression of ω in (3.36) involves the rotating parameter a coupling with $\partial f(u)/\partial u$ – both are non-zero quantities for a rotating Vaidya null radiating universe.

For non-rotating spacetime with $a = 0$, the total energy momentum tensor (3.35), will become the following form

$$
T_{ab} = T^{(\rm n)}_{ab} + T^{(\rm E)}_{ab} + \Lambda^* g_{ab} \tag{3.39}
$$

where $T^{(\rm E)}_{ab}$ is the energy momentum tensor for non-null electromagnetic field existing in the non-rotating Reissner-Nordstrom-de Sitter metric g_{ab} and $T^{(\rm n)}_{ab} =$

$\mu^* \ell_a \ell_b$ is the energy momentum tensor for the standard non-rotating Vaidya null fluid. The energy momentum tensor (3.39) may be considered as Guth's modification of T_{ab} [29] in non-stationary charged Vaidya-de Sitter universe.

This rotating Kerr-Newman-Vaidya-de Sitter metric (3.34) can be written in a Kerr-Schild form on the de Sitter background as

$$g_{ab}^{\rm KNVdS} = g_{ab}^{\rm VdS} + 2Q(u,r,\theta)\ell_a\ell_b \tag{3.40}$$

where $Q(u,r,\theta) = -\{r\,m - e^2/2\}R^{-2}$, and the vector ℓ_a is a geodesic, shear free, expanding as well as non-zero twist null vector of both $g_{ab}^{\rm VdS}$ as well as $g_{ab}^{\rm KNVdS}$ and given in (2.5). We can also write this solution (3.34) in another Kerr-Schild form on the Kerr-Newman background as

$$g_{ab}^{\rm KNVdS} = g_{ab}^{\rm KN} + 2Q(u,r,\theta)\ell_a\ell_b \tag{3.41}$$

where $Q(u,r,\theta) = -\{r f(u) + \Lambda^* r^4/6\}R^{-2}$. These two Kerr-Schild forms (3.40) and (3.41) certainly assure that the metric $g_{ab}^{\rm KNVdS}$ is a solution of Einstein's field equations, since the background rotating metrics $g_{ab}^{\rm KN}$ and $g_{ab}^{\rm VdS}$ are both solutions of Einstein's field equations. They have different stress-energy tensors $T_{ab}^{\rm (E)}$, $T_{ab}^{\rm (n)}$ and $T_{ab}^{\rm (C)}$ given in (3.36).

From the rotating solution (3.34), one can recover (i) the Kerr-Newman-de Sitter when $f(u) = 0$, (ii) the rotating charged Vaidya-de Sitter null radiating black hole if $m = 0$, (iii) the rotating Kerr-Newman-Vaidya metric (3.15) when $\Lambda^* = 0$. If one sets $f(u) = m = e = 0$, one will get the rotating de Sitter solution (3.25). All these rotating solutions mentioned here will be of interest to study the physical properties of embedded rotating solutions. One can also find that the rotating Kerr-Newman-de Sitter solution (3.34) with $f(u) = 0$ and its non-stationary extension ($f(u) \neq 0$) are different from the ones derived by Carter [30], Mallett [31] and Xu [32] in the terms involving Λ^*.

(vi) Kerr-Newman-monopole solution

Now we shall give another example of embedded solution of the Kerr-Newman black hole into the rotating monopole universe by choosing the Wang-Wu function as

$$q_n(u) = \begin{cases} m, & \text{when } n = 0 \\ b/2, & \text{when } n = 1 \\ -e^2/2, & \text{when } n = -1 \\ 0, & \text{when } n \neq 0, \pm 1, \end{cases} \tag{3.42}$$

where m and e are constants and b is identified as monopole constant [9]. Thus, the mass function becomes

$$M(u,r) = m + \frac{r\,b}{2} - \frac{e^2}{2r}$$

and other quantities are obtained from (3.6) and (2.13) as

$$\begin{aligned}
&\mu^* = \omega = 0,\\
&\rho^* = \frac{1}{K\,R^2\,\overline{R}^2}\Big(e^2 + b\,r^2\Big),\\
&p = \frac{1}{K\,R^2\,\overline{R}^2}\Big\{e^2 - b\,a^2\cos\theta)\Big\}, && (3.43)\\
&\Lambda = \frac{b}{12\,R^2}, && (3.44)\\
&\psi_2 = \frac{1}{\overline{R}\,\overline{R}R^2}\Big[e^2 - Rm - +\frac{b}{6}\Big(\overline{R}\,\overline{R} - 2ira\cos\theta\Big)\Big].\\
&\gamma = \frac{1}{2\overline{R}\,R^2}\,\Big[\{r(1-b) - m\}\overline{R} - \Delta^*\Big].
\end{aligned}$$

Then the embedded solution will be as follows

$$\begin{aligned}
ds^2 \;\; = \;\; & [1 - R^{-2}(2rm + b\,r^2 - e^2)]\,du^2 + 2du\,dr \\
& +2aR^{-2}(2rm + b\,r^2 - e^2)\sin^2\theta\,du\,d\phi \\
& -2a\sin^2\theta\,dr\,d\phi - R^2 d\theta^2 - \{(r^2 + a^2)^2 \\
& -\Delta^* a^2\sin^2\theta\}\,R^{-2}\sin^2\theta\,d\phi^2,
\end{aligned} \tag{3.45}$$

where $\Delta^* = r^2(1-b) - 2rm + a^2 + e^2$. Here m and e are the mass and the charge of Kerr-Newman solution, a is the non-zero rotation parameter and b represents the monopole constant. In this rotating solution (3.44), the matter field describing monopole particles is interacting with the non-null electromagnetic field. Thus, the total energy momentum tensor (EMT) for the rotating solution (3.45) takes the following form:

$$T_{ab} = T_{ab}^{\rm (E)} + T_{ab}^{\rm (m)}, \tag{3.46}$$

where the EMTs for the monopole matter field and electromagnetic field are given respectively

$$\begin{aligned}
T_{ab}^{\rm (m)} &= 2\{\rho^{*\rm (m)}\,\ell_{(a}\,n_{b)} + 2\,p^{\rm (m)}\,m_{(a}\overline{m}_{b)}\},\\
T_{ab}^{\rm (E)} &= 4\,\rho^{*\rm (E)}\{\ell_{(a}\,n_{b)} + m_{(a}\overline{m}_{b)}\},
\end{aligned} \tag{3.47}$$

where

$$\rho^{*(\mathrm{E})} = p^{(\mathrm{E})} = \frac{e^2}{K\,R^2\,R^2}, \quad \rho^{*(\mathrm{m})} = \frac{b\,r^2}{K\,R^2\,R^2},$$
$$p^{(\mathrm{m})} = -\frac{1}{K\,R^2\,R^2}\,b\,a^2\cos^2\theta. \tag{3.48}$$

Accordingly, for future use one can have a decomposition of Λ given in (3.43) as

$$\Lambda = \Lambda^{(\mathrm{E})} + \Lambda^{(\mathrm{m})} \tag{3.49}$$

where $\Lambda^{(\mathrm{E})}$ is the *zero* Ricci scalar for the electromagnetic field and $\Lambda^{(\mathrm{m})}$ is the *non-zero* monopole Ricci scalar with $\Lambda^{(\mathrm{m})} = b/(12R^2)$. One has also seen the interaction of the rotating parameter a with the monopole constant b in the expression of $p^{(\mathrm{m})}$, which makes different between the rotating as well as non-rotating monopole solutions.

The roots of the equation $\Delta^* = 0$ are found as

$$r_\pm = \frac{1}{(1-b)}\Big[m \pm \sqrt{\{m^2 - (1-b)(a^2+e^2)\}}\,\Big]. \tag{3.50}$$

From this we observe that the value of b must lie in $0 < b < 1$, with the horizons at $r = r_\pm$. The surface gravity of the horizon at $r = r_+$ is

$$\mathcal{K} = -\frac{1}{r_+R^2}\Big[r_+\Big\{r_+\Big(1-\frac{1}{2}\,b\Big) - m\Big\} + \frac{e^2}{2}\Big].$$

The entropy and the angular velocity of the horizon are found as

$$\mathcal{S} = \pi\{r_+^2 + a^2\},$$
$$\Omega_{\mathrm{H}} = \frac{a\{2rm + br - e^2\}}{(r^2+a^2)^2}\Big|_{r=r_+}.$$

We have also found that the solution (3.45) represents Petrov type D with the Weyl scalar ψ_2 given in (3.44) whose repeated principal null vector ℓ_a is shear free, rotating and non-zero twist. This Kerr-Newman-monopole solution (3.45) can be written in a Kerr-Schild form on the rotating monopole background as

$$g_{ab}^{\mathrm{KNm}} = g_{ab}^{\mathrm{m}} + 2Q(r,\theta)\ell_a\ell_b \tag{3.51}$$

where $Q(r,\theta) = -(rm - e^2/2)R^{-2}$, and the vector ℓ_a is a geodesic, shear free, expanding as well as non-zero twist null vector of both g_{ab}^{m} as well as g_{ab}^{KNm} and given in (2.5). We can also express this solution (3.45) in another Kerr-Schild ansatz on the Kerr-Newman background as

$$g_{ab}^{\mathrm{KNm}} = g_{ab}^{\mathrm{KN}} + 2Q(r,\theta)\ell_a\ell_b \tag{3.52}$$

where $Q(r,\theta) = -(b\,r/4)R^{-2}$. These two Kerr-Schild forms (3.51) and (3.52) assure that the metric g_{ab}^{KNm} is a solution of Einstein's field equations, since the background rotating metrics g_{ab}^{KN} and g_{ab}^{m} are both solutions of Einstein's field equations. They have different stress-energy tensors $T_{ab}^{(\mathrm{E})}$ and $T_{ab}^{(\mathrm{m})}$ given in (3.47).

