

A Machian Model of Dark Energy

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Abstract

Einstein believed that Mach's principle should play a major role in finding a meaningful spacetime geometry, though it was discovered later that his field equations gave some solutions which were not Machian. It is shown, in this essay, that the kinematical Λ models, which are invoked to solve the cosmological constant problem, are in fact consistent with Mach's ideas. One particular model in this category is described which results from the microstructure of spacetime and seems to explain the current observations successfully and also has some benefits over the conventional models. This forces one to think whether the Mach's ideas and the cosmological constant are interrelated in some way.

KEY WORDS: Mach's principle; cosmological constant; dark energy

Received an *Honorable mention* in the Essay Contest–2002 sponsored by the Gravity Research Foundation.

When Einstein introduced the cosmological constant Λ into his field equations to obtain a static solution, he was guided by Mach's principle, which argued that the distribution of matter determined the precise geometrical nature of spacetime and hence forbade the notion of empty universe. He believed that the presence of matter was essential for a meaningful spacetime geometry [1]. However, he had to discount his idea when deSitter discovered a cosmological model with Λ and no matter at all which had both static and dynamic representations. Later he also dismissed Λ when it was found that the universe was expanding. One however notices that if a dynamic $\Lambda(t)$ is introduced into Einstein's field equations, no solution is possible in the absence of matter. This is clear from the divergence of the field equations:

$$[R^{ij} - \frac{1}{2}Rg^{ij}]_{;j} = 0 = -8\pi G \left[T^{ij} - \frac{\Lambda(t)}{8\pi G} g^{ij} \right]_{;j}. \quad (1)$$

(I shall use the units with $c = 1$ throughout. However, c will be restored whenever needed.) Obviously a solution with a dynamic Λ is possible only if $T^{ij} \neq 0$ (and $T_{;j}^{ij} \neq 0$). In the absence of matter (or even if the matter is conserved), Λ has got to remain a constant. Thus the empty spacetime cannot be obtained as a solution of general relativity with a *dynamic* $\Lambda(t)$.

This way of introducing Λ into Einstein's equations gives it a status of a source term. Now it ($\Lambda/8\pi G$) represents the energy density of '*emptiness*' (vacuum) and hence invites particle physics to interact with general relativity via Λ . Note that the only possible covariant form for the energy momentum tensor of the quantum vacuum is $T_v^{ij} = -\rho_v g^{ij}$, which is equivalent to the cosmological constant. It behaves like a perfect fluid with the energy density $\rho_v = \Lambda/8\pi G$ and an isotropic pressure $p_v = -\rho_v = -\Lambda/8\pi G$. The conserved quantity is now the sum of matter and vacuum (and not the two separately), as is obvious from equation (1).

Here comes the problem: the value of vacuum energy at the Planck epoch comes out as $\approx 10^{76}$ GeV⁴, which is 123 orders of magnitude larger than its value predicted by the Friedmann equation

$$\frac{\dot{S}^2}{S^2} + \frac{k}{S^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}, \quad (2)$$

which gives $\Lambda_0 \approx H_0^2$ or equivalently $\rho_{v0} \approx 10^{-47}$ GeV⁴ [2]. (The subscript '0' denotes the value of the quantity at the present epoch.) It is fortunate for

general relativity that this predicted value of Λ by the theory is also consistent with the recent observations of type Ia supernovae [3] and the anisotropy measurements of the cosmic microwave background radiation (CMBR) [4], taken together with the complimentary observational constraints on matter density [5]; all indicate that the present constituent of the universe is dominated by some weird kind of energy with negative pressure, commonly known as ‘*dark energy*’. The simplest candidate for *dark energy* is the cosmological constant, though plagued with this so called the *cosmological constant problem*. Obviously the problem arises due to the incompatibility of general relativity and particle physics. The dynamical Λ was, in fact, invoked in an attempt (phenomenological in nature) to solve this problem (historically, it was not invoked to make the solutions of Einsein’s equations consistent with Mach’s ideas). The rationale behind this approach is that Λ was large during the early epochs and it decayed as the universe evolved, reducing to a small value at the present epoch.

