

# Constraints on $\Omega_B$ , $\Omega_m$ and $h$ from MAXIMA and BOOMERANG

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## ABSTRACT

We analyse the BOOMERANG and MAXIMA results in the context of models with  $\Omega_{\text{Total}} = 1$  and  $n_s = 1$ . We attempt to constrain three other parameters— $h$ ,  $\Omega_B$ , and  $\Omega_m$ —from these observations. We show that: (a) the value of  $\Omega_B h^2$  is too high to be compatible with primordial nucleosynthesis observations at 95% confidence level (b) universe with age greater than 12 Gyr is ruled out at 95% confidence level and (c) the value of  $\Omega_m h$  is too high to be compatible with the shape of the power spectrum of gravitational clustering. In effect, our analysis shows that models with  $\Omega_{\text{Total}} = 1$  and  $n_s = 1$  are ruled out by BOOMERANG and MAXIMA observations.

Precise determination of CMBR anisotropies has long been expected to give accurate values of cosmological parameters (see e.g. Bond, Efstathiou & Tegmark 1997 and references therein). These cosmological parameters include parameters of background FRW model ( $\Omega_{\text{Total}}$ ,  $\Omega_\Lambda$ ,  $h$ ,  $\Omega_B$ , etc.), parameters that determine the formation of structure in the universe ( $\sigma_8$ , scalar spectral index  $n_s$ , etc.), and the parameters related to the re-ionization of the universe (the optical depth to the last scattering surface,  $\tau$  etc.).

While the future experiments MAP and Planck<sup>1</sup>, largely owing to their all-sky coverage, are expected to determine most of these parameters with a few percent accuracy (Jungeman *et al.* 1996, Zaldarriaga *et al.* 1997, Prunet *et al.* 1999), recent observations have already begun to give important clues about some of these parameters (Miller *et al.* 1999, Mauskopf *et al.* 2000, Netterfield *et al.* 1997, for a summary of observation up to 1998 and parameter estimation from

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<sup>1</sup>For details see <http://map.gsfc.nasa.gov> and <http://astro.estec.esa.nl/SA-general/Projects/Planck>

these observations, see Lineweaver & Barbosa 1998). Recent balloon experiments BOOMERANG and MAXIMA reported CMBR anisotropy measurements at angular scales between  $\simeq 10^\circ$  and  $\simeq 10'$  (de Bernardis *et al.* 2000, Hanany *et al.* 2000). These experiments observed nearly 1% of the sky with angular resolution  $\simeq 10'$ . For both these experiments the cosmic variance was small enough (owing to the sky coverage) to determine precisely the position of the first Doppler peak ( $\ell \simeq 200$ ) of the CMBR anisotropies. Both BOOMERANG and MAXIMA results gave strong evidence that  $\Omega_{\text{Total}} = 1$  (de Bernardis *et al.* 2000, Hanany *et al.* 2000), which was already indicated by other observations (Netterfield *et al.* 1997).

While the position of the first Doppler peak give unambiguous evidence about the geometry of the universe, determination of other cosmological parameters is more difficult. This is because variation in several different parameters give the same change in measured anisotropies, e.g. the height of first Doppler peak is nearly degenerate in  $\Omega_B$ ,  $h$ ,  $\Omega_\Lambda$  and  $n_s$ . Some of this degeneracy can be lifted with the measurement of anisotropies at even smaller angular scales. BOOMERANG and MAXIMA probe with angular scales corresponding to multipoles  $\ell_{\text{max}} \simeq \{600, 700\}$ , respectively, which is up to or beyond the expected position of the second Doppler peak. Though the results of these experiments have not been able to find the position of the second Doppler peak, accurate measurement of anisotropies at such angular scales is expected to break some of the degeneracy which measurements near the first Doppler peak alone cannot.