From the rotating solution (3.45), we can recover (i) the Kerr-Newman metric when $b = 0$, (ii) the rotating charged monopole solution if $m = 0$, (iii) if one sets $m = e = 0$, one will get the rotating monopole solution. All these rotating solutions mentioned here will be of interest to study the physical properties of embedded rotating solutions. (iv) For $a = 0$, the metric (3.45) will reduce to the non-rotating Reissner-Nordstrom-monopole solution. (v) If $m = e = a = 0$, it will recover the non-rotating monopole solution [9].

4. HAWKING'S RADIATION ON THE VARIABLE CHARGED BLACK HOLES

In this section, as a part of discussion of the physical properties of the embedded solutions (3.15), (3.34) and (3.45), we shall discuss describe a scenario which is capable of avoiding the formation of negative mass naked singularity during Hawking radiation process in spacetime metrics, describing the life style of a rotating embedded radiating black hole. The formation of negative mass naked singularities in classical spacetime metrics is being shown in [1] after the complete evaporation of the masses of *non-embedded* rotating Kerr-Newman and non-rotating Reissner-Nordstrom, black holes due to Hawking electrical radiation.

Here we shall clarify two similar nomenclatures having different meaning like Hawking's radiation and Vaidya null radiation. Hawking's radiation is continuous radiation of energy from the black hole body thereby leading to the change in mass [1-5], whereas the Vaidya null radiation means that the stress-energy momentum tensor describing the gravitation in Vaidya spacetime metric is a null radiating fluid [33]. So Vaidya null radiation does not have any direct relation with Hawking's radiation of black holes [10].

(i) Variable-charged Kerr-Newman-Vaidya black hole

As mentioned earlier in the introduction, by electrical radiation of a charged black hole we mean the variation of the charge e with respect to the coordinate r in the stress-energy momentum tensor of electromagnetic field. This variation of e will certainly lead different forms or functions of the stress-energy tensor from that of Kerr-Newman-Vaidya black hole. To observe the change in the mass of black hole in the spacetime metric, one has to consider a different form or function of stress-energy tensor of a particular black hole. That is, in order to incorporate the Hawking's radiation in this black hole (3.15), we must have a different stress-energy tensor as the Kerr-Newman-Vaidya black hole with T_{ab} (3.16) does not have any direct Hawking's radiation effects. The consideration of different forms of stress-energy-momentum tensor in the study of Hawking's radiation effect in classical spacetime metrics here is followed from Boulware's suggestion [5] mentioned in introduction above.

It is noted that the Kerr-Newman-Vaidya black hole, describing the Kerr-Newman black hole embedded into the rotating Vaidya null radiating universe (3.4), is quite different from the standard Kerr-Newman black hole. That is, (i) the Kerr-Newman-Vaidya black hole is algebraically special in Petrov classification with the Weyl scalars $\psi_0 = \psi_1 = 0$, $\psi_2 \neq \psi_3 \neq \psi_4 \neq 0$, where as the standard Kerr-Newman black hole is Petrov type D with $\psi_0 = \psi_1 = \psi_3 = \psi_4 = 0$, $\psi_2 \neq 0$. (ii) the Kerr-Newman-Vaidya black hole possesses the total energy-momentum tensor (3.16), representing interaction of the null radiating fluid $T^{(\mathrm{n})}_{ab}$ with the electromagnetic field $T^{(\mathrm{E})}_{ab}$, i.e., the charged null radiating fluid; however the energy-momentum tensor of the Kerr-Newman black hole is that of electromagnetic field $T^{(\mathrm{E})}_{ab}$, simply a charged black hole. (iii) the Kerr-Newman-Vaidya black hole is a non-stationary extension of the stationary Kerr-Newman black hole. Due to the above differences between the two black holes, it is worth studying the physical properties of the embedded black hole. Hence, it is hoped that the study of Hawking electrical radiation in the rotating Kerr-Newman-Vaidya black hole will certainly lead to a different physical feature than that of *non-embedded* Kerr-Newman black hole.

Thus, we consider the line element of a variable-charged Kerr-Newman-Vaidya black hole with respect to the coordinate r as follows (by changing m to M for better understanding):

$$\begin{aligned} ds^2 \;=\; & [1 - R^{-2}\{2r(M + f(u)) - e^2(r)\}]\, du^2 \\ & +2du\, dr + 2aR^{-2}\{2r(M + f(u)) \\ & -e^2(r)\}\sin^2\theta\, du\, d\phi - 2a\sin^2\theta\, dr\, d\phi \\ & -R^2 d\theta^2 - \{(r^2 + a^2)^2 \\ & -\Delta^* a^2 \sin^2\theta\}\, R^{-2}\sin^2\theta\, d\phi^2, \end{aligned} \tag{4.1}$$

where $\Delta^* = r^2 - 2r\{M + f(u)\} + a^2 + e^2(r)$. Then, with the variable charge $e(r)$ in the above metric, there will be a different energy-momentum tensor from that of the original Kerr-Newman-Vaidya metric (3.15). Accordingly, we have to calculate the Ricci and Weyl scalars from the field equations with the functions $e(r)$. However, instead of calculating directly from the field equations, one can use the solutions given in (3.6) and (3.8) to obtain these scalars. For the completeness we present them as follows:

$$\begin{aligned} \phi_{11} =\; & \frac{1}{2\, R^2\, \overline{R}^2}\Big\{e^2(r) - 2r\, e(r) e(r)_{,r}\Big\} \\ & +\frac{1}{4R^2}\Big\{e^2(r)_{,r} + e(r)\, e(r)_{,rr}\Big\}, \end{aligned} \tag{4.2}$$

$$\begin{aligned} \phi_{22} =\; & -\frac{r}{2\, R^2\, \overline{R}^2}\Big\{2\, r\, f(u)_{,u} + a^2 \sin^2\theta\, f(u)_{,uu}\Big\}, \\ \phi_{12} =\; & \frac{i\, a \sin\theta}{\sqrt{2}\, K\, \overline{R}\, R^2}\, f(u)_{,u}, \end{aligned}$$

$$\Lambda = -\frac{1}{12\, R^2}\Big\{e^2(r)_{,r} + e(r)\, e(r)_{,rr}\Big\}, \tag{4.3}$$

$$\begin{aligned} \psi_2 =\; & \frac{1}{\overline{R}\,\overline{R}\, R^2}\Big[e^2(r) - R\{M + f(u)\} \\ & +\frac{1}{6}\,\overline{R}\,\overline{R}\Big\{e^2(r)_{,r} + e(r)\, e(r)_{,rr}\Big\} \\ & -\overline{R}\, e(r)\, e(r)_{,r}\Big], \end{aligned} \tag{4.4}$$

$$\begin{aligned} \psi_3 =\; & \frac{-i\, a \sin\theta}{2\sqrt{2}\overline{R}\,\overline{R}\, R^2}\Big\{(4\, r + \overline{R}) f(u)_{,u}\Big\}, \\ \psi_4 =\; & \frac{a^2 r \sin^2\theta}{2\overline{R}\,\overline{R}\, R^2\, R^2}\Big\{R^2 f(u)_{,uu} - 2r f(u)_{,u}\Big\}. \end{aligned}$$

From these we observe that the variable charge $e(r)$ has the effect only on the scalars ϕ_{11}, Λ and ψ_2. However, the non-vanishing of ϕ_{22}, ϕ_{12}, ψ_3,and ψ_4 indicate the different physical feature of non-statio nary Kerr-Newman-Vaidya black hole from the stationary standard Kerr-Newman black hole. One important feature of the field equations corresponding to the metric (4.1) with $e(r)$ is that the expressions for ϕ_{11} and Λ do not involve the Vaidya mass function $f(u)$. So it suggests the possibility to study the Hawking electrical radiation in this non-stationary black hole.

