A New Universal Local Feature in the Inflationary Perturbation Spectrum

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A model is developed in which the inflaton potential experiences a sudden small change in its second derivative (the effective mass of the inflaton). An exact treatment demonstrates that the resulting density perturbation has a quasi-flat power spectrum with a break in its slope (a step in n_s). The step in the spectral index is modulated by characteristic oscillations and results in large running of the spectral index localised over a few e-folds of scales. A field-theoretic model giving rise to such behaviour of the inflationary potential is based on a fast phase transition experienced by a second scalar field weakly coupled to the inflaton. Such a transition is similar to that which terminates inflation in the hybrid inflationary scenario. This scenario suggests that the observed running of the spectral index in the WMAP data may be caused by a fast second order phase transition which occurred during inflation.

1. INTRODUCTION

The present decade appears to have ushered in a golden age for precise cosmological observations. Observations of the CMB, most recently by the WMAP satellite, combined with measurements of the matter power spectrum from large scale structure, weak lensing surveys and Ly-α absorption, permit parameters of the 'standard cosmological model' to be determined to great accuracy. A consensus appears to be emerging that the late time behaviour of this model is 'close to' LCDM with an approximately scale invariant primordial spectrum for density perturbations such as those predicted by the simplest inflationary models. Indeed, the inflationary paradigm has been remarkably successful in providing explanations for some well known properties of the universe including its spatial flatness. It also provides a mechanism for seeding galaxies by generating an approximately flat (scale invariant) primordial perturbation spectrum. Both these predictions of the inflationary scenario have received considerable observational support from measurements of anisotropies in the cosmic microwave background (CMB) as detected by WMAP and other CMB experiments [1, 2, 3].

However, it is well known that, although primordial fluctuations spectra expected from inflation are likely to be approximately flat, or scale-invariant ($n_s(k) \equiv d\ln P_{\mathcal{R}}(k)/d\ln k \simeq 1$), exact scale invariance ($n_s = 1$) is achievable only for a very specific class of models [4], while a slightly red spectrum ($n_s \lesssim 1$) appears to be a generic prediction [5] of the simplest viable one-parameter family of inflationary models including, in particular, $R + R^2$ as well as new and chaotic inflation [6]. More sophisticated inflationary models belonging to the slow-roll class may also have n_s slightly exceeding unity, a notable example being hybrid inflation [7]. Still for all these models $|n_s(k) - 1| \ll 1$, and the running of the slope $\tilde{\alpha}(k) \equiv d\, n_s(k)/d\ln k$ is also expected to be small : $|\tilde{\alpha}(k)| \sim |n_s(k) - 1|^2 \ll 1$ (tilde is used here to avoid confusion with the Bogoliubov parameter below). Existing CMB and other observational data are just approaching the level of accuracy necessary to detect deviations from exact scale invariance and to distinguish between different inflationary models.

However, while on the one hand the recent WMAP data provide the first evidence for n_s being close to (but slightly less than) unity [1] (this result is at present inconclusive, see e.g. [8]), on the other, WMAP data [1] also suggest a rather large value of the running $|\tilde{\alpha}(k)| \sim |n_s(k) - 1|$, and the existence of local spikes like the 'Archeops feature' at $l \sim 40$ (first found in the Archeops data [3] and subsequently confirmed by WMAP [1]) which may indicate that inflation is altogether more complex than the simplest paradigms presented above. Therefore, though it is not yet clear if these features really exist in the primordial perturbation spectrum and are not foreground effects or statistical flukes, it is important to have a list of possible local 'features' such as bumps, wells, wiggles or spikes, superimposed on an approximately scale invariant smooth spectrum, which are expected in more complicated inflationary models. In particular, the large value of the running, if confirmed, should also be a local feature around the present Hubble scale since its persistence until the very end of inflation is incompatible with the requirement that the inflationary epoch be of sufficient duration (i.e. the number of e-folds $N \sim 50$) [9].

Such features in the primordial spectrum of fluctuations are likely to be measured to great accuracy with next generation CMB satellites such as Planck [10]. The precise nature of the primordial spectrum is of great importance to cosmology not only because it would lead to an in depth understanding of inflation but also because of its bearing on the values of the remaining cosmological parameters. (Cosmological parameters are usually estimated by means of a procedure such as a *Markov Chain Monte Carlo* (MCMC) scheme which assigns probabilities to cosmological parameters in a multi-dimensional parameter space. Better knowledge of one set of parameters values can therefore significantly influence the probabilities for the remaining.)

A generic way to obtain local deviations from the approximately flat spectrum in the inflationary scenario is to add additional scalar fields to the inflaton, either explicitly – which leads to multiple inflation [11, 12, 13] or implicitly, through a rapid change in the effective potential of the inflaton [14]. In both cases, at least one of these additional scalar fields should *not* be in the slow-rolling regime, otherwise the spectrum remains approximately flat [15]. In this paper we further investigate the second case with the aim of finding new characteristic features being grafted on the primordial inflationary spectrum which are less peculiar than those investigated previously (in view of the observational fact that there is not too much room for such features).

So, let some scalar field which is weakly coupled to the inflaton experience a *fast* phase transition during inflation, by fast is meant that its characteristic time is much less than the Hubble time $H^{-1}(t)$. This phase transition may be induced by the coupling of the scalar either to the inflaton directly, or to slowly changing space-time curvature. This requirement usually implies that the second field be much heavier than the inflaton as well as H. Then, assuming that this transition occurs adiabatically and the second field is always in the state of local thermodynamical equilibrium for fixed external values of the inflaton and space-time curvature, it can be integrated out leaving only some correction to the inflaton effective potential $V(\varphi)$ that may be non-analytic (see e.g. [16] for detailed derivation).

Following the classification in [17], let us consider possible local non-analytic features of $V(\varphi)$ arising at the point $\varphi = \varphi_0$ when the fast phase transition discussed above occurs during inflation. These features can be discontinuities (jumps) either in $V(\varphi)$ itself or in one of its derivatives. We denote by [] a jump in the relevant quantity, so that $[A] \equiv A(\varphi_0 + 0) - A(\varphi_0 - 0)$. The first three most interesting cases placed in the order of their decreasing peculiarity are:

1. $[V] \neq 0$ – a step in the effective potential $V(\varphi)$. This feature is the most peculiar one. Note that it may not arise from a fast quasi-equilibrium phase transition since that would contradict energy (more precisely, free energy) conservation. The only way to obtain such feature is to assume that duration of the phase transition is comparable to H^{-1}, so it is not too fast actually.

   Not unexpectedly, this results in a non-universal form of the corresponding local feature in the adiabatic perturbation spectrum which depends on detailed dynamics of the phase transition. In the most generic case, a bump modulated by strong oscillations appears in the power spectrum [14, 18]. Significant bumps in the observable part of the spectrum are excluded though one may introduce them at very large k, close to the end of inflation, to obtain a significant number of primordial black holes (PBH), see e.g. [19, 20]). If the size of the step is very small, the bump disappears and only a burst of oscillations remains. The latter case was studied in [21]. However, the most recent analysis shows that there is no definite evidence for such a feature in the observable part of the spectrum [22].

