NONCOSMOLOGICAL REDSHIFTS

(Invited Review)

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Abstract. Six decades ago Edwin Hubble found his velocity-distance relation that soon became the observational foundation of modern cosmology. According to the *Cosmological Hypothesis* (CH), all extragalactic objects – galaxies and the quasi-stellar objects – derive their redshifts from the expansion of the Universe.

This article reviews the evidence for and against the CH. To what extent is it universal? Does it provide the entire redshift of an extragalactic object? If an extra, noncosmological component of redshift is present, what is it due to? On the observational side the evidence presented here is of three kinds: (i) evidence that is *prima facie* consistent with the CH, (ii) neutral evidence that can be reconciled with the CH with a few epicycles, and (iii) discordant evidence which, if accepted, suggests that some objects at least possess substantial noncosmological redshifts.

The final part of this review discusses the various theories proposed to account for noncosmological redshifts and outlines further tests to establish the validity or otherwise of the CH.

Table of Contents

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1. Historical Background

Sixty yearts ago a paper with the somewhat unexciting title 'A relation between distance and radial velocity among extragalactic nebulae' by Edwin Hubble was communicated on January 17, 1929 to the Proceedings of the National Academy of Sciences, U.S.A. (Hubble, 1929). The author was cautious in attaching significance to his findings as he stated in his concluding remarks:

"New data to be expected in the future may modify the significance of the present investigation or, if confirmatory, will lead to a solution having many times the weight. For this reason it is thought premature to discuss in detail the obvious consequences of the present results..."

'New data' compiled by Humason and Hubble did confirm these initial findings and in a way launched cosmology as an observationally testable branch of science. The 'relation' discovered by Hubble became firmly established as Hubble's law. Stated simply as a linear equation

$$v = HD\,,$$

this law tells us that the radial velocity (v) of recession of an extragalactic nebula is proportional to its distance (D) from us. The constant of proportionality (H) is called the Hubble constant.

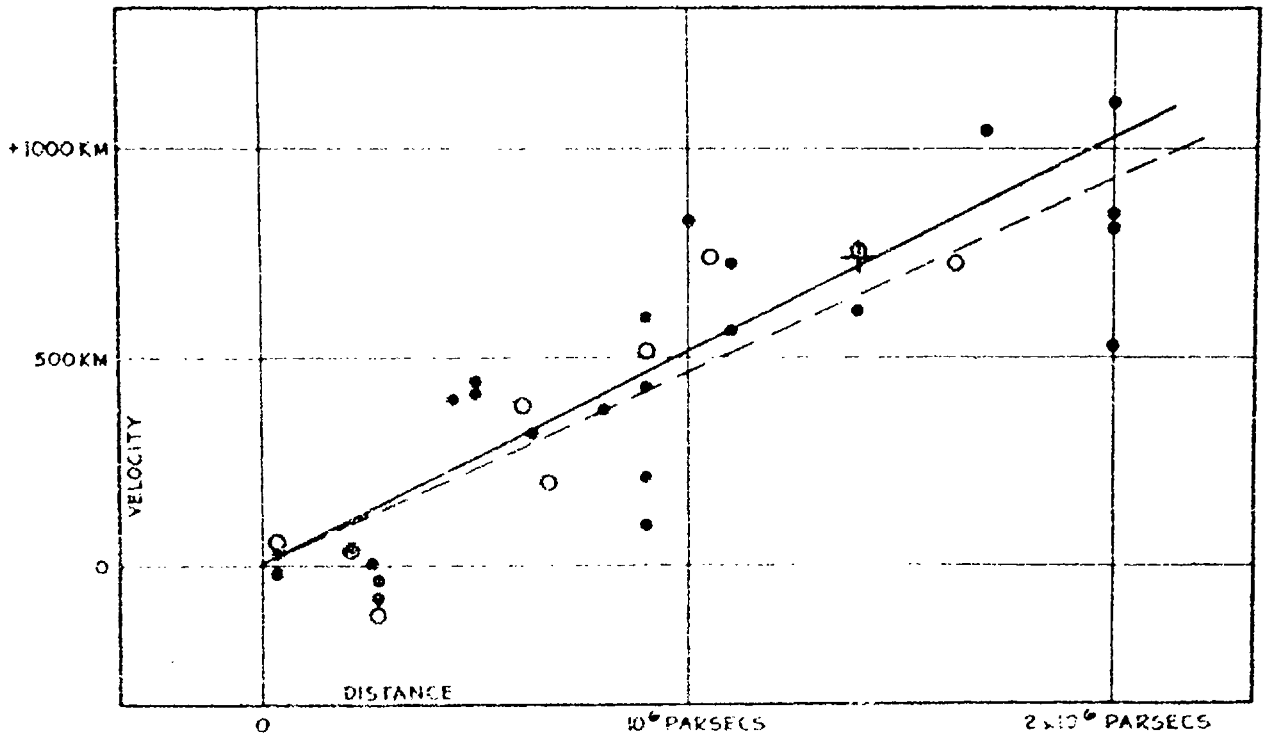

Fig. 1. The original velocity distance plot of Hubble reproduced from his paper in 1929 (Hubble, 1929). The black discs and continuous line represent the solution for nebulae taken individually while the circles and the broken line represent the solution combining the nebulae into groups. The cross represents the mean velocity corresponding to a mean distance of 22 nebulae whose distances could not be estimated individually.

Figure 1 is reproduced here from Hubble's original paper (*op. cit*). The radial velocities going up to a little above 1000 km s^{-1} are estimated from the 'redshift' z of the absorption lines in the spectra of galaxies. In a typical situation, the line is found at a wavelength λ instead of its familiar laboratory value λ_0, where

$$z = \frac{\lambda - \lambda_0}{\lambda_0} . \tag{1}$$

For $z \ll 1$, as was the case with Hubble's data, the Newtonian Doppler effect formula is used to calculate v:

$$v = cz , \tag{2}$$

where c is the speed of light.

The distance D was estimated from the standard magnitude measurements:

$$m - M = 5 \log D - 5 . \tag{3}$$

Here D is measured in parsecs while m and M are the apparent and absolute magnitudes of the galaxy. Hubble's own estimate for the constant (K instead of H in his paper!) was $\sim 530 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

Spectral shifts per se were not unfamiliar to the astronomers of the 1920s. Doppler shifts due to stellar motions in the Galaxy were known, while the idea of gravitational redshifts had just become accepted with the advent of the general theory of relativity. But the z-values in both cases were small (not exceeding 10^{-4}–10^{-3}), more than an order of magnitude smaller than those observed by Hubble and Humason. Moreover, while the stellar motions generated blueshifts as well as redshifts (i.e., $z < 0$ as well as $z > 0$) the nebular spectra almost invariably showed redshifts.

It appeared, therefore, that a new effect was at work, that called for a different theoretical explanation. Indeed, an explanation already existed in the literature but had gone unnoticed. Several years before Hubble's law became known, Friedman (1922, 1924) had constructed mathematical models of an 'expanding universe', in which the proper distance between any two galaxies increased systematically with time, leading thereby to a redshift precisely of the Hubble kind.

In modern terminology we may describe the effect with the help of the so called Robertson–Walker (R–W) line element that characterizes the spacetime geometry of a very simple model universe – one that is homogeneous and isotropic (Robertson 1929,

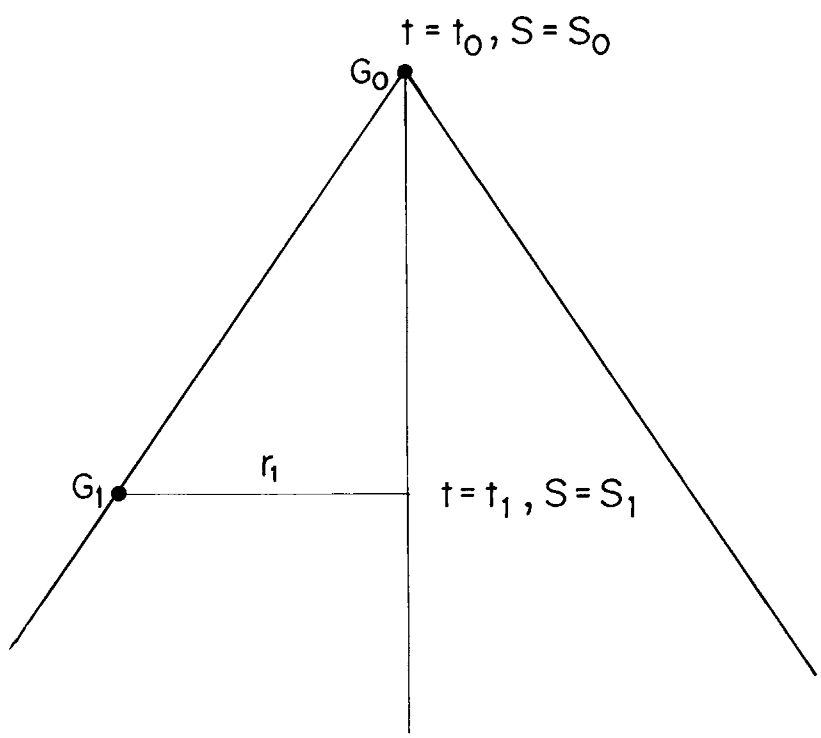

Fig. 2. The backward light cone from the observer in galaxy G_0. A galaxy G_1 at the coordinate distance $r = r_1$ is seen by the observer at the present time t_0 although the light left G_0 at the earlier epoch t_1. The observed cosmological redshift of G_1 can be related to the ratio of the scale factors of the Universe at these two epochs.

1935; Walker, 1936). In such a universe galaxies have well-ordered motion so that the world line of a typical galaxy may be denoted by three fixed parameters. It is convenient to choose these as the spherical polar coordinates (r, θ, ϕ). The symmetry (of homogeneity and isotropy) applies to space-like hypersurfaces labeled by a time-ordered sequence. The time (t) coordinate is chosen to be the proper time of a typical galaxy. Since this time has a global significance it is often called the cosmic time. In terms of (t, r, θ, ϕ) the R–W line element is written as:

$$ds^2 = c^2\,dt^2 - S^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]. \tag{4}$$

In the above expression the parameter k (= 0,1, or − 1) indicates whether the spaces t = constant are flat, closed, or open. The function $S(t)$ is the scale factor that tells us how the proper distance between two galaxies changes with time. In an expanding universe $S(t)$ increases with t.

Figure 2 shows a spacetime diagram in which a galaxy G_0 at $r = 0$ receives light at $t = t_0$ from a galaxy G_1 at $r = r_1 > 0$. This light would have left G_1 at an earlier epoch t_1 given by the general relativistic equation of null-ray propagation:

$$\int_0^{r_1} \frac{dr}{\sqrt{1-kr^2}} = \int_{t_1}^{t_0} \frac{c\,dt}{S(t)}\,. \tag{5}$$

It is then not too difficult to show that the light waves from G_1 to G will be redshifted by

$$z_1 = \frac{S(t_0) - S(t_1)}{S(t_1)}\,. \tag{6}$$

The redshift is called 'cosmological redshift'. For $z_1 \ll 1$ we get

$$z_1 = \frac{H}{c}\,D_1\,, \tag{7}$$

with

$$D_1 = r_1 S(t_0)\,, \qquad H = \left.\frac{\dot{S}(t)}{S(t)}\right|_{t=t_0}\,. \tag{8}$$

For a derivation of these relations see Narlikar (1983).

The expanding universe model thus not only accounts for the observed Hubble relation but it also conforms with the Copernican principle. Since the hypersurfaces t = constant are homogeneous, we can take any galaxy to have coordinate $r = 0$. Further, because of the isotropy of the R–W line element, the relation (7) does not depend upon direction. These two symmetries are known by the so called 'cosmological principle'.

The (r, θ, ϕ) coordinate frame is called the 'cosmological rest frame'. In general a galaxy may not have r, θ, ϕ strictly constant; this effect will show as small random motions of $\lesssim 300$ km s^{-1} with respect to the rest frame. The 'net' spectral shift of such a galaxy may be written as z, when

$$1 + z = (1 + z_C)(1 + z_D)\,. \tag{9}$$

Here z_C is the cosmological redshift derived in (6) and z_D is the additional Doppler shift due to the above mentioned random motions.

Notice that for very nearby galaxies z_C is small and could be swamped by the Doppler component which can be of either sign. This accounts for why Hubble found a few blueshifts in the nearby sample that he first examined.

In the early days it was customary to express z_C in velocity units, as cz_C, and call them 'velocity shifts'. This jargon is technically incorrect as we have seen that z_C arises from the geometrical effects of the curved spacetime and not from velocities. Unfortunately many astronomers still use the name 'velocity shift' for z_C when it should really be used for z_D. In discussing astronomical data of nearby objects we will use the velocity units for z_C wherever they have been so used by the observers. However, we will avoid the jargon 'velocity shift' for either z_C or z_D, calling the latter 'Doppler shift'.

Hubble's law happens to be the basis on which the entire superstructure of cosmology is erected. Since many profound conclusions about the early history and the large scale structures of the universe are drawn today, on the basis of Hubble's law, it is necessary to take stock of the situation from time to time. In this article we will review the modern evidence to assess the extent to which Hubble's law is valid. We begin by laying down the rules of the game.

2. Methods of Assessment

2.1. The Cosmological Hypothesis

The validity of Hubble's law can be tested in the form of a hypothesis which we shall call the 'cosmological hypothesis', CH in brief. It makes the following statement:

'The redshift of any extragalactic object is almost entirely due to the expansion of the Universe.'

The qualification 'almost entirely' allows for a small noncosmological component such as z_D in Equation (9). We may also allow a small component of gravitational redshift z_G and write

$$1 + z = (1 + z_C)(1 + z_D)(1 + z_G)\,. \tag{10}$$

The CH requires that *in all cases* of extragalactic objects $|z_D| \ll 1$, $|z_G| \ll 1$, and $z \simeq z_C$.

In case we find a counter-example to the CH in the form of an object for which z and z_C are not nearly equal we write

$$1 + z = (1 + z_C)(1 + z_{NC})\,. \tag{11}$$

In this case z_{NC}, the 'noncosmological' redshift is expected to be substantial, may even be comparable to z. It may happen that z_{NC} is entirely due to the Doppler or the gravitational effect. If not then one must allow for 'new' effects either missed by conventional physiscs or requiring new physics.

We next discuss possible ways of testing the CH: it is convenient to label them by letters A to F since they fall in different categories.

2.2. The distance-redshift relation (A)

The relation (6) or (7) contains the hall mark of CH, namely that the redshift z is due (and solely due) to the distance D of the extragalactic source from us. Our derivation of (7) was approximate, being valid only for small $z\,(\ll 1)$. For any z, we expect the cosmological model to provide a unique relation of the form $D = f(z)$ which, for $z \ll 1$ reduces to the linear relation $D = cz/H$.

Mattig (1958) gave such a relation for the dust-filled Friedman models. It is convenient to express the result in terms of the present value of Hubble's constant. Henceforth, we will use t_0 to denote the present epoch and the subscript (0) to indicate that the physical quantity has been measured at t_0.

Accordingly we write H_0 for the present value of H and denote by q_0 the present value of the 'deceleration parameter':

$$q = -\frac{1}{H^2}\frac{\ddot{S}}{S}\,. \tag{12}$$

The distance D is defined as the 'luminosity distance', i.e., it tells us that the total flux of radiation received from a source of luminosity L is given by

$$l = \frac{L}{4\pi D^2}\,. \tag{13}$$

With these definitions, Mattig's formula is:

$$D = \frac{c}{H_0}\frac{1}{q_0^2}\left[q_0 z + (q_0 - 1)\left\{\sqrt{1 + 2q_0 z} - 1\right\}\right]. \tag{14}$$

For an elementary derivation of this formula, see Narlikar (1983). In Figure 3, the $D - z$ curves are drawn for various q_0.

A test of Hubble's law is in principle provided by the formula (14). A relatively scatter-free $D - z$ diagram with a curve following (14) will not only establish Hubble's law out to large redshifts but will also tell us the value of the deceleration parameter and, hence, the curvature parameter k. For, it can be shown that $k = 0$, 1, or -1 according as $q_0 = \frac{1}{2}$, $>\frac{1}{2}$, or $<\frac{1}{2}$.

2.3. The effects of non-Euclidean geometry (B)

Observations of discrete source distributions out to large redshifts provide us (in principle) with a handle on the geometry of the large-scale structure of the Universe.

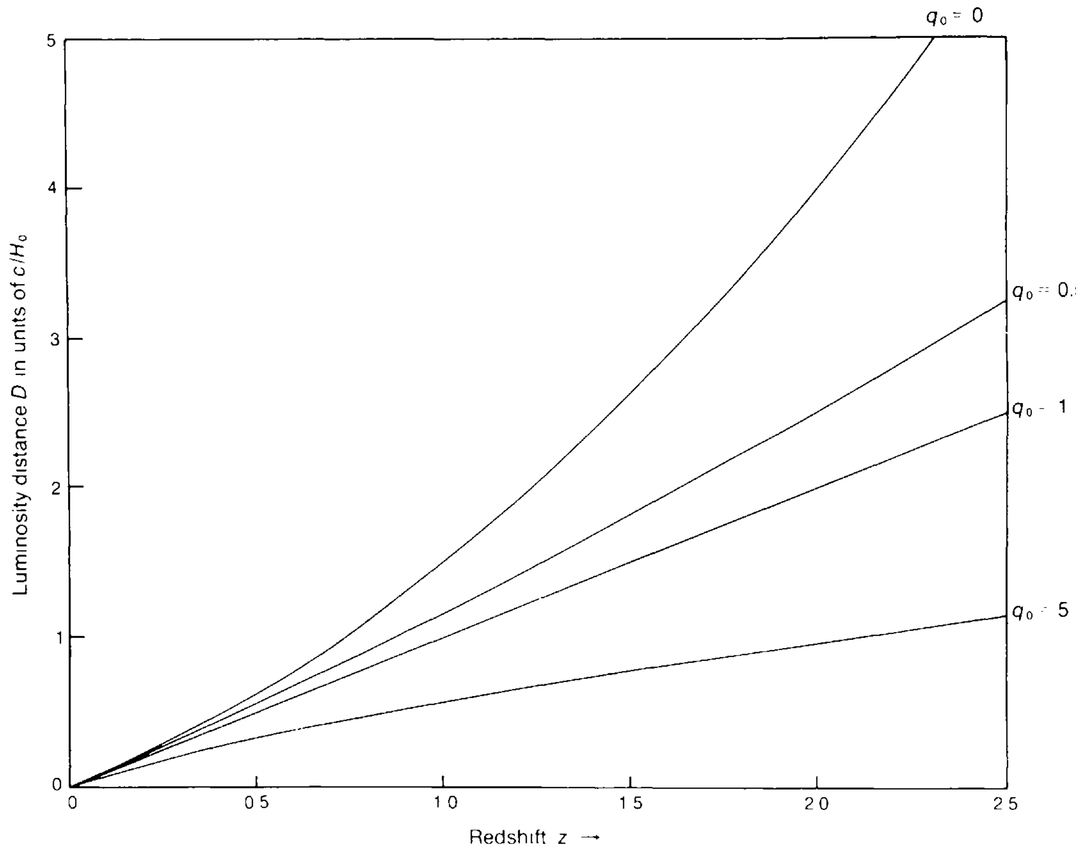

Fig. 3. The luminosity distance plotted as a function of redshift for various values of q_0 in Friedman models.