Now for an electromagnetic field, the Ricci scalar $\Lambda \equiv (1/24)g^{ab}R_{ab}$ (4.3) has to vanish leading to the solution

$$e^2(r) = 2rm_1 + C \tag{4.5}$$

where m_1 and C are real constants. Substituting this $e^2(r)$ in (4.2) and (4.4), and after identifying the constant $C \equiv e^2$, we obtain

$$\phi_{11} = \frac{e^2}{2\,R^2\,\overline{R}^2}, \tag{4.6}$$

$$\psi_2 = \frac{1}{\overline{R}\,\overline{R}\,R^2}\Big[e^2 - R\{M - m_1 + f(u)\}\Big]. \tag{4.7}$$

and ϕ_{12}, ϕ_{22}, ψ_3, and ψ_4 are remained unaffected as in (4.2) and (4.4) above. Accordingly, the Maxwell scalar ϕ_1 with constant e becomes

$$\phi_1 = \frac{e}{\sqrt{2}\,\overline{R}\,\overline{R}}, \tag{4.8}$$

which is the same Maxwell scalar of the Kerr-Newman-Vaidya metric (3.15) if once written out from (3.12). From the Weyl scalar (4.7) we have clearly seen *a change in the mass* M by some constant quantity m_1 (say) for the first electrical radiation in the embedded black hole. Then the total mass of Kerr-Newman-Vaidya black hole becomes $(M - m_1) + f(u)$ and the metric takes the form

$$\begin{aligned} ds^2 \;=\; & [1 - R^{-2}\{2r(M - m_1 + f(u)) - e^2\}]\,du^2 \\ & +2du\,dr + 2aR^{-2}\{2r(M - m_1 + f(u)) \\ & -e^2\}\sin^2\theta\,du\,d\phi - 2a\sin^2\theta\,dr\,d\phi - R^2d\theta^2 \\ & -\{(r^2+a^2)^2 - \Delta^* a^2\sin^2\theta\}\,R^{-2}\sin^2\theta\,d\phi^2, \end{aligned} \tag{4.9}$$

where $\Delta^* = r^2 - 2r\{M - m_1 + f(u)\} + a^2 + e^2$. Since the Maxwell scalar (4.8) remains the same for the first Hawking radiation, we again consider the charge e to be a function of r for the second radiation in the metric (4.9) with the mass $M - m_1 + f(u)$. This will certainly lead, by the Einstein-Maxwell field equations with the vanishing of Λ to reduce another quantity m_2 (say) from the total mass, *i.e.* the mass becomes $M - (m_1 + m_2) + f(u)$ after the second electrical radiation. Here again, we observe that the Maxwell scalar ϕ_1 remains the same form and also there is no effect on the Vaidya mass function $f(u)$ after the second radiation. Thus, if we consider n radiations, every time taking the charge e to be a function of r, the Einstein's field equations will imply that the total mass of the black hole will take the form

$$\mathcal{M} = M - (m_1 + m_2 + m_3 + m_4 + ... + m_n) + f(u) \tag{4.10}$$

without affecting the mass function $f(u)$. Taking Hawking's radiation of charged black hole embedded in the rotating Vaidya null radiating space, one might expect that the mass M may be radiated away, just leaving $M - (m_1 + m_2 + m_3 + m_4 + ... + m_n)$ equivalent to the Planck mass of about 10^{-5} g and $f(u)$ untouched; that is, M may not be exactly equal to $(m_1 + m_2 + m_3 + m_4 + ... + m_n)$, but has a difference of about a Planck-size mass. Otherwise, the mass M may be evaporated completely after continuous radiation, when $M = (m_1 + m_2 + m_3 + m_4 + ... + m_n)$, just leaving the Vaidya mass function $f(u)$ and the electric charge e only. Thus, we can show this situation of the black hole in the form of a classical spacetime metric as

$$\begin{aligned} ds^2 \;=\; & [1 - R^{-2}\{2rf(u) - e^2\}]\,du^2 + 2du\,dr \\ & +2aR^{-2}\{2rf(u) - e^2\}\sin^2\theta\,du\,d\phi \\ & -2a\sin^2\theta\,dr\,d\phi - R^2d\theta^2 - \{(r^2+a^2)^2 \\ & -\Delta^* a^2\sin^2\theta\}\,R^{-2}\sin^2\theta\,d\phi^2, \end{aligned} \tag{4.11}$$

where $\Delta^* = r^2 - 2rf(u) + a^2 + e^2$. The Weyl scalar of this metric becomes

$$\psi_2 = \frac{1}{\overline{R}\,\overline{R}\,R^2}\Big\{e^2 - R\,f(u)\Big\}. \tag{4.12}$$

and ψ_3, ψ_4 are unaffected as (4.4). From (3.4) we know that the remaining metric (4.11) is the rotating charged Vaidya null radiating black hole with $f(u) > a^2 + e^2$. The surface gravity of the horizon at $r = r_+ = f(u) + \sqrt{\{f(u)^2 - a^2 - e^2\}}$, is

$$\mathcal{K} = -\frac{1}{r_+R^2}\Big[r_+\sqrt{\{f(u)^2 - a^2 - e^2\}} + \frac{e^2}{2}\Big].$$

The Hawking's temperature on the horizon is $T_{\rm H} = (\mathcal{K}/2\pi)$. The entropy and angular velocity of the horizon are found

$$\mathcal{S} = 2\pi\, f(u)\Big\{f(u) + \sqrt{f(u)^2 - (a^2+e^2)}\,\Big\} - \frac{e^2}{4},$$
$$\Omega_{\rm H} = \frac{a\{2rf(u) - e^2(u)\}}{(r^2+a^2)^2}\Big|_{r=r_+}.$$

Thus, we may regard this left out remnant of the Hawking evaporation as the rotating charged Vaidya black hole. On the other hand, the metric (4.11) may be interpreted as the presence of Vaidya mass function $f(u)$ can avoid the formation of an 'instantaneous' naked singularity with zero mass. The formation of 'instantaneous' naked singularity with zero mass - *a naked singularity that exists for an instant and then continues its electrical radiation to create negative mass*, in *non-embedded* Reissner-Nordstrom and Kerr-Newman, black holes is unavoidable during Hawking's evaporation process, as shown in [1]. That is, if we set the mass function $f(u) = 0$, the metric (4.11) would certainly represent an 'instantaneous' naked singularity with zero mass, and at that stage gravity of the surface would depend only on electric charge, *i.e.* $\psi_2 = (e^2/\overline{R}\,\overline{R}\,R^2)$, and not on the mass of black hole. However, the Maxwell scalar ϕ_1 is unaffected. Thus, from (4.11) with $f(u) \neq 0$ it seems natural to refer to the *rotating* charged Vaidya null radiating black hole as an 'instantaneous' black hole - *a black hole that exists for an instant and then continues its electrical radiation*, during the Hawking's evaporation process of Kerr-Newman-Vaidya black hole.

The time taken between two consecutive radiations is supposed to be so short that one may not physically realize how quickly radiations take place. Immediately after the exhaustion of the Kerr-Newman mass, if one continues the remaining solution (4.11) to radiate electrically with $e(r)$, there may be a formation of new mass m_1^* (say). If this electrical radiation process continues forever, the new mass will increase gradually as

$$\mathcal{M}^* = m_1^* + m_2^* + m_3^* + m_4^* + ... \tag{4.13}$$

However, it appears that this new mass will never decrease. Then, the spacetime metric will take the following form

$$\begin{aligned} ds^2 \;\;=\;\; & [1 + R^{-2}\{2r(\mathcal{M}^* - f(u)) + e^2\}]\,du^2 \\ & +2du\,dr + 2aR^{-2}\{2r(f(u) - \mathcal{M}^*) \\ & -e^2\}\sin^2\theta\,du\,d\phi - 2a\sin^2\theta\,dr\,d\phi \\ & -R^2d\theta^2 - \{(r^2+a^2)^2 \\ & -\Delta^* a^2\sin^2\theta\}\,R^{-2}\sin^2\theta\,d\phi^2, \end{aligned} \tag{4.14}$$

where $\Delta^* = r^2 - 2r\{f(u) - \mathcal{M}^*\} + a^2 + e^2$. This metric will describe a black hole if $f(u) - \mathcal{M}^* > a^2 + e^2$, that is, when $f(u) > \mathcal{M}^* > a^2 + e^2$. Thus, we have shown the changes in the total mass of Kerr-Newman-Vaidya black hole in classical spacetime metrics without effecting the Maxwell scalar and the Vaidya mass function, for every electrical radiation during the primordial Hawking evaporation process. We have also observed that, when $f(u) > \mathcal{M}^*$, the presence of Vaidya mass $f(u)$ in (4.14) can prevent the direct formation of negative mass naked singularity. Otherwise, when $f(u) < \mathcal{M}^*$, this metric may describe a 'non-stationary' negative mass naked singularity, which is different from the 'stationary' one discussed in [1]. The metric (4.14) can be written in Kerr-Schild ansatze as

$$g_{ab}^{\rm NMV} = g_{ab}^{\rm V} + 2Q(r,\theta)\ell_a\ell_b,$$

where $Q(r,\theta) = (r\mathcal{M}^* + e^2/2)R^{-2}$, and

$$g_{ab}^{\rm NMV} = g_{ab}^{\rm NM} + 2Q(u,r,\theta)\ell_a\ell_b,$$

with $Q(u,r,\theta) = -rf(u)R^{-2}$. These Kerr-Schild forms show that the metric (4.27) is a solution of Einstein's field equations. Here the metric tensor $g_{ab}^{\rm V}$ is rotating Vaidya null radiating metric, and $g_{ab}^{\rm NM}$ is the metric describing the negative mass naked singularity.