2. $[V] = 0,\ [V'] \neq 0$. A kink in the potential $V(\varphi)$ leads to a step in its slope $V'(\varphi)$. The resulting feature in the power spectrum has a *universal* form which does not depend on the details of the fast phase transition, namely a step with superimposed oscillations which are not as pronounced as in the previous case [14]. As pointed in [17], to produce such a discontinuity in $V(\varphi)$, the phase transition should be of first order.

   Though initially such a feature was used to describe an apparent excess in the rich cluster power spectrum at $k \sim 0.05\, h^{-1}$ Mpc^{-1} [23] (h is the present Hubble constant in terms of 100 km s^{-1}Mpc^{-}1 and the present value of the scale factor is taken as unity), it seems now that the only place where such feature with a significant value of the step in the primordial spectrum may still exist is in the vicinity of the present Hubble scale [24]. Also, as in the previous case, such a feature may be introduced at scales close to the end of inflation to produce PBH [25].

3. $[V] = [V'] = 0, [V''] \neq 0$ and $|[V'']| \ll H^2$. A sudden small change in the slope of the potential (a kink) leads to a step in its second derivative $V''(\varphi)$. This, mildest of all discontinuities, can be caused by a fast second order phase transition during inflation [17]. The last inequality guarantees that slow-roll inflation continues during and after the phase transition, in contrast to the case of the hybrid inflation, or the case $|[V'']| \sim H^2$ considered in [13] where a second phase transition with the opposite sign of V'' had to be introduced to restore inflation after a short break (that required much fine tuning, of course). As a result, in contrast to previous cases, corrections to the primordial power spectrum $P(k)$ arise in the next to leading order only (at the same order as the Stewart-Lyth correction [26] in case of a smooth inflaton potential). However, due to the feature in $V(\phi)$, corrections to $(n_s(k) - 1)$ appear to be of the same order as the standard leading ones while corrections to the running of $n_s(k)$ dominate the smooth part of $\tilde{\alpha}(k)$ over a few e-fold interval of scales around the feature.

As in the previous case, they have a *universal* form, too[1] .

Most recent observational constraints on the inflaton potential $V(\phi)$ and the Hubble function $H(\phi)$ obtained using only the assumption that these functions may be Taylor–expanded up to the third order in the range of ϕ corresponding to the observable cosmological window of scales $(1-10^4$ Mpc) show that the first two slow-roll parameters ϵ and δ (defined in Sec. 2 below) are really small but the validity of the slow-roll expansion beyond them is not established, see [29] and references to previous papers therein. The last, third type of peculiarity is just the one satisfying these conditions. That is why in this paper we study it in detail and find the universal form of the corresponding feature. We shall show that a small step in V'' leads to a small step in the primordial spectral index n_s accompanied by oscillations with a decreasing amplitude.

The plan of the rest of the paper is as follows. In Sec. 2 small corrections to the background behaviour due to a jump in $V''(\phi)$ are calculated and their contribution to the Sasaki–Mukhanov equation for scalar perturbations is found. In Sec. 3 an exact solution for the resulting feature in $P_{\mathcal{R}}(k)$ is obtained. A microscopic model that can produce such a feature in the effective potential is considered in Sec.4, and the required values of its parameters are found. Sec. 5 contains conclusions and discussion.

2. BACKGROUND COSMOLOGY NEAR A FEATURE IN THE POTENTIAL

As discussed in the previous section, we shall examine a model in which the potential passes through a step-like discontinuity in its second derivative at time t_0 when $\varphi(t_0)=\varphi_0$. In practice the discontinuity will be smoothed in a small neighborhood of φ_0 which we denote by ε. In order to study the influence of this feature in $V(\varphi)$ on quantities such as $H(t)$ and $\varphi(t)$, we Taylor expand these quantities around t_0,

$$H(t) = H_0 + t\,\dot{H}_0 + \frac{t^2}{2!}\,\ddot{H}_0 + \frac{t^3}{3!}\,\dddot{H}_\pm + \cdots \tag{2.1}$$

similarly,

$$\varphi(t) = \varphi_0 + t\,\dot{\varphi}_0 + \frac{t^2}{2!}\,\ddot{\varphi}_0 + \frac{t^3}{3!}\,\dddot{\varphi}_\pm + \cdots \tag{2.2}$$

$$V'(\varphi) = V'(\varphi_0) + t\,\dot{\varphi}_0\,V''_\pm + \cdots \tag{2.3}$$

for simplicity we have shifted the origin of the time scale to $t_0=0$. The suffix $\pm$ denotes the value of a quantity at $t=t_0\pm\varepsilon\equiv\pm\varepsilon$.

The equation of motion of the inflaton is

$$\begin{aligned} \ddot{\varphi} &+ 3H\dot{\varphi} + V'(\varphi) = 0\ , \\ H^2 &= \frac{8\pi G}{3}\left(\frac{\dot{\varphi}^2}{2} + V(\varphi)\right)\ . \end{aligned} \tag{2.4}$$

The first of these equations leads to

$$\dddot{\varphi}_\pm = -3H_0\,\ddot{\varphi}_0 - 3\dot{H}_0\,\dot{\varphi}_0 - V''_\pm\,\dot{\varphi}_0\ . \tag{2.5}$$

Next consider the slow roll parameters ϵ, δ, ζ^2 which are usually defined as

$$\epsilon = 4\pi G\left(\frac{\dot{\varphi}}{H}\right)^2\ ,\quad \delta = \frac{\ddot{\varphi}}{\dot{\varphi}\,H}\ ,\quad \zeta^2 = \frac{1}{H^2}\left[\frac{\dddot{\varphi}}{\dot{\varphi}} - \frac{\ddot{\varphi}^2}{\dot{\varphi}^2}\right] \tag{2.6}$$

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[1] The second-order phase transition considered in [[27]] occurs during a time period $\delta t\sim H^{-1}$ that results in the temporal breaking of slow-roll during the transition, in a more peculiar behaviour of the effective mass in the equation (3.1) for ξ_k and in a step-like or even bump-like behaviour of the perturbation spectrum similar to those occurred in the cases 1 and 2.

