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Light Nuclei in the Quasi-Steady State Cosmological Model

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Abstract. A model for the decay of Planck particles is specified and the light elements synthesis resulting from it is described. The calculated values of $^4He/H$, $^7Li/H$, and $^{12}C/H$ are in close agreement with observations while those of D/H and $^3He/H$ are in agreement to within a factor of about 2. The model predicts a plateau under ^9Be but seemingly not under ^{11}B. The plateau under ^9Be corresponds to a freezing temperature $T_9 = 0.5$ whereas the calculated freezing temperature is $T_9 \simeq 0.62$.

1 Introduction

A series of papers (Hoyle, Burbidge and Narlikar, 1993, 1994a,b and 1995) have developed a cosmological model based on our belief that the gravitational theory must be scale invariant, along with the rest of physics. We have taken the view that the widely popular Big-Bang cosmology is logically flawed to an extent that we consider fatal. The gravitational theory on which Big-Bang cosmology is based, general relativity, assumes the world lines of particles considered classically to be unbounded. Then Big-Bang cosmology deduces the opposite, namely that world lines are bounded in the past at the Big-Bang. Our work referred to above is based on two requirements, one of resolving this contradiction and the other of constructing a scale invariant theory of gravitation.

In our theory particles at the past bounds of their world lines must be Planck particles, which subsequently decay into showers containing large numbers of familiar particles. The inverse process in which familiar particles come together, some 10^{19} of them into a region of dimension $\sim 10^{-33} cm$ can in principle lead to the termination of world lines in the future, but since this is a highly improbably configuration, it can be considered not to happen, thereby giving a one-sided time sense to the universe. Classical particles begin but they do not end.

A Planck particle is defined as one whose gravitational radius is comparable to its Compton wavelength, which requires in units with $c = \hbar = 1$, that the mass of the Planck particle is of the order of $G^{-1/2}$, G being the gravitational constant. When interactions other than gravitation are included Planck particles decay, ultimately into $\sim 10^{19}$ particles of familiar kinds. At least they do in our theory in which particles can be endowed at birth with properties defining the manner of this decay. Such particles expanding at a speed of order, but less than, c will be referred to as a Planck fireball.

The question to be addressed in this contribution is the synthesis of light elements in such fireballs.

2 The Model

Because the early stages in the development of a Planck fireball belong to the realm of unknown physics, it is necessary to begin with a specification of initial conditions. Fermions of familiar types are necessarily excluded by degeneracy conditions at early stages when the fireball dimension is only $\sim 10^{-33} cm$. Indeed, fermions of familiar types cannot appear until the interparticle spacing within the expanding fireball has increased to $\sim 10^{-13} cm$.

We take the view in specifying the model to be investigated that energy considerations discriminate against charmed, bottom and top quarks. We also take the view that degeneracy considerations, together with the need for electrical neutrality, prevent the strange quark from being discriminated against. When the up, down and strange quarks combine to baryons, equal numbers of N, P, Λ $\Sigma^{\pm}$, Σ^{0}, Ξ^{0}, Ξ^{-} are thus formed, with only a negligible amount of Ω^{-}. Because of the long lifetimes, $\sim 10^{-10}$ seconds, of Λ, $\Sigma^{\pm}$, Ξ^{0}, Ξ^{-}, the strange quark survives the effective stages in the expansion of the fireballs, although Σ^{0} goes to Λ plus a γ-ray at a stage proceeding the synthesis of the light elements. Finally, we consider that baryons containing the strange quark do not form stable nuclei. Ultimately they decay into N and P, but only after the particle density has fallen so far that the production of light elements has stopped. With N going on a much longer time scale (10 minutes) into P, six of the baryons of the octet go at last into hydrogen. Thus we see immediately that the fraction by mass of helium, Y, to emerge from Planck fireballs is given by

$$Y = 0.25(1 - y), \tag{1}$$

where $1 - y$ is the fraction of the original N and P to go to ^4He. Anticipating that y will be shown in the next section to be ~ 0.085, equation (1) gives $Y = 0.229$ somewhat lower than the value of ~ 0.237 obtained previously (Hoyle, 1992).