The BOOMERANG and MAXIMA data have been used to determine various cosmological parameters (Balbi *et al.* 2000, Jaffe *et al.* 2000, Lange *et al.* 2000, Tegmark & Zaldarriaga 2000). Combined with other independent measurements of cosmological parameters (e.g. measurement of  $\Omega_B h^2$  from element abundance, measurement of  $h$  from nearby observations or inference about the values of  $\Omega_\Lambda$  and  $\Omega_m$  from the SN1a data, etc.) these data are expected to lead to a unique picture. However, owing to degeneracies in parameter estimation, the value of estimated parameters and their errors depend sensitively on various assumption related to the assumed allowed range of parameters, i.e. on the priors on the parameters.

In this letter, we perform a likelihood analysis on the band-powers reported by the MAXIMA and BOOMERANG experiments. However, instead of of doing a multiple parameter analysis, we fix the values of most parameters from other considerations (and prejudices!) and then attempt to estimate just three parameters— $\Omega_B$ ,  $h$ , and  $\Omega_m$  assuming weak priors on their allowed values. In the next section, we explain our choice of parameters and the method we use in brief. In the third section, we present and summarise our results.

## 1. Cosmological parameters and CMBR data

Most generic models of inflationary scenario give two unique predictions:  $\Omega_{\text{Total}} = 1$  and  $n_s \simeq 1$  (see e.g. Steinhardt 1995). The first of these predications is confirmed by BOOMERANG and MAXIMA. The analysis of COBE-DMR data is consistent with  $n_s = 1$  (Bennett *et al.* 1996, Bunn

& White 1996). Therefore it is reasonable to believe that the current data is in good agreement with these predictions. We fix the value of these two parameters based on these considerations and use  $\Omega_{\text{Total}} = 1$  and  $n_s \simeq 1$  throughout. (It should be pointed out that  $\Omega_{\text{Total}} = 1$  is a stricter requirement of inflation than  $n_s = 1$ ; one gets  $n_s = 1$  only for exponential inflation; see e.g. Steinhardt 1995.) Also note that we are not concerned with the origin of the values  $\Omega_{\text{Total}} = 1$  and  $n_s = 1$ . For example, these could arise merely from the requirement of scale invariance for the background universe (giving  $\Omega_{\text{Total}} = 1$ ) and the perturbations (giving  $n_s = 1$ ) without invoking inflation — as was originally done by Harrison and Zeldovich, years before inflation was invented (Harrison 1970, Zeldovich 1972). But, of course, inflationary models made these parameter values fairly well accepted. Other parameters like  $\Omega_\Lambda$ ,  $h$  and  $\Omega_B$  cannot be fixed by theoretical considerations alone. In our analysis we assume a non-zero  $\Omega_\Lambda$  because recent high- $z$  SN1a observations suggest a non-zero cosmological constant (Perlmutter *et al.* 1999, Riess *et al.* 1998). We do not consider CMBR anisotropies from tensor perturbations as most models of inflation give negligible contribution from tensor perturbations for  $n_s = 1$  (Steinhardt 1995). Re-ionization of universe can also alter primary CMBR anisotropies significantly. Present observations suggest that the universe is ionized up to  $z \simeq 5$ . For the CMBR anisotropies to be significantly altered by re-ionization, the minimum re-ionization redshift should lie between 10 and 20 (see e.g. Bond 1996). Therefore, keeping the constraints from present observations in mind, we neglect the effect of re-ionization on the measured CMBR anisotropies.

We fix the value of  $\Omega_m + \Omega_\Lambda = \Omega_{\text{Total}} = 1$ , and compute the confidence levels on the best-fit values of  $\Omega_B$  and  $h$ , and  $\Omega_m$  from MAXIMA and BOOMERANG observations. The range of parameters in which the minimum of  $\chi^2$  is searched is:  $0.01 \leq \Omega_B \leq 0.15$ ,  $0.4 \leq h \leq 1.1$  and  $0.1 \leq \Omega_m \leq 0.95$ .