There is another phenomenological approach to solve this problem, which has become very popular since recent observations suggested the existence of a nonzero Λ . This invokes a slowly rolling down scalar field ϕ , commonly known as ‘*quintessence*’, with an appropriate potential $V(\phi)$ to explain the observations [6].

Note that though the quintessence fields also acquire negative pressure during the matter dominated phase and behave like dynamical Λ (with $\Lambda_{\text{effective}} \equiv 8\pi G\rho_\phi$), they are in general fundamentally different from the dynamical (kinematical) Λ . In the former case, quintessence and matter fields are assumed to be conserved separately (through the assumption of minimal coupling of the scalar field with the matter fields). However, in the latter case, the conserved quantity is $[T^{ij} + T_v^{ij}]$, as have been mentioned earlier. This implies that there is a continuous creation of matter from the *decaying* Λ as is clear from the following.

$$\rho = CS^{-3(1+\omega)} - \frac{S^{-3(1+\omega)}}{8\pi G} \int \dot{\Lambda}(t)S^{3(1+\omega)} dt, \quad C = \text{constant}, \quad (3)$$

which follows from (1) and suggests that there is a positive contribution to ρ from the *decaying* Λ ($\dot{\Lambda} < 0$). Here $\omega = p/\rho$ is the usual equation of state of the matter field. Obviously the quintessence models need not be consistent with Mach’s ideas.

It may also be noted that, for a given pair of $S(t)$ and $\rho(t)$, it is always

possible to find a $V(\phi)$ which explains the observations, as has been shown recently by Padmanabhan [7]. This result is irrespective of what the future observations reveal about the given $S(t)$ and $\rho(t)$, and hence makes these models trivial. Like the anthropic principle [8], these models also don't have any predictive power and lead to similar late time behaviour of the universe.

The true solution of the cosmological constant problem should be provided by a full theory of quantum cosmology, which is unfortunately not available at the moment. However, some arguments have been made, based on the quantum gravitational uncertainty principle and the discrete structure of spacetime at Planck length, which have made it possible to connect the cosmological constant with the microstructure of spacetime [7, 9]. By assuming that Λ is a stochastic variable arising from the quantum fluctuations and it is the rms fluctuation which is being observed in the cosmological context, it has been shown that the uncertainty in the value of Λ can be written as

$$\Delta\Lambda = \frac{1}{\sqrt{\mathcal{U}}}, \quad (4)$$

where \mathcal{U} is the four volume of the universe. If one estimates the 'radius' of the universe by $S \approx ct \approx cH^{-1}$, then this reduces to

$$\Delta\Lambda \approx H^2, \quad (\text{in units with } c = 1), \quad (5)$$

which matches exactly with the present observations.

There are also other ways which suggest $\Lambda \propto H^2$. Two such ways have been described in the following (two more have been described by Padmanabhan in his paper [7]).

(i) We know that a positive Λ introduces a force of repulsion between two bodies which increases in proportion to the distance between them. This force experienced by a test particle at the scale of the whole universe is $c\Lambda H^{-1}$. If this repulsive force roughly balances the gravitational attraction $4\pi Gc\rho/3H$ of the universe on the test particle, one finds $\Lambda \approx H^2$, provided $\Omega_m (\equiv 8\pi G\rho/3H^2)$ is of order unity.

(ii) From the dimensional considerations, it is always possible to write Λ in terms of Planck energy density times a dimensionless quantity [10]:

$$\Lambda \approx 8\pi G\rho_{\text{Pl}} \left[\frac{t_{\text{Pl}}}{t_H} \right]^\alpha \approx t_{\text{Pl}}^{-2} \left[\frac{t_{\text{Pl}}}{t_H} \right]^\alpha, \quad (6)$$

where $t_{\text{Pl}} \equiv (G\hbar/c^5)^{1/2}$ and $t_H \equiv H^{-1}$ are the Planck and Hubble times respectively and $\rho_{\text{Pl}} \equiv c^5/G^2\hbar$ is the Planck energy density. For $\alpha = 2$, which gives the right value of Λ at the present epoch, equation (6) leads to $\Lambda \approx H^2$.