equivalently

$$
\begin{aligned}
3-\epsilon &= 8\pi G\frac{V}{H^2} \ , \\
\delta+3 &= -\frac{V'}{H\,\dot\varphi} \ , \\
9-\zeta^2 &= 8\pi G\frac{3V}{H^2}+\frac{3V'}{H\,\dot\varphi}+\frac{V'^2}{H^2\,\dot\varphi^2}+\frac{V''}{H^2} \ ,
\end{aligned}
\tag{2.7}
$$

where the last equation can be rewritten as

$$
\zeta^2 = 3\epsilon - 3\delta - \delta^2 - \frac{V''}{H^2} \ . \tag{2.8}
$$

The slow roll condition $\delta \ll 1$ leads to

$$
\dot\varphi \simeq -\frac{V'}{3H} \ . \tag{2.9}
$$

However the presence of a feature in the inflaton potential will result in small corrections to this equation close to $\varphi \simeq \varphi_0$. Denoting by $\delta\dot\varphi$ the correction to (2.9) we find

$$
\begin{aligned}
\delta\dot\varphi &= -\dot\varphi_0\left\{\frac{\delta_0}{3}+tH_0\left[\epsilon_0-\delta_0+\frac{\epsilon_0\,\delta_0}{3}-\frac{V''_\pm}{3H_0^2}\right]\right.\\
&\left.-\frac{t^2H_0^2}{2}\left[\left(3\epsilon_0-3\delta_0-2\epsilon_0^2-2\epsilon_0\,\delta_0-\frac{2}{3}\,\epsilon\,\delta_0\,(\epsilon_0+\delta_0)\right)-(3-2\epsilon_0-\delta_0)\,\frac{V''_\pm}{3H_0^2}+\sqrt{\frac{2\epsilon_0}{8\pi G}}\,\frac{V'''_\pm}{3H_0^2}\right]\right\}
\end{aligned}
\tag{2.10}
$$

where ϵ_0, δ_0 are the slow roll parameters evaluated at t_0. The correction to (2.9) are therefore quite small.

Next we estimate corrections to the slow-roll parameters, which are found to be

$$
\begin{aligned}
\epsilon(t) &= \epsilon_0 + tH_0\left[2\epsilon_0^2+2\epsilon_0\delta_0\right]\\
&+t^2H_0^2\,\epsilon_0\left[3\epsilon_0-3\delta_0-3\epsilon_0^2+6\epsilon_0\delta_0+\delta_0^2-\frac{V''_\pm}{H_0^2}\right] \ ,
\end{aligned}
\tag{2.11}
$$

$$
\begin{aligned}
\delta(t) &= \delta_0 + tH_0\left[3\epsilon_0-3\delta_0+\epsilon_0\,\delta_0-\delta_0^2-\frac{V''_\pm}{H_0^2}\right]\\
&+\frac{t^2H_0^2}{2}\left[9\epsilon_0+9\delta_0+6\epsilon_0^2+3\epsilon_0\,\delta_0+9\delta_0^2+2\epsilon_0^2\,\delta_0+2\delta_0^3\right.\\
&\left.\qquad+(3-2\epsilon_0+2\delta_0)\,\frac{V''_\pm}{H_0^2}-\sqrt{\frac{2\epsilon_0}{8\pi G}}\,\frac{V'''_\pm}{H_0^2}\right] \ ,
\end{aligned}
\tag{2.12}
$$

$$
\begin{aligned}
\zeta^2(t) &= 3\epsilon_0-3\delta_0-\delta_0^2-\frac{V''_\pm}{H_0^2}\\
&+tH_0\left[-9\epsilon_0+9\delta_0+6\epsilon_0^2+9\delta_0^2-3\epsilon_0\,\delta_0-2\epsilon_0\,\delta_0^2+2\delta_0^3+(3-2\epsilon_0+2\delta_0)\frac{V''_\pm}{H_0^2}-\sqrt{\frac{2\epsilon_0}{8\pi G}}\,\frac{V'''_\pm}{H_0^2}\right]\\
&+\frac{t^2H_0^2}{2}\left[27\epsilon_0-27\delta_0-18\epsilon_0^2-63\delta_0ait^2+45\epsilon_0\,\delta_0+18\epsilon_0^3+6\epsilon_0^2\,\delta_0+36\epsilon_0\,\delta_0^2-6\epsilon_0^2\,\delta_0^2-36\delta_0^3\right.\\
&\qquad+4\epsilon_0\,\delta_0^3-6\delta_0^4-\left(9-12\epsilon_0+24\delta_0+6\epsilon_0^2+8\delta_0^2-4\epsilon_0\,\delta_0^2+\frac{V'''_\pm}{H_0^2}\right)\frac{V'''_\pm}{H_0^2}\\
&\left.\qquad+(3-4\epsilon_0+\delta_0)\sqrt{\frac{2\epsilon_0}{8\pi G}}\,\frac{V'''_\pm}{H_0^2}-\frac{2\epsilon_0}{8\pi G}\,\frac{V''''_\pm}{H_0^2}\right] \ .
\end{aligned}
\tag{2.13}
$$

It is well known that the perturbations in the inflaton field and perturbations in the space-time metric can be reduced to a single equation either for the gravitational potential Φ [28] or the quantity ξ [30]. We shall use the latter and remind the reader that $\xi = \delta\varphi - \frac{\dot\varphi}{6H}(\lambda+\mu)$, where λ and μ describe scalar perturbations of the metric in the synchronous reference frame [31]. The evolution of the fourier component ξ_k during inflation is described by the equation

$$\ddot{\xi}_k + 3H\,\dot{\xi}_k + \left(\frac{k^2}{a^2} + m_{eff}^2\right)\xi_k = 0 \tag{2.14}$$

where the effective mass m_{eff}^2 is

$$m_{eff}^2 = \frac{d^2V}{d\varphi^2} + 8\,\pi\,G\,\frac{\dot\varphi}{H}\,\frac{dV}{d\varphi} + H\,\frac{d}{dt}\left(\frac{\dot H}{H^2}\right) \tag{2.15}$$

Using the results obtained earlier in this section and omitting lengthy intermediate steps, we find the following expression for the effective mass

$$\begin{aligned}\frac{m_{eff}^2(t)}{H_0^2} \;=\;& \frac{V''_\pm}{H_0^2} + tH_0\sqrt{\frac{2\epsilon_0}{8\pi G}}\,\frac{V'''_\pm}{H_0^2}\\ &- 2\epsilon_0\,(3+\epsilon_0+2\,\delta_0)\\ &- 4\,\epsilon_0\,tH_0\left[\left(3\,\epsilon_0+\epsilon_0^2+3\,\epsilon_0\,\delta_0+\delta_0^2\right)+\frac{V''_\pm}{H_0^2}\right]\end{aligned} \tag{2.16}$$

The leading term in the right hand side of the above equation for $t\to 0$ ($\phi\to\phi_0$) which is of the first order in ϵ,δ is $V''_\pm/H_0^2 - 6\epsilon_0$. The perturbation equation, Eq. (2.14), with $m_-^2 = V''_- - 6\epsilon_0$ when $t<0$ and $m_+^2 = V''_+ - 6\epsilon_0$ when $t>0$ (so that $[m^2]=[V'']$ since ϵ_0 is continuous at $t=0$), therefore provides an excellent approximation to the dynamics. This is the main result of this section.