The  $\chi^2$  for the model comparison with observations is given by:

$$\chi^2 = \sum_{i=1}^N \left( \frac{C_\ell^{\text{obs}} - C_\ell^{\text{th}}}{\Delta C_\ell^{\text{obs}}} \right)^2 \quad (1)$$

Here  $N = 22$  (10 points from MAXIMA and 12 points from BOOMERANG),  $C_\ell^{\text{obs}}$  are the measured band-powers and  $C_\ell^{\text{th}}$  are the theoretical band-powers,  $\Delta C_\ell^{\text{obs}}$  are the errors on measured band-powers. We do not take into account the calibration uncertainties in our analysis. The theoretical band-powers are calculated using the CMBR Boltzmann code CMBFAST (Seljak & Zaldarriaga 1996), which gives COBE-normalized (normalized according to the fitting formula of Bunn & White (1996)) angular power spectra. Eq. (1) assumes that different band-powers are uncorrelated. This assumption is valid only if the anisotropies are measured over the entire sky (see e.g. Peebles 1993, Padmanabhan 1993). We assume lack of correlation in this work also because the covariance matrix of band-powers has not been made public yet.

Eq. (1) implicitly assumes that the likelihood function is Gaussian in band-powers near its maximum. While this assumption is true in principle, in practice there can be significant deviation from Gaussianity near the maximum. Bond, Jaffe, and Knox (2000) advocate using another variable

instead of band-powers for doing the maximum likelihood analysis. We do not use it here. However, while quoting errors we do not use the Fisher matrix approach which can give meaningful results only for the Gaussian case. Instead we directly give the confidence levels on  $\Delta\chi^2$ .

## 2. Results

Our results are shown in Figures 1 and 2. The  $\chi^2$  for 22 data points from BOOMERANG and MAXIMA with three fitted parameters (i.e. 19 degrees of freedom) is 22.4, which is an excellent fit (Goodness-of-fit probability,  $Q = 0.26$ ). The best fit values and  $1\sigma$  errors are:  $\Omega_B = 0.075_{-0.015}^{+0.003}$ ,  $h = 0.62_{-0.02}^{+0.08}$  and  $\Omega_m = 0.86_{-0.21}^{+0.04}$ . The best fit model along with BOOMERANG and MAXIMA data points is shown in Figure 3.

The range of allowed  $h$  is in fair agreement with the measurement of  $h$  from local observations (for a recent summary of these results, see Primack 2000). Recent SN1a suggest that that the universe is flat with  $\Omega_\Lambda + \Omega_m = 1$  with  $\Omega_m = 0.28_{-0.08-0.04}^{+0.09+0.05}$  ( $1\sigma$ ). This is within  $\Delta\chi^2 = 2$  of the value inferred by our analysis.

In Figure 1 we show the confidence levels in the  $\Omega_B$ - $h$  plane. The region bounded by the contours correspond to  $\Delta\chi^2 \leq 2.3$  and  $\Delta\chi^2 \leq 6.17$ , which, for Gaussian errors, corresponds to 68 % ( $1\sigma$ ) and 95.4 % ( $2\sigma$ ) for two-parameter fits. We also show the 95% region allowed by primordial nucleosynthesis observations (for a recent review see Tytler *et al* 2000). As seen in the figure, the region allowed by CMBR observations is at variance with the predictions of primordial nucleosynthesis at at least 95% level. This result is in agreement with other analyses on the same data set (Tegmark & Zaldarriaga 2000, Jaffe *et al* 2000).

The region corresponding to one and two  $\sigma$  in the  $\Omega_m$ - $h$  plane is shown in Figure 2. Our results show that the current CMBR observations favour a universe with age  $\leq 11$  Gyr and are incompatible with a universe of age  $\geq 12$  Gyr at  $\simeq 95\%$  level. Though this is on the lower side of the expected age of the universe from estimated ages of globular clusters, etc., this is not in disagreement with those observations (for a recent status report see Primack 2000). Another important constraint on the values of  $\Omega_m$  and  $h$  comes from the shape of the power spectrum of galaxy clustering (see e.g. Bond 1996). These observations, within the context of a flat model with cosmological constant, give  $\Omega_m h \simeq 0.25 \pm 0.05$ . We plot this region in the  $\Omega_m$ - $h$  plane in Figure 2. If we require that  $h \leq 0.75$ , as most recent measurements of  $h$  suggest (Primack 2000), then CMBR observations exclude the region required to satisfy the galaxy clustering observations by more than 95% confidence level.