(ii) Variable-charged Kerr-Newman-Vaidya-de Sitter black hole

Hawking radiation of black hole is due to the electrical radiation described by the energy-momentum tensor of electromagnetic field with variable charge $e(r)$. In order to incorporate the Hawking radiation in Kerr-Newman-Vaidya-de Sitter black hole, we consider the charge e of the electromagnetic field to be a function of radial coordinate. Accordingly, the metric (3.34) embedded into the de Sitter space will take the form (by changing the mass symbol m to M)

$$ds^2 \;\;=\;\; \Big[1 - R^{-2}\Big\{2r\Big(M + f(u)\Big) + \frac{\Lambda^* r^4}{3}$$

$$-e^2(r)\Big\}\Big]\,du^2 + 2du\,dr + 2aR^{-2}\Big\{2r\Big(M + f(u)\Big) + \frac{\Lambda^*\,r^4}{3} - e^2(r)\Big\}\sin^2\theta\,du\,d\phi - 2a\sin^2\theta\,dr\,d\phi - R^2 d\theta^2 - \Big\{(r^2+a^2)^2 - \Delta^* a^2\sin^2\theta\Big\}\,R^{-2}\sin^2\theta\,d\phi^2, \qquad (4.15)$$

where $\Delta^* = r^2 - 2r\{M + f(u)\} - \Lambda^* r^4/3 + a^2 + e^2(r)$ and Λ^* represents the cosmological constant of the de Sitter space. Now we can obtain the Ricci scalars and the Weyl scalars for this metric (4.15) from (3.6) and (3.8). However, the expression of Ricci scalars ϕ_{11} and Λ are purely matter dependent – the cosmological constant Λ^* and the electric charge $e(r)$ with its derivatives. As the cosmological object and the electromagnetic field are two different matter fields of different physical properties, it is, without loss of generality, possible to have a decomposition of ϕ_{11} and Λ in terms of the cosmological constant Λ^* and the electromagnetic field with charge e such that $\phi_{11}{=}\phi_{11}^{(C)} + \phi_{11}^{(E)}$ and $\Lambda{=}\Lambda^{(C)} + \Lambda^{(E)}$ with

$$\phi_{11}^{(C)} = -\frac{1}{2\,R^2\,\bar{R}^2}\,\Lambda^* r^2\,a^2\cos^2\theta, \qquad (4.16)$$

$$\phi_{11}^{(E)} = \frac{1}{2\,R^2\,\bar{R}^2}\Big\{e^2(r) - 2r\,e(r)e(r)_{,r}\Big\} + \frac{1}{4R^2}\Big\{e^2(r)_{,r} + e(r)\,e(r)_{,rr}\Big\}, \qquad (4.17)$$

$$\Lambda^{(C)} = \frac{\Lambda^* r^2}{6\,R^2}, \qquad (4.18)$$

$$\Lambda^{(E)} = \frac{-1}{12\,R^2}\Big\{e^2(r)_{,r} + e(r)\,e(r)_{,rr}\Big\}, \qquad (4.19)$$

and the non-vanishing Weyl scalars are given by

$$\psi_2 = \frac{1}{\bar{R}\,\bar{R}\,R^2}\Big[e^2(r) - R\{M + f(u)\} + \frac{1}{6}\,\bar{R}\,R\Big\{e^2(r)_{,r} + e(r)\,e(r)_{,rr}\Big\} - \bar{R}\,e(r)\,e(r)_{,r} + \frac{\Lambda^* r^2}{3}a^2\cos^2\theta\Big], \qquad (4.20)$$

$$\psi_3 = \frac{-i\,a\sin\theta}{2\sqrt{2}\bar{R}\,\bar{R}\,R^2}\Big\{(4\,r + \bar{R})f(u)_{,u}\Big\},$$

$$\psi_4 = \frac{a^2 r\sin^2\theta}{2\bar{R}\,\bar{R}\,R^2\,\bar{R}^2}\Big\{R^2 f(u)_{,uu} - 2r f(u)_{,u}\Big\}.$$

The decomposition of ϕ_{11} and Λ is followed from (3.36) for the two energy momentum tensors $T_{ab}^{(E)}$ and $T_{ab}^{(C)}$ admitted by Kerr-Newman-Vaidya-de Sitter solution. Now the vanishing Ricci scalar $\Lambda^{(E)}$ (4.19) for electromagnetic field will give

$$e^2(r) = 2rm_1 + C \qquad (4.21)$$

where m_1 and C are real constants. The substitution of this value in (4.17) and (4.20) with the identification of the constant $C \equiv e^2$ yields that

$$\phi_{11}^{(E)} = \frac{e^2}{2\,R^2\,\bar{R}^2}, \qquad (4.22)$$

$$\psi_2 = \frac{1}{\bar{R}\,\bar{R}\,R^2}\Big[e^2 - R\{(M - m_1) + f(u)\} + \frac{\Lambda^* r^2}{3}a^2\cos^2\theta\Big]. \qquad (4.23)$$

We have seen the change in the mass in (4.23) by a quantity m_1 (say) at the end of the first electrical radiation. However, due to the equation (4.22) the Maxwell scalar ϕ_1 with constant charge e remains the same as before $\phi_1 = e/(\sqrt{2}\,\bar{R}\,\bar{R})$ and also the cosmological Ricci scalar $\Lambda^{(C)}$ unchanged (4.18). Then the total mass of the Kerr-Newman-Vaidya-de Sitter black hole will take the form $(M - m_1) + f(u)$, and the line element will be of the form after the first radiation

$$ds^2 = \Big[1 - R^{-2}\Big\{2r\Big(M - m_1 + f(u)\Big) + \frac{\Lambda^*\,r^4}{3} - e^2\Big\}\Big]\,du^2 + 2du\,dr + 2aR^{-2}\Big\{2r\Big(M - m_1 + f(u)\Big) + \frac{\Lambda^*\,r^4}{3} - e^2\Big\}\sin^2\theta\,du\,d\phi - 2a\sin^2\theta\,dr\,d\phi - R^2 d\theta^2 - \Big\{(r^2+a^2)^2 - \Delta^* a^2\sin^2\theta\Big\}\,R^{-2}\sin^2\theta\,d\phi^2, \qquad (4.24)$$

where $\Delta^* = r^2 - 2r\{(M - m_1) + f(u)\} - \Lambda^* r^4/3 + a^2 + e^2$. For the second radiation we consider the charge e to be the function of r in Einstein's field equations with the mass $(M - m_1) + f(u)$. Here we again calculate the Ricci scalar of electromagnetic field $\Lambda^{(E)}$ which has to vanish to reduce another constant m_2 (say), such that after the second radiation of the black hole (4.24), the total mass will take the form $M - (m_1 - m_2) + f(u)$. However, the Maxwell scalar ϕ_1, the cosmological Ricci scalar Λ^* and the Vaidya mass $f(u)$ are unaffected by the second electrical radiation. Hence, if we consider n electrical radiations with the charge e to be function of r, the Einstein's field equations would yield that the total mass of the black hole will take the form $M - (m_1 + m_2 + m_3 + m_4 + ... + m_n) + f(u)$. Here it is emphasized that there may be two possibilities that (i) the mass of the Kerr-Newman black hole radiated away, just leaving the total M equivalent to the Planck mass and $f(u)$ and Λ^* unaffected by the electrical radiation process; M may not be exactly equal to the reduced mass $m_1+m_2+m_3+m_4+...+m_n$, leaving Planck-size mass; or (ii) the mass M is completely evaporated with $M=m_1+m_2+m_3+m_4+...+m_n$ just leaving the Vaidya mass function $f(u)$ behind, embedded into the de Sitter cosmological space and the electrical charge e of Kerr-Newman black hole. The remaining remnant will be of the form of spacetime metric as