Here Hubble's law enters indirectly in the sense that the theoretically expected results of measurements like the number density, projected angular size, etc., are based on its validity. Nevertheless, such tests are valuable inputs into deciding the validity of the CH. Indeed, it was Hubble himself who first made the attempt (cf. Hubble and Tolman, 1935; Hubble, 1936) to look for non-Euclidean effects in nebular counts. That effeort proved futile for reasons which we shall consider later (cf. Section 3.2).

2.4. The near-neighbour criterion (C)

The CH and its consequence a relation like (14) imply that two extragalactic objects in close physical proximity should have the same redshift.

This criterion can be used in different ways. Close physical proximity can be established through visible material connections (e.g., bridges, filaments, jets, etc.) linking the two objects. Physical superposition, like a QSO found in the nucleus of a galaxy can be another way. A third and less direct way is through statistics. Given the angular distribution of one type of object (say, a QSO) on the sky, one can calculate the probability of its being found within a given angular distance of the other object (say, a galaxy). If the probability turns out to be very small (say < 0.01), then one can discount the apparent proximity as being due to a chance projection effect. In other words, a low enough probability is an indication that the objects in question are real physical neighbours.

If physical nearness can be established then the redshifts of the two objects may be measured to check if they are equal as required by the CH (caution: very often this criterion is used the wrong way round. The redshifts are measured first and their equality or otherwise is used to state whether the objects are neighbours or not. This procedure cannot test the CH.)

If the redshifts of physical neighbours are discrepant, then it is usual to assume that the object with the larger redshift has $z_{NC}, > 0$. Such cases are referred to in the literature as 'anomalous redshift pairs' or 'discrepant redshift pairs'.

2.5. Background vs foreground systems (D)

If an object of high z is very distant then light coming from it will interact with systems located en route from the source to the observer. This could show up through absorption effects in the continuum or the line spectrum of the source or throught the gravitational bending of its light by massive deflectors.

In such cases if it can be established that the foreground system has a redshift according to the CH, and that it is less than the redshift of the source, then we get a result for the source that is consistent with the CH, and in some cases even establish Hubble's law for the source.

2.6. Physics and morphology of the source (E)

It may happen that the physics of the source or its structural details are easier to explain if the CH is correct. In that case we have indirect support for the CH. On the other hand, if the same considerations make it difficult to understand what is going on in the source, or require somewhat improbable epicycles to make everything consistent, then we should suspect the validity of the CH.

2.7. Extraordinary effects (F)

In science one must always be prepared for the unexpected. Astronomy has contributed its share to the list. There is no reason to believe that the science we know today is complete or that we have observed basically all the basic natural phenomena.

It is against this background that we have to look at observations that may be truly inexplicable in terms of the physics we know: observations that pose difficulties for the CH to explain. In this article we will look at such observations as well.

In Section 3.5 we will look at observations that come under three separate heads: (i) evidence consistent with the CH, (ii) neutral evidence that could be interpreted either way, and (iii) evidence not consistent with the CH. This type of categorization is due to Burbidge (1973). The testing methods A–F cut right across the three categories of evidence.

3. Evidence Consistent with the Cosmological Hypothesis

In this section we will examine the kind of evidence that is, *prima-facie*, consistent with the CH. That is, if Hubble's law is valid then the observations presented here receive a natural interpretation. In the language of logic, the observations are necessary, but not always sufficient for Hubble's law.

3.1. The magnitude-redshift relation for galaxies (A)

Coupling (3) with (14) we get the $m - z$ relation in Friedman models in the following form:

$$m - M = 5 \log \left\{ \frac{c}{H_0 q_0^2} \right\} - 5 + 5 \log [q_0 z + (q_0 - 1)\{\sqrt{1 + 2q_0 z} - 1\}] . \tag{15}$$

If we use the Taylor expansion for small z and write $H_0 = 100 h_0 \text{ km s}^{-1} \text{ Mpc}^{-1}$, then to order z we get (cf. Narlikar, 1983)

$$m - M = 42.38 - 5 \log h_0 + 5 \log z + 1.086 (1 - q_0) z + O(z^2) . \tag{16}$$

Figure 4 shows the theoretical curves drawn on the basis of (15). All curves with different q_0 fan out at large z, although for $z \to 0$ they all merge into the linear Hubble law.

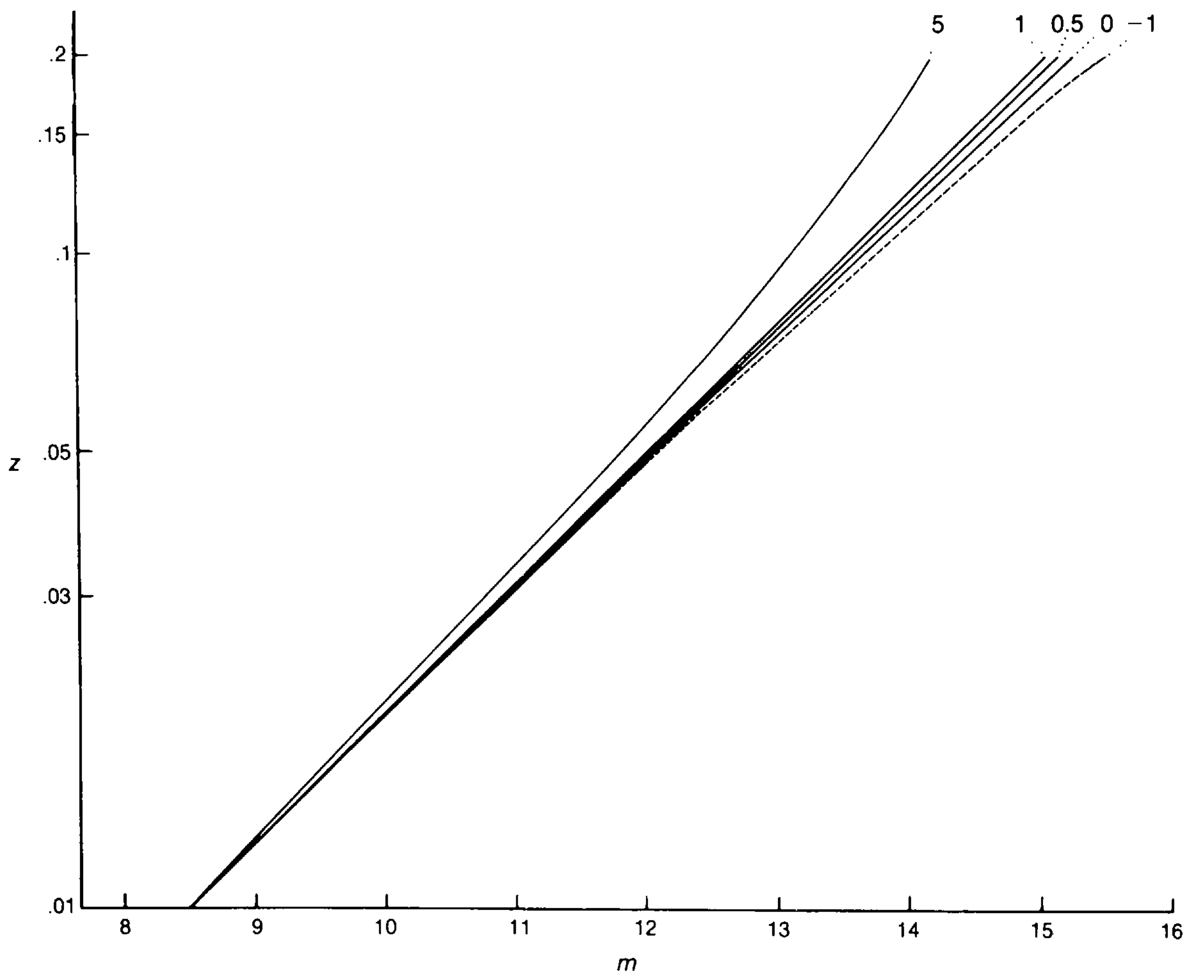

Fig. 4. Theoretical $m - z$ curves for different values of q_0.

Had the astronomer a direct way of measuring D, he would have preferred to plot the $D - z$ diagram of Figure 3 instead of the $m - z$ diagram of Figure 4. But such a method not being available, one has to do with Equations (15) and (16).

Sandage and his colleagues have made pioneering studies of the $m - z$ relation for galaxies in order to determine not only the true value of H_0 but also q_0. An excellent overview was given recently by Sandage (1987) highlighting the progress and the problems of this programme. Some important references to the series of attempts to measure q_0 are: Sandage (1961, 1968, 1972a, b, 1975a, b), Sandage and Hardy (1973), Gunn and Oke (1975), Gunn and Tinsley (1975), Kristian *et al.* (1978), Sandage and Tammann (1983).

Several uncertainties intervene to make the data less decisive than initially hoped for (a fuller discussion of these may be found in Narlikar, 1983). We simply enumerate them to caution the reader not to take too simplistic a view of the situation.

(i) *Local motions* relative to the cosmological rest frame which make it hard to pinpoint what exactly is that frame;

(ii) *the uncertainty of* h_0 with different observers favouring different values in the range $0.5 < h_0 < 1$ (we will elaborate on this at the end of this subsection);

(iii) *the aperture correticon* that corrects for the nebular (rather than point like) nature of galaxies as light sources;

(iv) *the K-correction* that allows for the shift in the wavelength band (due to redshift) over which radiation is measured;

(v) *the Malmquist bias* which truncates the observed luminosity distribution of a cluster of galaxies at progressively higher values for the lower limit as one looks at more remote clusters;

(vi) *the Scott effect* which makes the observer pick out intrinsically brighter galaxies at higher redshifts;

(vii) the possible but as yet inadequately understood effects of *intergalactic absorption*;

(viii) the model-dependent effects of *luminosity evolution* in galaxies due to the aging of stars in them; and

(ix) the scatter in m arising from the *scatter in M* for galaxies in a cluster (see Figure 5, for example).

On the last count, Sandage (1968) found that a good standard candle is provided by the first ranked member galaxies in clusters. Typically these are massive ellipticals whose luminosities do not appear to have much scatter when compared in different nearby clusters. The luminosity evolution aspect (ix) does however imply that even the standard candle might evolve with age.

Figure 6 illustrates the relatively scatter-free $m - z$ diagram obtained with first ranked cluster members. This lends considerable confidence to the view that at least for galaxies the Hubble law applies. The other uncertainties and the scatter in Figure 5 are such, however, as to make the deduction of the true value of q_0 almost impossible.

The use of radio galaxies has also been quite promising. Spinrad and Djorgovski (1987) have combined the $m - z$ data on the 3C-R radio galaxies with that on the birghtest cluster galaxies (with m measured as the photovisual magnitude). A com-

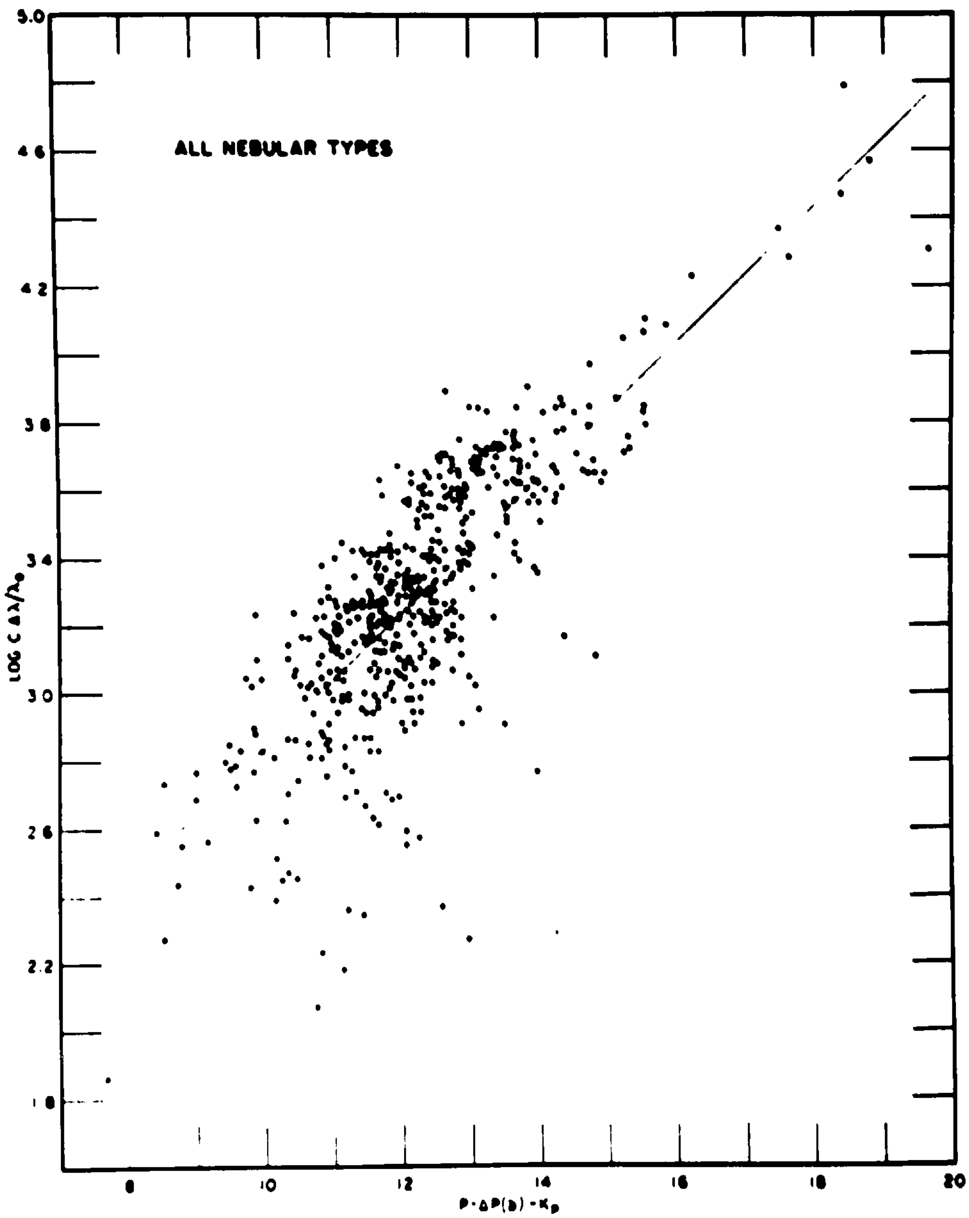

Fig. 5. The $m - z$ plot for bright galaxies from Humason *et al.* (1956).

parison with the theoretical curves for $q_0 = 0$ and $q_0 = 0.5$ and two models for light evolution in galaxies shows that it is not possible to discriminate between them. If the galaxies are observed in the near infrared (near 2μ) the light would be largely coming from old stars for whom the luminosity evolution is expected to be minimal. From this plot, Spinrad and Djorgovski find that $q_0 = 0.2$ and $\lambda = 0$ give the best theoretical fit.

On the other hand Wampler (1987) has advocated a much larger value of $q_0 (= 3)$, based on the observation that both radio galaxies and the QSOs give a well-defined Hubble relation if the Baldwin luminosity criterion (Baldwin, 1977) is used to correct the observed magnitudes of QSOs. Lilly and Longair (1984) also find that unless q_0 is

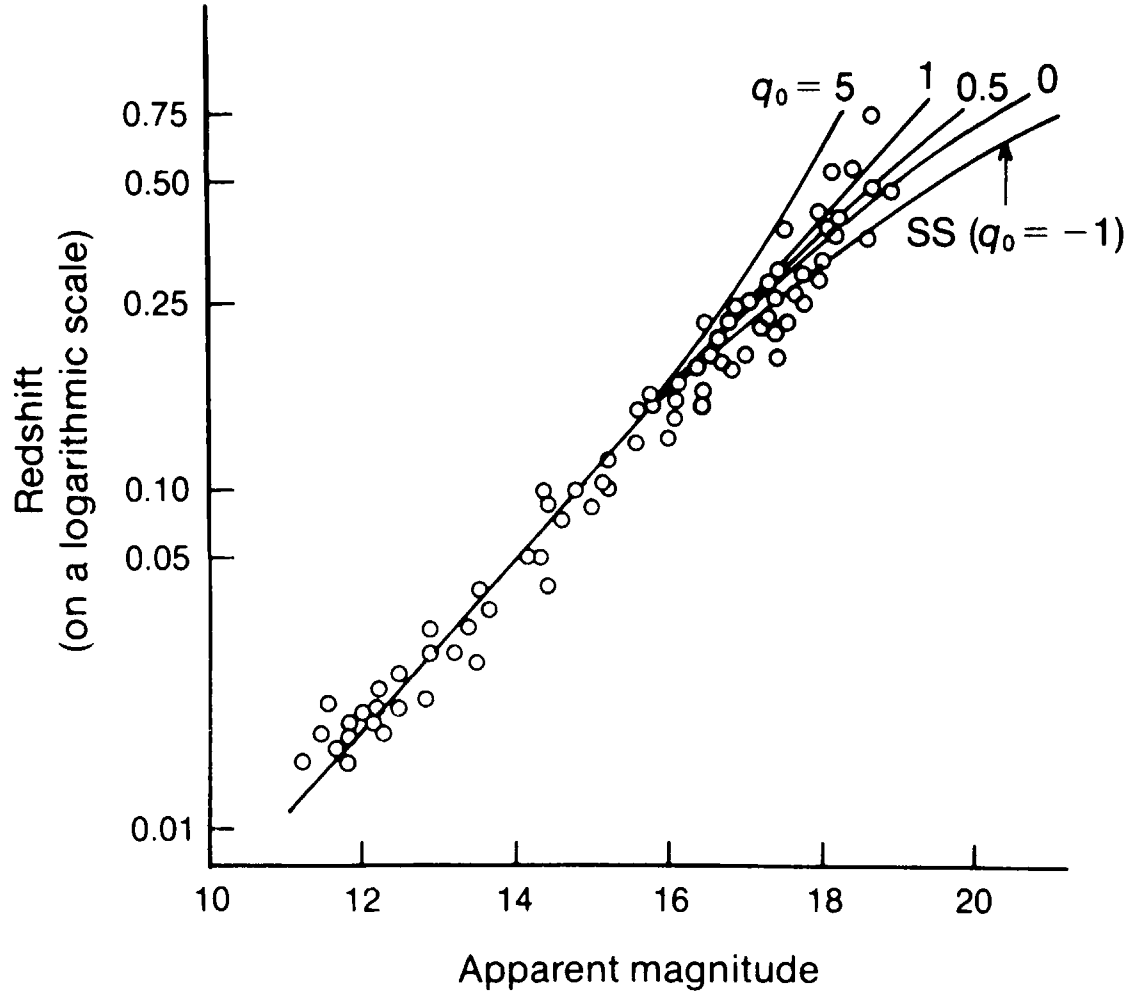

Fig. 6. The $m - z$ plot for first ranked cluster members: from the paper by Kristian *et al.* (1978). Theoretical curves for different q_0 are superposed on it. The SS stands for the steady-state model.

very high, the fit with the data in 2.2μ infrared photometry of 3CR galaxies requires strong luminosity evolution.