3. PERTURBATION SPECTRUM AND SPECTRAL INDEX

As demonstrated in the previous section, the jump in the effective mass is equal to the jump in the second derivative of the inflaton potential: $[\Delta m_{\rm eff}^2] = [\Delta V'']$. Next consider the motion of the inflaton as it rolls down its potential. If the feature is crossed by the inflaton at t_0, then at $t \ll t_0$ as well as $t \gg t_0$ the slow roll condition $|V''| \ll 24\pi G V$ remains valid, which permits us to solve (2.14) as if the effective mass were constant. In terms of the conformal time coordinate $\eta = \int dt/a(t)$ equation (2.14) acquires the form

$$\xi_k'' + 2\frac{a'}{a}\xi_k' + \left(k^2 + m_{\rm eff}^2 a^2\right)\xi_k = 0\ , \tag{3.1}$$

where the derivatives are with respect to η. The transformation $\xi_k = \chi_k/a$ results in an oscillator-type equation in which the frequency is time dependent

$$\chi'' + \left(k^2 + m_{\rm eff}^2 a^2 - \frac{a''}{a}\right)\chi = 0\ , \tag{3.2}$$

where we have dropped the suffix k in χ_k for simplicity. In passing note that equation (3.2) is equivalent to

$$\chi'' + \left(k^2 - \frac{z''}{z}\right)\chi = 0\ , \quad \text{where} \quad z = \frac{a\dot\phi}{H}\ , \tag{3.3}$$

which implies that on large scales ($k^2 \ll z''/z$), $\chi/z \to$ constant. In the following discussion we assume that the discontinuity in the second derivative of the potential is reached by the field $\varphi(t)$ at the time $t=t_0$ ($\eta=\eta_0$). The normalized solution to (3.2) corresponding to the adiabatic vacuum at early times ($t \ll t_0$, equivalently $\eta \ll \eta_0$) is

$$\chi_{\rm in}(\eta) = \frac{\sqrt{\pi\eta}}{2}H_{\mu_1}^{(2)}(k\eta)\ , \tag{3.4}$$

where $H_\mu^{(2)}(k\eta)$ is the Hankel function and $\mu_1 = \frac{3}{2} - \frac{V''_-}{3H_0^2} + 3\epsilon_0$, where $V'' \equiv \frac{d^2V}{d\varphi^2}$. (We assume that the expansion of the universe is quasi-exponential.) The behaviour of perturbations *after* the feature is crossed ($t \gg t_0$, $\eta \gg \eta_0$) will be described by a superposition of positive and negative frequency solutions of (3.2), namely

$$\chi_{\rm out}(\eta) = \frac{\sqrt{\pi\eta}}{2}\left(\alpha H_{\mu_2}^{(2)}(k\eta) + \beta H_{\mu_2}^{(1)}(k\eta)\right)\ , \tag{3.5}$$

$\mu_2 = \frac{3}{2} - \frac{V''_+}{3H_0^2} + 3\epsilon_0$ and α, β are the Bogoliubov coefficients. Note the following relationship between μ and the scalar spectral index n

$$\mu_{1,2} = 2 - \frac{n_{1,2}}{2} \ , \tag{3.6}$$

where $n_1(n_2)$ is the spectral index in the 'in' ('out') region.

In order to determine α and β we match $\chi_{\rm in} = \chi_{\rm out}$ and ${\chi'}_{\rm in} = {\chi'}_{\rm out}$ at $\eta = \eta_0$ to obtain

$$\alpha - \beta \;=\; -\frac{i\pi\Delta}{2} H^{(2)}_{\mu_1}(k\eta_0) J_{\mu_2}(k\eta_0) - \frac{i\pi k\eta_0}{2}\left[H^{(2)}_{\mu_1+1}(k\eta_0) J_{\mu_2}(k\eta_0) - H^{(2)}_{\mu_1}(k\eta_0) J_{\mu_2+1}(k\eta_0)\right] \ , \tag{3.7}$$

$$\alpha + \beta \;=\; \frac{\pi\Delta}{2} H^{(2)}_{\mu_1}(k\eta_0) Y_{\mu_2}(k\eta_0) + \frac{\pi k\eta_0}{2}\left[H^{(2)}_{\mu_1+1}(k\eta_0) Y_{\mu_2}(k\eta_0) - H^{(2)}_{\mu_1}(k\eta_0) Y_{\mu_2+1}(k\eta_0)\right] \ , \tag{3.8}$$

$$|\alpha|^2 - |\beta|^2 = 1 \ , \tag{3.9}$$

where $\Delta = \mu_2 - \mu_1$. The quantity $|\beta|^2$ corresponds to the number of scalar particle pairs carrying momenta $\vec{k}$, $-\vec{k}$ created due to the rapid variation in V'' as the inflaton φ crosses the feature at $\eta = \eta_0$. However, the quantity of interest is related to the late time behaviour of $\xi_k(t \to \infty)$, namely $\xi_k(\eta \to 0) \equiv \frac{\chi_{\rm out}(\eta \to 0)}{a} \propto (\alpha - \beta)$, which contributes to the growing mode of scalar adiabatic perturbations. The corresponding power spectrum for the curvature perturbations is simply $P_{\mathcal{R}}(k) = \left(\frac{H}{\dot\phi}\right)^2 \left|\xi_k(\eta_{\to 0})\right|^2$. It is important to note that

$$P_{\mathcal{R}}(k) \propto \mathcal{P}_{\mathcal{R}0}(k) \times |\alpha - \beta|^2 \tag{3.10}$$

where $P_{\mathcal{R}}(k) \propto k^{n_s - 1}$ and $\mathcal{P}_{\mathcal{R}0}(k)$ is the power specrum of the background model on which the transfer function $|\alpha - \beta|^2$, describing the feature, has been overlayed. In our case $\mathcal{P}_{\mathcal{R}0}(k) \propto k^{n_2 - 1}$ where n_2 is the spectral index in the 'out' region; see (3.6). Now, substituting $H^{(2)}_\mu(z) \simeq -H^{(1)}_\mu(z) \simeq \frac{i}{\pi}\Gamma(\mu)\left(\frac{z}{2}\right)^{-\mu}$ as $z \to 0$, into (3.5) we get,

$$\mathcal{P}_{\mathcal{R}0}(k) = \frac{2^{2\mu_2 - 3}}{\pi^3}\Gamma^2(\mu_2)(1-\epsilon)^{2\mu_2 - 1}\left(\frac{H^2}{|\dot\phi|}\right)^2\Bigg|_{aH=k} \ , \tag{3.11}$$

when $\eta \to 0$. It is clear that the above expression is in agreement with Eq. (60) of [26] (Stewart-Lyth correction). The transfer function $|\alpha - \beta|^2$ differs from unity by terms of order $\mu_2 - \mu_1$ only. Indeed, from (3.9) one readily finds, for the transfer function

$$\begin{aligned}\frac{4}{\pi^2}|\alpha - \beta|^2 \;&=\; \Delta^2 J^2_{\mu_2}\left(Y^2_{\mu_1} + J^2_{\mu_1}\right) \\ &+\; (k\eta_0)^2\left\{J^2_{\mu_2}\left(Y^2_{\mu_1+1} + J^2_{\mu_1+1}\right) + J^2_{\mu_2+1}\left(Y^2_{\mu_1} + J^2_{\mu_1}\right) - 2J_{\mu_2}J_{\mu_2+1}\left(Y_{\mu_1}Y_{\mu_1+1} + J_{\mu_1}J_{\mu_1+1}\right)\right\} \\ &+\; 2\Delta\,(k\eta_0)J_{\mu_2}\left\{J_{\mu_2}\left(Y_{\mu_1}Y_{\mu_1+1} + J_{\mu_1}J_{\mu_1+1}\right) - J_{\mu_2+1}\left(Y^2_{\mu_1} + J^2_{\mu_1}\right)\right\}\end{aligned} \tag{3.12}$$

where the Bessel functions are evaluated at $x = k/k_0$, k_0 is the mode just leaving the Hubble radius at the time of the transition in V''. The functional dependence of $|\alpha - \beta|^2$ on x is shown in figure 1 for model parameters $\mu_1 = 1.49, \mu_2 = 1.52$. The expression for $|\alpha - \beta|^2$ in (3.12) has the following useful asymptotic forms:

1. For $x = k/k_0 \gg 1$

$$|\alpha - \beta|^2 \simeq 1 + \frac{3\,(\mu_2 - \mu_1)}{2x^2}\left[\cos(2x) - \frac{\sin(2x)}{x}\right] \ , \tag{3.13}$$

2. For $0.01 \lesssim x \lesssim 1$

$$\alpha - \beta \simeq 1 + (\mu_2 - \mu_1)\left[\log\left(\frac{x}{2}\right) + \frac{1}{3} - \psi(5/2)\right] \ , \tag{3.14}$$

here $\psi(z)$ is the digamma function, $\psi(z) = \frac{d}{dz}\log\Gamma(z)$, $\psi(5/2) = \frac{8}{3} - 2\log 2 - \gamma$, where $\gamma \simeq 0.5772$ is Euler's constant. And,

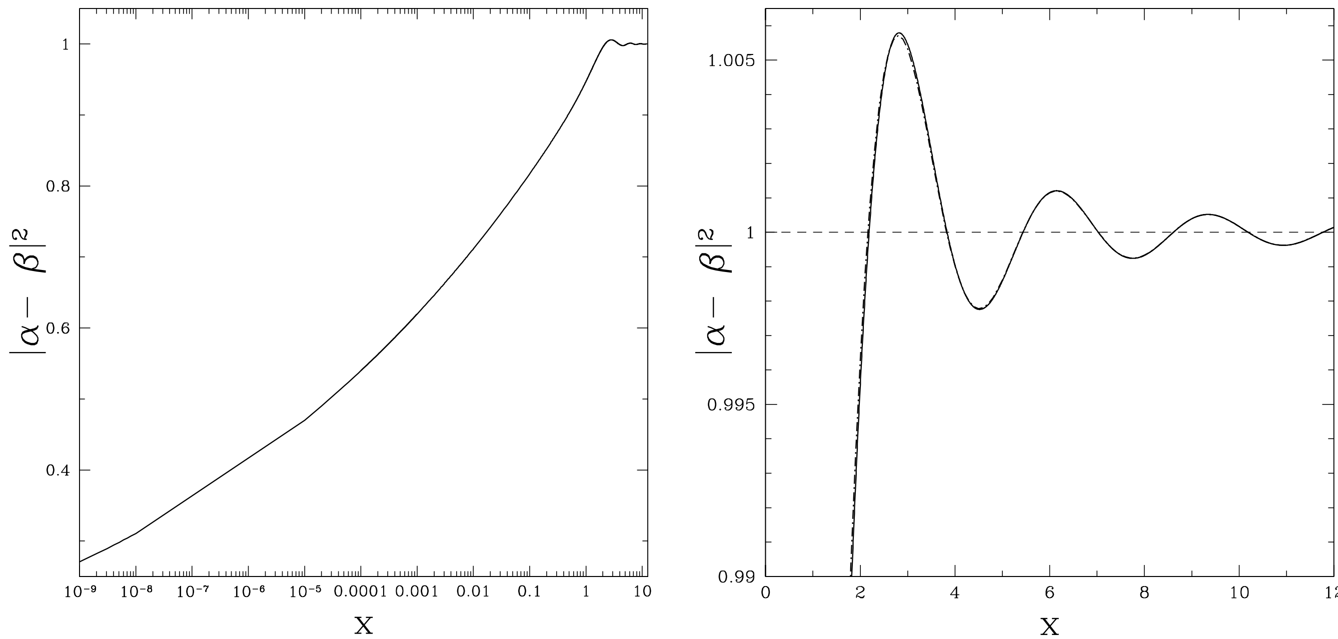

FIG. 1: The transfer function $|\alpha-\beta|^2$ is shown as a function of $x = k/k_0$. The exact expression for $|\alpha-\beta|^2$ given in (3.12) is represented by the solid line in the left and right panels while the asymptotic expression (3.13) is shown dot-dashed in the right panel which shows the oscillations in the transfer function in greater detail. Note that the asymptotic expression provides an excellent approximation to the results for $x = k/k_0 \gtrsim 2$. The feature associated with the step in $V''(\phi)$ occurs at $x \sim 1$. The relevant values of the parameters are $\mu_1 = 1.49$, $\mu_2 = 1.52$.

3. For $x \ll 1$

$$|\alpha-\beta|^2 \simeq \left(\frac{x}{2}\right)^{2(\mu_2-\mu_1)} \ . \tag{3.15}$$

The effective spectral index $n_s(k) \equiv d\ln P_{\mathcal{R}}(k)/d\ln k$ can be determined from the power spectrum, (3.10) as follows

$$n_s(k) - 1 = n_2(k) - 1 + \frac{d\log\left(|\alpha-\beta|^2\right)}{d\log k} \ , \tag{3.16}$$

where the asymptotic forms for $|\alpha-\beta|^2$ in (3.13), (3.14) and (3.15) lead to the following useful approximations

1. For $x = k/k_0 \gg 1$

$$\frac{d\log\left(|\alpha-\beta|^2\right)}{d\log k} \simeq -\frac{3\,(\mu_2-\mu_1)}{x}\left[\sin(2x) + \frac{2\cos(2x)}{x}\right] \ , \tag{3.17}$$

From (3.6), (3.11) and (3.17) we find, for the spectral index

$$n_s(k) \simeq 1 - 4\epsilon_0 - 2\delta_0 - 3\,(\mu_2-\mu_1)\,\frac{\sin(2x)}{x} \ . \tag{3.18}$$

2. For $x \ll 1$

$$\frac{d\log\left(|\alpha-\beta|^2\right)}{d\log k} \simeq 2\,(\mu_2-\mu_1) \ , \tag{3.19}$$

so that

$$n_s \simeq n_2 + 2\,(\mu_2-\mu_1) = n_1 \ . \tag{3.20}$$

The preceeding discussion has been quite general. In order to explore our model in more detail we need to give values to its parameters. Accordingly we set $\mu_1 = 1.49, \mu_2 = 1.52$ which correspond to $n_1 = 1.02$, $n_2 = 0.96$ – see eqn. (3.6). The functional form of $n_s(k)$ for this model is shown in figure 2. Our choice of the spectral indices is largely for a descriptive purpose, other values can easily be accomodated by the model. In the final analysis, a judicious choice of n_1 and n_2 must stem from a comparison of this model with observations that lies outside the scope of the present paper. From figure 2 we see that a discontinuity in the second derivative of the inflaton potential (a step) leads to a