$$\begin{aligned} ds^2 &= \Big[1-R^{-2}\Big\{2rf(u)+\frac{\Lambda^*\,r^4}{3}-e^2\Big\}\Big]\,du^2 \\ &\quad +2du\,dr+2aR^{-2}\Big\{2r+f(u)+\frac{\Lambda^*\,r^4}{3} \\ &\quad -e^2\Big\}\sin^2\theta\,du\,d\phi-2a\sin^2\theta\,dr\,d\phi-R^2d\theta^2 \\ &\quad -\Big\{(r^2+a^2)^2-\Delta^*a^2\sin^2\theta\Big\}\,R^{-2}\sin^2\theta\,d\phi^2, \end{aligned} \tag{4.25}$$

where $\Delta^*=r^2-2rf(u)-\Lambda^*\,r^4/3+a^2+e^2$. The Weyl scalar describing the curvature of this black hole remnant is calculated as

$$\psi_2=\frac{1}{\overline{R}\,\overline{R}\,R^2}\Big[e^2+R\,f(u)+\frac{\Lambda^*r^2}{3}a^2\cos^2\theta\Big] \tag{4.26}$$

and the other two Ricci scalars ψ_3 and ψ_4 are the same as given in (4.4) indicating that the curvature of algebraically special gravitational field of rotating Vaidya-de Sitter space is unaffected during the Hawking electrical radiation process of the mass of Kerr-Newman black hole. The surface gravity of the horizon at $r=r_{++}$ is

$$\mathcal{K}=-\Big[\frac{1}{rR^2}\Big\{r\Big(r-f(u)-\frac{\Lambda^*r^3}{6}\Big)+\frac{e^2}{2}\Big\}\Big]_{r=r_{++}}$$

with the Hawking's temperature $T_{\rm H}=(\mathcal{K}/2\pi)$ on it. The entropy and angular velocity of the horizon are obtained as

$$\mathcal{S}=\pi\{r^2+a^2\}\Big|_{r=r_{++}},$$
$$\Omega_{\rm H}=\frac{a\{2rf(u)+(\Lambda^*r^4/3)-e^2\}}{(r^2+a^2)^2}\Big|_{r=r_{++}}.$$

Here the value of r_{++} may be obtained from appendix (A5). If one sets $f(u)=\Lambda^*=0$ in (4.25), one will obtain the rotating 'instantaneous' naked singularity with zero mass, whose surface gravity depends only on the electric charge e with the Weyl scalar $\psi_2=(e^2/\overline{R}\,\overline{R}\,R^2)$. The formation of 'instantaneous' naked singularity with zero mass can be prevented in the case of Kerr-Newman-Vaidya-de Sitter black hole by the presence of Vaidya mass $f(u)$ and the cosmological constant Λ^* during Hawking's radiation process. Since the metric (4.25) describes the rotating Vaidya-de Sitter black hole, it can be referred to the rotating charged Vaidya-de Sitter black hole as an 'instantaneous' black hole; that is, it exists for an instant and then continues its electrical radiation to create negative mass, during the Hawking's evaporation process of Kerr-Newman-Vaidya-de Sitter black hole. However, we find that the Maxwell scalar ϕ_1 and the cosmological constant Λ^* are unaffected during the radiation process, i.e., the metric (4.25) still admits the total energy momentum tensor (3.35).

It is emphasized that the time taken between two consecutive radiations is supposed to be so short that it may not be possible to realize physically how quickly radiation take place. Immediately after the exhaustion of the Kerr-Newman mass, if the remaining solution (4.25) continues to radiate electrically with the variable charge $e(r)$, the Einstein's field equations with the vanishing Ricci scalar Λ will lead to create a new mass m_1^* (say). If the electrical radiation process of black hole (4.25) continues forever, the new mass might increase gradually as $\mathcal{M}^*=m_1^*+m_2^*+m_3^*+m_4^*+...$. This will lead the classical spacetime metric with this mass

$$\begin{aligned} ds^2 &= \Big[1+R^{-2}\Big\{2r\Big(\mathcal{M}^*-f(u)\Big) \\ &\quad -\frac{\Lambda^*\,r^4}{3}+e^2\Big\}\Big]\,du^2+2du\,dr \\ &\quad +2aR^{-2}\Big\{2r\Big(f(u)-\mathcal{M}^*\Big)+\frac{\Lambda^*\,r^4}{3} \\ &\quad -e^2\Big\}\sin^2\theta\,du\,d\phi-2a\sin^2\theta\,dr\,d\phi \\ &\quad -R^2d\theta^2-\Big\{(r^2+a^2)^2 \\ &\quad -\Delta^*a^2\sin^2\theta\Big\}\,R^{-2}\sin^2\theta\,d\phi^2, \end{aligned} \tag{4.27}$$

where $\Delta^*=r^2-2r\{f(u)-\mathcal{M}^*\}-\Lambda^*r^4/3+a^2+e^2$. Thus, we have seen the changes in the total mass of Kerr-Newman-Vaidya-de Sitter black hole in classical spacetime metrics without effecting the Maxwell scalar, the Vaidya mass function and the cosmological constant, during the Hawking evaporation process of electrically radiating black hole. The metric (4.27) can be expressed in Kerr-Schild ansatz as

$$g_{ab}^{\rm NMVdS}=g_{ab}^{\rm VdS}+2Q(r,\theta)\ell_a\ell_b,$$

where $Q(r,\theta) = (r\mathcal{M}^* + e^2/2)R^{-2}$, and

$$g_{ab}^{\rm NMVdS} = g_{ab}^{\rm NM} + 2Q(u,r,\theta)\ell_a\ell_b,$$

with $Q(u,r,\theta) = -\{rf(u) + \Lambda^* r^4/6\}R^{-2}$. These Kerr-Schild forms show that the metric (4.27) is a solution of Einstein's field equations. Here the metric tensors $g_{ab}^{\rm VdS}$ and $g_{ab}^{\rm NM}$ are rotating Vaidya-de Sitter metric and the negative mass naked singularity metric respectively.

(iii) Variable-charged Kerr-Newman-monopole black hole

Here we shall study the Hawking radiation of variable charged Kerr-Newman-monopole black hole derived above, when the electric charge e is considered to be a function of radial coordinate r in the field equations. The line element (3.45) with $e(r)$ takes the form:

$$\begin{aligned} ds^2 &= [1 - R^{-2}\{2rM + b\,r^2 - e^2(r)\}]\,du^2 \\ &+2du\,dr + 2aR^{-2}\{2rM + b\,r^2 \\ &-e^2(r)\}\sin^2\theta\,du\,d\phi - 2a\sin^2\theta\,dr\,d\phi \\ &-R^2 d\theta^2 - \{(r^2+a^2)^2 \\ &-\Delta^* a^2\sin^2\theta\}\,R^{-2}\sin^2\theta\,d\phi^2, \end{aligned} \tag{4.28}$$

where $\Delta^* = r^2(1-b) - 2rM + a^2 + e^2(r)$ and the monopole constant b lies in $0 < b < 1$. This metric will reduce to Kerr-Newman solution when e becomes constant initially and $b = 0$, Then, the Einstein-Maxwell field equations for the metric (4.28) with $e(r)$ can be solved to obtain the following quantities:

$$\begin{aligned} \phi_{11} &= \frac{1}{2\,R^2\,\overline{R}^2}\Big\{e^2(r) - 2r\,e(r)e(r)_{,r}\Big\} \\ &+\frac{1}{4R^2}\Big\{e^2(r)_{,r} + e(r)\,e(r)_{,rr}\Big\} \\ &+\frac{1}{4\,R^2\,\overline{R}^2}\,b\Big(r^2 - a^2\cos^2\theta\Big), \end{aligned} \tag{4.29}$$

$$\begin{aligned} \Lambda &= -\frac{1}{12\,R^2}\Big\{e^2(r)_{,r} + e(r)\,e(r)_{,rr}\Big\} \\ &+\frac{b}{12\,R^2}, \end{aligned} \tag{4.30}$$

$$\begin{aligned} \psi_2 &= \frac{1}{\overline{R}\,\overline{R}\,R^2}\Big[e^2(r) - RM - \overline{R}\,e(r)\,e(r)_{,r} \\ &+\frac{1}{6}\,\overline{R}\,\overline{R}\Big\{e^2(r)_{,r} + e(r)\,e(r)_{,rr}\Big\} \\ &-\frac{b}{6}\Big\{RR + 2ia\,r\cos\theta\Big\}\Big]. \end{aligned} \tag{4.31}$$