In conclusion, recent BOOMERANG and MAXIMA observations, within the context of simplest inflationary models ( $\Omega_{\text{Total}} = 1$  and  $n_s \simeq 1$ ), imply: (1) too large a value of  $\Omega_B h^2$  to be compatible with primordial nucleosynthesis observations, (2) an age of universe  $t_0 \leq 12$  Gyr, and (3) too large a value of  $\Omega_m h$  to be in agreement with the shape of the power spectrum of galaxy clustering. In view of this, it is safe to conclude that the models with  $\Omega_{\text{Total}} = 1$  and  $n_s = 1$  are ruled out by these observations.

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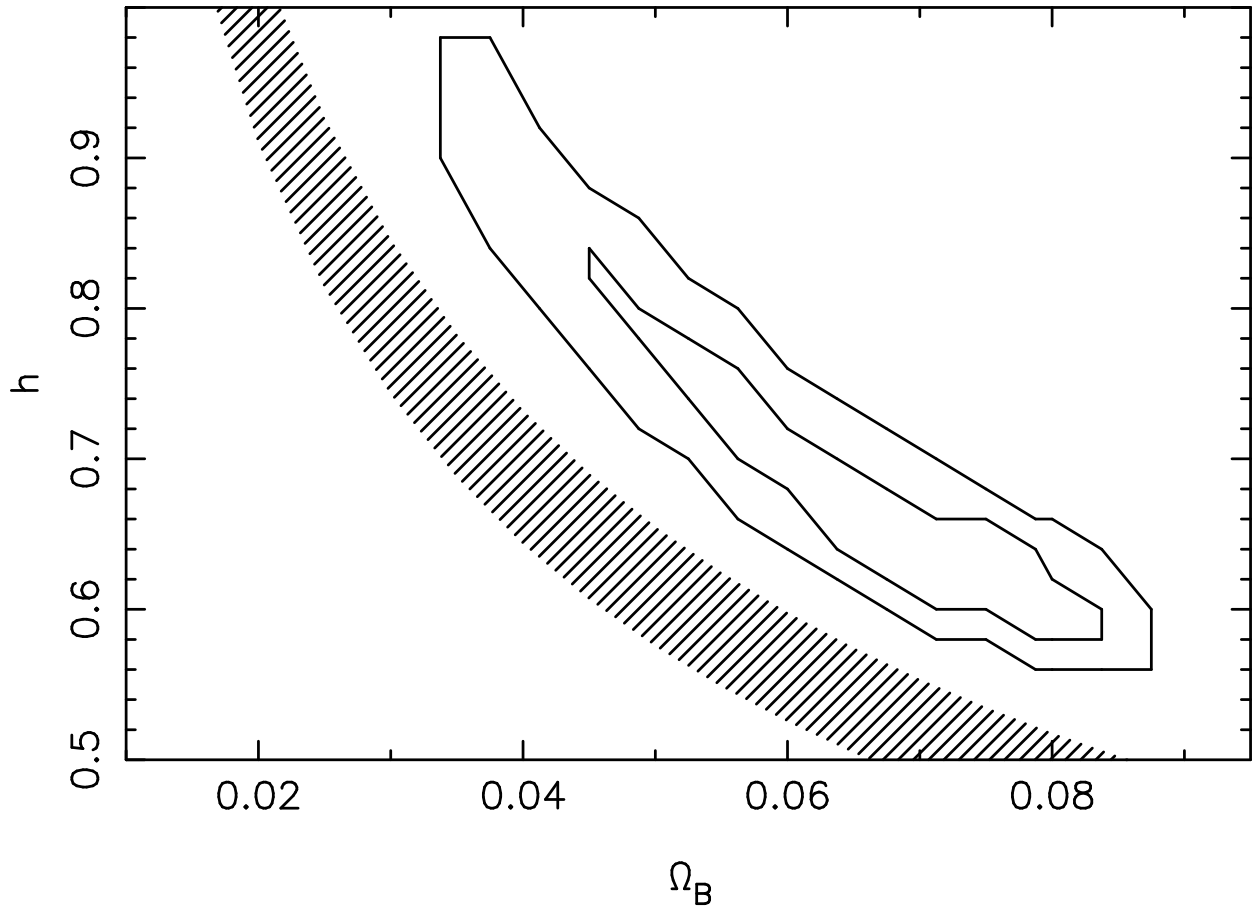


Fig. 1.— The contours correspond to allowed 1 and 2  $\sigma$  regions by CMBR observations (see text for detail). The cross-hatched region correspond to the 95% region ( $\simeq 2\sigma$ ) from primordial nucleosynthesis (Tytler *et al.* 2000).

