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PROBING THE QUANTUM MICROSTRUCTURE OF SPACE–TIME

T. PADMANABHAN*
IUCAA, Post Bag 4, Ganeshkhind, Pune 411007, India

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The question of how tightly one can constrain the microscopic theory of quantum gravity from the known features of low energy gravity is addressed. To begin with, from the very fact that our universe made a transition from a quantum regime to classical one, it is possible to conclude that infinite number of degrees of freedom had to be integrated out from the fundamental theory to obtain the low energy Einstein Lagrangian. Further constraints can be imposed from the fact that the quantum state describing a black hole has to possess certain universal form of density of states, in any microscopic description of space–time, which can be ascertained from general considerations. Since a black hole can be formed from the collapse of any physical system with a low energy ($E \ll E_p$) Hamiltonian H, it is possible to obtain the form the effective high energy ($E \gg E_p$) Hamiltonian from general consideration. These results provide the physical reasons for some of the mathematical features underlying string theories and other models for quantum gravity.

The fact that there will exist violent space–time fluctuations at small scales $[l \lesssim (G\hbar/c^3)^{1/2}]$ suggests that the macroscopic, continuum, description of space–time can only be approximated and valid when quantum fluctuations are averaged over large scales. The description of continuum space–time in terms of classical Einstein's equation is similar to the description of a solid by elastic constants or the description of a gaseous system by an equation of state. While the knowledge of microscopic quantum theory of atoms and molecules will allow us, in principle, to construct the description in terms of elastic constants, the reverse process is unlikely to be unique. Given the above paradigm, is it possible to describe the key features which must be present in any future, successful, theory of quantum gravity? We believe this can be done by taking clues from well designed thought experiments, thereby identifying some key generic features of the microscopic theory.

We start with the minimal assumption that the microscopic description is in terms of certain (as yet unknown) variables q_i and that the conventional space–time metric is obtained from these variables in some suitable limit. Such a process will necessarily involve coarse-graining over a class of microscopic descriptors of

*E-mail: paddy@iucaa.ernet.in

geometry. We will now outline an argument which suggests that there are *infinite* number of microscopic descriptors which are "integrated out" in proceeding from the fundamental description to space–time description.[1] The argument proceeds in three steps:

1. Among all systems dominated by gravity, the universe possesses a very peculiar feature. If the conventional cosmological models are reasonable, it then follows that *our universe proceeded from quantum mechanical behavior to classical behavior in the course of dynamical evolution defined by some intrinsic time variable.* It can be easily shown that a system with bounded Hamiltonian can never make such a transition if classicality is defined in terms of behavior of a suitable Wigner function. It follows that the quantum cosmological description of our universe, as a Hamiltonian system, should contain at least one unbounded degree of freedom. This feature, of course, is well known in the low energy limit of the theory described by Einstein's equations: in the superspace of all homogeneous space–times, for example, the overall scale factor of three-geometry (defined, say, in terms of the three-volume) has a "kinetic energy" term which is unbounded from below. It can also be shown that the unbounded mode will go classical first: For example, in the evolution of two degrees of freedom $C(t)$ and $q(t)$, described by a Lagrangian,

$$L = -\frac{1}{2}\dot{C}^2 - \frac{1}{2}\omega^2 C^2 + \frac{1}{2}\dot{q}^2 - \frac{1}{2}\nu^2 q^2 - V(C,q)\,, \qquad V(C,q) \geq 0 \tag{1}$$

it is straightforward to show that the Wigner function of the unbounded mode $C(t)$ will become progressively peaked around the classical trajectory in the phase space while the bounded mode $q(t)$ will remain quantum mechanical. This is precisely what is experienced in the evolution of the universe, in which the unbounded mode corresponds to the expansion factor. *The strange behavior of our universe, as a system which evolved from quantum mechanical behavior to classicality, is completely understandable in terms of the above argument.* What is more, it is also clear why the expansion factor is the single most dominant mode governing the behavior of the classical universe.

2. Let us next address the task of obtaining such an unbounded Hamiltonian for an effective theory when the original theory contained a larger set of dynamical variables. It can again be shown that, if one starts with a bounded Hamiltonian for a system with *finite* number of quantum fields and integrate out a *finite* subset of them, the resulting Hamiltonian for the low energy theory cannot be unbounded. This result can intuitively be understood as follows: Consider a theory described by a set of fields $[C, q_1, \ldots, q_N]$ with a bounded Euclidean action. If the modes $q_1, \ldots, q_N$ are integrated out in a path integral of the type

$$K[C] = e^{-A_{\text{eff}}(C)} \equiv \int \mathcal{D}q_1 \cdots \mathcal{D}q_N \cdots \exp[-A(C, q_1, \ldots, q_N)]\,, \tag{2}$$

then the resulting effective low energy theory for C, described by $A_{\text{eff}}(C)$, will be bounded from below. This result, of course, need not hold if an infinite number of degrees of freedom are integrated out, since, in such a case, the configuration space itself has changed.