According to the total stress-energy momentum tensor (3.46) we shall, without loss of generality, have the following decompositions

$$\begin{aligned} \phi_{11}^{(\rm E)} &= \frac{1}{2\,R^2\,\overline{R}^2}\Big\{e^2(r) - 2r\,e(r)e(r)_{,r}\Big\} \\ &+\frac{1}{4R^2}\Big\{e^2(r)_{,r} + e(r)\,e(r)_{,rr}\Big\}, \end{aligned} \tag{4.32}$$

$$\phi_{11}^{(\rm m)} = \frac{1}{4\,R^2\,\overline{R}^2}\,b\Big(r^2 - a^2\cos^2\theta\Big), \tag{4.33}$$

$$\Lambda^{(\rm E)} = \frac{-1}{12\,R^2}\Big\{e^2(r)_{,r} + e(r)\,e(r)_{,rr}\Big\}, \tag{4.34}$$

$$\Lambda^{(\rm m)} = \frac{b}{12\,R^2}. \tag{4.35}$$

For electromagnetic field, the Ricci scalar $\Lambda^{(\rm E)}$ given in (4.34) must vanish. This yields

$$e^2(r) = 2rm_1 + C \tag{4.36}$$

with m_1 and C real constant of integration. Then we substitute this results in (4.32) to get

$$\phi_{11}^{(\rm E)} = \frac{C}{2\,R^2\,\overline{R}^2}. \tag{4.37}$$

Now replacing C by a real constant e^2 in (4.37) we obtain the Maxwell scalar

$$\phi_1 = \frac{e}{\sqrt{2}\,\overline{R}\,\overline{R}} \tag{4.38}$$

with the charge e. Then using the relation (4.36) in (4.31), we find the changed Weyl scalar

$$\begin{aligned} \psi_2 = \frac{1}{\overline{R}\,\overline{R}\,R^2}\Big[e^2 - R(M - m_1) \\ -\frac{b}{6}\Big\{RR + 2ia\,r\cos\theta\Big\}\Big]. \end{aligned} \tag{4.39}$$

which shows the reduction of the mass M by some quantity m_1. Thus, we have the line element with change of mass as

$$\begin{aligned} ds^2 &= [1 - R^{-2}\{2r(M - m_1) + b\,r^2 - e^2\}]\,du^2 \\ &+2du\,dr + 2aR^{-2}\{2r(M - m_1) + b\,r^2 \\ &-e^2\}\sin^2\theta\,du\,d\phi - 2a\sin^2\theta\,dr\,d\phi \\ &-R^2 d\theta^2 - \{(r^2+a^2)^2 \\ &-\Delta^* a^2\sin^2\theta\}\,R^{-2}\sin^2\theta\,d\phi^2, \end{aligned} \tag{4.40}$$

with $\Delta^* = r^2(1-b) - 2r(M - m_1) + a^2 + e^2$. The introduction of the constant m_1 in the metric (4.40) suggests that the first electrical radiation of Kerr-Newman-monopole black hole has reduced the original mass M by a quantity m_1. For the next radiation, we again consider the charge e to be the function of r with the mass $M - m_1$ in (4.40). Then the Einstein-Maxwell equations yield to reduce the mass by another constant quantity m_2 (say), i.e., after the second radiation, the mass becomes $M - (m_1 + m_2)$. Thus , if we consider n radiations, every time considering the charge e to be a function of r, the Maxwell scalar ϕ_1 will be the same as in (4.38). However the mass will become $M - (m_1 + m_2 + m_3 + m_4 + ... + m_n)$ without affecting monopole constant b. Taking Hawking's radiation of charged black holes into account, one might expect that the mass M may be radiated away, just leaving $M - (m_1 + m_2 + m_3 + m_4 + ... + m_n)$ equivalent to the Planck mass. Otherwise, the mass M may be evaporated completely after continuous radiation, when $M = (m_1 + m_2 + m_3 + m_4 + ... + m_n)$, just leaving the monopole constant b and the electric charge e. Thus, we can show this situation of the black hole in the form of a classical spacetime metric as

$$\begin{aligned} ds^2 \;=\; & [1 - R^{-2}\{b\,r^2 - e^2\}]\,du^2 + 2du\,dr \\ & +2aR^{-2}\{b\,r^2 - e^2\}\sin^2\theta\,du\,d\phi \\ & -2a\sin^2\theta\,dr\,d\phi - R^2 d\theta^2 - \{(r^2+a^2)^2 \\ & -\Delta^* a^2\sin^2\theta\}\,R^{-2}\sin^2\theta\,d\phi^2, \end{aligned} \quad (4.41)$$

where $\Delta^* = r^2(1-b) + a^2 + e^2$. At this stage the gravity of the surface would depend on electric charge and monopole constant, not on the mass. This metric has the non-zero Weyl scalar

$$\psi_2 = \frac{1}{\bar{R}\,\bar{R}\,R^2}\Big[e^2 - \frac{b}{6}\Big\{RR + 2ia\,r\cos\theta\Big\}\Big], \quad (4.42)$$

describing the curvature of the remaining remnant. However, the Maxwell scalar ϕ_1 is unaffected. The metric describes the rotating charged monopole black hole with the horizons at $r_\pm = \pm\frac{1}{(1-b)}[\surd\{(b-1)(a^2 + e^2)\}]$. As the electrical radiation has to continue, this black hole will remain only for an instant. Hence, one can refer to the solution (4.41) as an 'instantaneous' charged black hole with the surface gravity

$$\mathcal{K} = -\frac{1}{r_+ R^2}\Big[r_+\Big\{r_+\Big(1 - \frac{1}{2}\,b\Big)\Big\} + \frac{e^2}{2}\Big]$$

and the Hawking's temperature $T_{\rm H} = (\mathcal{K}/2\pi)$. The entropy and angular velocity of the horizon are given as

$$\mathcal{S} = \pi\{r^2 + a^2\}\Big|_{r=r_+},$$

$$\Omega_{\rm H} = \frac{a\{br^2 - e^2(u)\}}{(r^2+a^2)^2}\Big|_{r=r_+}.$$

Immediately, after the exhaustion of the Kerr-Newman mass, if the remaining solution (4.41) continues to radiate electrically with $e(r)$, there will be a formation of new mass m_1^* (say). If this electrical radiation process continues forever, the new mass will increase gradually as

$$\mathcal{M}^* = m_1^* + m_2^* + m_3^* + m_4^* + ... \quad (4.43)$$

However, it appears that this new mass will never decrease. Then, the spacetime metric will take the following form

$$\begin{aligned} ds^2 \;=\; & [1 + R^{-2}\{2r\mathcal{M}^* - b\,r^2 + e^2\}]du^2 + 2du\,dr \\ & +2aR^{-2}\{b\,r^2 - 2r\mathcal{M}^* - e^2\}\sin^2\theta\,du\,d\phi \\ & -2a\sin^2\theta\,dr\,d\phi - R^2 d\theta^2 - \{(r^2+a^2)^2 \\ & -\Delta^* a^2\sin^2\theta\}\,R^{-2}\sin^2\theta\,d\phi^2, \end{aligned} \quad (4.44)$$

where $\Delta^* = r^2(1-b) + 2r\mathcal{M}^* + a^2 + e^2$. This metric has the Weyl scalar

$$\psi_2 = \frac{1}{\bar{R}\,\bar{R}R^2}\Big[R\mathcal{M}^* + e^2 - \frac{b}{6}\{RR + 2iar\cos\theta\}\Big] \quad (4.45)$$

describing the gravity of the surface with negative sign. This metric (4.44) expresses the negative mass naked singularity embedded into the rotating monopole solution i.e., the metric can be expressed in Kerr-Schild ansatz based on different backgrounds as

$$g_{ab}^{\rm NMm} = g_{ab}^{\rm m} + 2Q(r,\theta)\ell_a\ell_b \quad (4.46)$$

where $Q(r,\theta) = (r\mathcal{M} + e^2/2)R^{-2}$, and

$$g_{ab}^{\rm KNm} = g_{ab}^{\rm NM} + 2Q(r,\theta)\ell_a\ell_b \quad (4.47)$$

with $Q(r,\theta) = -(b\,r/4)R^{-2}$. Here $g_{ab}^{\rm m}$ is the rotating monopole solution and $g_{ab}^{\rm NM}$ represents the negative mass naked singularity metric tensor. The vector ℓ_a is a geodesic, shear free, expanding as well as non-zero twist null vector which is one of the repeated principal null vectors of both $g_{ab}^{\rm m}$ as well as $g_{ab}^{\rm NM}$. These two Kerr-Schild forms indicate that the metric (4.44) is a solution of Einstein's field equations.