The numerical values used in the detailed calculations of later sections are given in the Table 1.

Table 1. Densities and Temperatures at $1 < \tau < 4$ in the expansion of a Planck Fireball

Here N is the number per cm^3 of each baryon type, the values in the table being such that N declines with increasing r as r^{-3}. The unit of r depends on a specification of the total number of baryons in the fireball. Thus for a total of 5.10^{18} the unit of r is $5.10^{-7} cm$. However, since this total is uncertain, because the Planck mass, usually given as $(3\hbar c/4\pi G)^{1/2}$, is theoretically uncertain to within factors such as 4π, we prefer to leave the unit of r unspecified – we shall not need it in the calculations. Suffice it that there will always be a unit for r such that N has the values in the table.

Taking the expansion of the fireball to occur at a uniform speed v, the time t of the expansion to radius r is proportional to r, $t \propto r$. In specifying the model we take the factor of proportionality here to be 10^{-16} seconds. With the unit of r chosen as $5.10^{-7} cm$ this requires $v = 5.10^{9} cm\ s^{-1}$, a rather low speed. But for a Planck mass increased by 4π above $(3\hbar c/4\pi G)^{1/2}$ the expansion speed is raised by $(4\pi)^{1/3}$ to $1.16 \times 10^{10} cm\ s^{-1}$. Thus

$$t = 10^{-16} r \text{ seconds}, \tag{2}$$

thereby relating t to N and T_9 through the values in Table 1. The numerical coefficient of equation (2) can be regarded as a parameter of the theory, but it is not a parameter that can be varied by more than a small factor, unlike the parameter η in Big-Bang nucleosynthesis which could be varied by many orders of magnitude for all one knows from the theory.

The temperature values in Table 1 are calculated from a heating source which comes into play at $r = 1$, i.e. at $t = 10^{-16} s$. The heating source is from the decay of π^0 mesons with a mean life of $8.4 \times 10^{-17} s$. The temperature values in Table 1 correspond to a π^0 meson concentration of $2/3N\ cm^{-3}$, which is to say one π meson to each neutron and each proton, with π^0, $\pi^{\pm}$ in equal numbers.

The decay of a π^0 meson into two 75 Mev γ-rays does not immediately deposit energy into the temperature T_9 of the heavy particles. It does not even lead to more than a limited production of $e^{\pm}$ pairs, because at these densities this is prevented by electron degeneracy. Thus the energy of π^0 decay is at first stored in the form of relativistic particles, quanta and some $e^{\pm}$, the latter being adequate, however, to prevent the γ-rays from escaping out of the fireball.

As the fireball expands, confined relativistic particles lose energy proportional to $1/r$, the energy loss going to the heavy particles, for each type of which there is a conservation equation of the form $dQ = dE + PdV$, viz

$$-\alpha d(1/r) = 3/2kdT + 3kTdV/V, \tag{3}$$

an equation that integrates to give

$$T_9 = \frac{2}{3}\frac{\alpha}{k}\ \frac{r-1}{r^2}, \tag{4}$$

the constant of integration being chosen to given $T_9 = 0$ at $r = 1$. The constant α in (3) and (4) is easily determined from the energy yield of the π^0 mesons. Sharing the energy communicated to the heavy particles equally among all of them, leads to the values of T_9 in Table 1.

The energy is considered to have all gone to the heavy particles by the stage of the expansion when r reaches 4, after which T_9 declines as r^{-2}, i.e. adiabaticly, the heavy particles being non-relativistic in their thermal motions. Thus for $r > 4$ we have

$$T_9 = 16.3.\left(\frac{4}{r}\right)^2 = \frac{260.8}{r^2}, \tag{5}$$

$$t = 10^{-16} r = \frac{1.62 \times 10^{-15}}{T_9^{1/2}} \text{seconds}, \tag{6}$$

while the particle densities decline as r^{-3}.