Sandage (1987) has aptly summarized the basic difficulty in this test, in the dictum of Walter Baade:

'You must understand the galaxies before you can get the geometry right.'

Perhaps the same dictum is responsible for the continuing uncertainty about the value of H_0. In the relations (15) or (16) the uncertainty shows up through the parameter h_0 whose value may push the $m - z$ curve up or down along the m-axis. Two schools have persisted, one led by Sandage favouring $h_0 \sim 0.5$, the other by de Vaucouleurs favouring $h_0 \sim 1$. For arguments on both sides see, for example, Tammann (1987) and Aaronson (1987) and also de Vaucouleurs (1986).

The situation may in fact turn out to be less simple than presently believed. There is increasing evidence of departure from the cosmological principle on the scale ranging from clusters to supercluster. The nonuniformity of motion shows itself in the value of

Hubble's constant. Thus H_0 seems to depend on the type of sample used. On smaller scales (cluster) the lower value $h_0 \sim 0.5$ seems to emerge while on the larger scale (supercluster), h_0 appears to be closer to unity. We will return to this aspect of the Hubble's constant in Section 5.1.

In spite of these problems and uncertainties we can conclude that by and large the $m - z$ relation for galaxies, at least for non-local regions and for first ranked cluster members is in reasonable agreement with Hubble's law.

3.2. The number-distance relation (B)

As mentioned earlier Hubble (1953) himself had made unsuccessful attempts to decide the geometry of spacetime by counting galaxies out to large distances. His efforts were inconclusive because galaxies are far too many to count out to distances over which significant geometrical effects might be expected. Moreover, their intrinsic properties vary considerably over the entire population and one is also faced with the uncertain evolutionary effects if one wants to extend the surveys to very long look-back times.

To what extent have more recent surveys been successful in this respect? Several galaxy-counting programmes down to $m \sim 24$ have been attempted (see, for example, Tyson and Jarvis, 1979; Karachentsev and Kopylov, 1977; Peterson *et al.*, 1979, etc.). The attempt is, as in Hubble's case, to look for deviation, if any, from the Euclidean result

$$\frac{\mathrm{d}\,\log N(<m)}{\mathrm{d}m} = 0.6\,. \qquad (17)$$

A theoretical exercise by Narlikar and Burbidge (1981) to fit the data points with the empty Friedman model ($q_0 = 0$: this is in fact a flat space model) showed that a reasonable fit *without evolution* can be obtained (see Figure 7).

Again, several questions intervene before one can really assert that the fit proves anything. Are the objects being counted down to 24^{m} (or even fainter ones) really galaxies and not stars or some artifacts? Is the spectrum of a typical galaxy in its rest frame really known, especially in the ultraviolet part? For, this information is essential for the K-correction. To what extent do we know the luminosity function of the galaxies? Is it really affected by evolution? If yes, in what way?

So far as number counts of QSOs are concerned Green (1986) and Schmidt (1987) have argued that strong evolutionary effects are present. They use complete samples of QSOs down to different magnitudes from different surveys and derive luminosity functions that turn out to have strong z-dependence. The thrust of these arguments is not, however, directed towards testing Hubble's law. Rather, the law is taken as correct and evolutionary parameters are chosen to give the best fit to the data.

'Evolution' is thus an issue that has consistently dogged cosmology, preventing the observational probes from diagnosing the real geometry of the Universe. Nowhere has this fact been so effectively demonstrated than in the counting of radio sources – the history of which goes back to the mid-50s.

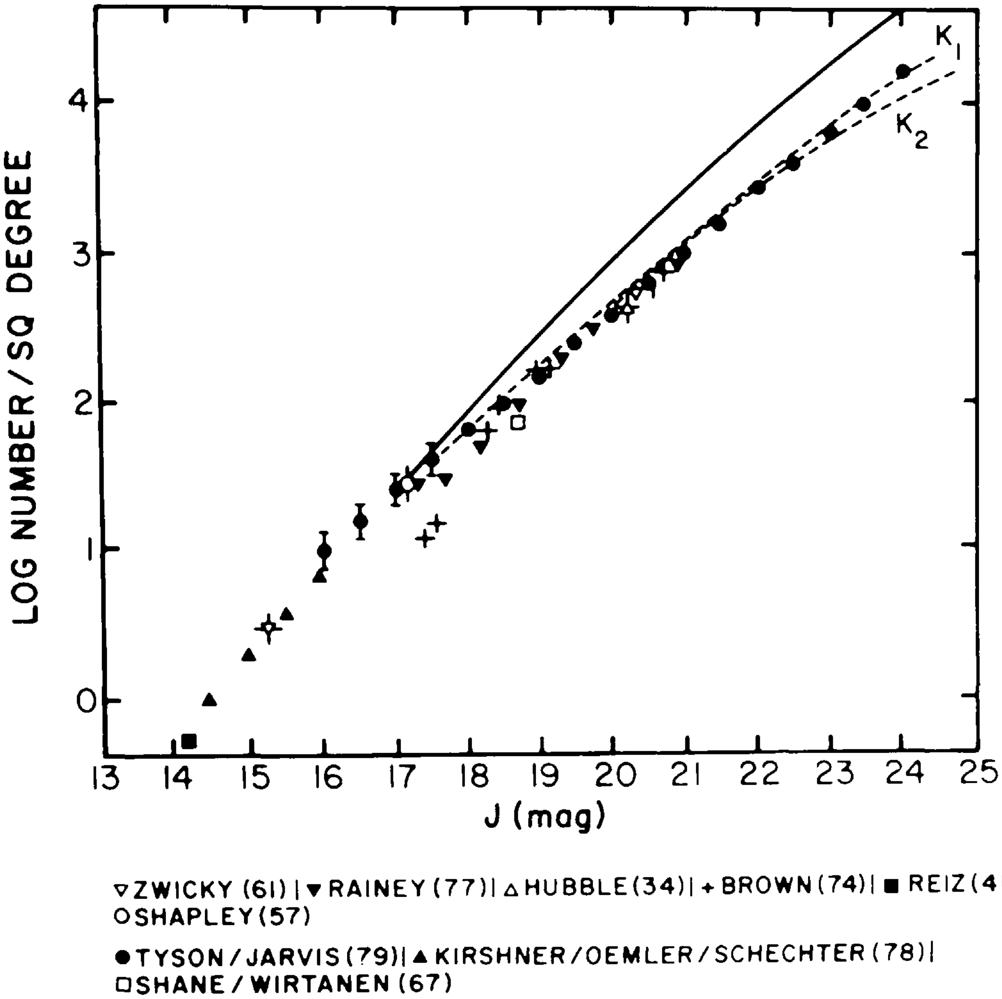

Fig. 7. The $N(m)$ curve for no evolution in a $q_0 = 0$ Friedmann model drawn through the various data points. For details of the plot see Narlikar and Burbidge (1981). The two dotted curves arise from applying two different types of K-correction to the theoretical curve shown by continuous line.

The property of being a strong radio source is not universal in all galaxies. Indeed, one galaxy in a few ten thousands could, on average, be a radio source. Therefore, the radio astronomers felt that radio source counts would more easily reveal the geometry of the Universe than galaxy counts. In the 1950s Mills in Australia and Ryle in Cambridge initiated radio source count programmes on a large scale.

Early claims of cosmological significance have now been replaced by more cautious appraisals of the numerous difficulties, starting with Baade's dictum! How well do we understand the basic phenomenon that makes a galaxy or a QSO a radio source? Do the structural details of the Galaxy (e.g., spiral or elliptical), the time-scale of the phenomenon, the role of the environment (e.g., in a cluster or isolated), the spectral index of the radio source, etc., enter into the source-count problem in subtle ways?

There is another fundamental difficulty. Radio astronomers have often claimed that their surveys cover more remote regions of the Universe than do the optical surveys. Yet, a radioastronomer cannot really know the distance of a radio source unless he has the source optically identified, its redshift measured and Hubble's law applied to it. So the only radio sources whose distances can be confidently asserted are those accessible to optical astronomy.

To do a radio source count properly, it is, therefore, necessary to know the source redshifts. It is worth noting that the only complete sample of radio sources that can claim a near-100% determination of redshifts is the 3CR catalogue (Bennett, 1962). After over two decades this catalogue has now most redshifts determined (Spinrad *et al.*, 1985).

In a radio source survey it is customary to count all sources brighter than a specified flux density S, for various values of S, The number count in a Euclidean universe filled with a uniform distribution of sources must satisfy the relation

$$N_0(>S) \sim S^{-3/2}. \qquad (18)$$

It is customary to plot the ratio $\Delta N/\Delta N_0$ (of differential source counts actually observed in the range $(S, S + \Delta S)$ and the expected Euclidean values) as a function of S. Figure 8 shows a plot of this kind for several surveys put together. The number count is 'super-Euclidean' if $\Delta N/\Delta N_0 > 1$ and 'sub-Euclidean' if $\Delta N/\Delta N_0 < 1$.

Cosmological implications of the source counts have been discussed from time to time. For a recent survey of the situation see Kellermann and Wall (1987). These authors conclude that except for the strongest sources, the observed counts are consistent with a homogeneous distribution in the relativistic Friedman model. So far as strong sources are concerned there appears to be a deficiency of a few nearby sources. This point has

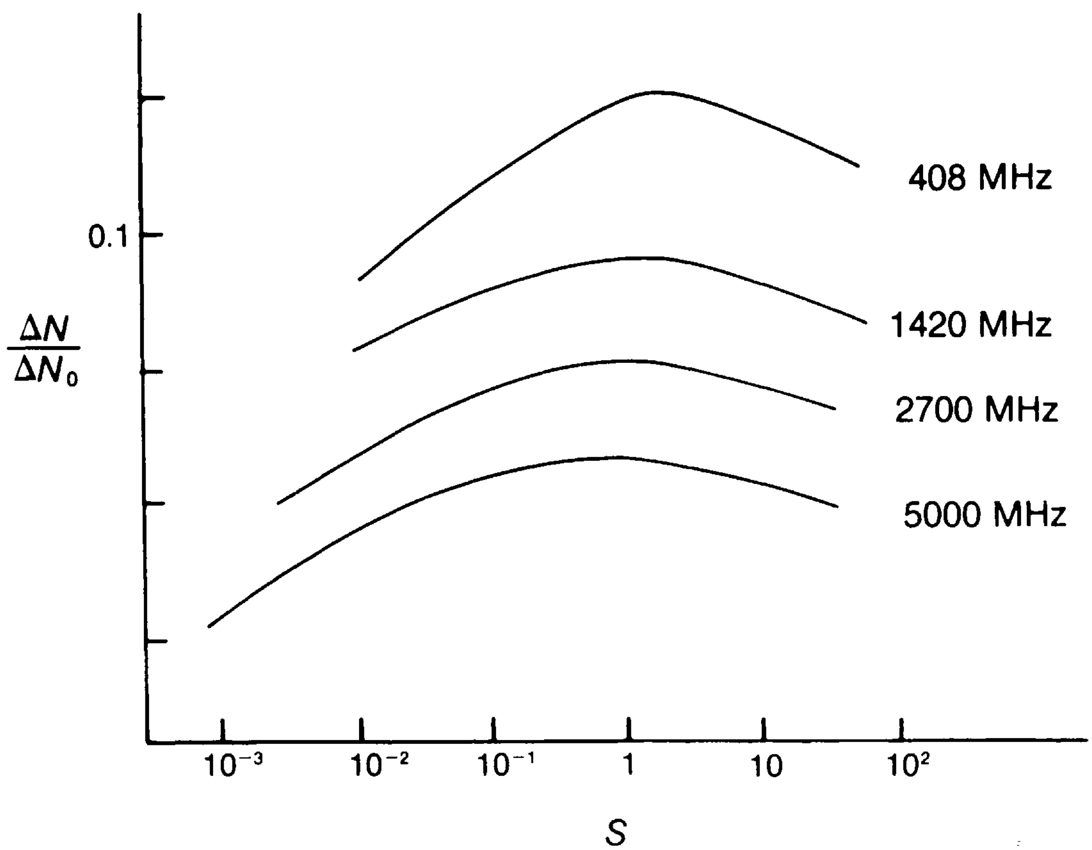

Fig. 8. The $\Delta N/\Delta N_0$ plot for several different surveys. The vertical scale shows a change of factor 10 in $\Delta N/\Delta N_0$ within a given survey. The relative placing of the curves for different surveys is arbitrary.

been made in different ways by Hoyle (1968), Kellermann (1972), and Jauncey (1967). Jauncey estimates that a local hole with 35 missing sources over the whole sky (i.e., <3 per steradian) would account for the apparently super-Euclidean slope at high flux densities.

A local hole implies large scale inhomogeneity in the Universe. Indeed, the suggestion that source counts may fluctuate considerably depending on the observer's location in a homogeneous universe was first made by Hoyle and Narlikar (1961, 1962) while fitting the source count data to the steady state cosmology. The inhomogeneity considered by these authors was on the scale of ~ 50–100 Mpc, characteristic of superclusters. The idea of an inhomogeneous universe on this scale was considered preposterous in 1962; today it is taken as an accepted fact, thanks to the discovery of superclusters and voids.

Nevertheless, where redshifts are known, there have been attempts to piece together the radio luminosity function (RLF) from the source counts and then to ask whether the RLF is redshift dependent, i.e., evolving. A strong affirmative has come from Peacock (1985) who has obtained a best fit multiparameter RLF. Peacock has adapted the Kolmogrov–Smirnov test (see Kendal and Stewart, 1961) to a two-dimensional format suitable for the redshift-flux density plane. Using this test he finds that the RLF has a strong dependence on the redshift. However, using a similar Kolmogorov–Smirnov test, Das Gupta *et al.* (1988) have shown that a non-evolving RLF in a Friedman universe is concistent with the data. Das Gupta (1988) further finds that a non-evolving RLF fits the data even better within the framework of the steady state model, vis-à-vis the Kolmogorov–Smirnov test.

We end this subsection with a quote from the Kellermann–Wall paper:

"Thus in its current form the argument for evolution depends on the cosmological interpretation of redshifts. While we do not wish to raise the issue of non-cosmological redshifts, we do want to emphasize that the source counts by themselves do not necessarily require departure of the radio source population from homogeneity on a cosmological scale."

3.3. Absorption line redshifts (D)

Absorption lines in the spectrum of an extragalactic object like a QSO could arise either within the source itself (intrinsic absorption) or in an intervening gas cloud (en-route absorption). In the former case the absorption line redshift could exceed, equal or be less than the emission line redshift depending on whether the absorbing material is infalling, stationary or outflowing relative to the light source. For en-route absorption, however, the absorption line redshift is necessarily less than the emission line redshift.

The evidence for absorption line redshifts in QSOs has been reviewed earlier by several authors. See for example, Strittmatter and Williams (1976) and Weymann *et al.* (1981). The two reviews reflect the shift in the balance of opinion from the intrinsic to the en-route absorption. Of course, intrinsic absorption does not tell us anything about the distance of the QSO while the en-route absorption gives a handle on the distance: the QSO has to be farther away than the intervening material. This latter conclusion is consistent with the CH although it does not quantitatively establish it.

Weymann *et al.* (*op. cit.*) single out many cases wherein $z_{\text{emission}} \cong z_{\text{absorption}}$, say,

$$\frac{|z_e - z_a|}{1 + z_e} < 0.01 \; ; \tag{19}$$

for which they concede that the absorption is intrinsic. This criterion also includes a few case with $z_e < z_a$ which in any case cannot be explained by en-route absorption.

The en-route absorption hypothesis is believed to offer explanation for the sharp absorption line systems containing metals (C II, Si II, Al II, Fe II, Mg II, Si IV, C IV, etc.) and for the Lα forest seen shortward of the Lα line. Basically, the line of reasoning is as follows: assume that the QSO is at the cosmological distance. Given the spectroscopic data on its absorption lines calculate the physical parameters required of the clouds. Then verify whether these parameters are consistent with the current understanding (or lack of it!) of galaxies, their halos and the intergalactic medium. The general consensus is that such calculations give an overall consistent picture.

Young *et al.* (1982) have made a statistical study of the C IV, $\lambda\lambda$1548, 1550 absorption systems in a homogeneous sample of 33 QSOs. Of these 3 have absorption troughs that are easily explainable as due to ejection (in the QSOs itself) of the absorbing material. The remaining 30 have the C IV doublets uniformly distributed in the spectra. Young *et al.* argue that this would be expected if the absorption is due to intervening galaxies since their distribution en route to the different QSOs would be uncorrelated. Thus these authors consider the evidence as favouring the en-route absorption hypothesis.

Direct evidence of the type wherein the absorption line redshift of a QSO matches that of an intervening galaxy situated well away from the QSO but approximately along the line of sight would clinch the issue. But such evidence thas not yet come. A few examples enumerated below come close to it, however.

Haschick and Burke (1975) examined several QSO – galaxy pairs of this kind and found only one case with positive esults. This case is of 3C 232 – NGC 3067 pair. The QSO spectrum showed a 21 cm absorption in the halo of the Galaxy. Boksenberg and Sargent (1978) found the Ca II, H and K absorption lines in the spectrum of the QSO at the galaxy redshift 0.0050.

Other cases are of $z_a = 0.02865$ in the spectrum of the QSO, PKS 2030 – 370 (Boksenberg *et al.*, 1980) and $z_a = 0.0669$ in the spectrum of the QSO, 0446 – 208 (Blades *et al.*, 1981). Unfortunately, in all these cases the absorption is occurring in a very nearby galaxy and the QSOs in question could very well be nearby.

Brown and Mitchell (1983) have found a 21 cm absorption line at $z_a = 0.4366$ ($z_e = 0.871$) with the radio observations of the QSO, 3C 196 at 988.7 MHz. There is a thin optical wisp perpendicular to the separation of the radio lobes of 3C 196 which could be an intervening galaxy whose halo is causing the absorption. The redshift of the optical object has not, however, been measured to see if it agrees with z_a. It is also a curious fact (pointed out by the authors) that $z_a \cong \frac{1}{2} z_e$ within measurement errors!

The gravitational lens candidates (cf. Section 3.4) are ideally suited to provide intervening galaxies for absorption. However, none of the lens candidates to date show absorption lines at the redshift of the intervening galaxy. In fact Young *et al.* (1981) have argued that the absorption line system in the lensed QSO, 0957 + 561 A, B (z_e = 1.4) observed at z_a = 1.1249 is best explained by the intrinsic hypothesis.