3. Assuming that the original theory is describable in terms of a bounded Hamiltonian for some suitable variables, it follows that an infinite number of fields have to be involved in its description and an infinite subset of them have to be integrated out in order to give the standard low energy gravity. This feature is indeed present in one form or the other in the descriptions of quantum gravity based on strings[2] or Ashtekar variables.[3] Our argument suggests that this is indeed inevitable.

If the description in terms of continuum space–time is like theory of elasticity, and we do not know the fundamental descriptors of space–time, is there any way of bridging the gap between the two? It turns out that this is possible by using the properties of macroscopic space–time near the trapped surfaces. If the space–time has certain quantum mechanical micro-structure at Planck scales and the modes of an external field can probe scales comparable to Planck length in some region of space–time, then we cannot describe physics around that region using Einstein's equations. Normally, *virtual* modes of arbitrarily high energy of matter fields do interact strongly with the quantum micro-structure of space–time; but this is of no consequence unless such a virtual process can manifest as a *real* ones in some way. This is exactly what happens in space–times with an infinite redshift surface (event horizon), like that of a black hole. We have given detailed arguments elsewhere[4,5] to show that the event horizon of a Schwarzschild black hole acts as a magnifying glass, allowing us to probe Planck scale physics. Two specific features are described below.

Consider any physical system with a low energy Hamiltonian $H(\Gamma)$ which depends on some suitably defined phase space variables denoted symbolically by Γ. When the system has energy E, let the density of states be given by

$$g(E) = \int d\Gamma\, \delta_D[E - H(\Gamma)] \propto E^N\,, \tag{3}$$

where N is related to the degrees of freedom of the system. We now let this system collapse in a spherically symmetric manner to form a black hole with the same energy E. Once the black hole is formed, the density of states *must* have the characteristic form $g_{\rm BH}(E) = \exp(4\pi E^2/E_{\rm P}^2)$ for $E \gg E_P$. If E remains conserved during the collapse, then this can *only* happen if the formation of the event horizon necessitates a change in the description of the physical system. The actual, full description of the black hole will require knowledge of the microscopic description. But if we try to describe the situation in the familiar language of an effective Hamiltonian, then the effective Hamiltonian of the system must have a unique relationship to the original Hamiltonian in the form

$$H^2_{\rm true} = A^2 \ln\left(1 + \frac{H^2_{\rm low}}{A^2}\right)\,, \qquad A \propto E_P(N+1)^{1/2}\,. \tag{4}$$

Of course, the description at trans-Planckian energies cannot be in terms of the original variables in the rigorous theory. The above formula should be interpreted as giving the mapping between an effective field theory (described by $H_{\rm true}$) and a conventional low energy theory (described by $H_{\rm low}$) such that the black hole entropy will be reproduced correctly.

In fact, one can do better and construct a whole class of effective field theories[4] such that the one-particle excitations of these theories possess the same density of states as a Schwarzschild black hole. All such effective field theories are nonlocal in character and possess a universal two-point function at small scales. The nonlocality appears as a smearing of the fields over regions of the order of Planck length thereby confirming ones intuition about microscopic structures, trapped surfaces and black hole entropy.

The simplest Lagrangian describing such an effective field $\phi(t, \mathbf{x})$ will be

$$\begin{aligned} L &= \frac{1}{2}\int d^D\mathbf{x}\,\dot{\phi}^2 - \frac{1}{2}\int d^D\mathbf{x}\,d^D\mathbf{y}\,\phi(\mathbf{x})F(\mathbf{x}-\mathbf{y})\phi(\mathbf{y}) \\ &= \int \frac{d^D\mathbf{k}}{(2\pi)^D}\frac{1}{2}\left[|\dot{Q}_\mathbf{k}|^2 - \omega_\mathbf{k}^2|Q_\mathbf{k}|^2\right]. \end{aligned} \tag{5}$$

The Lagrangian is nonlocal in the space coordinates $\mathbf{x}$ which is taken to be D-dimensional; the corresponding Fourier space coordinates are labeled by $\mathbf{k}$. The quadratic nonlocality allows us to describe the system in terms of free harmonic oscillators with a dispersion relation $\omega(\mathbf{k})$ related to $F(\mathbf{r})$ by

$$\omega^2(\mathbf{k}) = \int d^D\mathbf{r}\,F(\mathbf{r})e^{i\mathbf{k}\cdot\mathbf{r}}. \tag{6}$$