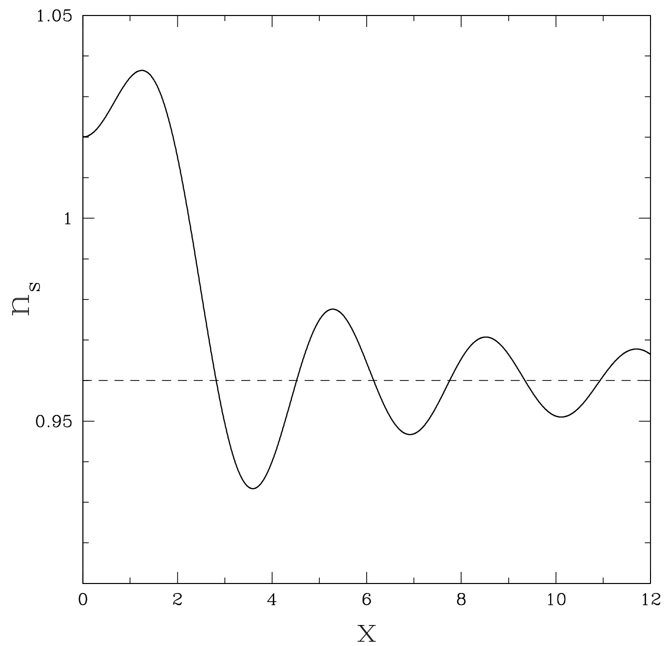

FIG. 2: The primordial spectral index n_s is shown as a function of $x = k/k_0$ for an inflationary model in which the potential has a sudden change in its second derivative. Such a discontinuity in V'' leads to step in n_s at $x \sim 1$ which is followed by oscillations with decreasing amplitude described by (3.18). The parameters of our model are $\mu_1 = 1.49$, $\mu_2 = 1.52$ which correspond to $n_1 = 1.02$, $n_2 = 0.96$.

step in the spectral index, which in our case drops from $n_s = 1.02$ at $k/k_0 \ll 1$ to $n_s = 0.96$ at $k/k_0 \gg 1$. The step in n_s is accompanied by 'ringing' – slowly decreasing oscillations in n_s about the mean (asymptotic) value of $n_s = 0.96$. From (3.18) we also find the following expression for the running of the spectral index

$$\tilde{\alpha} \equiv \frac{dn_s}{d\log k} = \frac{dn_2}{d\log k} - 6\left(\mu_2 - \mu_1\right)\cos\left(2x\right) \ , \quad x = \frac{k}{k_0} \gg 1 \ . \tag{3.21}$$

From figure 2 we find that the running $\tilde{\alpha}$ is of the order of $n_s - 1$ at $x \sim 1$. For $x \gg 1$ equation (3.21) informs us that $\tilde{\alpha}$ has two components, of which the second is of order $n_s - 1$ but oscillates, while the first is smooth and of order $(n_s - 1)^2$, as is usually the case in the slow-roll regime. The following simple relationship between the CMB multipole ℓ and the comoving wavenumber k helps relate the feature in our potential with a corresponding angular scale θ

$$\ell \sim \theta^{-1} \simeq k(\eta_0 - \eta_{\rm ls}) \ , \tag{3.22}$$

where

$$\eta_0 - \eta_{\rm ls} = \frac{c}{H_0}\int_0^{z_{\rm ls}} \frac{dz}{h(z)} \ , \tag{3.23}$$

and $z_{\rm ls}$ is the redshift of the last scattering surface. This leads to

$$k \simeq \ell \times 10^{-4} h \ {\rm Mpc}^{-1} \ , \tag{3.24}$$

in a spatially flat LCDM cosmology with $\Omega_m \simeq 0.3$ and $\Omega_\Lambda \simeq 0.7$.

4. INFLATIONARY MODEL WITH A STEP-LIKE DISCONTINUITY IN THE EVOLUTION OF THE EFFECTIVE MASS

As an example of a microscopic field-theoretic model which can give rise to the feature in the inflaton potential discussed in this paper, let us consider the standard model used in many occasions, in particular, in the hybrid inflationary scenario [7]:

$$V(\psi,\phi) = \frac{1}{4\lambda}\left(M^2 - \lambda\psi^2\right)^2 + \frac{1}{2}m^2\phi^2 + \frac{g^2}{2}\phi^2\psi^2 \ . \tag{4.1}$$

From (4.1) we find that, near $\psi = 0$, the effective mass of the field ψ is given by

$$m_\psi^2 \equiv \frac{d^2V}{d\psi^2} = g^2\phi^2 - M^2 \ , \tag{4.2}$$

so that $m_\psi^2 > 0$ if $\phi > \phi_c$ and $m_\psi^2 < 0$ if $\phi < \phi_c$, where $\phi_c = M/g$ is the critical value of the field ϕ at which the curvature of the potential $V(\psi,\phi)$ along the ψ direction vanishes. The change in the sign of m_ψ^2 is a crucial ingredient of this model: $m_\psi^2 > 0$ ensures that at early times the field ψ rolls towards $\psi = 0$, whereas $m_\psi^2 < 0$ at late times, destabilizes the $\psi = 0$ configuration resulting in a rapid cascade (waterfall) of ψ towards the minimum of its potential. Thus, just before the (weakly second order) phase transition, $\phi > M/g$, $\psi = 0$ so that

$$V(\phi) = \frac{M^4}{4\lambda} + \frac{m^2\phi^2}{2} \ , \tag{4.3}$$

and $\partial^2 V/\partial\phi^2 = m^2$. Introducing the parameter $\kappa = 2\lambda m^2/g^2M^2$, we write $V(\phi)$ at the instant of transition as

$$V(\phi_c) = \frac{M^4}{4\lambda}(1+\kappa) \ . \tag{4.4}$$

A large value $\kappa > 1$ implies that the correction from the $m^2\phi^2$ term to the vacuum energy density $V(0,0) = M^4/4\lambda$ is significant, while the opposite is true for $\kappa < 1$. As we shall discover, $\kappa \lesssim 1$ will be more relevant to the scenario which we are considering. It is easy to see that prior to the transition the slow-roll condition $|V''|/H^2 \ll 1$ implies

$$M^2 \gg \sqrt{\frac{3\lambda}{2\pi}}\,\frac{m\, m_P}{(1+\kappa)^{1/2}} \ , \tag{4.5}$$

where $m_P^2 = G^{-1}$. Soon after the transition, $\phi < M/g$, $\psi^2 = (M^2 - g^2\phi^2)/\lambda$ and

$$V(\phi) = \frac{1}{2}(m^2 + \frac{g^2M^2}{\lambda})\phi^2 - \frac{g^4\phi^4}{\lambda} \ . \tag{4.6}$$

