

# CMB Anisotropy Due to Tangled magnetic fields in re-ionized models

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## Abstract

Primordial tangled cosmological Magnetic Fields source rotational velocity perturbations of the baryon fluid, even in the post-recombination universe. These vortical modes in turn leave a characteristic imprint on the temperature anisotropy of the Cosmic Microwave Background (CMB), if the CMB photons can be re-scattered after recombination. Observations from WMAP indicate that the Universe underwent a relatively early re-ionization ( $z_{ri} \sim 15$ ), which does indeed lead to a significant optical depth for re-scattering of CMB photons after the re-ionization epoch. We compute the resulting additional temperature anisotropies, induced by primordial magnetic fields in the post-recombination universe. We show that in models with early re-ionization, a nearly scale-invariant spectrum of tangled magnetic fields which redshift to a present value of  $B_0 \sim 3 \times 10^{-9}$  Gauss, produce vector mode perturbations which in turn induce additional temperature anisotropy of about 0.3 to 0.4  $\mu\text{K}$  over very small angular scales, with  $l$  upto  $\sim 10000$  or so.

98.62.En, 98.70.Vc, 98.80.Cq, 95.30.Qd

## I. INTRODUCTION

There are two possible processes which might explain the origin of large scale cosmic magnetic fields. Both have, however, potential difficulties. One possibility is that some high-energy process in the early universe (like inflation or a cosmological phase transition) generated primordial magnetic fields which manifest today as galactic or cluster fields [1]. The problem here is that this involves speculative physics and there is as yet no compelling mechanism to generate fields of the required strength [2–4]. Alternatively, seed magnetic fields can get amplified by a large scale dynamo, to produce fields as observed today, coherent over galactic or cluster scales [5,6]. There are, however, constraints from helicity conservation and/or suppression of lagrangian chaos, due to which the efficacy of this process is unclear (see for example [6] and references therein). The effects of weak primordial magnetic fields (whose strength today is of order  $10^{-9}$  Gauss), and which are tangled on galactic scales can affect galaxy formation [7–11] Hence, it is of considerable interest, to find different ways of limiting or detecting such primordial fields (see [2,3,12] for reviews).

In several earlier papers the consequences of tangled primordial cosmological magnetic fields on the observable signatures on the CMBR anisotropy and polarization have been investigated in a cosmological scenarios with no re-ionization [13–18]. A detailed numerical investigation of CMB signals due to tangled magnetic fields has also been undertaken in [19]. The First year results from WMAP satellite observations however indicate that the universe could have undergone an early stage of re-ionization [20,21], with a large optical depth  $\kappa \sim 0.17$  to the re-scattering of CMB photons. In this paper we follow up our earlier work by investigating the consequences of a tangled primordial magnetic field, for CMB anisotropies, in a scenario with early re-ionization. In particular we focus on the rotational perturbations, that are produced by primordial fields in the post-recombination universe, and the additional CMB anisotropy signals that they induce in a universe with early re-ionization. Note that compressive velocity modes could also induce CMB anisotropies; however they also have non magnetic sources and more importantly suffer larger cancellation effects due to the thickness of the last scattering surface around the re-ionization epoch. Hence, our focus is only on rotational modes here.

In section II the general formulation of the problem and the parameters of re-ionization relevant for our calculation are derived. Semi-analytic estimates of the additional CMB signals are made in section III. In Section IV we present numerical calculations and in Section V we discuss our results.

## II. GENERAL FORMULATION

The equations for the evolution of temperature anisotropy for scalar, vector and tensor modes have been derived by Hu and White [22] (hereafter referred to as HW97). We concentrate here on the additional contributions which arise in a re-ionized universe, due to the vector modes induced by inhomogeneous magnetic fields. From equation (74) and (56) of HW97, the angular power-spectrum of CMB anisotropy corresponding to vector modes is given by,

$$C_l = 4\pi \int dk \frac{k^2}{2\pi^2} \frac{l(l+1)}{2} \langle | \int_0^{\tau_0} d\tau g(\tau_0, \tau) V(k, \tau) \frac{j_l(k(\tau_0 - \tau))}{k(\tau_0 - \tau)} |^2 \rangle \quad (2.1)$$

Here,  $V(k, \tau)$  is the magnitude of the vorticity, generated by tangled primordial magnetic fields, in Fourier space. Also,  $k$  is the co-moving wave number,  $\tau$  is conformal time,  $\tau_0$  its present value, and  $j_l(z)$  is the spherical Bessel function of order  $l$ . We have ignored a small polarization correction to the source term and also a metric perturbation term which are in general sub-dominant (cf. [13,16]). The 'visibility function',  $g(\tau_0, \tau)$ , is given by,

$$g(\tau_0, \tau) = n_e(\tau)\sigma_T a(\tau) \exp \left[ - \int_{\tau}^{\tau_0} n_e(\tau')\sigma_T a(\tau') d\tau' \right], \quad (2.2)$$

where  $g(\tau_0, \tau)d\tau$  is the probability that a photon that reaches us at epoch  $\tau_0$  was last scattered between the epochs  $(\tau, \tau + d\tau)$ . We assume a flat universe throughout, with a total matter density parameter  $\Omega_m$  and a non-zero cosmological constant density parameter  $\Omega_\Lambda$  today.