The energy levels of the system are built out of elementary excitations with energy $\hbar\omega(\mathbf{k})$ and the density of states corresponding to the one-particle state of the system is:

$$g(E) = \int d^D k\,\delta_D[\omega(\mathbf{k}) - E] \tag{7}$$

which is just the Jacobian $|d^D\mathbf{k}/d\omega|$. It is straightforward to see that if the dispersion relation $\omega(\mathbf{k})$ has the form

$$\omega^2(\mathbf{k}) = \frac{E_P^2 D}{8\pi}\ln\left(1 + \frac{8\pi k^2}{DE_P^2}\right), \tag{8}$$

then the density of states has the required form for that of a black hole.

In this class of models, the black hole is treated as the one-particle state of a *nonlocal* field theory. The dispersion relation is arranged so as to give a normal massless scalar field at low energies. It is certainly of interest to explore the new features of this field theory. Since nonlocality is the key new feature let us begin by studying the form of $F(\mathbf{x})$ in real space which is given by using an integral representation for ln and interchanging the orders of integration:

$$F(\mathbf{x}) = \int \frac{d^D\mathbf{k}}{(2\pi)^D} A^2 \ln\left(1 + \frac{k^2}{A^2}\right) e^{i\mathbf{k}\cdot\mathbf{x}} = -\frac{2A^{2+D}}{(2\pi)^{D/2}} \left(\frac{1}{A^2x^2}\right)^{D/4} K_{D/2}(Ax)\,, \qquad A^2 = \frac{D}{8\pi}\frac{1}{L_P^2}\,, \tag{9}$$

where $K_\nu(z)$ is the MacDonald function. This function vanishes exponentially for large values of the argument, showing that the effective correlation length of the nonlocal field is of the order of $A^{-1} = (8\pi/D)^{1/2}L_P$. For small z, we have $K_\nu(z) \to z^{-\nu}$ and the correlation function goes as a power law $F(x) \propto x^{-D}$. The short distance behavior of the correlation function is universal and depends only on the asymptotic form of the dispersion relation. In fact, this is the only feature of $F(\mathbf{x})$ which is needed to reproduce the density of states leading to the correct theory of black hole thermodynamics. When $L_P \to 0$, the function $F(x)$ is proportional to the second derivative of Dirac delta function as can be seen from the fact that, as $L_P \to 0$, $\omega^2(k) \to k^2$. In this limit, we recover the standard local field theory.

If the physical description above Planck energy (or equivalently below Planck length) changes drastically, how can one modify the low energy description such that the singularities in space–times and the perturbative divergences in quantum field theory are removed? This question cannot be answered rigorously without knowing the microscopic structure of space–time. However — among theories which are *not* perturbatively renormalizable — two broad classes of theories can be distinguished in terms of a general criterion. In the first class of theories, the low energy ($E \ll E_P$) and high energy ($E \gg E_P$) behavior are not related by any manner and the high energy sector of the theory *does* affect the low energy behavior significantly. If nature is built along these lines, then we cannot predict much without knowing the full theory. On the other hand, one can think of another class of theories in which the high energy and low energy descriptions are related in a *specified* manner and are not completely independent. The simplest form of such a relation will be a "duality" in which the behavior at a scale E is related to a behavior at scale (E_P^2/E), or — equivalently — the behavior at length scales l and (L_P^2/l) are related. Implementing this duality in the path integral representation for a propagator, say, leads to a remarkable result[6]: *The effect of this duality is the same as assuming that the space–time possesses a "zero-point-length" and replacing the flat space–time interval* $(x-y)^2$ *by* $(x-y)^2 + L_P^2$. The converse is also true: if the structure of the theory is such that Planck length acts as a minimal length to the space–time, then the theory will possess a duality between length scales l and (L_P^2/l). String theories do show related — though not the same — features. If nature is built along these lines, then trans-Planckian physics is dual to the low energy theory and must possess a description in terms of some effective field theory.

The key conclusions which emerge from all these are the following: Given the unlikely event of experimental confirmation of quantum gravity, it is necessary to attempt a top–down approach (classical gravity → effective field theory → microscopic space–time descriptors) using, say, well-defined thought experiments. In this regard, space–times with trapped surfaces will be valuable. It is also important to worry, at a conceptual level, the effect of trans-Planckian physics on low energies. If this effect is not to be unreasonably strong, thereby killing predictability, it is necessary that the low energy theory is protected by some kind of duality mapping relating trans-Planckian energies to low energies. A program for quantum gravity, along these lines, holds promise.

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