Shaver and Robertson (1983) have examined close pairs of QSOs with nearly equal redshifts and argued that large galactic halos containing the QSOs cause their absorption lines. In particular, for the pair Q 0307 – 195 A, B separated by 58″ the circumstance that the absorption line seen in A has redshift almost identical to the emission redshift of B has led the authors to estimate the size of the halo cloud to be as large as 0.5–1 h_0^{-1} Mpc.

Oort (1981) had argued that the Lα absorption lines arise from gas distributed in a supercluster. This should lead to cross-correlation in the Lα lines seen in the absorption spectra of QSOs with nearly the same angular direction. Sargent *et al.* (1982), however, find that this conjecture is false for the close pair of QSOs 1623 + 269 and 1623 + 268 separated by 173″.

One difficulty of the en-route absorption hypothesis, first quantitatively stated by Burbidge *et al.* (1977) is the apparent rarity of many intervening systems at different redshifts. The chance of a galaxy intercepting light from a QSO to the observer becomes large only if the Galaxy (+ its halo) have a cross section much larger than seen optically. In quite a few QSOs there are multiple absorption line redshifts, requiring this interception to happen that many times. Weymann *et al.* (1979) and Sargent *et al.* (1979) who did a similar analysis using more homogeneous sets of data find that the required halo radii (for explaining the observed incidence of the C IV and Mg II systems) are in the range 100–180 kpc for $h_0 = \frac{1}{2}$.

Libby *et al.* (1984) examined the absorption lines in the spectra of 13 QSOs. On an average there were 73 lines per QSO. They reduced the wavelengths of all lines to the rest frames of the QSOs by using their emission line redshifts. They found that for five or more QSOs, 55 lines fell within 1 Å wavelength interval of each other. On a random distribution of lines only 13 $\pm$ 2 would have fallen in 1 Å band, according to the Monte-Carlo techniques used by the authors. So they conclude that the lines must be due to intrinsic absorption.

To summarize then, if one accepts the CH to be valid and argues that the bulk of the absorption line systems not satisfying (19) come from intervening galaxies then one can make a fairly plausible case for the observations to date. There are a few problems which we have referred to here and one may choose to ignore them at the cost of reduction in confidence in the entire structure.

3.4. Gravitationally lensed QSOs (D)

The striking similarity between the spectral features of the close pair 0957 + 561 A, B (Walsh *et al.*, 1979) presented for the first time the strong possibility that the two objects are in fact image of one source, produced by a gravitational lens. The idea of lensing had been thought of much earlier by Barnothy and Barnothy (1965) but its demonstration had to wait for more than a decade. The case for a lens in the above system was made much stronger by the identification by Young *et al.* (1980) of a galaxy at z = 0.39

along with a cluster as a candidate lens. Thus, according to the lensing hypothesis the QSO with $z = 1.41$ must be farther away than the lens galaxy at $z = 0.39$ – a result consistent with the CH.

Since then nearly ten possible candidate QSO groups (most pairs, one triplet and one quartet) have turned up for gravitational lensing. For a typical lensing system there should be multiple objects with same redshifts, small angular separations and a plausible line-of-sight galaxy. The separations in the above cases range from 2″–7″. Only three QSO pairs have identified lens systems. In other cases theoretical calculations giving 'best set' parameters for a lens galaxy are given but no object satisfying those values is seen. For recent surveys of gravitational lens observations and theory see Burke (1986) and Canizares (1987).

The similarity of spectra and (as in the case of the first lensed pair discovered) the similarity of the VLBI radio features in these objects are striking enough to lend plausibility to the lens hypothesis. An observation to clinch the issue should demonstrate that the light variation in the original object are seen in the observed images at different times – there being a prescribed time-delay corresponding to the light travel times for these images. The time-delay can be computed but it depends on the lens parameters assumed in the model and the somewhat uncertain value of Hubble's constant.

The issue of time delay is important since the lens models are, in principle, scalable. Thus, if the QSO being lensed is not at its cosmological distance but much closer, the lensing sysem would be less massive and nearby and the time delay much less. Thus confirmation of time delay will not only establish the lens hypothesis on a firmer footing, it will also help settle the validity or otherwise of the CH.

There are two reservations about the lens hypothesis that must be mentioned here, however. First, there are some rigorous mathematical theorems (Subramanian and Cowling, 1986; Subramanian and Padmanabhan, 1988) that conclude that the number of images produced by a gravitational lens should be odd. In practice even number of images are more commonly seen (only one triplet is seen so far!). This discrepancy requires one epicycle to save the lens hypothesis: viz., that the lens parameters are such that the odd image is too faint to be seen.

The second point was first made by Burbidge (1981): that although several close pairs of QSOs are known, how one looks at them depends on one's prejudice. Thus the close pair 1548 + 114 a, b (5.5″ separation) happens to have different redshifts and is considered a case of chance projection effect (cf. Section 5.5), while the pair 0957 + 561 A, B (6″ separation) is treated as a lensed pair because of the equality of redshifts. Likewise, if QSOs with same redshift are too far apart, as in the case of 1146 + 111 A, B (157″ separation), then they are looked upon as two distinct objects. Indeed in this particular case, attempts were made to describe the pair as a lensed system with very massive and esoteric deflectors like $10^{15}\ M_{\odot}$ supercluster, cosmic string, etc. Later it was realized that the lens models would be hard to sustain.

Thus with all the ifs and buts apart, if we accept that gravitational lensing has been demonstrated, then we have another example of the QSO being shown to be far enough away to be behind the lensing galaxy – a result that is consistent with the CH.

3.5. QSO-GALAXY ASSOCIATION (C)

The $m - z$ relation for galaxies is tight enough to generate confidence in the applicability of the CH to galaxies (cf. Section 3.1). This fact has been used by some workers to show that the CH applies to QSOs also by picking out QSO-galaxy pairs wherein the 'near neighbour' criterion and the equality of redshifts of the two members in a pair apply. The approach was outlined in Section 2.4.

Gunn (1971) and Robinson and Wampler (1972) used the statistical method to argue that the QSO PKS 2251 + 11 was close to a galaxy (28″ away) at a common redshift of $z = 0.324$. More isolated instances have been produced since then; but the only systematic attack on this problem to date seems to have been due to Stockton (1978).

Stockton chose a sample of 27 QSOs with $z < 0.45$ and looked for galaxies within 45″ from each member of the sample. The search was a 'success' if a galaxy with redshift within 1000 km s^{-1} of the QSO redshift could be detected. As we mentioned earlier, a velocity of this order could be permitted to contribute a small Doppler redshift.

Stockton registered 8 successes. The chance of so many successes on the basis of a random distribution of galaxies would be $< 1.5 \times 10^{-6}$ – small enough to support the conclusion that the members of a successful pair were neighbours.

This work, often cited as establishing the CH for QSOs is open to one criticism. Apart from the 8 successes, there were also 12 failures wherein a galaxy was found within 45″ of the QSO but its redshift differed considerably from that of the QSO. These cases were totally ignored as being due to chance projection. Thus, the issue was in a sense prejudged.

A less direct statistical study was concluded by Yee and Green (1984), in which fields of galaxies around 108 QSOs with redshifts between 0.05 and 2.05 were examined and compared with control fields 1° north of each QSO. Simple counting as well as two-point correlation analysis showed that galaxies tend to cluster around low redshift QSOs ($z < 0.5$) only. Where clustering appeared, the apparent magnitude distributions of the excess objects were found to be consistent with those galaxies being at the same redshifts as the corresponding QSOs.

These data, therefore, suggest that these QSOs are at the cosmological distances implied by their redshifts. There is, however, one case of a high redshift QSO, PG 1138 + 04, with $z = 1.9$ in the above study where the two-point correlation amplitude is comparable to that for the low redshift QSOs in the sample. Yee and Green attribute 'chance projection' to this exceptional case.

In all the cases where the method has worked in favour of the CH the QSOs in question have relatively modest redshifts (< 0.5). The implicit assumption behind this prior selection of the QSO sample is that if it is too far, i.e., if its redshift is too large, then it will be hard to find galaxies near it. This assumption is correct provided the CH is valid and, hence, cannot be considered independent of the conclusion one wants to establish.

To get out of this circular argument one should choose a QSO sample without reference to its redshift – say by a magnitude-limited criterion or (for radio loud QSOs) by a flux density limited criterion. Then, after establishing nearness to a galaxy for a particular QSO one should compare the two redshifts. If the two agree (say, within $cz < 1000 \text{ km s}^{-1}$) then the CH is validated. Likewise if they do not agree, the CH is disproved.

This philosophy (due to Karl Popper) sets a stringent criterion for a theory to be considered scientific: it must, in principle, be disprovable. If only those associations which are consistent with the CH are to be accepted (the others being dismissed as chance coincidences) then the CH can never be disproved – and as such fails the above criterion for a scientific theory.

In Section 5 we will encounter situations where the 'nearness' is established regardless of redshifts.

3.6. QSOs and AGNs (E)

On the basis of the CH a fairly plausible case can be made for an evolutionary connection between the QSOs and the active galactic nuclei. In this argument one looks at the relative brightness of the nuclear region of a galaxy and the rest of it. In a normal galaxy one sees the rest more prominently as the nebulosity. If the nucleus of the galaxy becomes energetically active as in *N*-type and Seyfert galaxies, then it becomes more noticeable against the nebulosity. Further, as a sign of activity one may see emission lines from the nuclear region as one sees in QSOs.

Proceeding in the same sequence, a QSO may be looked upon as the next stage where the nucleus completely outshines the rest of the Galaxy. If such a QSO is at large distance (with cosmological redshift > 1, say), then one would only see the nucleus part and not the nebulosity.

This idea developed from the early work of Sandage (1973) on *N*-systems which were modelled by him as a combination of a QSO component and an elliptical galaxy component. Sandage found that if one allows the former then the luminosities of the *N*-systems agree with those of normal galaxies on the $m - z$ plot.

There has been some evidence to show nebulosities (fuzzes) round low redshift QSOs. Morton *et al.* (1978) and Hutchings *et al.* (1981) report evidence that the fuzz could be galactic disc or bulge while Cowie *et al.* (1981) find evidence for starlight in the fuzz. Wyckoff *et al.* (1981) find that the absolute magnitudes of the fuzz in several QSOs average to $M_R \sim -21.8 \pm 0.8$ ($h_0 = 0.6$), consistent with galaxies of typical luminosities. The magnitudes appear to range from those of the CD galaxies at the bright end (for 3C 273 the estimated magnitude of the fuzz is $M_R \sim -23.2$) to a possible unresolved dwarf galaxy at the faint end (e.g., for PKS 1510 – 089, $M_R > -17.4$). Apart from the faint unresolved cases the data are consistent with the fuzz being a Seyfert galaxy also.

While, for reasons stated earlier, the fuzz is more likely to be seen, if CH is right, around low redshift QSOs, there has been no systematic attempt to look for it around high redshift QSOs. This should be done, if only to test the CH critically. This line of evidence would also be made stronger by establishing the stellar nature of the fuzz, and by measuring the redshift of the fuzz independently of the QSO in each such case.

As emphasized by Rees (1986), the relationship of QSOs to host galaxies raises the question: What is the relationship of QSOs to radio galaxies (normally ellipticals) and to Seyferts (normally spirals)? The physics of QSOs needs to be understood much more thoroughly than it is today to be able to answer this question.

3.7. Summary

From the evidence so far discussed it looks as if the CH is valid out to large redshifts and that a reasonably tight $m - z$ relation consistent with Hubble's law exists for carefully selected samples of galaxies. The evidence on absorption lines in the QSO spectra and gravitational lensing is also consistent with the cosmological interpretation of redshifts. The number magnitude relation now going down to very faint galaxies also does not throw up any surprises. The morphological arguments linking QSOs with the AGNs in an evolutionary sequence make sense if the CH is valid. So far as the near-neighbour criterion is concerned, Stockton's work provides the best argument for the cosmological nature of QSO redshifts.

None of these checks, however, fulfill the objective that Hubble was striving to achieve: to determine the large scale geometry of the Universe from the studies of discrete objects of large redshifts. All attempts in this direction have been vitiated by the observational uncertainties and model-dependent evolutionary effects.

For reasons described in this section, the CH is the most favoured hypthesis for extragalactic redshifts. A hypothesis enjoying such a status must, therefore, be subjected to more critical scrutiny than any other alternative theories are. For this reason, in the rest of this article we will highlight the shortcomings of the CH as revealed by the present data.

4. Neutral Evidence

Under this head we will discuss evidence that *prima facie* does not suggest the CH as the most likely theory but which can nevertheless be made consistent with it by the injection of a few (reasonable looking?) epicycles.

4.1. The magnitude-redshift relation for QSOs (A)

Figure 9 shows the $m - z$ plot for some 3000 QSOs taken from the Hewitt–Burbidge (1987) catalogue. No one (Hubble included, had he been alive to see it!) would be able to deduce a velocity distance relation from this scatter diagram. Notice that scatter was present in the $m - z$ diagram for galaxies (Figure 5) but one could make out a trend there that conformed with the CH. No such trend is seen in Figure 9.

From Figure 9 we can draw two alternative conclusions: (i) The redshifts are distance indicators but the magnitudes are not. (ii) The redshifts are uncorrelated with distance but the magnitudes are distance indicators.

To a stellar astronomer the second alternative would seem reasonable. To the cosmologist, however, it is imperative to accept (i) because otherwise the QSOs cease to have cosmological significance as very distant objects. This alternative therefore

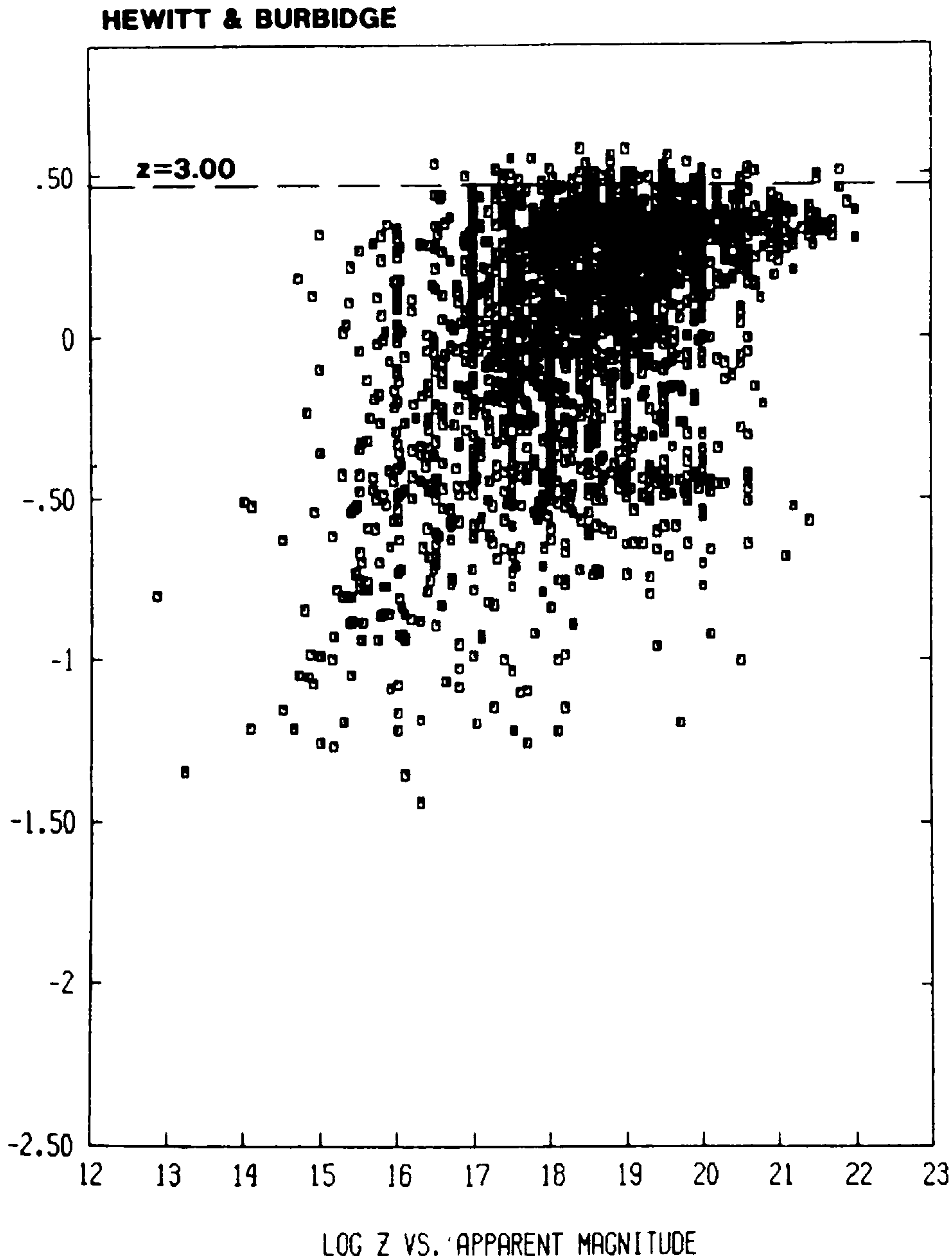

Fig. 9. The $m - z$ plot for QSOs in the Hewitt–Burbidge catalogue.

requires us to assume that there is considerable dispersion in the intrinsic luminosity of the QSOs at the same epoch.

So the first step in this direction is to use a criterion (like Sandage's standard candle for galaxies) to reduce the scatter. Bahcall and Hills (1973) undertook such an exercise and found that for a sample of optically most luminous QSOs corrected for selection effects, a tight Hubble relation emerges with a slope $dm/d \log z$ slightly greater than 5. For a Friedman model with $q_0 = 1$ the slope is exactly 5 out to large z, as can be seen from (15). However, a reanalysis of the data by Burbidge and O'Dell (1973) led to a discrepant result. These authors found the slope to be 4.3 ± 0.4 if the abnormally bright QSO, 3C 273 was included and otherwise as low as 2.1 ± 0.6.

Since for $q_0 \neq 1$, Equation (15) does not predict a curve of uniform slope, can one look upon the above slope as an average over the entire m-log$z$ range? For $q_0 = 0.5$ the average slope is 5.5 while for $q_0 = 0$ it is even higher. Thus the observed slope appears to be too low.

Lang *et al.* (1975) used a somewhat different approach. They constructed composite Hubble diagram for 663 normal galaxies, 235 radio galaxies and 265 QSOs, and computed regression lines in the m-log$z$ diagram. They found that the uncertainty in the slope d log$z$/dm for QSOs was comparable to the uncertainty for normal galaxies over a similar magnitude range. On that basis these authors argued that the scatter in the Hubble diagram for QSOs could not by itself be used as an argument against the CH. However, curiously enough, Lang *et al.* base their argument on the slope of the regression line of log$z$ on m instead of the other way round since, technically the redshifts have no errors whereas magnitudes have fluctuations due to variations of luminosity.