The exact form of the visibility function is determined by the ionization history of the universe. For standard recombination  $g(\tau_0, \tau)$  has only one peak around the epoch of recombination. However re-ionization can modify this further by making  $g$  significant after the universe has been re-ionized, the exact modification depending on the complex ionization history of the baryons. The modifications of the visibility function will show up in the power spectrum of the anisotropy. In this paper we will not be focusing our attention on the physical mechanism responsible for re-ionization. We will, however, assume that whatever be the source, it results in a sharp transition to a re-ionized situation. We assume this transition to be a step function at a redshift  $z_{ri}$  corresponding to a conformal time  $\tau_{ri}$ . The value of  $\tau_{ri}$  is fixed in this simple model by the optical depth indicated by the WMAP results. The visibility function will then have two peaks, one at  $\tau_{rec}$  and the other at  $\tau_{ri}$ .

The free electron number density in the era after standard recombination is modelled as,

$$n_e = \frac{n_{b0}}{a^3} \Theta(\tau - \tau_{ri}) \quad (2.3)$$

where  $n_{b0} = \Omega_b 3H_0^2 / (8\pi G m_b)$  with  $\Omega_b$  being the density parameter of the Baryons. (We have neglected the small residual electron density after recombination). The optical depth is given by

$$\kappa = c\sigma_T \int_t^{t_0} n_e(t') dt' = c\sigma_T \int_{\tau}^{\tau_0} n_e(\tau') a(\tau') d\tau'. \quad (2.4)$$

Here the proper time  $t$  and conformal time  $\tau$  are related by using  $dt = a d\tau$  and  $(t_0, \tau_0)$  correspond to the present epoch. The integration over  $t$  can be converted into an integration over redshift by the substitution,  $dt = da / (Ha) = -(H(z)(1+z))^{-1} dz$ . Here  $H(z)$  is the Hubble expansion rate and is given by

$$H(z) = H_0 [\Omega_\Lambda + \Omega_m (1+z)^3]^{1/2}. \quad (2.5)$$

The optical depth up to re-ionization can now be expressed as,

$$\kappa_{ri} = c\sigma_T \frac{\Omega_b H_0}{8\pi G m_b} \frac{\sqrt{\Omega_\Lambda}}{\Omega_m} \int_{z=0}^{z=z_{ri}} dy \frac{1}{\sqrt{1+y}} \quad (2.6)$$

where  $y$  is defined as  $y = (1+z)^3 (\Omega_m / \Omega_\Lambda)$ . On integration, this gives,

$$\kappa_{ri} = \frac{cH_0\Omega_b}{4\pi Gm_b} \frac{\sqrt{\Omega_\Lambda}}{\Omega_m} \sigma_T \left[ \sqrt{1 + (1 + z_{ri})^3 \frac{\Omega_m}{\Omega_\Lambda}} - \sqrt{1 + \frac{\Omega_m}{\Omega_\Lambda}} \right] \quad (2.7)$$

Adopting,  $\kappa_{ri} = 0.17$ ,  $h = 0.71$ ,  $\Omega_b = 0.044$ ,  $\Omega_m = 0.27$  and  $\Omega_V = 0.73$ , we get,  $z_{ri} = 14.57$ . (This value is close to the one obtained from equation 24.79 given in Peebles [23]. We have neglected a small correction due to helium fraction. This, however, does not significantly affect the results.) The conformal time of re-ionization is given by,

$$\tau_{ri} = \frac{2cH_0^{-1}}{\sqrt{\Omega_m}} \left( \sqrt{a_{ri} + a_{eq}} - \sqrt{a_{eq}} \right) \quad (2.8)$$

where,  $a_{ri}^{-1} = 1 + z_{ri}$  and  $a_{eq}^{-1} = 1 + z_{eq}$  specifies the scale factor at matter-radiation equality. The current epoch is given by

$$\tau_0 = \frac{2cH_0^{-1}}{\sqrt{\Omega_m}} [1 - 0.0841 \ln(\Omega_m)] \left( \sqrt{1 + a_{eq}} - \sqrt{a_{eq}} \right) \quad (2.9)$$