By writing this law as $\Lambda = nH^2$, where n is a constant parameter, the dynamics of the resulting model can be obtained, from equations (2) and (3), as

$$\rho \propto \Lambda \propto H^2 \propto t^{-2}, \quad S \propto t^{2/[3-n(1+\omega)]}, \quad n < 3, \quad (7)$$

where we have considered $k = 0$, as has been suggested by the recent CMBR observations [4]. The cases $n \geq 3$ (where $\rho \leq 0$) are either unphysical or not compatible with $\Lambda = \Lambda(t)$. Note that the ansatz $\Lambda = nH^2$ is equivalent to assuming that Ω_Λ ($\equiv \Lambda/3H^2 = n/3$) is a constant and, hence, so is Ω_m ($= 1 - n/3$) in a flat model. Hence $\rho_v/(\rho + \rho_v) = n/3$ is also a constant. The deceleration parameter, in the model, is obtained as

$$q = \frac{(3-n)(1+\omega)}{2} - 1, \quad (8)$$

which is also constant and implies that $q \gtrless 0$ according as $n \lesseqgtr (1+3\omega)/(1+\omega)$. Thus two different values of the parameter n , viz., one with $n < (1+3\omega)/(1+\omega)$ (say, n_1) and the other with $n > (1+3\omega)/(1+\omega)$ (say, n_2) can make the universe shift from deceleration to acceleration. This is interesting in view of the result obtained by Turner and Riess [11], which shows that the supernovae data favour a past deceleration followed by a recent acceleration, independent of the content of the universe. In fact, this is exactly the case in this model, as can be checked from the constraints on n coming from various observations. Freese et al [12], who derived this model by assuming $\rho_v/(\rho + \rho_v) = \text{constant}$, found that the element abundances from the primordial nucleosynthesis require $\rho_v/(\rho + \rho_v) \leq 0.1$. In terms of the parameter n , this translates to an $n \leq 0.3$ in the early radiation era, implying a deceleration. Let us now see how the present observations constrain the model. It has already been shown that the model fits the high redshift supernovae Ia data (including SN 1997ff at $z \approx 1.7$) very well [13]. Additionally, it also fits the data on the angular size and redshift of the compact radio sources very well [14]. Both the observations require $n \approx 1.5$ and hence predict an accelerating expansion at the present epoch.

Interestingly these constraints are also consistent with the CMBR anisotropy observations which, especially the first peak in the angular power spectrum curve which has been confirmed by various observations, require n to change at a redshift of a few. Thus if the expansion dynamics switches over from deceleration to acceleration at $z = z_1$, the *angular diameter distance* to the last scattering surface (at $z = z_{\text{dec}}$) is given by

$$d_A = \frac{1}{(1 + z_{\text{dec}})} \left[\int_0^{z_1} \frac{dz}{H(n_2; z)} + \int_{z_1}^{z_{\text{dec}}} \frac{dz}{H(n_1; z)} \right]. \quad (9)$$

For the dynamics of the model given by equation (7), this yields

$$d_A = \frac{1}{H_0(1 + z_{\text{dec}})} \left[\int_0^{z_1} (1 + z)^{(n_2-3)/2} dz + \int_{z_1}^{z_{\text{dec}}} (1 + z)^{(n_1-3)/2} dz \right]. \quad (10)$$

If one considers $n_2 = 1.5$ (from the SN and the radio sources data) and $z_1 = 5$ (to be on the safe side in view of the future higher redshift observations), then a value of $n_1 = 0.15$ gives the angle subtended by the Hubble radius $d_H(z_{\text{dec}}, n_1)$ (with $z_{\text{dec}} = 1100$) at the observer as $\approx 0.9^\circ$ which is equivalent to a peak at a Legendre multipole size $\ell \approx 200$. This is exactly what the CMBR anisotropy observations have measured. Note that the parameter space (n_1, z_1) is wide enough which makes the model robust.