O'Dell and Roberts (1976) criticized the above work and its conclusion arguing that the least square technique employed by Lang *et al.* was technically unsound. Using the proper estimator of error in the slope, O'Dell and Roberts concluded that the Hubble diagram for the QSOs was formally incompatible with the linear m-log$z$ relation for $q_0 = 1$, at 3.6σ. On the other hand, if one took the CH to be correct and used the apparent cut off in the redshifts of the QSO sample at $z = 2.2$ then the observed least-square slope might be reconciled with the expected slope.

A more promising approach towards a scatter free diagram was suggested by Baldwin (1977) who found a relatively tight correlation between the equivalent widths at half maximum of C IV λ1548 emission line and the luminosities of QSOs. The Baldwin relation is an empirical one but it provides a way of selecting QSOs in a narrow band of luminosities by their C IV line widths.

Using this relation Wampler (1987) obtained a relatively tight $m - z$ relation for complete samples of the radio quiet QSOs including those of Schmidt and Green (1983), Osmer and Smith (1980) and a few additional QSOs. His $m - z$ plot is shown in Figure 10.

It is a composite plot including these QSOs together with the 3CR radio galaxies. To make the absolute luminosities of QSOs comparable to those of galaxies all QSO magnitudes were shifted by $6^m.75$. Not only is there a relatively scatter free $m - z$ plot but it also shows a continuity from the galaxy population (at lower end of the z-scale) to the QSO population (at the higher end). However, there is one difficulty! Wampler points out that this plot is fitted closest with the theoretical $m - z$ curve for the $q_0 = 3$ Friedman model. Such a model, as we shall see in the next sub-section, leads to difficulties with the age of the Universe.

If one wants to bring down the value of q_0 for this composite $m - z$ diagram, one needs to introduce *ad hoc* evolution. But then it becomes difficult to see why the same physical evolution should apply to both the QSOs and the galaxies. The radiation from the former is non-thermal while from the latter is thermal (of stellar origin). Thus we do not expect the two populations to evolve in the same way.

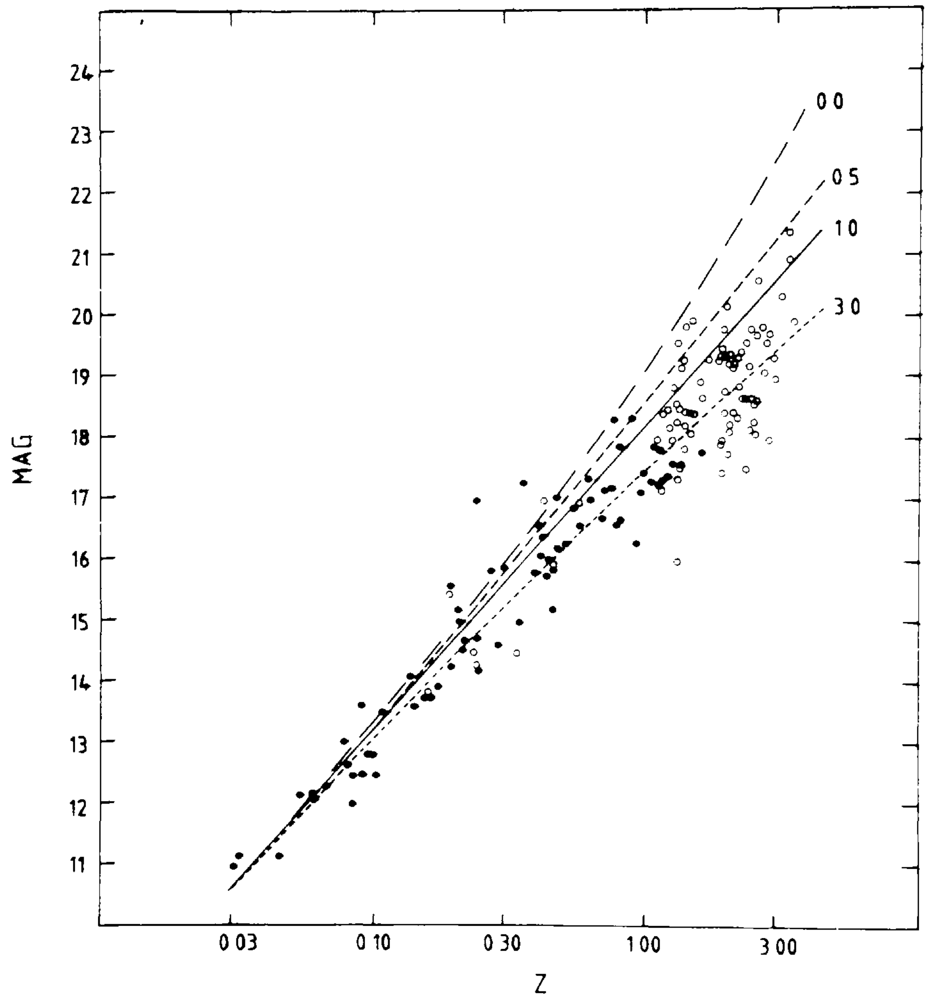

Fig. 10. Wampler's composite $m - z$ plot (see text for the source). Here radio galaxies are filled circles and QSOs open ones.

4.2. The age of the universe (E)

The age t_0 of the Friedman universe is expressible in units of H_0^{-1} as an analytical function of q_0. The expressions for $k = 0, 1, -1$, respectively, are:

$$t_0 = \frac{1}{H_0}\begin{cases} \frac{2}{3} \quad \text{for} \quad k = 0\,, \quad q_0 = \frac{1}{2}\,, \\ \frac{q_0}{(2q_0 - 1)^{3/2}}\left(\sin^{-1}\frac{q_0 - 1}{q_0} + \frac{\pi}{2}\right) - \frac{1}{2q_0 - 1}\,, \quad \text{for} \quad k = 1, q_0 > \frac{1}{2}\,, \\ \frac{1}{1 - 2q_0} - \frac{q_0}{(1 - 2q_0)^{3/2}} \ln\left(\frac{1 - q_0}{q_0} + \frac{\sqrt{1 - 2q_0}}{q_0}\right), \quad \text{for} \quad k = -1, q_0 < \frac{1}{2}\,. \end{cases} \tag{20}$$

Figure 11 shows the theoretical plot of $t_0\,(q_0)$ as a shaded band reflecting the uncertainty about the value of H_0. There is a simple relation connecting q_0 to the matter density ρ_0 of the Universe. The result is most conveniently expressed in terms of the so-called

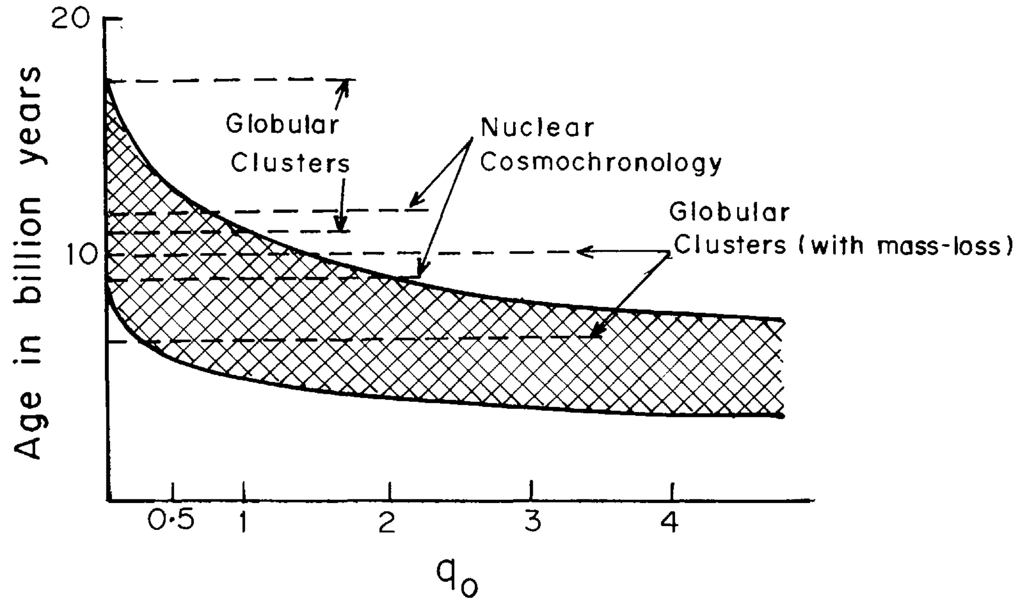

Fig. 11. The age t_0 of the Universe as a function of q_0. The dotted lines enclose the band of uncertainty in the ages of globular clusters and also in the ages computed by nuclear cosmochronology. The hatched portion describes the uncertainties in t_0 due to H_0.

'critical' or 'closure' density:

$$\rho_c = \frac{3H_0^2}{8\pi G}\,, \tag{21}$$

corresponding to the $k = 0$ model. For $\rho_0 > \rho_c$ the Universe is spatially closed, i.e., $k = 1$ while for $\rho_0 < \rho_c$, the Universe is open, i.e., $k = -1$. We then have

$$\rho_0 = \Omega_0 \rho_c = \frac{3H_0^2}{8\pi G}\,\Omega_0\,, \quad \Omega_0 \equiv 2q_0\,. \tag{22}$$

The dotted lines parallel to the q_0-axis describe the present uncertainty ($\sim$12–18 billion years) in the ages of globular clusters. These values could come down to $\sim$7–10 billion years if mass loss during stellar evolution is taken into account (for a review see Fowler, 1987). Nuclear cosmochronology yields the age band $(11 \pm 1.6) \times 10^9$ yr (Fowler, 1987).

It is clear that $H_0 = 100$ km s^{-1} Mpc^{-1} would rule out all Friedman models. The other end of the uncertainty range of H_0 does allow some models with $q_0 < \frac{1}{2}$. Clearly, the $q_0 = 3$ model found by Wampler from his analysis of the $m - z$ relation (cf. the preceding subsection) is ruled out in all cases. There is, however, a loophole that allows larger values of q_0 by introducing the λ-term.

The age apart, baryonic density of the Universe as indicated by the observed deuterium abundance (cf. Wagoner, 1980, for a review) also poses a problem for models with large q_0, H_0. Wampler and Burke (1987) find that by going to $\lambda > 0$, acceptable fits to the age and abundance data are possible if $0.65 < H_0 t_0 < 0.80$.

Further epicycles in the problem are introduced by theorizing about non-baryonic dark matter like massive neutrinos, photinos, gravitinos, axions, etc. We will not go into those details. The purpose of this subsection is to draw attention to the constraints imposed on the Hubble constant by physical cosmology.

4.3. The angular-size redshift relation (B)

The most dramatic demonstration of the non-Euclidean geometry of the expanding universe is given by the variation of angular sizes with redshifts. Figure 12 illustrates how the angle θ subtended by a sphere of diameter d of redshift z at the observer varies with z. The formula giving this variation for Friedman models and the steady state model was computed by Hoyle (1959). For each Friedman model there is a critical redshift $z_m(q_0)$ at which the angular size θ_m minimum. We see from Figure 12 that $z_m(\frac{1}{2}) = 1.25$ and $z_m(1) = 1$.

By now we have a lot of QSOs with redshifts exceeding these values. Even galaxies with redshifts > 1.25 are known. Do we see the predicted behaviour of $\theta(z)$ for such objects? If we do, then we have a proof of the validity of the CH.

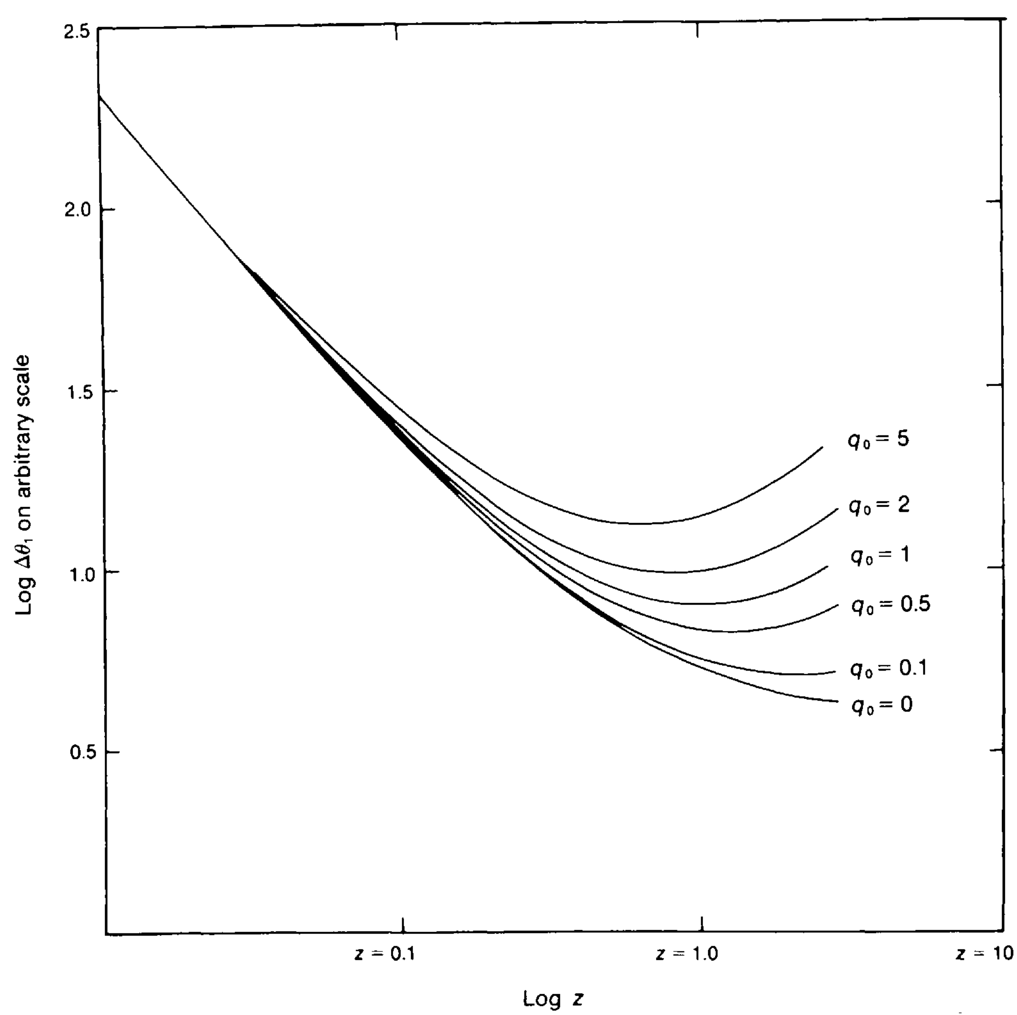

Fig. 12. Theoretical $\theta - z$ curves for different values of q_0 without including any evolution. For convenience the plots are on logarithmic scales.

This test has been tried for several samples of galaxies, radio sources and QSOs. Djorgovski and Spinrad (1981) looked for this effect in first ranked galaxies in clusters. The found no indication of a turn up above the Euclidean $\theta \sim z^{-1}$ relation for redshifts up to $z < 1$.

For radio sources the angular separation between the two radio components can be measured. An extensive study was made by Kapahi (1975) who used the occultation data from the Ooty radio telescope along with the angular sizes available for 3CR sources. Since redshifts were not available for most sources, a plot was made of angular size with flux density S. Because a linear source may be arbitrarily oriented with respect to the observer, there is a scatter in θ from this circumstance alone. Besides there is no fixed linear size associated with a typical source. Also, sources come with various intrinsic luminosities, leading to a scatter in the S-values. The $\theta - S$ diagram of Kapahi therefore has considerable scatter. To derive a meaningful statistic he, therefore, took the median value θ_{med} for θ in specified S-bins. If S decreases with z and becomes small enough to allow z to exceed z_m, then we would expect an upturn in θ_{med} at the low flux density end.

No such effect was found. To reconcile the result with the CH Kapahi assumed that source sizes evolve with epoch as $(1 + z)^{-n}$ where $1 < n < 2$. This would imply that sources with large z were intrinsically smaller and hence their θ-values small enough, not to rise above the minimum predicted.

An alternative (non-evolutionary) interpretation of these data was given by Narlikar and Chitre (1977) who were able to reproduce a $(\theta - S)$ distribution within permitted statistical fluctuations of the observed one, on the hypothesis that there is a correlation between power and physical extent of a typical source. Later Masson (1980) had also come to a similar conclusion.

The scatter can be reduced in a $\theta - z$ plot, of course, if all redshifts are known. Studies of this kind for QSOs were made by Miley (1971), Macdonald and Miley (1971), Katgert-Merkelijn *et al.* (1980), and more recently by Kapahi (1987). The earlier work looked for the largest angular size θ_L in a redshift bin (to get closest to lateral projection) and found that $\theta_L \sim z^{-1}$. This correlation with z^{-1} seemed at first to suggest that the CH was at work: otherwise why should sources at larger z look smaller? However, the $\theta_L \sim z^{-1}$ effect is essentially an Euclidean effect that is expected to be valid at small z. At the large z-values one should see the turn-up effect predicted by Hoyle. That is not seen.

Kapahi (1987) has compared the median angular size at different redshifts drawn from four different samples of radio galaxies selected from 1.4 GHz surveys to be of similar luminosities. (The luminosities of these galaxies vary over a relatively small factor of 30.) The redshifts are not known for all sources: only the brightest sample has measured redshifts. In other samples they are estimated from their optical magnitudes. Figure 13 shows Kapahi's results.