With  $2cH_0^{-1} = 6000 h^{-1}\text{Mpc}$ ,  $z_{eq} = 3233$  and  $z_{ri} = 14.57$ , we get  $\tau_0 = 12595h^{-1}\text{Mpc}$ ,  $\tau_{ri} = 2741h^{-1}\text{Mpc}$  and  $\tau_0 - \tau_{ri} = 9854h^{-1}\text{Mpc}$ . Using the functional form of the number density of the electrons as given in equation (2.3) and the  $\tau_{ri}$  determined above, we can calculate the form of the visibility function. We will use this form for the visibility function in the numerical computation in section IV. However, to begin with, we approximate the visibility function  $g_2(\tau_0, \tau)$  after recombination, to be a truncated exponential, and estimate the CMB anisotropy in a semi-analytic manner.

Specifically, we adopt,

$$g_2(\tau_0, \tau) = \frac{N_2}{\alpha} e^{-\frac{\tau - \tau_{ri}}{\alpha}} \Theta(\tau - \tau_{ri}) \quad (2.10)$$

Here the Heavyside  $\theta(x)$  function, is zero for  $x < 0$  and 1 for  $x > 0$ . It takes account of the fact that before re-ionization,  $n_e$  is negligible. Further,  $N_2$  is a normalization constant and  $\alpha$  gives the spread of the exponential. By appropriately choosing  $\alpha$ , we can set the width of the reionized last scattering surface. Also note that  $g(\tau_0, \tau)$  has the interpretation of probability; so its integral over  $\tau$  from  $\tau = 0$  to  $\tau = \tau_0$  should be normalized to unity. This determines the normalization factor  $N_2$ . For a sufficiently early epoch of re-ionization, we generally have  $(\tau_0 - \tau_{ri})/\alpha \gg 1$ . In this case, the condition that the integral of  $g(\tau_0, \tau)$  over  $\tau$  should be unity implies  $N_2 + e^{-\kappa_{ri}} = 1$ , or  $N_2 = 1 - \exp(-\kappa_{ri})$ . So  $N_2$  measures the probability of at least one scattering between  $\tau_0$  and  $\tau_{ri}$ , due to the re-ionization. For small  $\kappa_{ri} \ll 1$ , we have  $N_2 \sim \kappa_{ri}$

The constant  $\alpha$  is determined by the (conformal) time, say  $\tau_m$ , after which  $g_2$  drops to  $1/e$  times its peak value (at  $\tau_{ri}$ ). Thus if

$$\frac{g_2(\tau_0, \tau_m)}{g_2(\tau_0, \tau_{ri})} = \frac{1}{e} \quad (2.11)$$

then,  $\alpha = \tau_m - \tau_{ri}$ . To determine  $\tau_m$ , we use the exact form of  $g_2$  and calculate the epoch when  $[g(\tau_0, \tau_{ri})/g(\tau_0, \tau_m)] = e$ . Using equation (2.3) and the expression for the visibility function in equation (2.2), we get,

$$\frac{g(\tau_0, \tau_{ri})}{g(\tau_0, \tau_m)} = e = \frac{n_e(\tau_{ri})a(\tau_{ri})}{n_e(\tau_m)a(\tau_m)} \exp \left[ - \int_{\tau_{ri}}^{\tau_m} n_e(\tau'') \sigma_T a(\tau'') d\tau'' \right] \quad (2.12)$$

In the interval between  $\tau_{ri}$  and  $\tau_m$ , the universe is in general matter dominated. Hence  $a(\tau) \propto \tau^2$ . Also for  $\tau > \tau_{ri}$ , we have,  $n_e \propto a^{-3}$ . Hence, we can simplify the above equation to

$$\frac{\tau_m^4}{\tau_{ri}^4} \exp \left[ -c\sigma_T \frac{3\Omega_{b0}H_0^2}{8\pi Gm_b} \int_{\tau_{ri}}^{\tau_m} \frac{d\tau}{a^2} \right] = e \quad (2.13)$$

Substituting for the scale factor as  $a = \tau^2/\tau_0^2$ , we get,

$$4 \ln \left( \frac{\tau_m}{\tau_{ri}} \right) - \frac{\Omega_{m0}H_0^2 \sigma_T c \tau_{ri} (1 + z_{ri})^2}{8\pi Gm_b} \left( 1 - \frac{\tau_{ri}^3}{\tau_m^3} \right) = 1 \quad (2.14)$$

This gives  $\tau_m = 3519h^{-1}\text{Mpc}$  for  $\tau_{ri} = 2741h^{-1}\text{Mpc}$  giving the value of  $\alpha$  as  $778h^{-1}\text{Mpc}$ .