3 The Abundance of ^4He

It will be shown in this section that neutrons and protons are in statistical equilibrium with ^4He up to $r = 3$ in Table 1, but not for $r > 3$. Defining a parameter ζ by

$$\log \zeta = \log N - 34.07 - \frac{3}{2} \log T_9 \tag{7}$$

it was shown by Hoyle (1992) that the fraction y of neutrons and protons remaining free at temperature T_9 and particle density N for each nucleon type is given in statistical equilibrium by

$$\log \frac{1-y}{y^4} = 0.90 + 3 \log \zeta + \frac{142.6}{T_9}, \tag{8}$$

the values of T_9 and N in Table 1 at $r = 3$ giving $y = 0.085$, leading to the value $Y = 0.229$ given above. A similar calculation at $r = 2.5$ yields $y = 0.083$, much the same as at $r = 3$. For $r < 2.5$ the values of y fall away to ~ 0.06. Thus in moving to the right in the table the values of y increase towards $r = 3$, where the falling value of T_9 eventually freezes the equilibrium.

The condition for freezing is that the break-up of ^4He by $^4He(2N, T)T$, followed by the break-up of T and D into neutrons and protons should just be capable of supplying the densities of P and N, $n(P) = n(N) \simeq 5.10^{33} cm^{-3}$ for the range of r from 2.5 to 3 and $y \simeq 0.085$. The time available for this break-up of ^4He is that for r to increase from ~ 2.5 to ~ 3, i.e. 5.10^{-17} seconds. In this time the break-up of $n(A) \simeq 2.9 \times 10^{34} cm^{-3}$ using the reaction rates of Fowler, Coughlan and Zimmerman (1975) we verify that this is so, viz

$$\frac{1.67 \times 10^9}{T_9} . \frac{3.28 \times 10^{-10}}{T_9^{2/3}} \exp - \frac{4.872}{T_9^{1/3}} . \exp - \frac{131.51}{T_9}$$

$$(1 + 0.086 T_9^{1/3} - 0.455 T_9^{2/3} - 0.271 T_9 + 0.108 T_9^{4/3} + 0.225 T_9^{5/3}$$

$$\left(\frac{n(N)}{6.022 \times 10^{23}}\right)^2 n(A).\ 5 \times 10^{-17} \tag{9}$$

is required to be $\frac{1}{2} n(N) = 2.5 \times 10^{33} cm^{-3}$. Taking $T_9 \simeq 20$ for the range of r from 2.5 to 3, and putting $n(N) = 5.10^{33} cm^{-3}$, $n\ (A) = 2.9 \times 10^{34} cm^{-2}$, the value of (9) is $2.85 \times 10^{33} cm^{-3}$, adequately close to the required value of $2.5 \times 10^{33} cm^{-3}$.

This is already an astonishing result. That so complicated an expression as (9) should combine so exactly to produce such an outcome is not a consequence of the parametric choice of the model. The freedom of choice of the numerical coefficient in (2) is entirely dwarfed by the factors 10^{34}, 10^{33}, 10^{9}, 10^{-10}, 10^{-17} in (9), while even some variation in the parameter α in (4), as it affects the value of the factor $\exp -131.51/T_9 \approx 2.5 \times 10^{-3}$, is also dwarfed by the much larger powers in (9). The most license that can be permitted to a critic would be to accept the above result as model-dependent to the extent that it already consumes essentially all the available degrees of freedom of the model, leaving all further results to be judged as effectively parameter independent.

4 The Abundances of D and ^3He

Because of space restrictions, we have omitted the analysis which leads to the values(given in the summary Table 2)

$$D/H =^3 He/H \simeq 5 \times 10^{-5}$$

5 The Abundance of ^7Li

Writing $n(P)$, $n(A)$ for the densities of protons and alpha particles we have

$$n(P) = 1.58 \times 10^{33} \left(\frac{T_9}{16.3}\right)^{3/2} cm^{-3}, n(A) = 8.5 \times 10^{33} \left(\frac{T_9}{16.3}\right)^{3/2} \qquad (10)$$

The ratio of the abundance of ^7Li to ^8Be established in statistical equilibrium at temperature T_9 is given by

$$\log \frac{^7Li}{^8Be} = \frac{3}{2} \log \frac{7}{8} + \log 4 - \log n(P) + 34.07$$

$$+\frac{3}{2} \log T_9 - \frac{5.04}{T_9} \times 17.35.$$

$$= 3.20 - \frac{87.44}{T_9}, \qquad (11)$$

with

$$\log \frac{^8Be}{^4He} = \frac{3}{2} \log 2 + \log n(A) - 34.67 - \frac{3}{2} \log T_9 = -2.11 \qquad (12)$$

also given by statistical equilibrium.