Over the redshifts range of (0.007–2) the above plot again leads to a result of the kind $\theta_{\text{med}} \sim z^{-1}$. For reasons stated earlier, this redshift dependence cannot be reconciled with the CH from purely geometrical effects. Kapahi (1987), therefore, introduces a

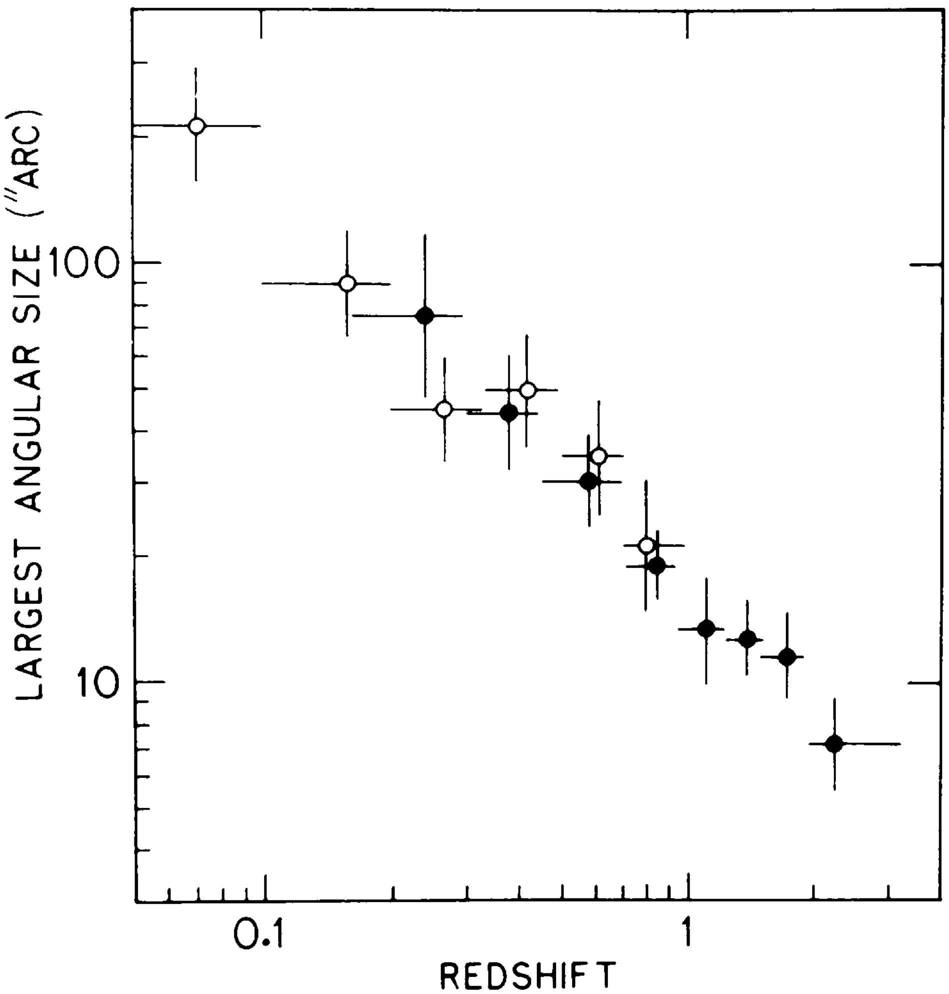

Fig. 13. Kapahi's plot of θ_{med} against redshift. (For source see the text.) Here QSOs are shown by filled circles and radio galaxies by unfilled ones.

linear size evolution with redshift:

$$d(z) \sim (1 + z)^{-n}, \quad n = 1.5 \pm 0.5 . \tag{23}$$

This makes the data consistent with theory, although robbing it of the predictive power it had otherwise.

Das Gupta (1988) has offered an evolution-free explanation of Kapahi's data by suggesting that a weak inverse correlation between luminosity and size would reproduce Kapahi's results. In a flux limited sample the more remote sources have to be more powerful radiators and, hence, smaller in size. Using the radio luminosity function derived for source counts (Das Gupta *et al.*, 1988, see Section 3.2) he finds that the size variation produced by the luminosity spread over a factor 30 is sufficient to prevent the upturn beyond the theoretical minimum. However, like the evolutionary hypothesis, this assumption also props up the CH but robs it of its predictive power for cosmic geometry.

4.4. LYMAN-α ABSORPTION (D)

In the early days of the discovery of QSOs it was suggested that if the intergalactic medium is largely made of neutral hydrogen then it would absorb radiation at the neutral hydrogen wavelengths, in particular depressing the continuum in the QSO spectrum blue-ward of Lα. This test would be applicable to QSOs with redshifts $z > 2$, since their spectrum would be sufficiently redshifted to bring the Lα line into the visible range.

Suppose $\lambda_L (\equiv 1216$ Å) is the rest wavelength of Lα, and let λ_0 be the lower end of the visible band of radiation. For a QSO of redshift z, the Lα line will be observed at $\lambda = \lambda_L(1 + z)$. A line with wavelength $\lambda < \lambda_L$ will at some intermediate point attain the wavelength λ_L (through cosmological redshift) provided λ lies in the range $\lambda_L(1 + z)^{-1} < \lambda < \lambda_L$. Thus the continuum in the range $[\lambda_0, \lambda_L(1 + z)]$ of wavelengths in the reference frame should show a dip. If most of the matter density $\rho_0 = \Omega_0 \rho_c$ in the Universe is made up of neutral hydrogen then a significant dip should be seen.

The actual observations (Gunn and Peterson, 1965) of 3C-9 hardly showed any dip in the continuum blue-ward of Lα. The QSO has redshift 2.012. The dip observed, if any, was consistent with a null result within the observational accurcy. This either meant that the QSO is not at the cosmological distance or that the density of neutral hydrogen is very low ($< 10^{-5} \rho_c$).

To preserve the CH, the latter conclusion was drawn and it was argued that the intergalactic matter is made of hot ionized hydrogen (Weymann, 1967). Although the X-ray observations of clusters of galaxies tend to support the existence of hot intergalactic gas, the amount so estimated is hardly enough to make up the closure density. Rather, it is sufficient to give $\Omega_0 = 0.2$. The proponents of closed or inflationary models have to look for dark matter to reach $\Omega_0 = 1$.

4.5. SUPERLUMINAL MOTIONS (E)

The VLBI studies of QSOs begun in the early 1970s have now yielded several cases of apparent superluminal motions. For a review of these cases see, for example, Cohen (1986). Typically in such an observation the angular separation between two radio emitting blobs is found to increase with time. If the angular speed observed is ω, then it translates to a linear speed $v > D\omega$, where D is the distance of the source from the observer. The inequality allows for projection perpendicular to the line-of-sight.

If the CH is valid then $v > D\omega > c$ for many sources. To avoid conflict with special relativity and all the modern physics based on it, the simplest resolution of the difficulty is to abandon the CH and argue that the sources are much closer than their redshift-distances.

However, to preserve the CH also under these circumstances calls for ingenuity! In Figures 14 and 15 are illustrated two scenarios that produce the illusion of superluminal motions without violating either the special theory of relativity or the cosmological hypothesis.

Figure 14 describes the most popular scenario, the so-called relativistic beaming model. Here the blob A is fixed while blob B is travelling almost directly towards the

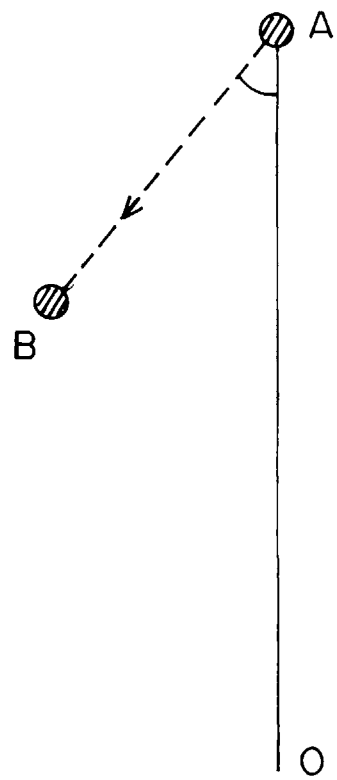

Fig. 14. The scenario of beaming model described in the text is illustrated here. For appreciable superluminal motion, the angle BAO has to be very small.

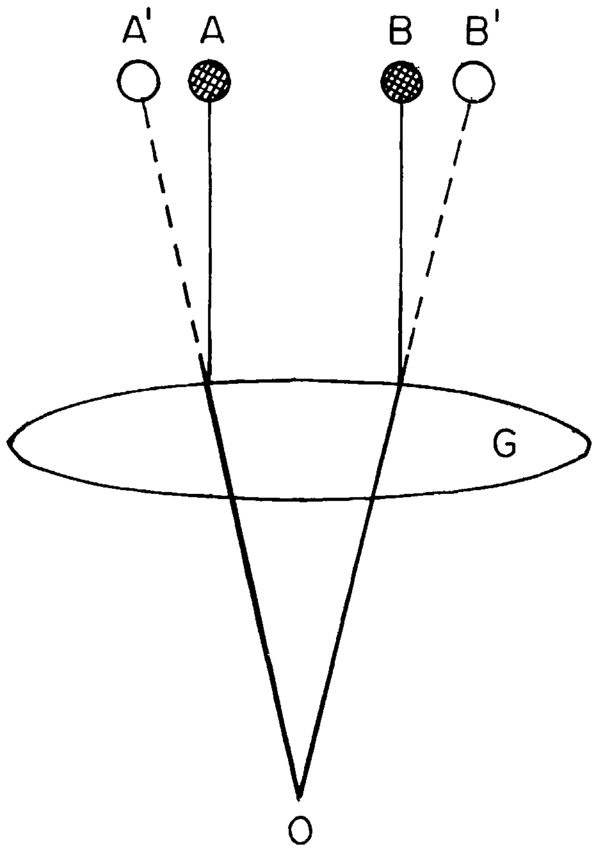

Fig. 15. The gravitational bending scenario. For effective superluminal motion, the observer *O* should be close to the conjugatge point of the source with respect to the lensing galaxy *G*.

observer O with speed $V < c$. A fairly simple kinematical argument based on ideas first proposed by Rees (1967) leads to an apparent velocity of separation

$$v \leq \frac{V}{\sqrt{1 - \frac{V^2}{c^2}}} . \tag{24}$$

For equality, the beaming angle BAO must be given by $\cos^{-1}(V/c)$. Thus for a well beamed source with $V < c$, we can have $v > c$.

In Figure 15 we see a model proposed by Chitre and Narlikar (1979) wherein radio waves from the blobs A and B are gravitationally bent by their passage through an intervening galaxy G to the observer O, in such a way that O sees virtual images of the blobs in the directions to A' and B'. If G is appropriately located to make O close to the conjugate point of the source, the magnification $A'B'$ produced would be large compared to AB, leading to an apparent velocity $v \gg$ the real velocity of separation V. For $V < c$ it may thus be possible to arrive at $v > c$.

For a comparison of the merits and demerits of the two scenarios see a review by Narlikar and Chitre (1984). Both explanations share the common criticism of being contrived and having somewhat low probability ($\sim 10^{-4}$). Detailed morphological studies of both explanations are desirable in view of the importance of the superluminal motions for the disproof of the CH. For example, one should search for intervening masses that could serve the role of G in Figure 15. Likewise in Figure 14 one needs to determine V and the beaming angle *independently* of what is required to get the desired result.

4.6. Energy problems (E)

The viability of the CH is also linked up with the problem of generating sufficient luminosity in a comparatively small volume of a QSO. (The problem of energy does not arise for galaxies whose luminosity is seen to be of stellar origin.)

If observed flux of radiation from the source is l, then its estimated luminosity must be

$$L = 4\pi D^2 l , \tag{25}$$

where D is the distance of the source from the observer. With D given by the CH, the L for a QSO ranges between 10^{43} to 10^{47} ergs s^{-1}. These luminosities are comparable to or even brighter than galactic luminosities. Yet, because of its starlike appearance, the emission from the QSO must come from a relatively compact region. Moreover, the time-variability of radiation in different wavebands imposes further restrictions as we shall see below.

Theoretical studies of size limits set by flux variations were made in the early days of the discovery of QSOs by Terrell (1964, 1967), Williams (1965), and Noerdlinger (1966). These conclusions were based on the argument that the light emitting surface of the object is moving nonrelativistically and the characteristic size limits must, therefore, be

$$R < cT, \tag{26}$$

where T is the time-scale observed in flux variation. This limit is, however, modified if the expansion of the object is occurring at relativistic speeds ($v \approx c$):

$$R < cT\gamma, \qquad \gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} . \tag{27}$$

This result is due to Rees (1966).

Nevertheless, the difficulty of producing large quantity of energy from a small volume remains. The first observations to highlight the problem came from radioastronomy when Dent (1965) found a 40% increase in the flux from 3C273 at 8000 MHz within $2\frac{1}{2}$ years. Several reports of short-term time variability at other wavelengths have since been reported. One of the shortest time scales comes from X-ray astronomy. The X-ray flux from the QSO, OX-169 showed a significant flux variation over ~ 100 min (Giacconi, 1980). The specific issues raised by such small sizes of QSOs are briefly discussed next.

4.6.1. *Synchrotron Self-Absorption*

Synchroton emission is believed to be the main mechanism of QSO radiation, because it is polarized to some extent. The observations of Dent coupled with the assumption (usually made) that the synchrotron emission is occurring in a uniform magnetic field imply that synchrotron self absorption should set in at frequencies considerably higher than those actually observed in the radio spectra of QSO. (For detailed calculations see Burbidge and Burbidge, 1967). The difficulty can be alleviated to some extent by postulating a non-uniform magnetic field, higher in the central core of the QSO and lower in the outer regions (Rees and Sciama, 1965; Hoyle and Burbidge, 1966).

However, the minimum energy requirements in a synchrotron source are usually estimated on the assumption of equipartition of energy between particles and the magnetic field (Burbidge, 1959). This condition is not satisfied in some of the variable sources and thus their total energy requirements go up considerably above the minimum value. Thus if the CH is to be retained one must assume that the central energy machine in a QSO is considerably more powerful than in a typical radio galaxy or that some other radiation mechanism besides the synchrotron is operating there.

4.6.2. *Compton Losses*

The emission of large energy in a small volume increases the radiation energy density

$$U_{\mathrm{rad}} = \frac{L}{\pi R^2 c} \tag{28}$$

to such an extent that U_{rad} may exceed the magnetic field energy $B^2/8\pi$. Under such circumstances the high-energy electrons in the source tend to lose energy from the inverse Compton effect rather than through the synchrotron process. Hoyle *et al.* (1966) found that in order to make the synchrotron losses dominate over the Compton losses, B must be so large that a typical high energy electron cannot travel the entire length R of the source. Therefore, the idea of a central energy source supplying all the fast electrons does not work. Rather, one needs several such energy centres all over the QSO or that in some way electrons are energized *in situ*.

This difficulty can be relieved only partially by the Rees time-scale formula (27) with a large γ-factor. For, that prescription is able to account for rapid *increases* in flux received from the QSO, not for rapid *decreases* which are also recorded.

4.6.3. *The Eddington Limit*

The general consensus today seems to converge on a highly collapsed object – a massive black hole – as the key to the central engine of a QSO. The idea of powering QSOs through the gravitational energy of a supermassive collapsed object was first suggested by Hoyle and Fowler (1963) soon after the discovery of the first two QSOs 3C273 and 3C48. At that time skeptics had voiced disbelief about such objects with masses as high as 10^6–$10^8\,M_\odot$ existing as bound units. Today several scenarios are proposed leading to the formation of a massive black hole, although the detailed model has not yet been successfully proposed (Rees, 1978). These scenarios are outlined in Figure 16. An accretion disc then forms around the black hole and radiates (Rees *et al.*, 1981).

The luminosity arising from an accretion disc around a supermassive black hole is subject to a limit earlier discussed by Eddington (1926) in connection with stars whose equilibrium is provided by a balance between their self-gravity and their radiation pressure. The latter can become very high and disrupt the object if too much luminosity comes out of a small volume. The limiting luminosity, called the 'Eddington luminosity' is given by

$$L_\mathrm{E} = \frac{4\pi GMmc}{\sigma}\,, \tag{29}$$

where M is the mass of the object; m, the mass of the proton; and σ, the cross section for Thomson scattering of photons by electrons. For $L > L_\mathrm{E}$, the gravity of the object is unable to hold it together against radiative pressures.

Setting $M = 10^8\,M_\odot$ in the above formula we get the critical (i.e., maximum) luminosity as 2×10^{45} ergs s^{-1}. Thus we may rewrite (29) as

$$L < 2 \times 10^{45}\left(\frac{M}{10^8\,M_\odot}\right)\ \text{ergs s}^{-1}\,. \tag{30}$$

Take for example, the case of OX-169 which shows a 100 min time-scale of variability in X-rays. This limit sets a size limit from (26) as $R \lesssim 2 \times 10^{14}$ cm for the accretion disc radius. Assuming that the black hole radius is one tenth of this radius, its mass must be $\sim 6 \times 10^7\,M_\odot$. The limiting luminosity given by (30) is then 1.2×10^{44} erg s^{-1}. The actual luminosity of OX-169 is about half this value!

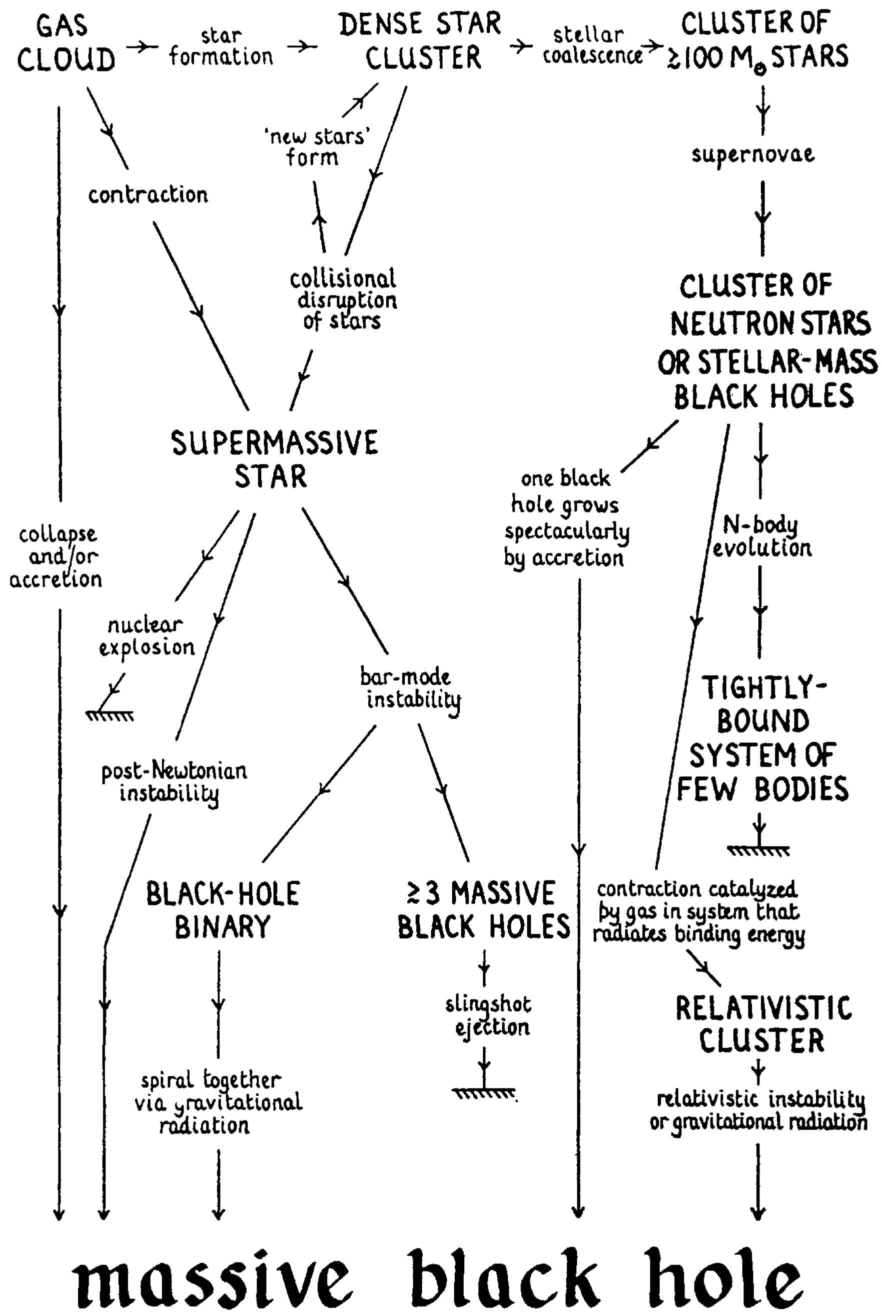

Fig. 16. Scenarios leading to the formation of a massive black hole are illustrated here. (For source, see the text.)