### III. RESULTS OF SEMI-ANALYTIC APPROXIMATION

Consider first the limiting case when  $j_l$  is more sharply peaked than  $g_2$ . This implies the limit  $l \gg (\tau_0 - \tau_{ri})/\alpha$ . On using the values of  $\tau_0$ ,  $\tau_{ri}$  and  $\alpha$  determined above this limit translates to  $l \gg 12$ . We will also assume that the source term  $V$  varies slower than either the variation of  $g_2$  or that of  $j_l$ . These approximations will help in understanding the effect of the model with re-ionization in a semi-analytic manner. Results of numerical calculations which do not make these approximations are given in the section IV.

Since we wish to focus on the effects of re-ionization we take the visibility function to be  $g_2$  in the integral appearing in equation (2.1), which then becomes,

$$\int_0^{\tau_0} d\tau g_2(\tau_0, \tau) V(k, \tau) \frac{j_l(k(\tau_0 - \tau))}{k(\tau_0 - \tau)}. \quad (3.1)$$

The function,  $j_l$  peaks at  $l \sim k(\tau_0 - \tau)$  or  $\tau \sim \tau_0 - l/k$ . So in the above integral we evaluate  $V(k, \tau)$  and  $g_2(\tau_0, \tau)$  at  $\tau = \tau_0 - l/k$  and replace  $k(\tau_0 - \tau_{ri})$  by  $l$  and move them out of the integral. We also use the identity,

$$\int_0^\infty j_l(x) dx = \sqrt{\frac{\pi}{2l}}. \quad (3.2)$$

Substituting the final result in the expression for  $C_l$  in equation (2.1), we get,

$$C_l = \frac{N_2^2}{\alpha^2} \frac{l(l+1)}{2l^3} \int_0^\infty dk k^2 \langle |V(k, \tau_0 - l/k)|^2 \rangle e^{-2(\tau_0 - \tau_{ri} - l/k)/\alpha} \Theta(\tau_0 - \tau_{ri} - l/k) \quad (3.3)$$

$$= \frac{N_2^2}{2\alpha^2 l} \int_0^{(\tau_0 - \tau_{ri})} dx \frac{k^4}{l} \langle |V(k, \tau_0 - l/k)|^2 \rangle e^{-(2x/\alpha)} \Theta(x) \quad (3.4)$$

where, we have substituted  $x = (\tau_0 - \tau_{ri} - \frac{l}{k})$ . The exponential term peaks at  $x = 0$ . So we will evaluate the rest of the integrand at  $x \rightarrow 0$  or  $k \rightarrow l/(\tau_0 - \tau_{ri})$  and move it out of the integrand. The integration of the exponential factor then gives,

$$C_l = \frac{N_2^2 2\pi^2}{4\alpha l^2} \left( \frac{\tau_0 - \tau_{ri}}{l} \right) \left[ \frac{k^3 \langle |V(k, \tau_0 - l/k)|^2 \rangle}{2\pi^2} \right]_{k=l/(\tau_0 - \tau_{ri})} \quad (3.5)$$

$$\frac{l(l+1)}{2\pi} C_l = \frac{N_2^2 \pi}{4\alpha} \left( \frac{\tau_0 - \tau_{ri}}{l} \right) \Delta_V^2(k, \tau_{ri})|_{k=l/(\tau_0 - \tau_{ri})} \quad (3.6)$$

for  $l \gg 1$  and  $\tau_0 - \tau_{ri} \gg \alpha$ . Here we have defined the power spectrum associated with rotational velocity perturbations,

$$\Delta_V^2(k, \tau) = \frac{k^3 \langle |V(k, \tau)|^2 \rangle}{2\pi^2}. \quad (3.7)$$

We assume that the magnetic field which induces vortical perturbations is initially a Gaussian random field. On large enough scales, the induced velocity is generally so small that it does not lead to any appreciable distortion of the initial field [10,24,25]. So, the magnetic field simply redshifts away as  $\mathbf{B}(\mathbf{x}, t) = \mathbf{B}_0(\mathbf{x})/a^2$ . The Lorentz force associated with the tangled field is then  $\mathbf{F}_L = (\nabla \times \mathbf{B}_0) \times \mathbf{B}_0/(4\pi a^5)$ , which pushes the fluid and creates rotational velocity perturbations. These can be estimated as in [8] by using the Euler equation for the baryons and we give a detailed derivation in Appendix A.