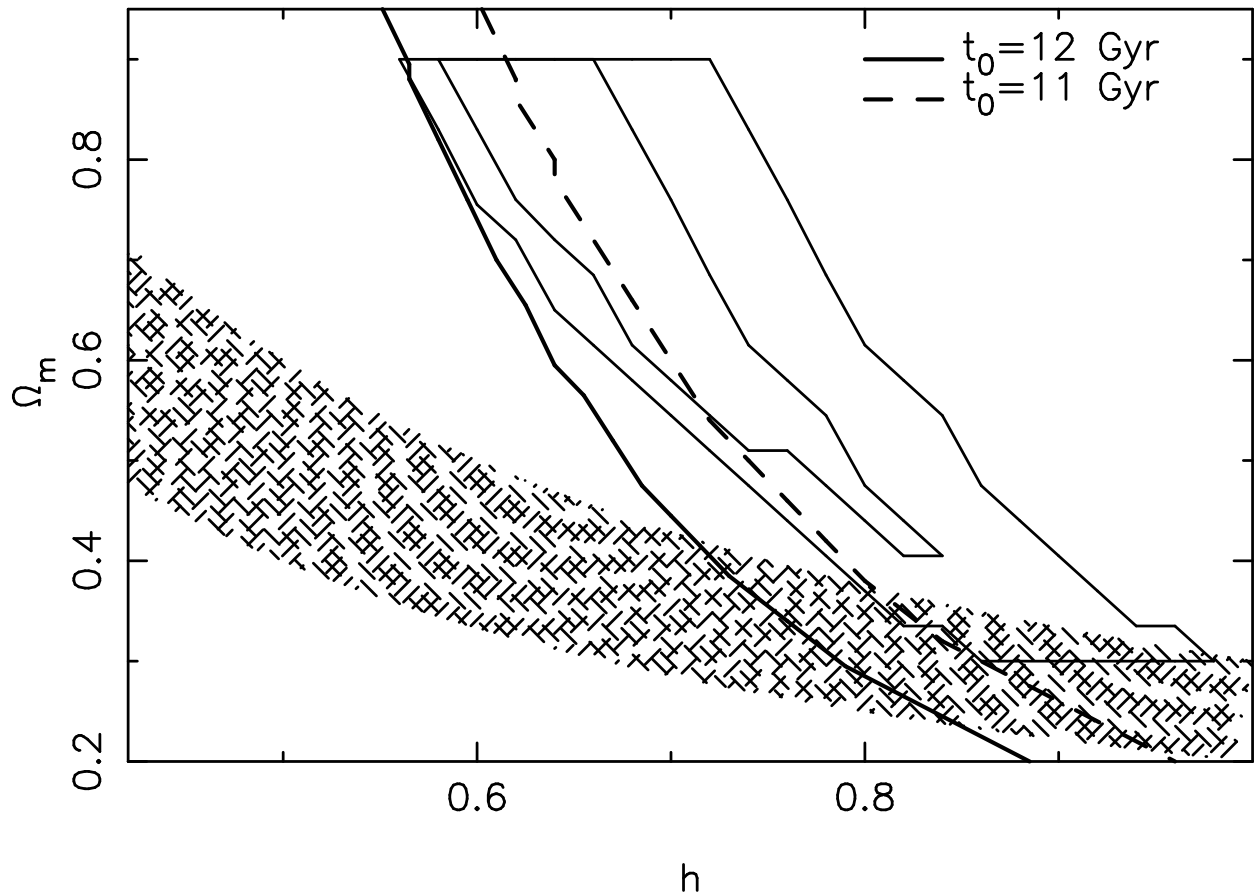


Fig. 2.— The contours correspond to allowed 1 and 2  $\sigma$  regions by CMBR observations (see text for detail). The hatched region corresponds to  $\Omega_m h = 0.25 \pm 0.05$ . Other curves show the age of the universe  $t_0$  (value indicated in the figure legend)