5. CONCLUSION

In this paper, we have presented NP quantities for a rotating spherically symmetric metric with three variables in an appendix. We find that the general expressions in NP quantities can be used to discuss the general properties of the spacetimes. For example, the metric (2.3) with three variables, in general possesses a geodesic, shear free, rotating and expanding null vector ℓ_a as shown in (2.8) and (2.9). The non-vanishing ψ_2, ψ_3, ψ_4, presented in appendix (A2) suggests that the spacetime metric (2.3) is algebraically special in the Petrov classification. With the help of these NP quantities, we have first given a class of rotating solutions like, rotating Vaidya-Bonnor, rotating Vaidya, Kerr-Newman and rotating de Sitter. Then, with the help of Wang-Wu functions, we come to the unpublished examples of rotating metrics that we combined them with other rotating solutions in order to generate new embedded rotating solutions like Kerr-Newman-Vaidya, Kerr-Newman-Vaidya-de Sitter, Kerr-Newman-monopole, and studied the gravitational structure of the solutions by observing the nature of the energy momentum tensors of respective spacetime metrics. The embedded rotating solutions have also been expressed in terms of Kerr-Schild ansatze in order to indicate them as solutions of Einstein's field equations. These ansatze show the extensions of those of Glass and Krisch [7] and Xanthopoulos [8].

The remarkable feature of the analysis of rotating solutions in this paper is that all the rotating solutions, stationary Petrov type D and non-stationary algebraically special possess the same null vector ℓ_a, which is geodesic, shear free, expanding as well as non-zero twist. From the studies of the rotating solutions we find that some solutions after making rotation have disturbed their gravitational structures. For example, the rotating monopole solution (3.45) with $m = e = 0$ possesses the energy momentum tensor with the monopole pressure p, where the monopole charge b couples with the rotating parameter a. Similarly, the rotating de Sitter solution (3.25) becomes Petrov type D spacetime metric, where the rotating parameter a is coupled with the cosmological constant. After making rotation in (3.4), the Vaidya metric with $e(u) = 0$ becomes algebraically special in Petrov classification of spacetime metric with a null vector ℓ_a which is geodesic, shear free, expanding and non-zero twist. The Wang-Wu functions in the rotating metric (3.10) play a great role in the derivation of the rotating embedded solutions discussed here. The method adopted here with Wang-Wu functions might be another possible version for obtaining *non-stationary* rotating black hole solutions with visible energy momentum tensors describing the interaction of different matter fields with well-defined physical properties like Guth's modification of T_{ab} (3.39) etc. It is believed that such interactions of different matter fields as in (3.16), (3.35), (3.39) and (3.46) have not seen published before. We have also found the direct involvement of the rotation parameter a in each expression of the surface gravity and the angular velocity, which shows the important of the study of rotating, embedded and non-embedded, black holes in order to understand the nature of different black holes located in the universe.

In section 4, we find that the changes in the masses of embedded black holes take place due to the vanishing of Ricci scalar of electromagnetic fields with the charge $e(r)$. It is also shown that the Hawking's radiation can be expressed in classical spacetime metrics, by considering the charge e to be the function of the radial coordinate r of Kerr-Newman-Vaidya, Kerr-Newman-Vaidya-de Sitter and Kerr-Newman-monopole black holes. That is, every electrical radiation produces a change in the mass of the charged objects. These changes in the mass of black holes embedded into Vaidya, Vaidya-de Sitter and monopole spaces, after every electrical radiation, describe the relativistic aspect of Hawking's evaporation of masses of black holes in the classical spacetime metrics. Thus, we find that the black hole evaporation process is due to the electrical radiation of the variable charge $e(r)$ in the energy momentum tensor describing the change in the mass in classical spacetime metrics which is in agreement with Boulware's suggestion [3]. The Hawking's evaporation of masses and the creation of embedded negative mass naked singularities are also due to the continuous electrical radiation. The formation of embedded naked singularity of negative mass is also Hawking's suggestion [2] mentioned in the introduction above. This suggests that, if one accepts the continuous electrical radiation to lead the complete evaporation of the original mass of black holes, then the same radiation will also lead to the creation of new mass to form negative mass naked singularities. This clearly indicates that an electrically radiating embedded black hole will not disappear completely, which is against the suggestion made in [2,3,5]. It is noted that we observe the different results from the studies of *embedded* and *non-embedded* black holes. In the embedded cases here above, the presence of Vaidya mass in (4.11), the Vaidya mass and the cosmological constant in (4.25) and the monopole charge in (4.41) completely prevent the disappearance of embedded radiating black holes during the radiation process and thereby, the formation of 'instantaneous' charged black holes. In *non-embedded* cases in [1], the disappearance of a black hole during radiation process is unavoidable, however occurs for an instant with the formation of 'instantaneous' naked singularity with zero mass, before continuing its next radiation. It is also noted that the 'instantaneous' black holes (4.11), (4.25) and (4.41) admit the total energy momentum tensors (3.16), (3.35) and (3.46) respectively as these tensors are not affected by the Hawking's radiation.

It appears that (i) the changes in the mass of black holes, (ii) the formation of 'instantaneous' naked singularities with zero mass and (iii) the creation of 'negative mass naked singularities' in *non-embedded* Reissner-Nordstrom as well as Kerr-Newman black holes [1] are presumably the correct formulation in classical spacetime metrics of the three possibilities of black hole evaporation suggested by Hawking and Israel [34]. However, the creation of 'negative mass naked singularities' may be a violation of Penrose's cosmic censorship hypothesis [19]. It is found that (i) the changes in the masses of embedded black holes, (ii) the formation of 'instantaneous' charged black holes (4.11), (4.25) and (4.41), and (iii) the creation of embedded 'negative mass naked singularities' in Kerr-Newman-Vaidya, Kerr-Newman-Vaidya-de Sitter, Kerr-Newman-monopole black holes might presumably be the mathematical formulations in classical spacetime metrics of the three possibilities of black hole evaporation [34]. All embedded black holes discussed here can be expressed in Kerr-Schild ansatze, accordingly their consequent negative mass naked singularities are also expressible in Kerr-Schild forms showing them as solutions of Einstein's field equations. It is also observed that once a charged black hole is embedded into some spaces, it will continue to embed forever through out its Hawking evaporation process. For example, Kerr-Newman black hole is embedded into the rotating Vaidya null radiating universe, it continues to embed as 'instantaneous' charged black hole in (4.11) and embedded negative mass naked singularity as in (4.14). There Hawking's radiation does not affect the Vaidya mass through out the evaporation process of Kerr-Newman mass. Similarly, in the cases of Kerr-Newman-Vaidya-de Sitter as well as Kerr-Newman-monopole black holes we find that the Vaidya mass, the cosmological constant and the monopole charge remain unaffected. This means that the embedded negative mass naked singularities (4.14), (4.27) and (4.44) possess the total energy momentum tensors (3.16), (3.35) and (3.46) respectively, as the Kerr-Newman mass does not involved in these tensors, and the change in the mass due to continuous radiation does not affect them. Thus, it may be concluded that once a black hole is embedded into some universes, it will continue to embed forever without disturbing the nature of matters present. If one accepts the Hawking continuous evaporation of charged black holes, the loss of mass and creation of new mass are the process of the continuous radiation. So, it may also be concluded that once electrical radiation starts, it will continue to radiate forever describing the various stages of the life of radiating black holes.

Also, we find from the above that the change in the mass of black holes, *embedded* or *non-embedded*, takes place due to the Maxwell scalar ϕ_1, remaining unchanged in the field equations during continuous radiation. So, if the Maxwell scalar ϕ_1 is absent from the space-time geometry, there will be no radiation, and consequently, there will be no change in the mass of the black hole. Therefore, we cannot, theoretically, expect to observe such *relativistic change* in the mass of uncharged Schwarzschild as well as Kerr black holes. This suggests that these uncharged Schwarzschild as well as Kerr black holes will forever remain the same without changing their life styles. Therefore, as far as Hawking's radiation effect is concerned, they may be referred to as *relativistic death black holes*.

From the study of Hawking's radiation above, it is also found that, as far as the embedded black holes are concerned, the Kerr-Newman black hole has relations with other rotating black holes, like the charged Vaidya black hole (4.11), the charged Vaidya-de Sitter (4.25) and the rotating charged monopole (4.41). There the later ones are 'instantaneous' black holes of the respective embedded ones. It is also noted that the rotating charged de Sitter, when $f(u) = 0$ in (4.25), may be regarded as an instantaneous cosmological black hole of Kerr-Newman-de Sitter. It is observed that the classical spacetime metrics discussed above would describe the possible life style of radiating embedded black holes at different stages during their continuous radiation. These *embedded* classical spacetimes metrics describing the changing life style of black holes are different from the *non-embedded* ones studied in [1] in various respects shown above. Here the study of these embedded solutions suggests the possibility that in an early universe there might be some black holes, which might have embedded into some other spaces possessing different matter fields with well-defined physical properties.

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Appendix

Here we shall present the NP quantities for the metric (2.3), calculated from the Cartan's first and second equations of structure developed by McIntosh and Hickman [11] in Newman-Penrose formalism [10].