Further, in order to compute the ensemble average  $\Delta_V^2$  and hence the  $C_l$ s, we need the magnetic spectrum  $M(k)$ . This is defined using  $\langle b_i(\mathbf{k})b_j(\mathbf{q}) \rangle = \delta_{\mathbf{k},\mathbf{q}} P_{ij}(\mathbf{k})M(k)$ , where  $\delta_{\mathbf{k},\mathbf{q}}$  is the Kronecker delta which is non-zero only for  $\mathbf{k} = \mathbf{q}$ . Here  $\mathbf{b}(\mathbf{k})$  is the Fourier transform of  $\mathbf{B}_0$ , the present day value of the tangled magnetic field. Also  $P_{ij}(\mathbf{k}) = (\delta_{ij} - k_i k_j/k^2)$  is the projection operator which ensures that magnetic field has zero divergence. This gives  $\langle \mathbf{B}_0^2 \rangle = 2 \int (dk/k) \Delta_b^2(k)$ , where  $\Delta_b^2(k) = k^3 M(k)/(2\pi^2)$  is the power per logarithmic interval in  $k$  space residing in magnetic tangles, and we replace the summation over  $k$  space by an integration. It is convenient to define a dimensionless spectrum,  $m(k) = \Delta_b^2(k)/(B_0^2/2)$ , where  $B_0$  is a fiducial constant magnetic field. The baryonic Alfvén velocity,  $V_A$ , for this fiducial field is,

$$V_A = \frac{B_0}{(4\pi\rho_{b0})^{1/2}} \approx 1.5 \times 10^{-5} B_{-9}, \quad (3.8)$$

where  $\rho_{b0}$  is the present day Baryon density, and  $B_{-9} \equiv (B_0/10^{-9}\text{Gauss})$ .

We will also consider as in [15], power-law magnetic spectra,  $M(k) = Ak^n$ . We will cut-off this spectra at the scale where the perturbations are no longer linear, say at  $k = k_N$ . We expect  $k_N$  to be of order galactic scales, or  $k_N \sim (fhMpc)^{-1}$  with  $f \sim 1$ , for the range of redshifts which make a non-zero contribution to the visibility function and for the field strengths that we consider [10,25]. We fix  $A$  by demanding that the field smoothed over a scale,  $k_N$ , (using a sharp  $k$ -space filter) is  $B_0$ , giving a dimensionless spectrum for  $n > -3$  of

$$m(k) = (n+3)(k/k_N)^{3+n}. \quad (3.9)$$

Assuming such a spectrum we have from Appendix A,

$$\frac{k^3 \langle |V(k, \tau)|^2 \rangle}{2\pi^2} = \left( \frac{2c}{H_0} \right)^2 \left( \frac{1+z}{\Omega_{m0}} \right) \left[ \frac{kV_A^2}{\sqrt{8}} I(k) \right]^2. \quad (3.10)$$

Here  $I(k)$  is a mode coupling integral which has been worked out in detail in [18,13]. In particular for the case  $n < -3/2$  which we consider below it is given by,

$$I^2(k) = \frac{8}{3}(n+3)\left(\frac{k}{k_N}\right)^{6+2n}. \quad (3.11)$$

Using equation (3.11) in equation (3.10) and substituting the resulting expression for velocity power-spectrum in equation (3.6) we get,

$$\frac{l(l+1)}{2\pi}C_l = \frac{\pi N_2^2}{4} \left(\frac{2c}{H_0}\right)^2 \frac{1+z_{ri}}{8\Omega_m} V_A^4 \frac{8}{3}(n+3)\frac{k_N}{\alpha} \left(\frac{k}{k_N}\right)_{k=\frac{l}{\tau_0-\tau_{ri}}}^{7+2n} \quad (3.12)$$

We define  $\Delta T = T_0[l(l+1)C_l/2\pi]^{1/2}$  as a measure of the temperature anisotropy, where  $T_0 = 2.73 K$  is the present CMB temperature. Taking the square root of the above expression and using the values of  $V_A$ ,  $k_N$ ,  $\Omega_m$  and  $z_{ri}$  mentioned earlier we get,

$$\Delta T = 0.52 N_2 B_{-9}^2 (n+3)^{1/2} f^{-(n+3)} \left(\frac{l}{9854}\right)^{3.5+n} \mu K \quad (3.13)$$

(Note that  $k_N(\tau_0 - \tau_{ri}) = f9854$ ). For a field of 3 nano-gauss ( $B_{-9} = 3$ ), a nearly scale invariant spectrum for the magnetic field ( $n = -2.9$ ) and for an optical depth of  $\sim 0.17$  or  $N_2 \sim .17$ , we get,

$$\Delta T = 0.25 \left(\frac{l}{9854}\right)^{0.6} \quad (3.14)$$

A similar analysis in the opposite limit ( $g_2$  more sharply peaked than  $j_l$  and hence  $l \ll (\tau_0 - \tau_{ri})/\alpha \sim 12$ ) gives

$$\Delta T = 5.3 \times N_2 B_{-9}^2 (n+3)^{1/2} \left(\frac{l}{9854}\right)^{4+n} \mu K \quad (3.15)$$