The abundance of ^7Li established at T_9 according to (23) will, however, be subject to attenuation as the temperature declines from T_9, according to an attenuation factor

$$A \int_0^{T_9} \exp - \frac{30.443}{T_9} dt. \qquad (13)$$

with

$$dt = \frac{8.1 \times 10^{-16}}{T_9^{3/2}} dT_9 \tag{14}$$

as before and A a numerical coefficient obtained from the reaction rate for $^7Li(P, A)^4He$ given by FCZ, viz

$$A = 1.7 \times \frac{1.05 \times 10^{10}}{T^{3/2}} \cdot \frac{2.40 \times 10^{31}}{6.022 \times 10^{23}} T^{3/2} = 7.25 \times 10^{17} s^{-1} \tag{15}$$

The factor 1.7 here arises from an estimate of the combined effect of various terms adding to the rate of $^7Li(P, A)^4He$, the rest of A being the main term. Evaluating (13) leads to

$$\sim 19.3 T_9^{1/2} \exp - \frac{30.443}{T_9} \tag{15}$$

as the attenuation factor to be applied to the abundance of ^7Li given by (11). With $\log \frac{^4He}{H} = -1.08$ we thus have

$$\log \frac{^7Li}{H} = 3.20 - 2.11 - 1.08$$

$$-19.3 \times 0.4343 T_9^{1/2} \exp - \frac{30.443}{T_9} \tag{16}$$

$$= 0.01 - 8.38 T_9^{1/2} \exp - \frac{30.443}{T_9} \tag{17}$$

which has a maximum of -9.60 at $T_9 \simeq 12$. Thus the surviving lithium abundance is

$$\frac{^7Li}{H} \simeq 2.50 \times 10^{-10}, \tag{18}$$

a result in good agreement with the observational requirement, again calculates from highly complicated formulae, again without any model adjustment.

6 The Abundance of ^{11}B

A similar calculation for ^{11}B leads to $^{11}B/H \simeq 10^{-18}$, below the observational detection limit. This is significantly lower than the value calculated by Hoyle (1992) who used an attenuation factor that was not sufficient. From an observational point of view the model therefore predicts that there is effectively no 'plateau' under boron. Such boron as exists is required to come from cosmic-ray spallation on ^{12}C, ^{16}O.

7 The Abundance of ^9Be

As noted in Hoyle 1992, the nucleus of ^9Be is exceptionally fragile, leading to a particularly low freezing temperature. Statistical equilibrium at higher temperatures establishes

$$\log \frac{^9Be}{H} = \frac{3}{2}\log\frac{9}{8} + \log\frac{4}{3} - 0.15 + \log D/H$$
$$+ \log \frac{^8Be}{^4He} + \log \frac{^4He}{n(P)} - \frac{3.28}{T_9} \tag{19}$$

with respect to the reaction $^9Be(P,D)2^4He$. Using $\log D/H = -4.30$ already calculated, $\log {}^8Be/{}^4He = -2.11$, $\log^4 He/n(P) = 0.73$, gives $-5.63 - 3.28/T_9$ for the right hand side of (19). Because $^9Be(P,A)^6Li$ contributes equally with $^9Be(P,D)2^4He$ to the destruction of ^9Be, whereas at $T_9 \simeq 1$ it contributes essentially nothing to the production of ^9Be, the equilibrium concentration of ^9Be is lowered by a further factor 2, so that

$$\log \frac{^9Be}{H} = -5.93 - \frac{3.28}{T_9}. \tag{20}$$

Freezing of the equilibrium condition at $T_9 = 0.50$ for ^9Be would thus give

$$\log \frac{^9Be}{H} = -12.5 \tag{21}$$

in satisfactory agreement with the apparent observed plateau under ^9Be (A.M. Boesgaard, this conference).