The example illustrates the central issue to the energy problem. On the one hand the time-scales of variation limit the source size while the luminosity is constrained not to exceed L_E. The above example works, but only just! We have not allowed for efficiency of conversion of one form of energy to another – we do not have much room for manoeuvre in the above case. This is in contrast to other processes in astrophysics where low-efficiency factors are the rule. For example, the conversion of nuclear energy in the Sun has an efficiency of only 0.7%.

It will be noticed that all three issues raised here come from the use of the redshift distance for D in Equation (25). The constraints ease considerably if the distance is less than this value, say, by a factor 100 to 1000. In other words, the energy mechanisms in QSOs are easier to understand if they are not a cosmological distances.

4.7. Summary

The evidence presented in this section could be argued either way. The supporter of the CH can argue that with suitable epicycles all of the evidence can be made constistent with the CH. It is true that the epicycles may be empirical (requiring high demands on efficiency of physical processes), or esoteric and contrived. Nevertheless they do not transgress the bounds of established physics.

The opponent of the CH, on the other hand, may wonder, whether the need for such epicycles itself is not suggesting that a critical look at that hypothesis is now needed. After all, the Greeks did not worry when each time they had to add another circle to make discrepant observations consistent with the geocentric theory: but the entire structure became so elaborate that finally it had to be replaced by another theory.

5. Discordant Evidence

Unlike the evidence presented in Section 4, where attempts are made to understand what is going on within the framework of the CH by invoking epicycles from conventional physics, the evidence to be presented now is such that accepting it at face value means discarding the CH. It is not surprising therefore that the data are either dismissed as wrong or are considered to be of no particular significance, by those who have implicit faith in the validity of the CH.

It is characteristic of science that even after a theory has become well established, discordant data continue to come. After a critical examination the data are either shown to be spurious or false, or, if they are accepted as genuine than there are two possible alternatives, which we illustrate with respect to Newton's law of gravitation:

(i) The discrepancy between theory and observation can be traced to an incomplete perception of the observational situation. The anomalous motion of planet Uranus could be understood only when the perturbing planet Neptune was found. Indeed, here the prediction of the existence of such a planet was made by J. C. Adams and U. J. J. Leverrier in order to resolve the discrepancy between Newton's theory and the observed facts.

(ii) The discrepancy between theory and observation needs a drastic modification of the existing theory or a totally new theory. When anomalies were discovered in the orbit of planet Mercury, Leverrier attempted to explain it in terms of a new intramercurial planet (Vulcan): but no such planet was found. This was a case where the alternative (i) failed and the discrepancy could be explained finally by replacing Newton's theory by the general theory of relativity.

We have to keep all these alternatives in mind as we look at the discrepant data. It is worth noting that contrary to early expectations that better observing instruments will resolve the discrepancies, the set of discrepant observations has steadily grown over the last two decades.

5.1. A nonlinear Hubble relation (A)

Figure 17 shows a figure reproduced from an analysis of Hubble's original velocity distance relation by Hewitt and Burbidge (Burbidge, 1981). The upper plot is according to modern measurements of redshifts and magnitudes while the lower one is that of Hubble (shown earlier in Figure 1). The reader may draw his own conclusion as to whether the revised plot could have suggested a linear relation to Hubble!

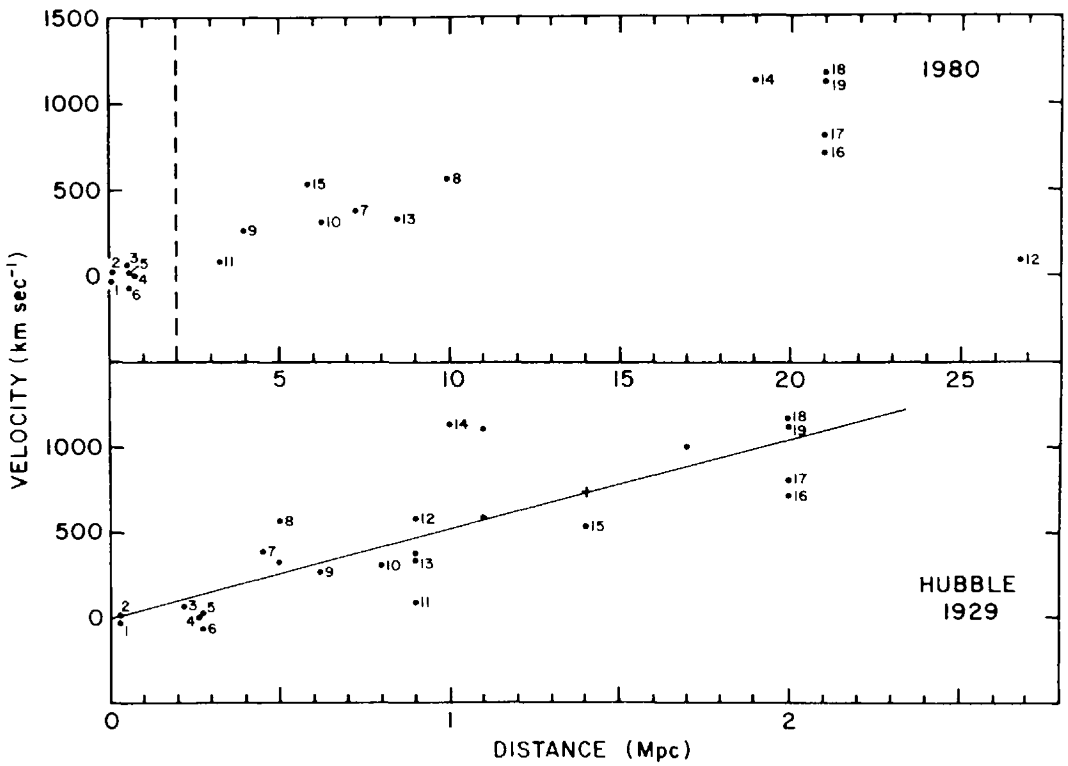

Fig. 17. Hubble's original plot is shown below the modern measurements for the same galaxies. (For source, see the text.)

Others have raised doubts concerning the linearity of the velocity distance relation for bright galaxies. For example, Hawkins (1962) has claimed that the velocity distance relation is in fact quadratic. He points out that the 430 galaxies brighter than 14^m that constitute 91% of the sample of Humason *et al.* (1956) show a redshift-magnitude relation of the form

$$m = 2.26 \log z + \text{constant}, \tag{31}$$

instead of (16). A quadratic law would in fact predict a slope $dm/d(\log z) = 2.5$ and Hawkins claims that the 'best fit' slope of 2.26 in (31) is closer to this value than to 5 required by the linear Hubble law. According to Hawkins there are systematic observational biases that can change the 'true' quadratic law to a 'false' linear law.

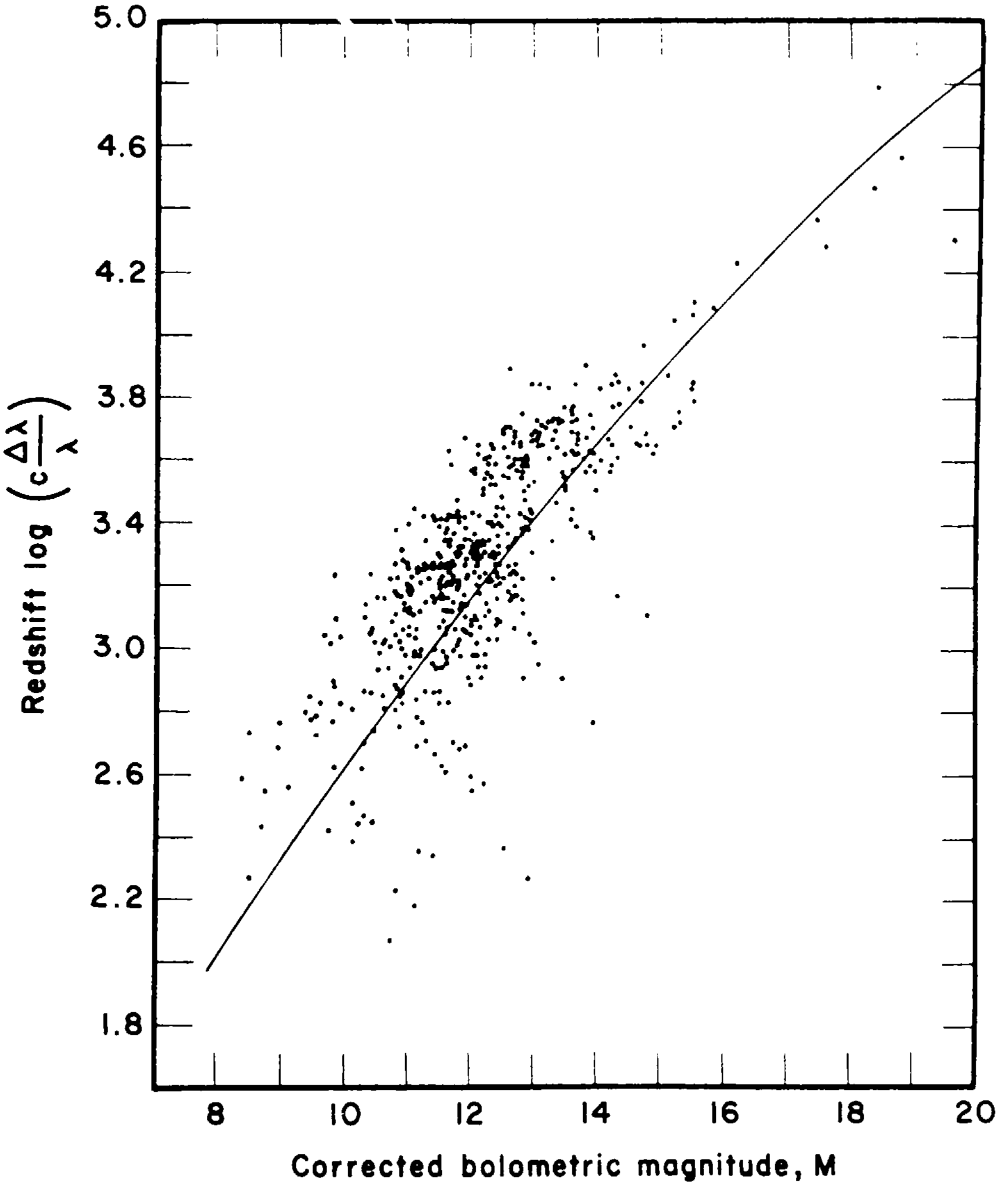

Fig. 18. The $m - z$ plot of bright galaxies adapted by Hawkins from the original plot of Humason *et al.* (1956) shown in Figure 5. The best fit line has a slope of 2.26, as opposed to 5 expected from Hubble's law. (For source, see the text.)

De Vaucouleurs (1972) has also noted that the redshift magnitude relation for galaxies in the redshift range $cz < 3000 \text{ km s}^{-1}$ is quadratic in nature. He has related this observation to a hierarchical model of the Universe.

Segal (1980) has given a detailed statistical analysis of the $m - z$ data on bright galaxies in the range $[500 < cz < 2500 \text{ km s}^{-1}, m < 12.5^m]$ from the work of Sandage and Visvanathan (1978). Figure 19 shows his $m - z$ plot with the lines for linear and quadratic laws superposed on the data points. Again, it appears that the quadratic law gives a better fit than the linear law.

These claims have, however, been refuted by Sandage and Tammann (1975) and Soneira (1979) who show that the linear relation gives a better fit. They attribute the

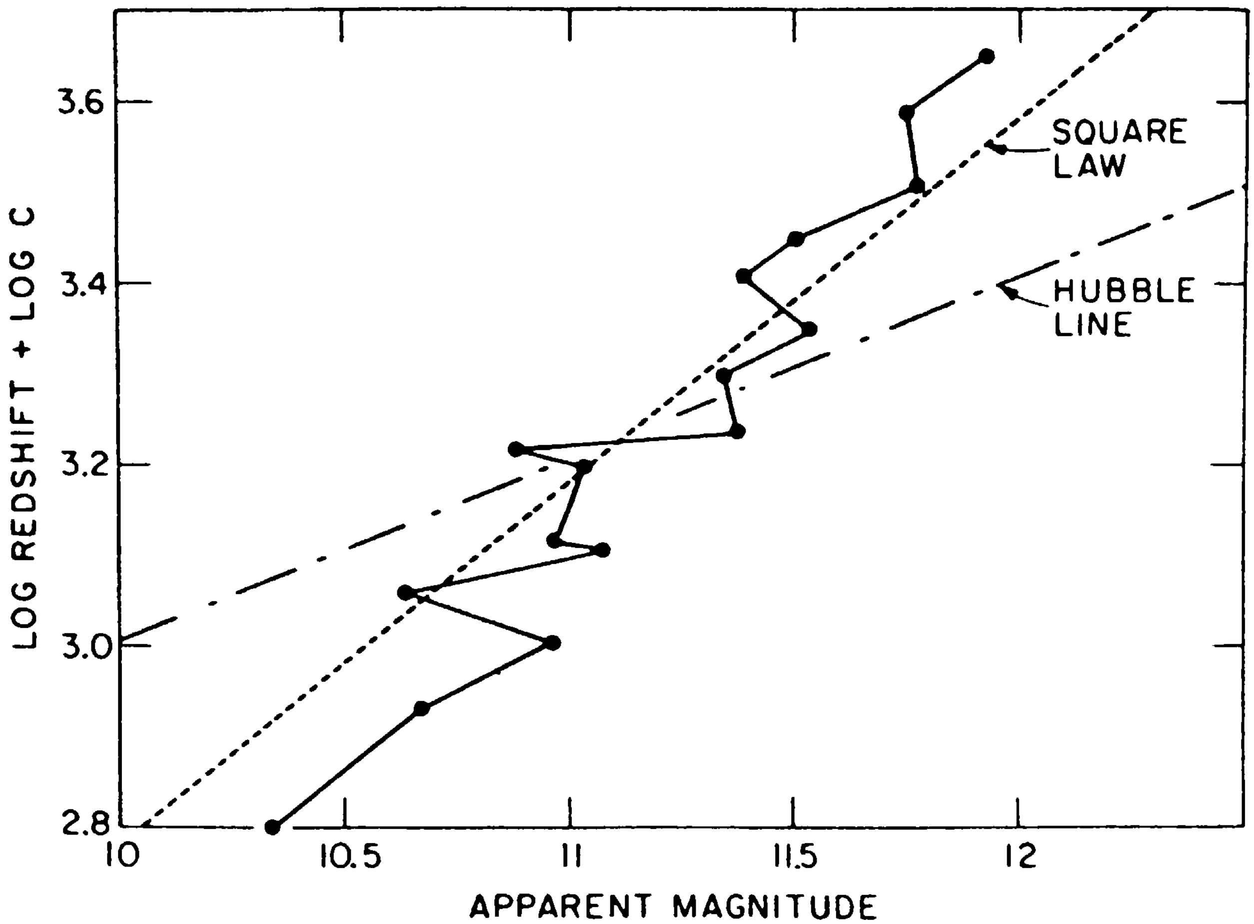

Fig. 19. Segal's $m - z$ plot of bright galaxies with the lines corresponding to the linear and quadratic laws superposed on it. (For source, see the text.)

apparent quadratic law to the Malmquist bias. The Sandage–Tammann plot based on the sizes of H II regions in a sample of nearby SC galaxies is given in Figure 20. Nicoll and Segal (1980) have challenged this claim giving further analysis of the data. In the next section we will refer to Segal's chronometric cosmology that makes precisely a prediction of quadratic law for bright galaxies.

In Section 3.1. we mentioned the growing evidence which suggests that the value of Hubble's constant increases from ~ 50 km s^{-1} Mpc^{-1} to ~ 100 km s^{-1} Mpc^{-1} as we sample fainter (and more remote) galaxies. With a quadratic law $v \sim D^2$, the Hubble 'constant', V/D itself becomes proportional to D. Thus the quadratic law may be the outcome of inhomogeneity on different scales, from galaxies to superclusters, while for scales beyond the supercluster the linear Hubble law does seem to hold. Nevertheless, the current controversy about the status of Hubble's law for nearby regions has not yet been resolved.

5.2. Periodicities in the Redshift Distribution (F)

In a homogeneous and isotropic universe we expect the redshift distribution of extragalactic objects to approximate to a continuous and aperiodic distribution. In the

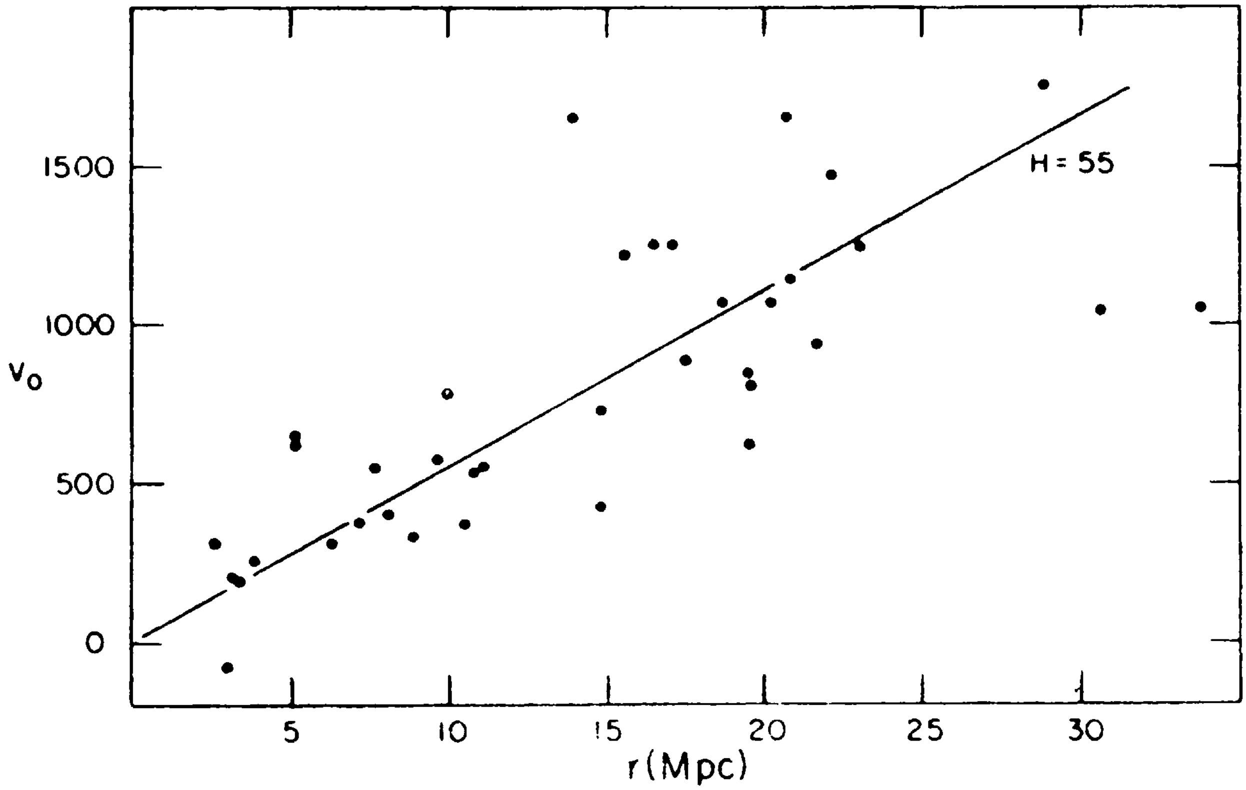

Fig. 20. The velocity-distance plot by Sandage and Tammann (1975) using the sizes of the H II regions in a sample of bright SC galaxies.