For a field of 3 nano-gauss ( $B_{-9} = 3$ ), a nearly scale invariant spectrum for the magnetic field ( $n = -2.9$ ) and for an optical depth of 0.17 or  $N_2 \sim .17$ , we now get,

$$\Delta T = 2.56 \left(\frac{l}{9854}\right)^{1.1} \mu K \quad (3.16)$$

For example at  $l \sim 10$  one predicts  $1.3 \times 10^{-3} \mu K$ , which is very small compared to other signals expected at these low  $l$  values.

#### IV. NUMERICAL RESULTS

While the calculations in the semi-analytic approximation in the last section provide us with rough estimates, to make concrete predictions we need to numerically compute the temperature anisotropy. In this section we give the results of the numerical evaluation. Specifically, we numerically evaluate the integral in equation (2.1). As discussed earlier,

the visibility function appearing in this integral has a dominant contribution at the epoch of standard recombination and another at the epoch of re-ionization. The contribution at the epoch of re-ionization we have denoted by  $g_2$  and we are interested in the additional contribution to  $C_l$  resulting from  $g_2$ . Unlike the semi-analytic case, (where we approximated  $g_2$  as an exponential decay for epochs later than the re-ionization epoch) we use the form given in equation (2.2) with the number density of the electrons as given in equation (2.3). The resulting expression for  $g_2$  is,

$$g_2(\tau_0, \tau) = \frac{3\Omega_{m0}H_0^2\sigma_T}{8\pi Gm_b} \frac{\tau_0^4}{\tau^4} \Theta(\tau - \tau_{ri}) \exp \left[ -\frac{3\Omega_{m0}H_0^2\sigma_T}{8\pi Gm_b} \int_{\tau_{ri}}^{\tau_0} d\tau' (\tau_0/\tau')^4 \right] \quad (4.1)$$

Further, as the universe is believed to have been matter dominated after  $\tau = \tau_{ri}$ , the redshift and the conformal time are related by,  $1+z = (\tau_0/\tau)^2$ . We can neglect the accelerated phase of the universe, as this phase is believed to have set in at a low redshift ( $z \sim 1$ ) by which time, the visibility function would have decayed sufficiently. Hence, this will not introduce any significant error in our computation. With these simplifying assumptions, we have computed  $\Delta T$  by evaluating the  $\tau$  and  $k$  integrals numerically in Eq (2.1). While evaluating this we have retained the analytical expression for  $I(k)$  given in equation (3.11). For  $B_0 \sim 3$  nano Gauss and a nearly scale invariant spectrum ( $n = -2.9$ ) we find  $\Delta T = 0.33\mu K$  for  $l = 10000$ . For higher  $n$  the fluctuations are larger. The results of the numerical calculation are shown in Figure 1, for the magnetic spectral index  $n = -2.9, -2.8$  and  $-2.7$ . We can compare this with the semi-analytic results from equation (3.14). For  $n = -2.9$ , our semi-analytic calculations give  $\Delta T = 0.25$  for  $l = 10000$ . So we see that although the semi-analytic calculation marginally underestimates the temperature anisotropy, we do get the correct order. This is the observed trend for all  $n$ . For  $n = -2.8$  and  $n = -2.7$ ,  $\Delta T$  turns out to be  $0.47 \mu K$  and  $0.58 \mu K$  respectively for  $l \sim 10000$ . Indeed both the amplitude of  $\Delta T$  and its  $l$  dependence, computed using the semi-analytic calculation, agrees reasonably well with the more exact numerical integration.

## V. DISCUSSION AND CONCLUSIONS

We have investigated the additional CMB temperature anisotropies that are generated by tangled cosmological magnetic fields in a Universe that underwent a relatively early re-ionization. We are motivated by WMAP results, which indicate that the universe could have been re-ionized as early as  $z \sim 15$ . We have focused for the present on rotational velocity perturbations, which can be sustained only by cosmological magnetic fields. These modes also suffer a milder damping due to the finite thickness of the re-ionized last scattering surface, than compressional modes. Our results supplement earlier work obtained in the context of a Universe that does not undergo re-ionization.