The estimated freezing temperature according to the model can be obtained by requiring that the product of the expansion time scale, $1.62 \times 10^{-15}/T_9^{1/2}$ seconds at temperature T_9 and the sum of the reaction rates terms for $^9Be(P,D)2\,^4He$ and of those for $^9Be(P,A)^6Li$ be unity, viz

$$2.\frac{1.03\times 10^9}{T_9}.\frac{2.40\times 10^{31}}{6.033\times 10^{23}}.\frac{1.62\times 10^{-15}}{T_9^{1/2}}T_9^{3/2}.\exp -\frac{3.046}{T_9} = 1. \tag{22}$$

The factor 2 on the left of this formula comes from the circumstance that at the values of T_9 in question the highly complicated non-resonant contribution given by FCZ about doubles the resonant reaction rates. Equation (22) determines a freezing temperature $T_9 = 0.623$, reasonably close to the required value of 0.5.

8 The Abundances of ^{12}C and ^{16}O

The reaction rate on ^9Be from $^9Be(A,N)^{12}C$ as given by FCZ is

$$\sim \frac{2.40\times 10^8}{T_9^{3/2}}\frac{n(A)}{6.023\times 10^{23}}.\exp -\frac{12.732}{T_9}. \tag{23}$$

Using (10) for $n(A)$ and putting $T_9 \simeq 10$, at which temperature most of the production of ^{12}C takes place, gives 1.44×10^{16} for (23). Multiplying by

Table 2. Summary of Results

$^4He/H = Y \simeq 0.229$
$\frac{D}{H} \simeq \frac{^3He}{H} \simeq 5.10^{-5}$
$\frac{^7Li}{H} = 2.5 \times 10^{-10}$
$\frac{^{11}B}{H}$ very small
$\frac{^{12}C}{H} \simeq \frac{^{16}O}{H} \simeq 4.1 \times 10^{-6}$

the time-scale $1.62 \times 10^{-15}/T_9^{1/2}$ then gives ~ 7.4, implying that an abundance $^9Be/H \simeq 5.5 \times 10^{-7}$ given by (20) is converted 7.4 times over to ^{12}C, leading to

$$\frac{^{12}C}{H} \simeq 4.1 \times 10^{-6} \tag{24}$$

The value of $^{16}O/H$ is of a similar order.

9. The External Medium

All of the above followed from just the N and P members of the baryon octet. The other six baryons are considered not to form stable nuclei. They decay in $\sim 10^{-10}$ seconds, by which time a Planck fireball has effectively expanded into its surroundings, which according to the QSSC model (Hoyle et al. 1993,1994a,b) is necessarily a strong gravitational field in which the decay products of Λ, $\Sigma^{\pm}$, Ξ^0 and Ξ^- may be expected rapidly to lose energy. The Ξ^0 baryon decays to Λ and π^0 in a mean life of $3.0 \times 10^{-10} s$, Σ^+ which decays in a mean life of $8.0 \times 10^{-11} s$, gives a π^0 meson in about a half of the cases, so that together with Λ, which decays in a mean life of $2.5 \times 10^{-10} s$, there is a late production of about $2.5\pi^0$ per baryon octet, yielding ~ 5 late γ-rays per octet, typically with energies $\sim 100\ Mev$. It is these γ-rays and their products that are expected to be subjected to energy loss in strong gravitational fields.

The production of Planck particles near large masses of the order of galactic clusters occurs typically in an environmental density $\sim 10^{-16} g\ cm^{-3}$ at which density γ-rays of 100 Mev have path lengths of $\sim 10^{18} cm$, ample for considerable redshifting effects to occur, when quanta and particles in the $1 - 100\ kev$ range would arise. Although such particles and quanta are readily shielded against, it is an interesting speculation that pathways into the external universe may be briefly opened and that the mysterious γ-ray bursts arise in such situations.

10. Summary of Abundances and Conclusions

The calculations outlined here are more accurate than those described earlier (Hoyle, et al. 1993, Hoyle 1992). They lead to the abundances and results shown in the following table.

To obtain a ratio $\frac{^9Be}{H} \simeq 3.10^{-13}$ requires a freezing temperature $T_9 \simeq 0.5$ which is close but not equal to the calculated freezing temperature $T_9 \simeq 0.62$.