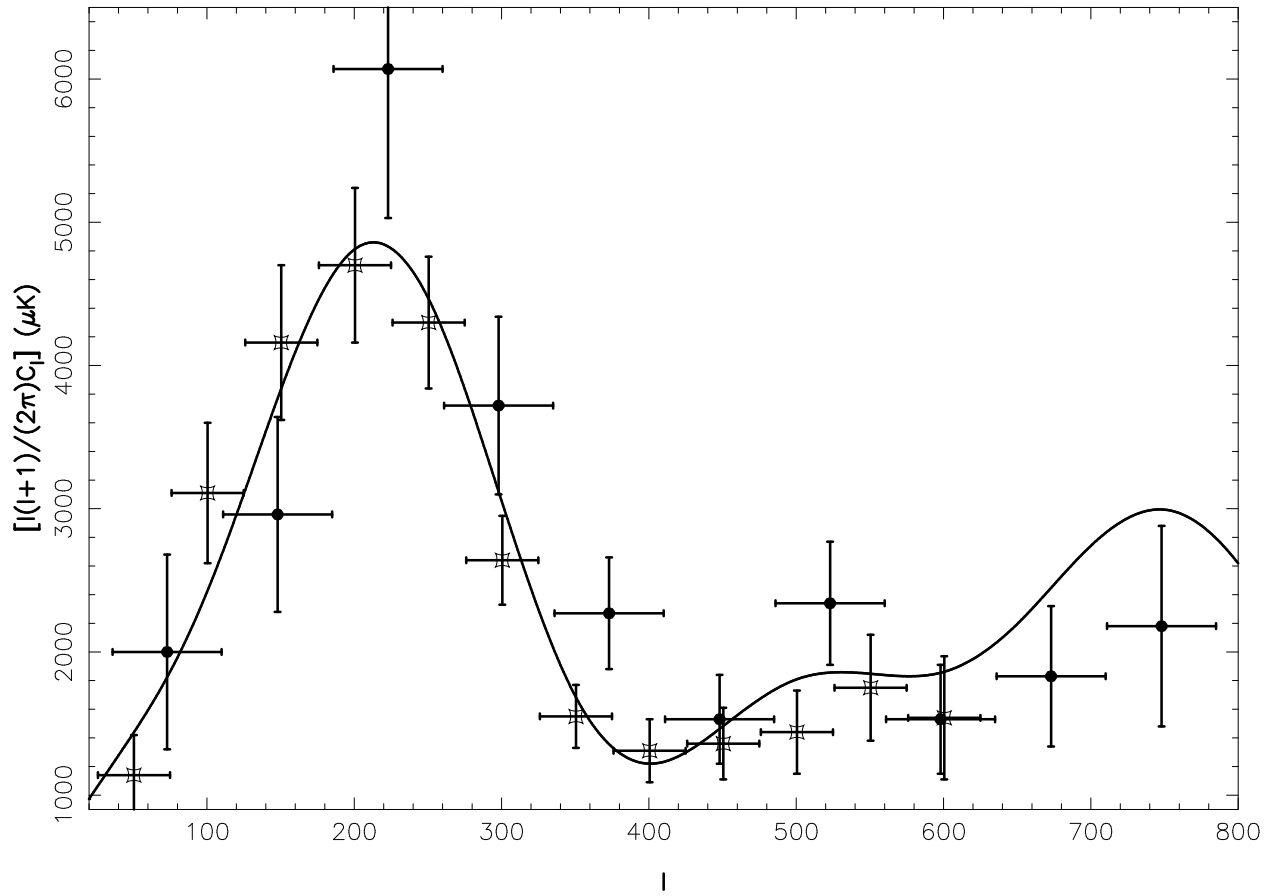


Fig. 3.— The best fit model is plotted along with BOOMERANG and MAXIMA data points, which correspond to empty and filled polygons, respectively.