The requirement that slow-roll remain valid immediately after the transition gives

$$M \gg g m_P \ . \tag{4.7}$$

The product of (4.5) and (4.7) results in the following constraint

$$M^3 \gg \sqrt{\frac{3\lambda}{2\pi}}\,\frac{g m m_P^2}{\sqrt{1+\kappa}} \ . \tag{4.8}$$

Unlike the field ϕ which slowly rolls down the potential $V(\phi,\psi)$, the motion of ψ is rapid and the condition $\frac{|\partial^2 V/\partial\psi^2|}{H^2} \gg 1$ is valid all time apart from a very small interval $\Delta t \ll H^{-1}$ around the transition if

$$M^3 \ll \frac{\lambda m\, m_P^2}{1+\kappa} \ . \tag{4.9}$$

It is easy to see that (4.8) and (4.9) imply $g^2 \ll \lambda$ in our model, i.e. self-coupling of the ψ field should be much more than its coupling to the inflaton ϕ. The value of the spectral index before (n_1) and after (n_2) the transition can be determined quite simply, by applying the well known formula

$$n - 1 = -\frac{3\, m_P^2}{8\pi}\left(\frac{V'}{V}\right)^2 + \frac{m_P^2}{4\pi}\left(\frac{V''}{V}\right) \ , \tag{4.10}$$

to (4.3) and (4.6). Consequently

$$n_1 - 1 \;=\; \frac{1}{2\pi}\left(\frac{gm_P}{M}\right)^2 \frac{\kappa\,(1-2\kappa)}{(1+\kappa)^2}\;, \tag{4.11}$$

$$n_2 - 1 \;=\; -\frac{1}{2\pi}\left(\frac{gm_P}{M}\right)^2 \frac{4+3\kappa+2\kappa^2}{(1+\kappa)^2}\;. \tag{4.12}$$

From (4.11) we find that the inflationary spectrum on large scales has a red (blue) tilt if $\kappa > 1/2$ ($\kappa < 1/2$); $\kappa = 1/2$ results in precise scale-invariance for the initial spectrum: $n_1 = 1$. The total change in the spectral index during the course of the transition is given by

$$\Delta n \equiv n_1 - n_2 = \frac{2}{\pi(1+\kappa)}\left(\frac{gm_P}{M}\right)^2\;. \tag{4.13}$$

Clearly, in order to make contact with observations the value of κ must not be too large since otherwise $n_1 \simeq n_2$, and it will be difficult to test the predictions of this model rigorously. From (4.11) and (4.12) we find that n_1 and n_2 are

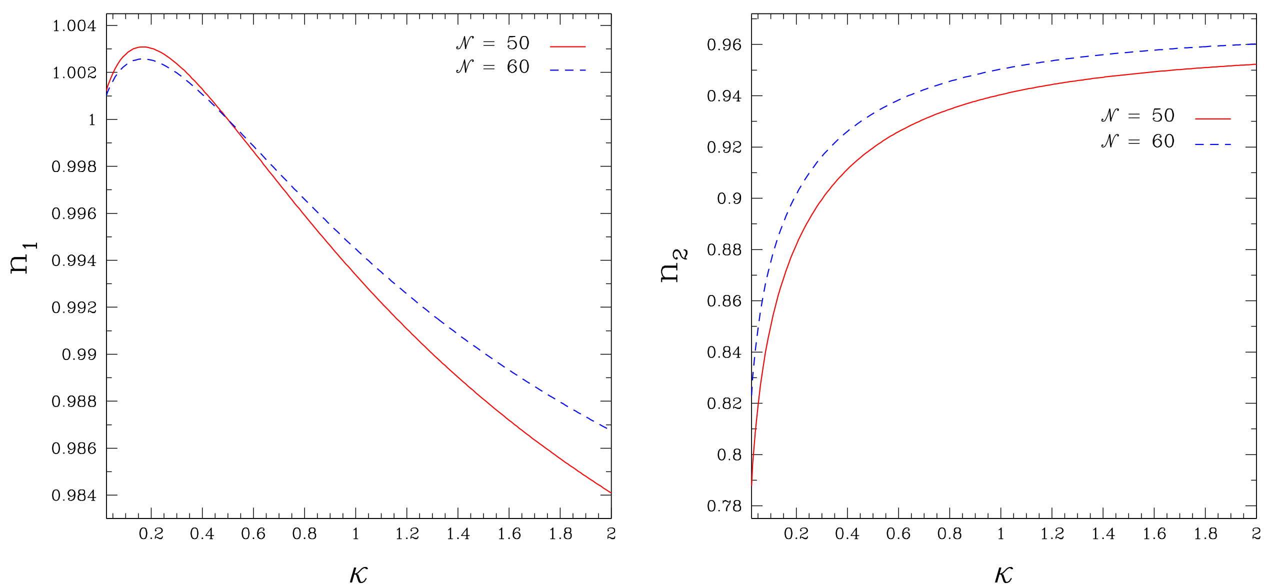

FIG. 3: The spectral index just before (n_1) the phase transition in hybrid inflation and immediately after it (n_2), is shown as a function of the parameter $\kappa = 2\lambda m^2/g^2M^2$ in the left and right hand panel of this figure. The red (solid) line corresponds to 50 e-folds of inflationary expansion occuring after the phase transition in hybrid inflation, while the dashed (blue) line corresponds to 60 e-folds.

completely determined if we know the value of κ ($\equiv 2\lambda m^2/g^2M^2$) as well as gm_P/M. κ and gm_P/M are also related to the number of inflationary e-folds which take place after the phase transition has occurred,

$$\mathcal{N} = \pi\left(\frac{M}{gm_P}\right)^2\left[1 + \left(1+\frac{\kappa}{2}\right)\log\frac{2+\kappa}{\kappa}\right]\;. \tag{4.14}$$

The exact value of $\mathcal{N}$ can be chosen as one of free parameters of the model. It is uniquely defined by the preferred location k_0 of the feature and the length of the reheating period after inflation. Below we use $\mathcal{N} = 60$ as an estimate which roughly corresponds to k_0 being of the order of the inverse present Hubble scale, leaving the determination of the most probable location of k_0 needed to explain running in the WMAP data for future work. As follows from Fig. 2 the transition, from the slope value $n_s = n_1$ to $n_s = n_2$, mainly occurs in the range $k \approx (1-3)k_0$ (neglecting small oscillations for higher values of k).