Another attractive feature of the model is that it supplies a sufficiently large age of the universe, which is very remarkable in view of the fact that the age of the universe in the FRW model with a constant Λ is uncomfortably close to the age of the globular clusters $t_{\text{GC}} = 12.5 \pm 1.2$ Gyr [15]. The *quintessential* models give even lower age. In Figure 1, we have plotted the expansion age of the universe t_0 as a function of $\Omega_{\Lambda 0}$ in the present model, together with the favoured quintessence model ($\omega_\phi \equiv p_\phi/\rho_\phi = -0.8$) and the FRW model with a constant Λ ($\omega_\phi = -1$). Note that if the required mass density $\Omega_{\text{m}0}$ of the universe was smaller, one could get higher age in these models, as is clear from the figure. This does not, however, seem likely, as the recent measurements give very narrow range of $\Omega_{\text{m}0}$ as $\Omega_{\text{m}0} = 0.330 \pm 0.035$ at one sigma level [5]. By using $H_0 = 72 \pm 7$ km s⁻¹ Mpc⁻¹ (which is recently measured by the *Hubble Space Telescope* key project and is also consistent with a host of other experiments [16]), this value of $\Omega_{\text{m}0}$ gives $t_0 = 12.7 \pm 1.6$ Gyr in the FRW model with a constant Λ . This is roughly consistent with the

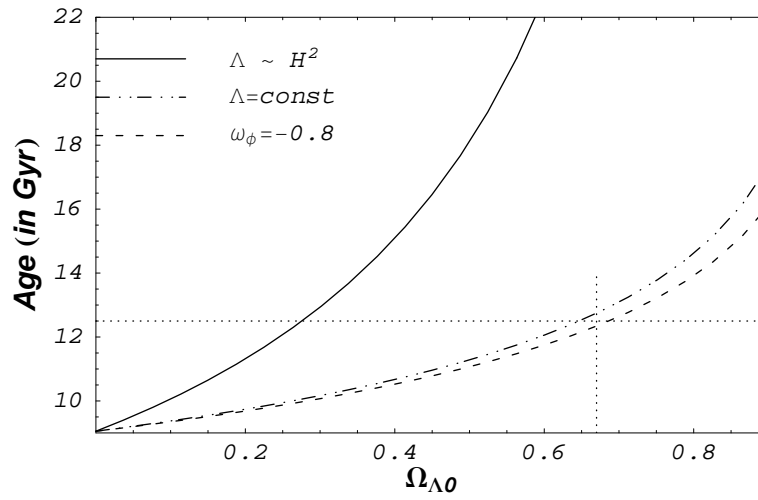


Figure 1: The age of the universe is plotted as a function of $\Omega_{\Lambda 0}$ in some flat models, by using $H_0 = 72 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The horizontal dotted line represents the age of the globular clusters $t_{\text{GC}} = 12.5 \text{ Gyr}$. The vertical dotted line corresponds to the mass density of the present universe ($\Omega_{\text{m}0} = 0.33$).

value $t_0 = 14 \pm 0.5$ Gyr estimated from the CMBR observations, which has been claimed to give more accurate age of the universe [17]. The value of t_0 in the favoured quintessence model is obtained as $t_0 = 12.3 \pm 1.5$ Gyr, which seems in real trouble in view of $t_{GC} = 12.5 \pm 1.2$ Gyr. In this connection it is very encouraging that the model $\Lambda \propto H^2$, where the expression for the age of the universe yields $t_0 \approx 2/(3\Omega_m)H_0^{-1}$, gives $t_0 \approx 27.4 \pm 5.6$ Gyr which is remarkably high.

In light of the successes and achievements stated above of this model, one is inclined to ask if it is just a matter of coincidence that the model is consistent with Mach's ideas and at the same time it solves the cosmological constant problem (at least phenomenologically). Should Mach's principle play some fundamental role in solving the cosmological constant problem? Two concepts (Mach's principle and the cosmological constant), invoked by Einstein, and later dismissed by himself, seem to be unavoidable. Are they really interlinked in some intricate way? Only the future will answer these questions.

ACKNOWLEDGEMENTS

The author thanks Professor J. V. Narlikar for useful discussion and the Department of Atomic Energy, India for support of the *Homi Bhabha post-doctoral fellowship*.

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