

**SPONTANEOUS SYMMETRY BREAKING IN NON-INERTIAL FRAMES AND CURVED SPACE-TIME**

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The conformally coupled Higgs scalar field (with  $\lambda\phi^4$  quartic term) is studied in a class of non-lorentzian background metrics. It is shown that the spontaneous symmetry breaking depends crucially on the choice of the background metric. Various implications are discussed.

*1. Introduction.* Consider a massless, conformally invariant scalar field described by the action

$$S = \frac{1}{2} \int \sqrt{-g} d^4x (g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{6} R \phi^2 - \frac{1}{6} \lambda \phi^4), \tag{1}$$

where  $R$  is the scalar curvature of the background space-time. To develop the formalism further, assume the space-time metric to have the form

$$ds^2 = a^2(t)[dt^2 - d\chi^2 - \sinh^2\chi (d\theta^2 + \sin^2\theta d\phi^2)], \tag{2}$$

with an arbitrary  $a(t)$ . Let the vacuum expectation value (VEV) of the Higgs field be,

$$\langle 0 | \phi(x, t) | 0 \rangle = \eta(x, t). \tag{3}$$

In the standard formalisms of field theory (in flat space, inertial coordinate frames) the lagrangian will be translationally invariant which implies that  $\eta(x, t)$  must be a constant independent of space-time coordinates [1]. In our lagrangian, however, external potentials are present which spoil this invariance. However, the metric in eq. (2) has spatial homogeneity and isotropy implying that  $\eta$  can only be of function of time,

$$\eta(x, t) = \eta(t). \tag{4}$$

The exact dependence can be easily evaluated at the tree level by taking the expectation value of the field equation,

$$\nabla_\mu \nabla^\mu \phi + \frac{1}{6} R \phi + \frac{1}{3} \lambda \phi^3 = 0 \tag{5}$$

and using (at tree level)

$$\langle 0 | \phi^3 | 0 \rangle \approx \langle 0 | \phi | 0 \rangle^3. \tag{6}$$

Substituting for the metric components and  $R$  leads to (dot denotes derivative with respect to  $t$ )

$$\ddot{\eta} + (2\dot{a}/a)\dot{\eta} + (\ddot{a}/a - 1)\eta + \frac{1}{3} \lambda a^2 \eta^3 = 0. \tag{7}$$

Clearly  $\eta = 0$  is a solution; but we have to find the most stable solution. Substituting,

$$\eta(t) = \sqrt{3/\lambda} [1/a(t)] f(t) \tag{8}$$

gives,

$$\ddot{f} + f^3 - f = 0. \tag{9}$$

A simple analysis shows that the stable solution is  $f^2 = 1$  rather than  $f^2 = 0$ . Thus the coupling to the metric induces the VEV,

$$\langle 0 | \phi | 0 \rangle = \sqrt{3/\lambda/a(t)}. \tag{10}$$

We shall now consider two physically relevant metrics in which eq. (10) holds.

*2. Open Friedmann model.* The open Friedmann model is described by a metric of the form eq. (2) with

$$a(t) = a_0 (\cosh t - 1) \tag{11}$$

(dust source). From eq. (10) it follows that the sym-

metry is *broken* in the early phase of the universe and is restored for large  $a$  (present epoch). Thus this effect goes against the standard high temperature restoration of symmetry. It is interesting to see which effect predominates. Let us assume that symmetry breaking occurs due to this mechanism. Then it can be shown, by a simple analysis, that the temperature required (at any time  $t$ ) to restore the symmetry is given by,

$$T_{st} \approx \sqrt{3/\lambda}/a(t). \tag{12}$$

In a radiation filled model, the actual temperature goes as

$$T_r = B/a(t), \quad B = \text{const.}, \tag{13}$$

leading to,

$$T_{st}/T_r = \text{const.} \tag{14}$$

Thus the universe is always hotter (or colder) than the temperature required to break (or retain) the symmetry. Hence no high temperature phase transition can take place in this model.

We have not yet specified the source for the background metric. It may be an externally specified matter distribution; or one can generate the metric completely from the vacuum energy density of the broken symmetric phase. Since the vacuum has negative energy density this leads to a non-singular evolution,

$$a(t) = (4\pi G/3\lambda)^{1/2} \cosh t. \tag{15}$$

All our previous results are still applicable (for details see ref. [2]).

3. *The non-inertial frame Milne universe.* Our result in eq. (10) is independent of the form chosen for  $a(t)$ . However for

$$a(t) = Ae^t \tag{16}$$

the space-time described by the metric in eq. (2) is flat. The transformations,

$$r = Ae^t \sinh \chi; \quad T = Ae^t \cosh \chi \tag{17}$$

lead back to the inertial coordinates,

$$ds^2 = dT^2 - dr^2 - r^2 (d\theta^2 + \sin^2\theta d\phi^2). \tag{18}$$

However, the scalar field still has a nonzero *vacuum expectation value* in this frame,

$$\langle 0|\phi|0\rangle = \lambda^{-1/2} e^{-t}/A. \tag{19}$$

This shows that the process of symmetry breaking is frame dependent and not generally covariant.

A “reverse” result exists in the Rindler universe with a metric

$$ds^2 = (1 + gx)^2 dt^2 - dx^2 - dy^2 - dz^2, \tag{20}$$

which is related to the inertial frame  $(T, R)$  by the transformation,

$$(1 + gX) = (1 + gx) \cosh gt, \\ gT = (1 + gx) \sinh gt. \tag{21}$$

This frame does not *introduce* any symmetry breaking. However, if one starts with a lagrangian,

$$L = \frac{1}{2}(g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \mu^2 \phi^2 - \frac{1}{6} \lambda \phi^4) \sqrt{-g} \tag{22}$$

(with the sign of  $\mu^2$  reversed, so that the symmetry is broken) then the symmetry can be restored for values of acceleration,

$$g \geq 4\pi\lambda^{-1/2} (\mu c^3/\hbar) \tag{23}$$

(for a detailed derivation see ref. [2]). This result can be anticipated from the conventionally quoted result that a uniformly accelerated frame is equivalent to a thermal bath with temperature [3]

$$T = g/2\pi, \tag{24}$$

and the fact that the symmetry is restored at high temperatures.

4. *Conclusions.* Detailed investigation is necessary to ascertain the practical implications of this result. It is likely that one has to take into account these effects while considering phase transitions in the early universe. Also the behaviour of the system in a non-inertial frame seems to indicate non-covariance of the symmetry breaking scheme.

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*References*

[1] E.S. Abers and B.W. Lee, Phys. Rep. 9 (1973) 1.  
 [2] T. Padmanabhan, TIFR-TAP Preprint 21 (Nov. 1981).  
 [3] See e.g. DeWitt, in: General relativity, eds. S.W. Hawking and W. Isrel (Cambridge U.P., London, 1979).