We plot the behaviour of the spectral index as a function of κ for two different values of $\mathcal{N}$ in figures 3, 4. Values of parameters (M, m, g) in our model can be very simply estimated by comparing the inflationary curvature fluctuation

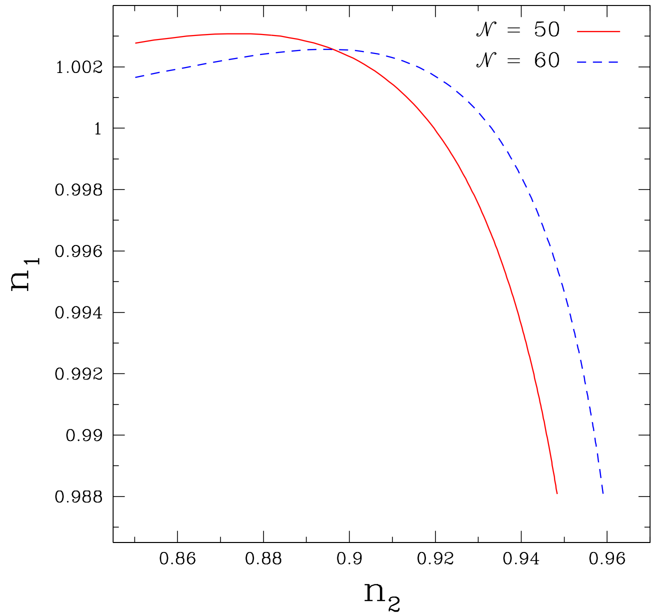

FIG. 4: Spectral indices for perturbations generated just before (n_1) and immediately after (n_2) the phase transition in hybrid inflation are shown. Note that distinct values of the pair $\{n_1, n_2\}$ correspond to different values of the parameter κ, as shown in figure 3. Results are shown for two possible number of e-folds after the phase transiton: $\mathcal{N} = 50$ (red, solid), and $\mathcal{N} = 60$ (blue, dashed).

on large scales with the observed CMB fluctuation measured by COBE or WMAP. The perturbation spectrum can be approximated as

$$P_{\mathcal{R}}(k) = \frac{128\pi}{3m_P^6}\frac{V^3}{V'^2} = \frac{8\pi}{3}\frac{g^4}{\lambda}\left(\frac{M}{gm_P}\right)^6\frac{(1+\kappa)^3}{\kappa^2} , \tag{4.15}$$

which is related to the dimensionless density contrast $\delta_H(k)$ via

$$\delta_H^2(k) = \frac{4}{25}\left(\frac{g(\Omega_m)}{\Omega_m}\right)^2 P_{\mathcal{R}}(k) , \tag{4.16}$$

where [32]

$$g(\Omega_m) = \frac{5}{2}\Omega_m\left(\frac{1}{70} + \frac{209\Omega_m - \Omega_m^2}{140} + \Omega_m^{4/7}\right)^{-1} , \tag{4.17}$$

and the 4 year COBE data implies, for an LCDM Universe [33, 34],

$$\delta_H = 1.91\times 10^{-5}\,\frac{\exp\left[1.01(1-n)\right]}{\sqrt{1+f(\Omega_m)r}}\Omega_m^{-0.8-0.05\log\Omega_m}\times\{1-0.18(1-n)\Omega_\Lambda - 0.03r\Omega_\Lambda\} , \tag{4.18}$$

where the ratio of tensor to scalar power spectrum denoted by $r \simeq 16\,\epsilon_0$, $f(\Omega_m) = 0.75 - 0.13\Omega_\Lambda^2$. Substituting, for $V(\phi)$ from (4.3) we obtain, after setting $\mathcal{N} = 60$ in (4.14), and $\lambda = 0.1$, for a spatially flat LCDM universe with $\Omega_m = 0.26$ and $\Omega_\Lambda = 0.74$, parameter values κ, g, M, m given in table 1. We therefore conclude that, for this model, the value of M lies in the GUT range while the inflaton mass $m \ll M$ is of the order of (though slightly less than) that in the simplest inflationary model with $V = m^2\phi^2/2$. Note that for these values of parameters, all inequalities (4.5), (4.7) and (4.9) are satisfied (actually, in the case of (4.7) the quantity that should be sufficiently large is the number of e-folds after transition $\mathcal{N}$ (4.14)).

TABLE I: Typical parameter values for the hybrid inflationary model (4.1) with a step in the spectral index.

κ	g	M/m_p	m/m_p
1	3×10^{-4}	8×10^{-4}	5.3×10^{-7}
0.5	2.9×10^{-4}	7.2×10^{-4}	3.3×10^{-7}
0.25	2.6×10^{-4}	6.1×10^{-4}	1.8×10^{-7}

5. CONCLUSIONS AND DISCUSSION

The rapid advance and sophistication of cosmology experiments, most notably those associated with measuring fluctuations in the cosmic microwave background and which purport to throw light on the physics of the very early universe, necessitate a close examination of different possibilities for the generation of primordial fluctuations responsible for the CMB signal. In this paper we have demonstrated the possibility of a new kind of perturbation spectrum generated during inflation. We have shown that, if during inflation, the effective mass of the inflaton changes rapidly, then this change results in a universal local feature being imprinted onto the primordial spectrum of density perturbations. Namely, a sudden change in $m^2_{\rm eff}$ satisfying the condition $|[m^2_{\rm eff}]| \ll H^2$ leads to a break in the spectral slope – equivalently – to a step in the value of the primordial spectral index n_s. This break is accompanied by rapid oscillations decaying both in $P_{\mathcal{R}}(k)$ and $n_s(k)$ away from the transition point. These oscillations are rather small in magnitude as compared to all exact solutions for perturbation spectra with features considered before. The amplitude of the running of the spectral index is rather large, $\sim (n_s - 1)$, at the transition point but decays away from it, too. The precise location of the step in n_s and its amplitude are free parameters of this model whose values must be set after comparing the predictions of this scenario with observations. Note also we have not specified the form of the background power spectrum $\mathcal{P}_{\mathcal{R}0}(k)$ on which our feature is superimposed. The form of $\mathcal{P}_{\mathcal{R}0}(k)$ must clearly be derived from a concrete physical model. (One might even conjecture, for arguments sake, that $\mathcal{P}_{\mathcal{R}0}(k)$ could contain additional features generated by other physical effects such as the existence of a radiative epoch prior to inflation, etc.) We have also demonstrated that a field–theoretic model which can give rise to a step in $V''(\phi)$ is similar to that used to end inflation in the hybrid inflationary model, though different values of its parameters are required in our case which should satisfy the inequalities (4.5), (4.7) and (4.9). It describes a fast second–order phase transition during inflation that occurs in some other scalar field weakly coupled to the inflaton. Some estimates of the values of the spectral index in this model are given and analytical formulae relating n_s to the fundamental parameters in the inflationary potential are derived; see (4.11) & (4.12). The reader may also like to note that while the treatment in section 3 is quite general and allows the running of the spectral index to be positive ($\tilde{\alpha} > 0$) as well as negative ($\tilde{\alpha} < 0$), the microphysical model in section 4 favours $\tilde{\alpha} < 0$ which is suggested by (4.13). To conclude, the scenario discussed in this paper suggests that the observed running of the spectral index in the WMAP data may be caused by a fast second order phase transition which occurred during inflation. A detailed comparison of this model with observations remains an important problem for further study.

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