We conclude that a certain model of the decay of Planck particles leads to interesting values for the abundances of the light elements. The work is deductive, and in this sense the model used is not subject to negotiation, any more than the axioms on which a mathematical theorem is proved are subject to negotiation. Or any more than supporters of Big-Bang nucleosynthesis regard the choice of their parameter η as a matter of negotiation. Thus the only basis for judging the situation is to assess how good, or bad, are the results. Our assessment is the following:

(i) Our result for $^4He/H$ is very good.

(ii) The ratios D/H, $^3He/H$ too high by factor ~ 2. A more detailed calculation might well lower $^3He/H$ to its observational value. But at the expense of a further increase in D/H, necessitating an epicycle for the theory in which the observed $D/H \simeq 1.5 \times 10^{-5}$ is due to environmental effects.

(iii) The ratio $^7Li/H$ is very good.

(iv) The prediction of essentially no 'floor' under ^{11}B is subject to test. The 'floor' under ^9Be requires a freezing temperature $T_9 \simeq 0.50$, whereas the calculated freezing temperature was $T_9 \simeq 0.62$. Considering the very complicated expressions of FCZ, especially that involved in a cut-off procedure for non-resonant contributions, this correspondence is adequately close.

Finally we may ask how this situation for the synthesis of light elements from Planck particles compares with the situation in Big-Bang nucleosynthesis. In that case

(a) The classic choice $\eta = 3.10^{-10}$ for the baryon to photon ratio is good for $^3He/H$ but is too low for $^7Li/H$ and too high for Y and D/H.

(b) While reducing η brings Y and $^7Li/H$ into good agreement with observation the value of D/H becomes so large that the theory requires an astration epicycle to save itself.

(c) Raising η to $\sim 6.10^{-10}$ gives good results for D/H, $^3He/H$ and $^7Li/H$ but the resulting value $Y = 0.25$ is too high, and hardly savable by any epicycle or combination of epicycles.

(d) The theory predicts no plateau under ^9Be, which seems wrong. A recourse to inhomogeneous cosmological models would be to make the theory wildly epicyclic.

(e) Big-bang nucleosynthesis, but not the present model, predicts a present-day average baryon density in the universe much below the cosmological closure value, either forcing a change to a so-called open model (when galaxy formation is made difficult or impossible) or leading to the proposal that most of the material in the universe must be dark and non-baryonic. It is this argument that has led to the proposal that non-baryonic matter dominates the universe. None has so far been found and we find the argument far from overwhelming.

In view of these points (a) to (e) it is difficult to understand why supporters of Big-Bang cosmology claim that the synthesis of the light elements strongly supports their theory. Considering also the arbitrariness of the choice of η, one might rather say the reverse.

On several occasions in making our calculations above we have referred to our surprise at finding highly complex expressions for reaction rates, containing numerical factors involving high powers of 10, turning out to yield numbers agreeing with observation to within factors ~ 2. When one experiences complexities that simplify repeatedly into agreement with observations, the tendency is to make an inversion of logic. Instead of arguing from hypotheses to conclusions, the temptation is to argue in the contrary direction, from the conclusions to the hypotheses, i.e. to argue that because the conclusions are correct, or nearly so, the hypotheses must be true. Undeniably, science has made its most progressive steps from inversions of this kind. But also undeniably, science goes most wrong from them as well. Science goes wrong, because invalid inversions lead to a theological style of thinking. A theological style of thinking is one in which we begin from an implicit belief in the correctness of some hypothesis that has not been explicitly proved by observation or experiment, as many people believe today in the Big-Bang. When departures in deductive logic then appear, as in (a) to (e) above, epicycles are invented and the epicycles then become 'true' in the eyes of the believers. Escape from theological thinking is relatively easy when only a score or so of people are effectively involved, as it was for physics in the 1920s. But when thousands are involved as in modern science, or millions as in medieval theology, escape becomes a lengthier and more fraught procedure. The historical caution against this syndrome is usually attributed to William of Ockham, who said one should not invent hypotheses to 'save' appearances, which meant that one should not invent epicycles to bolster one's belief, a perceptive statement that is often misquoted and misunderstood.

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