We find that a nearly scale-invariant spectrum of tangled magnetic fields (with  $n = -2.9$  to  $n = -2.7$ ), which redshift to a present day value of about 3 nano Gauss can produce anisotropies at the level of about  $0.3\mu K$  to  $0.5\mu K$  for  $l \sim 10000$ . Even larger signals would obtain if we were to consider models with larger  $n$ . We have simply stopped at this large  $l$  because we cannot use linear theory at present to calculate the expected signals for  $l$  larger than  $k_N(\tau_0 - \tau_{ri}) \sim f9854$ . It is also interesting to note that, at these large  $l$ , the above signals

are comparable to primary signals due to tangled fields, arising from the usual recombination epoch [26]. CMBR anisotropy experiments which probe such very small angular scales can thus be useful to measure or impose bounds on the magnitude as well as the spectral index of these magnetic fields. We have also considered here only the simplest first order effects due to re-ionization. It would be of interest also to estimate the effects of an inhomogeneous re-ionization on the CMB anisotropy from vector modes.

## APPENDIX A

We need to calculate the rotational velocity field of the baryons in the post-recombination era, in particular around the epoch of re-ionization. We shall follow the formalism developed by Wasserman [8]. Here we summarize briefly the essential features of the derivation based on Wasserman's paper.

The Fourier space, linearized Euler equation for the rotational component of the baryon velocity in the post recombination era is given by,

$$\frac{\partial v_i}{\partial t} + \frac{\dot{a}}{a}v_i = \frac{P_{ij}F_j}{4\pi\rho_b(t)a^5} \quad (\text{A1})$$

Here  $v_i(k, t)$  is the Fourier component of the rotational velocity perturbation,  $F_j$  is the Fourier component of the vector  $[(\nabla \times \mathbf{B}_0) \times \mathbf{B}_0]_j$  and  $P_{ij} = \delta_{ij} - k_i k_j / k^2$  as usual projects out its rotational component. Note that  $V(k, t)$  is the magnitude of the vector  $v_i(k, t)$ , that is  $|V|^2 = v_i v_i^*$ . We have also assumed here, (as mentioned in the text) that on large enough scales, larger than  $k_N^{-1} \sim \text{Mpc}$ , the velocities are so small that the magnetic field does not get significantly distorted, but simply redshifts away as  $\mathbf{B}(\mathbf{x}, t) = \mathbf{B}_0/a^2$ .

The above equation can be solved in a straightforward fashion to obtain the rotational component of the velocity field. Note that in the post-recombination era, we can neglect the radiation density compared to the matter density. Further, the dark energy component dominated over matter at late times, *i.e.*, at redshift less than unity, whereas the results from WMAP can be interpreted to mean that the Universe underwent a re-ionization phase at an earlier time. Thus from the epoch of recombination ( $t_{rec}$ ), till the epoch of re-ionization ( $t_{ri}$ ), pressureless matter was the dominant component of the Universe. Hence, for  $t_{rec} \leq t \leq t_{ri}$  we can take  $a(t) = a_{rec}(t/t_{rec})^{2/3} = (1+z)^{-1}$ . The solution for Eq. A1 is then given by

$$v_i = \frac{3t_{rec}(P_{ij}F_j)}{4\pi\rho_{b0}a_{rec}^2} \left[ \left(\frac{t_{rec}}{t}\right)^{1/3} - \left(\frac{t_{rec}}{t}\right)^{2/3} \right]. \quad (\text{A2})$$

Here  $\rho_{b0}$  is the present day Baryon density, and we have assumed that the rotational velocity was negligible at recombination. (Any small rotational velocity at recombination contributes to a faster decaying term in the above solution than the term we will retain below). Since  $a(t) \propto t^{2/3}$ ,  $H_{rec} = 2/(3t_{rec})$ . Also we have from Einstein equation  $H_{rec}^2 = 8\pi G\rho_{rec}/3 = H_0^2 \Omega_{m0} (1+z_{rec})^3$ . Thus

$$t_{rec} = \frac{2}{3H_0} \frac{(1+z_{rec})^{-3/2}}{\sqrt{\Omega_{m0}}} \quad (\text{A3})$$

Substituting this expression for  $t_{rec}$  in Eq. A2, and noting that  $a(t) = 1/(1+z)$ , we have,

$$v_i = \frac{2}{H_0} \frac{1}{\sqrt{\Omega_{m0}}} \frac{P_{ij} F_j}{4\pi \rho_{b0}} \times (1+z)^{1/2} \quad (\text{A4})$$

For a power spectrum of  $\mathbf{B}_0$  as given in the text, a tedious but straightforward computation gives the power spectrum of the rotational component of the Lorentz force. We have