Newman-Penrose spin coefficients:

$$\begin{aligned}
&\kappa = \sigma = \lambda = \epsilon = 0,\\
&\rho = -\frac{1}{\overline{\overline{R}}}, \quad \mu = -\frac{H(u,r,\theta)}{2\overline{R}},\\
&\alpha = \frac{(2ai - R\cos\theta)}{2\sqrt{2}\overline{R}\,\overline{R}\sin\theta}, \quad \beta = \frac{\cot\theta}{2\sqrt{2}R},\\
&\pi = \frac{i\,a\sin\theta}{\sqrt{2}\overline{R}\,\overline{R}}, \quad \tau = -\frac{i\,a\sin\theta}{\sqrt{2}R^2}, \qquad \text{(A1)}\\
&\gamma = \frac{1}{4}\Big\{H_{,r} - \frac{2ia\cos\theta\, H}{R^2}\Big\},\\
&\nu = \frac{1}{2\sqrt{2}\overline{R}}\Big\{H_{,\theta} - ia\sin\theta H_{,u} - \frac{2a^2 H\cos\theta\sin\theta}{R^2}\Big\},
\end{aligned}$$

where $\Delta^* = r^2 - 2rM(u,r,\theta) + a^2 + e^2(u,r,\theta)$ and the function $H(u,r,\theta)$ is given in (2.7).

The Weyl scalars:

$$\begin{aligned}
\psi_0 &= \psi_1 = 0,\\
\psi_2 &= \frac{1}{\overline{R}\,\overline{R}\,R^2}\Big\{(-RM + e^2) + \overline{R}\,(rM_{,r} - ee_{,r})\Big\}\\
&+\frac{1}{6R^2}\Big(- 2M_{,r} - rM_{,rr} + e^2_{,r} + ee_{,rr}\Big),\\
\psi_3 &= \frac{-1}{2\sqrt{2}\overline{R}\overline{R}\,R^2}\Big[4\Big\{i\,a\sin\theta\,(rM_{,u}\\
&-ee_{,u}) - (rM_{,\theta} - ee_{,\theta})\Big\}\\
&+\overline{R}\Big\{i\,a\sin\theta(r\,M_{,u} - e\,e_{,u})_{,r}\\
&-(r\,M_{,\theta} - e\,e_{,\theta})_{,r}\Big\}\Big], \qquad \text{(A2)}\\
\psi_4 &= \frac{\{a\,i(1+2\sin^2\theta) - r\cos\theta\}}{2\overline{R}\,\overline{R}\,\overline{R}\,R^2\sin\theta}\Big[i\,a\sin\theta\,(rM_{,u}\\
&-ee_{,u}) - (rM_{,\theta} - ee_{,\theta})\Big] - \frac{1}{\sqrt{2}\overline{R}}i\,a\sin\theta\,\nu_{,u}\\
&+\frac{1}{\sqrt{2}\overline{R}}\,\nu_{,\theta} - \frac{\sqrt{2}}{\overline{R}\,R^2}\,a^2\nu\sin\theta\cos\theta,
\end{aligned}$$

where ν is given in (A1).

The Ricci scalars:

$$\begin{aligned}
\phi_{00} &= \phi_{01} = \phi_{10} = \phi_{20} = \phi_{02} = 0,\\
\phi_{11} &= \frac{1}{4\,R^2\,R^2}\Big[2\,e^2 + 4r(r\,M_{,r} - ee_{,r})\\
&+R^2\Big(- 2M_{,r} - r\,M_{,r\,r} + e^2_{,r} + ee_{,r\,r}\Big)\Big],\\
\phi_{12} &= \frac{1}{2\sqrt{2}R^2R^2}\Big[ia\sin\theta\Big\{(RM_{,u} - 2ee_{,u})\\
&-(rM_{,r} - ee_{,r})_{,u}\overline{R}\Big\}\\
&+\Big\{(RM_{,\theta} - 2\,ee_{,\theta}) - (rM_{,r} - ee_{,r})_{,\theta}\overline{R}\Big\}\Big],\\
\phi_{21} &= \frac{-1}{2\sqrt{2}R^2R^2}\Big[ia\sin\theta\Big\{(\overline{R}M_{,u} - 2ee_{,u})\\
&-(rM_{,r} - ee_{,r})_{,u}R\Big\}\\
&+\Big\{(\overline{R}M_{,\theta} - 2ee_{,\theta}) - (rM_{,r} - ee_{,r})_{,\theta}R\Big\}\Big],\\
\phi_{22} &= \frac{1}{\sqrt{2}\,R}i\,a\sin\theta\,\nu_{,u} + \frac{1}{\sqrt{2}\,R}\,\nu_{,\theta}\\
&-\frac{\sqrt{2}}{R\,R^2}\,a^2\nu\sin\theta\cos\theta\\
&-\frac{(a^2\sin\theta + \overline{R}\cot\theta)}{2\overline{R}\,R^2\,R^2}\Big\{i\,a\sin\theta\,(r\,M_{,u}\\
&-e\,e_{,u})\Big\} - \frac{r^2+a^2}{\overline{R}\,R^2\,R^2}\Big(r\,M_{,u} - e\,e_{,u}\Big)\\
&-\frac{(a^2\sin\theta - \overline{R}\cot\theta)}{2\overline{R}\,R^2\,R^2}\Big(r\,M_{,\theta} - e\,e_{,\theta}\Big),
\end{aligned}$$

$$\Lambda = \frac{1}{12 R^2} \Big(2M_{,r} + r\, M_{,r\,r} - e_{,r}^2 - e e_{,r\,r} \Big). \quad \text{(A3)}$$

According to Carter [30] and York [35], we shall introduce a scalar $\mathcal{K}$ defined by the relation $n^b \nabla_b n^a = \mathcal{K}\, n^a$, where the null vector n^a in (2.5) above is parameterized by the coordinate u, such that $d/du = n^a \nabla_a$. Then this scalar can, in general, be expressed in terms of NP spin coefficient γ (A1) as follows:

$$\mathcal{K} = n^b\, \nabla_b\, n^a \ell_a = -(\gamma + \overline{\gamma}). \quad \text{(A4)}$$

On a horizon, the scalar $\mathcal{K}$ is called the surface gravity of a black hole.

The biquadratic equation for the solution (4.24)

$$\Delta^* \equiv r^2 - 2r\{m + f(u)\} - \Lambda^* r^4/3 + a^2 + e^2 = 0$$

has the following four roots for non-zero cosmological constant Λ^*:

$$\begin{aligned} \Big[r_+\Big]_{\pm} = & +\frac{1}{2}\sqrt{\Gamma} \pm \frac{1}{2}\sqrt{\Big[\frac{4}{\Lambda^*} - 32^{1/3}\chi} \\ & - \frac{1}{32^{1/3}\Lambda^*}\Big\{P + \sqrt{P^2 - 4Q}\Big\}^{1/3} \\ & - \frac{12}{\Lambda^*\sqrt{\Gamma}}\{m + f(u)\}\Big], \end{aligned} \quad \text{(A5)}$$

$$\begin{aligned} \Big[r_-\Big]_{\pm} = & -\frac{1}{2}\sqrt{\Gamma} \pm \frac{1}{2}\sqrt{\Big[\frac{4}{\Lambda^*} - 32^{1/3}\chi} \\ & - \frac{1}{32^{1/3}\Lambda^*}\Big\{P + \sqrt{P^2 - 4Q}\Big\}^{1/3} \\ & + \frac{12}{\Lambda^*\sqrt{\Gamma}}\{m + f(u)\}\Big] \end{aligned} \quad \text{(A6)}$$

where

$$P = 54\Big[18\Lambda^*\{m + f(u)\}^2 - 12\Lambda^*(a^2 + e^2) - 1\Big],$$

$$Q = \Big\{9 - 36\Lambda^*(a^2 + e^2)\Big\}^3,$$

$$\chi = \frac{1 - 4\Lambda^*(a^2 + e^2)}{\Lambda^*\Big\{P + \sqrt{P^2 - 4Q}\Big\}^{1/3}},$$

$$\Gamma = \frac{2}{\Lambda^*} + 32^{1/3}\chi - \frac{32^{-1/3}}{\Lambda^*}\Big\{P + \sqrt{P^2 - 4Q}\Big\}^{1/3}.$$

The calculation of these roots has been carried out by using 'Mathematica'. The area of a horizon of black hole can be calculated as follows [19]:

$$\mathcal{A} = \int_0^{\pi}\int_0^{2\pi} \sqrt{g_{\theta\theta} g_{\phi\phi}}\, d\theta\, d\phi\Big|_{\Delta^*=0}, \quad \text{(A7)}$$

depending on the values of the roots of $\Delta^* = 0$. Then the entropy on a horizon of a black hole may be obtained from the relation $\mathcal{S} = \mathcal{A}/4$ [36].

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