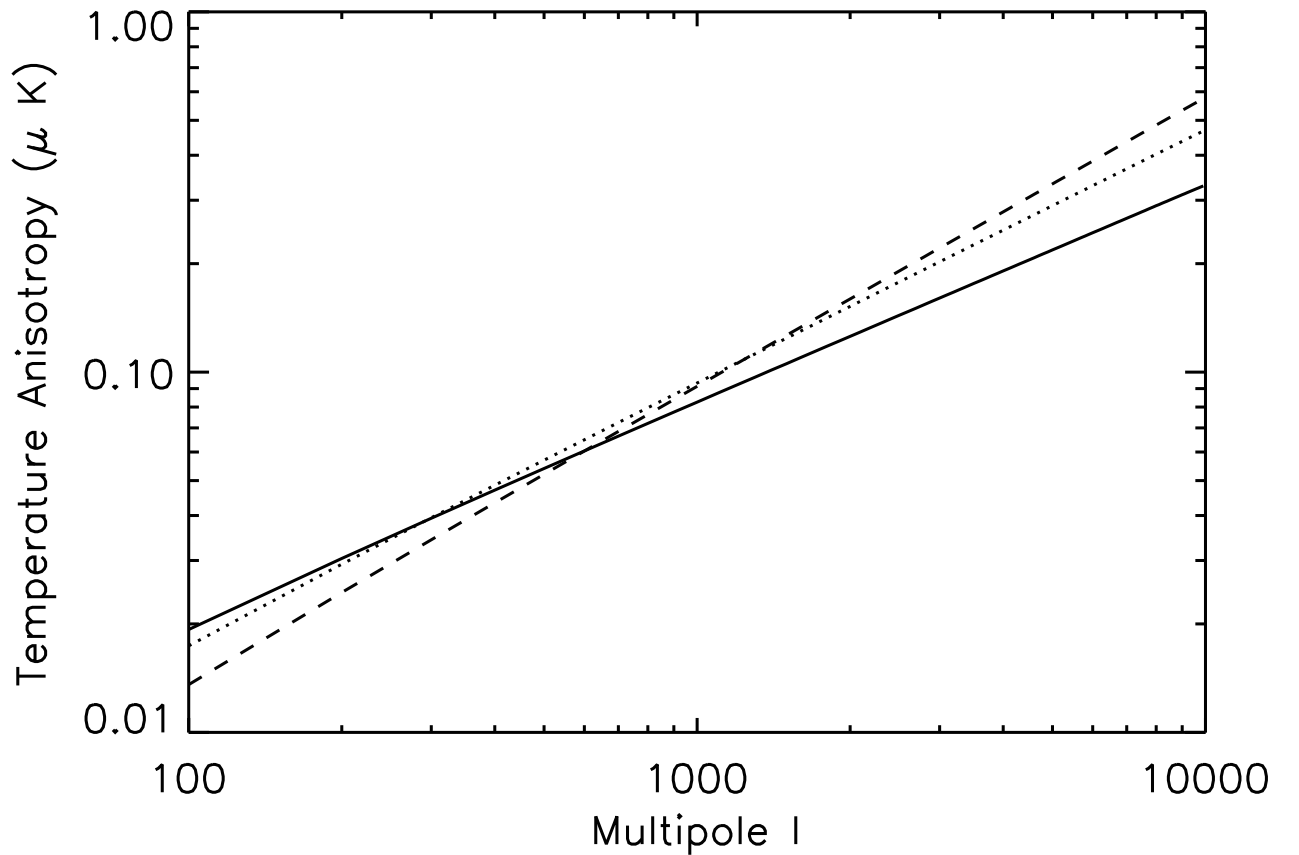
$$\frac{k^3}{2\pi^2} \langle | \frac{P_{ij} F_j P_{il} F_l^*}{4\pi \rho_{b0}} | \rangle = \frac{k^2 V_A^4}{8} I^2(k) \quad (\text{A5})$$

where  $I(k)$  is a mode coupling integral whose explicit form is given in [10,15]. The magnetic field enters through the baryon Alfvén velocity  $V_A$ . The power-spectrum of the  $V$  (or the magnitude of  $v_i$ ) required in the text is then given by

$$\frac{k^3}{2\pi^2} \langle | V(k, z) |^2 \rangle = \left( \frac{2}{H_0} \right)^2 \frac{1+z}{\Omega_{m0}} \left[ \frac{k V_A^2}{\sqrt{8}} I(k) \right]^2 \quad (\text{A6})$$

## FIGURES

FIG. 1. The figure shows the temperature anisotropy due to vector type modes induced by tangled cosmological magnetic fields that are of a strength of 3 nano Gauss today. The magnetic power spectrum is assumed to be a power-law characterized by an index  $n$ . The figure shows the results for  $n = -2.9$  (solid line),  $-2.8$  (dotted line) and  $-2.7$  (dashed line).



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