Friedman universe, the coordinate volume of the shell sandwiched between radial coordinates r and $r + \mathrm{d}r$ is given by

$$\mathrm{d}v \sim \frac{r^2 \,\mathrm{d}r}{\sqrt{1 - kr^2}} \,, \tag{32}$$

since there is a unique relationship between r and z, this translates to (cf. Das Gupta *et al.*, 1988).

$$\mathrm{d}v \sim \frac{[q_0 z + (q_0 - 1)(\sqrt{1 + 2q_0 z} - 1)]^2 \,\mathrm{d}z}{(1 + z)^3 \sqrt{1 + 2q_0 z}} \,. \tag{33}$$

Unless there is an epoch dependent evolution, the observed redshift distribution of discrete sources will simulate (33) which is a continuous and aperiodic distribution. If a survey is magnitude/flux density limited then the above distribution will be truncated appropriately by folding in the luminosity function (Das Gupta *et al.*, 1988). Again, this further input will not change the expected continuous nature of the observed redshift distribution. Various observers, however, have reported results that suggest discreteness or periodicity in the redshift distribution, contrary to the above expectation. We group the data under two headings: (i) nearby galaxies and (ii) quasi stellar objects.

5.2.1. *Nearby Galaxies*

Studies of non-dynamical correlations between redshift, magnitude, and morphology in clusters of galaxies led Tifft in the mid-1970s to indications of periodicity within $m - z$ bands (Tifft, 1974, 1977). The characteristic redshift interval came out to be $\sim 72 \text{ km s}^{-1}$ (for cz). However, to establish periodicity at this level it was essential to reduce the error bars on the cz-measurements below the $\sim 25 \text{ km s}^{-1}$ limit then existing.

By 1982 high quality measurements with 21 cm wavelength became available for galaxies in small groups with the accuracy $\Delta(cz) < 9 \text{ km s}^{-1}$. Using these data Tifft (1983) confirmed that the earlier effect still held quite unambiguously.

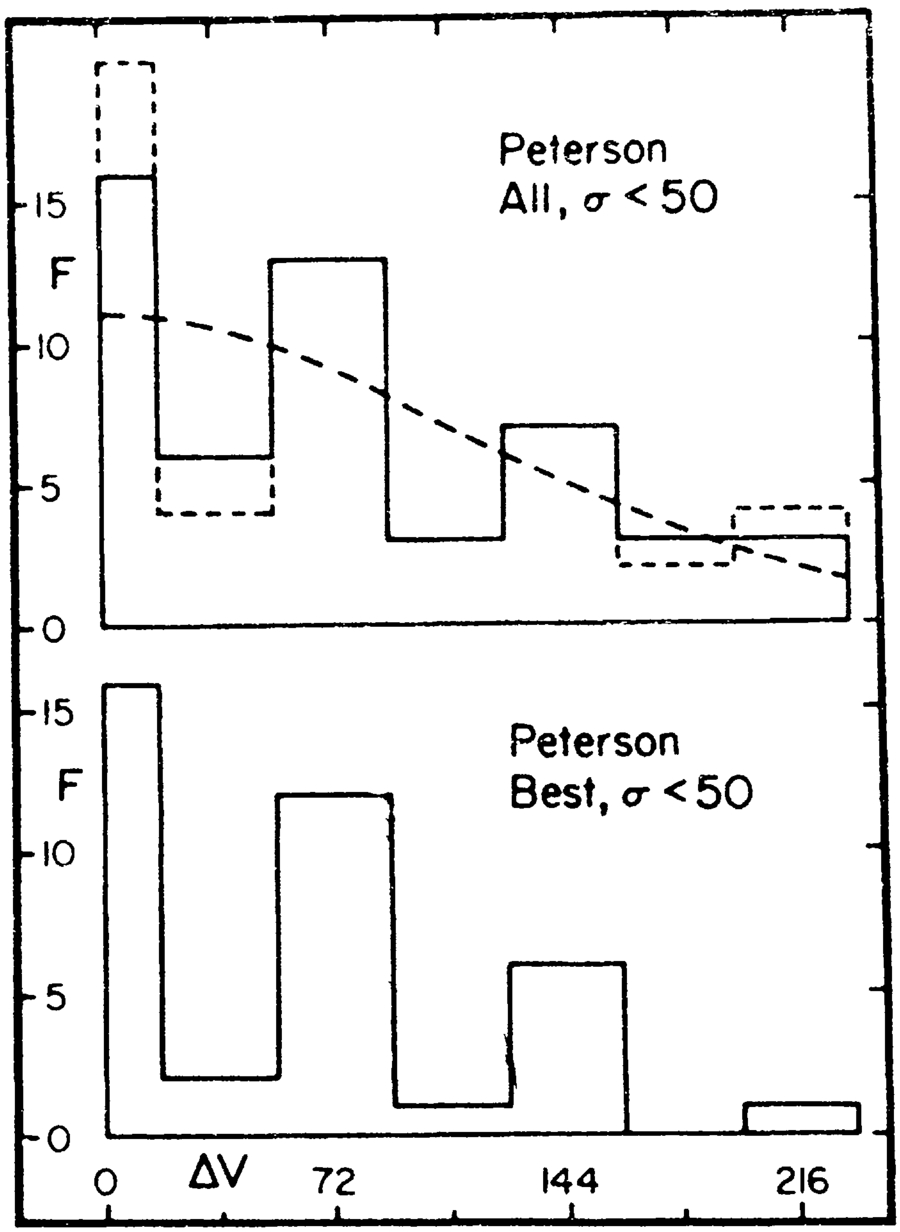

Fig. 21. The histogram of velocity differentials plotted by Tifft based on the data of Peterson (1979) on double galaxies. (For details see the text.)

Tifft's analysis considers all possible non-negative redshift differences $\Delta(cz)$ within individual groups of galaxies. For each $\Delta(cz)$ is defined a phase ϕ_{72} as follows:

$$\phi_{72} = \frac{\Delta(cz)}{72} + \frac{1}{4} - \left[\frac{\Delta(cz)}{72} + \frac{1}{4}\right], \quad (cz \text{ in km s}^{-1}), \tag{34}$$

where $[x]$ for a number x denotes its integral part. Thus ϕ_{72} lies between 0 and 1. Had there been no 'discrete effect', the numbers ϕ_{72} would be randomly distributed over the interval (0, 1), with $\langle \phi_{72} \rangle = \frac{1}{2}$. If there is a discrete clumping of values of $\Delta(cz)$ close to the integral multiples of 72 km s^{-1}, then there should be a significantly larger number of the ϕ's in the interval $0 < \phi_{72} < \frac{1}{2}$ than in the interval $\frac{1}{2} < \phi_{72} < 1$. Out of a sample of 75, Tifft found 47 in the former interval and 28 in the latter. Under a random situation this (or a more lopsided) distribution would arise with a probability 0.021 ± 0.002.

How the data have improved in quality over the years is shown in Figures 21 and 22. In Figure 21 we see Tifft's analysis of double galaxy data showing the distribution of Δcz in double galaxies (Peterson, 1979), ranging up to 250 km s^{-1} with $\sigma < 50$ km s^{-1}. The numbers are counted in cells 36 km s^{-1} wide, centred at values of $N \times 36.07$ km s^{-1} for $N = 0$–6. The normal distribution curve of same area and dispersion is shown by the dotted line in the upper histogram.

In Figure 22 we see the result obtained by Arp and Sulentic (1985) for 260 galaxies from more than 80 groups. Of these 160 galaxies are taken from the Rood (1982) catalogue where the accuracy of $\Delta(cz)$ measurements (with the 21 cm line) is as good as 8 km s^{-1}. The remaining 100 are as accurate as $\Delta(cz) < 4$ km s^{-1}. The typical system

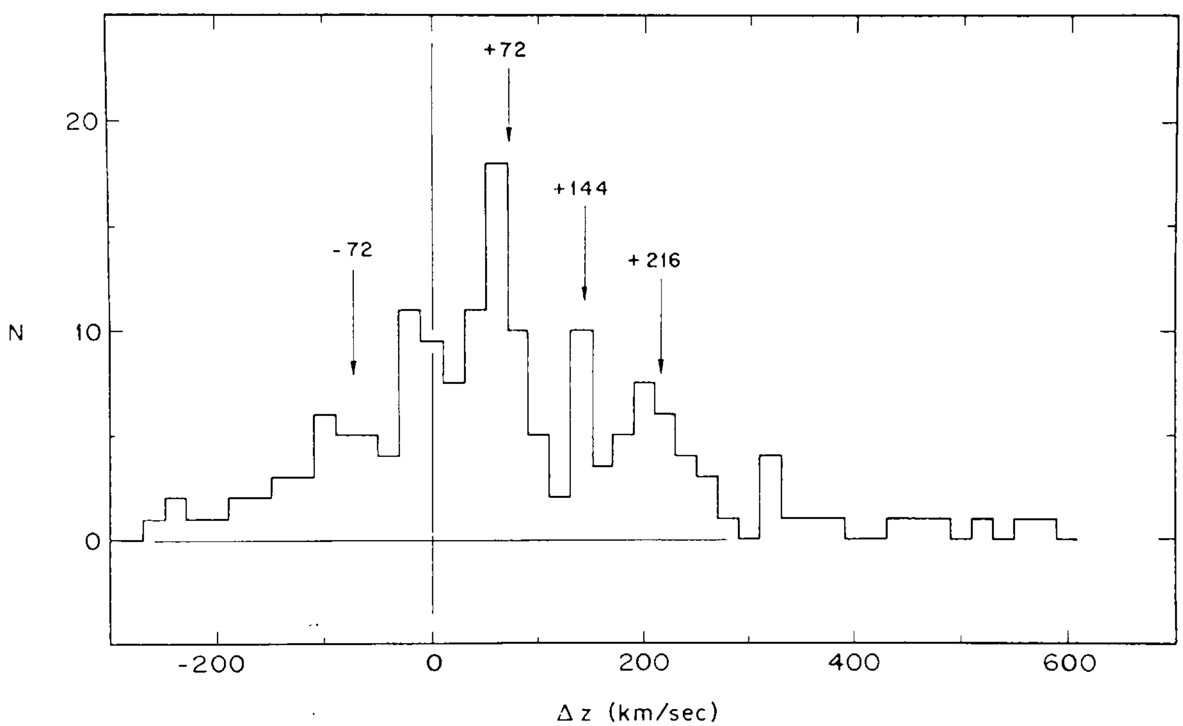

Fig. 22. The peaks at multiples of 72 km s^{-1} stand out in the above data collected by Arp and Sulentic (1985). (See text for details.)

has a large galaxy with a satellite companion usually with excess redshift $\Delta(cz) > 0$. With such improved accuracy the peaks stand out at multiples of 72 km s^{-1}.

For a review of all his previous work see Tifft's more recent paper (Tifft, 1988). It is worth remarking that the Tifft effect has survived critical examination by statisticians. Sharp (1984) confirmed the effect after an independent examination, although he does not consider the effect incompatible with conventional dynamics. More recently, Napier *et al.* (1988) have subjected these data to power spectrum analysis based on the classical theory of random walks. For the sample of 21 galaxies in the M31–M32 region where a periodicity has been claimed by Arp (1987a) the analysis reveals a period of 71.7 km s^{-1} with a confidence level of ~ 0.996. This period was not prespecified: it was indicated by the technique itself. Likewise, the analysis regarded as a test for the hypothesis of periodicity in the range $70 < \Delta cz < 75$ km s^{-1} (as proposed by Tifft from various samples) leads to a confidence level of 0.999.

Guthrie and Napier (1988) also examined the H I redshifts of galaxies in the Virgo cluster. They found periodicity with a period of 71.1 km s^{-1} at the confidence level > 0.999, for spirals in the low-density region of the cluster; but not for other spirals, or for dwarf irregular galaxies in either low or high density region. Where it works, the effect holds to over 40 multiples of 71.1 km s^{-1}. However, it is likely that the treatment of solar motion as free parameter in this work may have boosted the confidence level artificially.

Apart from the fact that the effect has survived improved data and independent statistical checks, its most disturbing aspect for conventional physics is that there appears to be no explanation for it within the known physical framework. Taken at face value it implies that if the CH is valid the space has some discrete structure on the scale of $\sim 0.72 h_0^{-1}$ Mpc!

We will return to the theoretical aspects of the Tifft-effect in Section 6.

5.2.2. *Quasi-Stellar Objects*

Another line of evidence at the higher redshifts of QSOs has been reported from time to time. Burbidge (1978) suggested that the peaks in the observed distribution of redshifts of QSOs continued to persist in spite of addition of new redshifts and that they indicated a periodicity. The issue had been discussed many times before. The earlier work of Burbidge and O'Dell (1972) involving 346 redshifts had concluded that although more sources were seen with $z = 1.95$ and 0.061 than expected by chance, the statistical significance of the result was not certain. They did find, however, that there was an underlying periodicity of $\Delta z = 0.031$ ($\sim \frac{1}{2} \times 0.061$) in the redshift distribution that was significant.

Subsequently, Karlsson (1977) made a power spectrum analysis of QSO redshifts to conclude that the peaks in the redshift distribution were significant and occurred with a period

$$\Delta \log(1 + z) = 0.089 . \tag{35}$$

The peaks occur at $z = 0.30, 0.60, 0.96, 1.41, 1.96$. They have been analysed by Depaquit *et al.* (1985) for any possible selection effects, (e.g., the role of strong emission lines like [O III], Mg II, C III, C IV, Lα, etc.). They conclude that in the case of radio loud QSOs the selection effects cannot account for the observed periodicity. These authors do point out, however, that so far as optically selected QSOs are concerned, the presence of strong emission lines in the QSO spectrum might introduce an observational bias in favour of certain redshifts.

Hewitt and Burbidge (1986) have highlighted the fact that even in the enlarged Hewitt–Burbidge (1987) catalogue of QSOs the peaks in the redshift distribution cannot

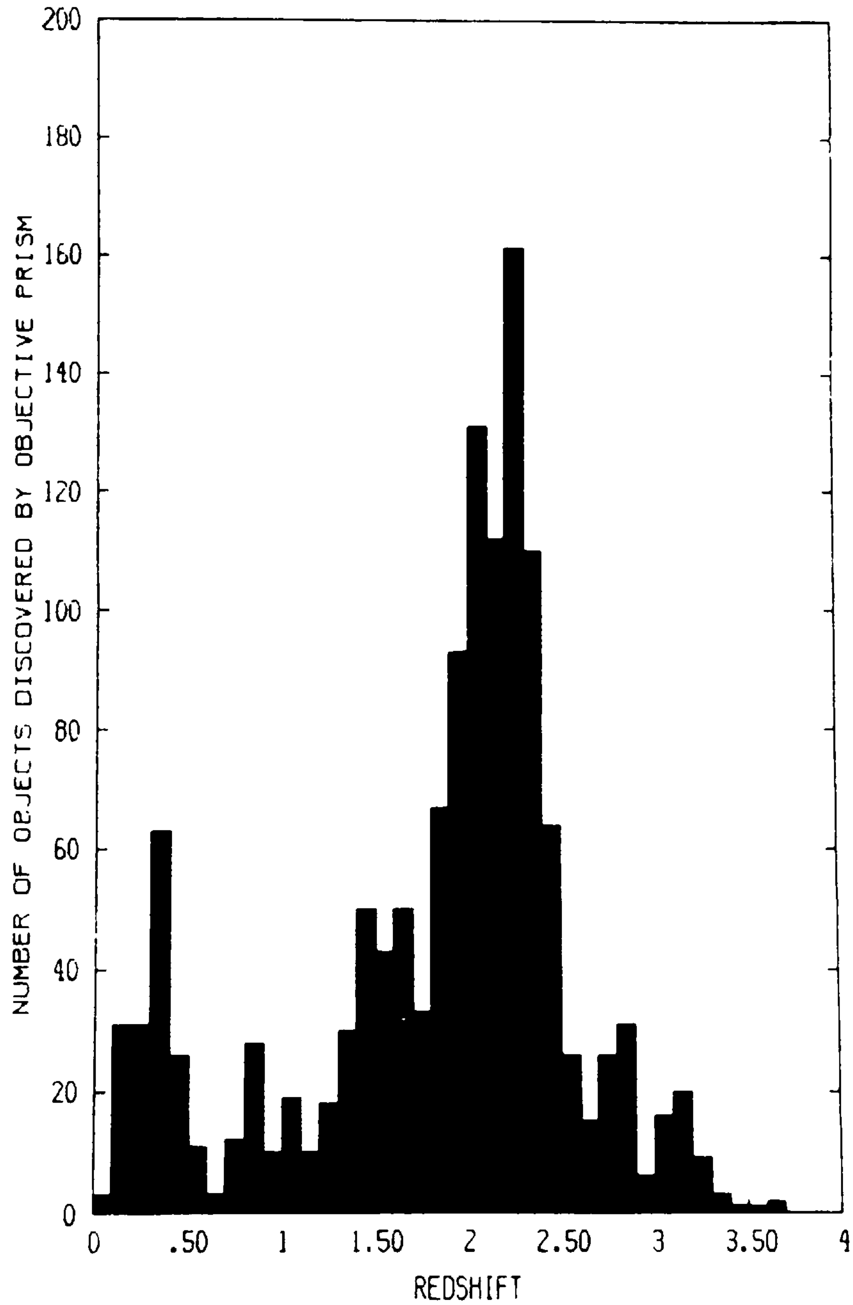

Fig. 23. The peaks in the redshift distribution of QSOs detected by the objective prism method, as plotted by Hewitt and Burbidge (1986).

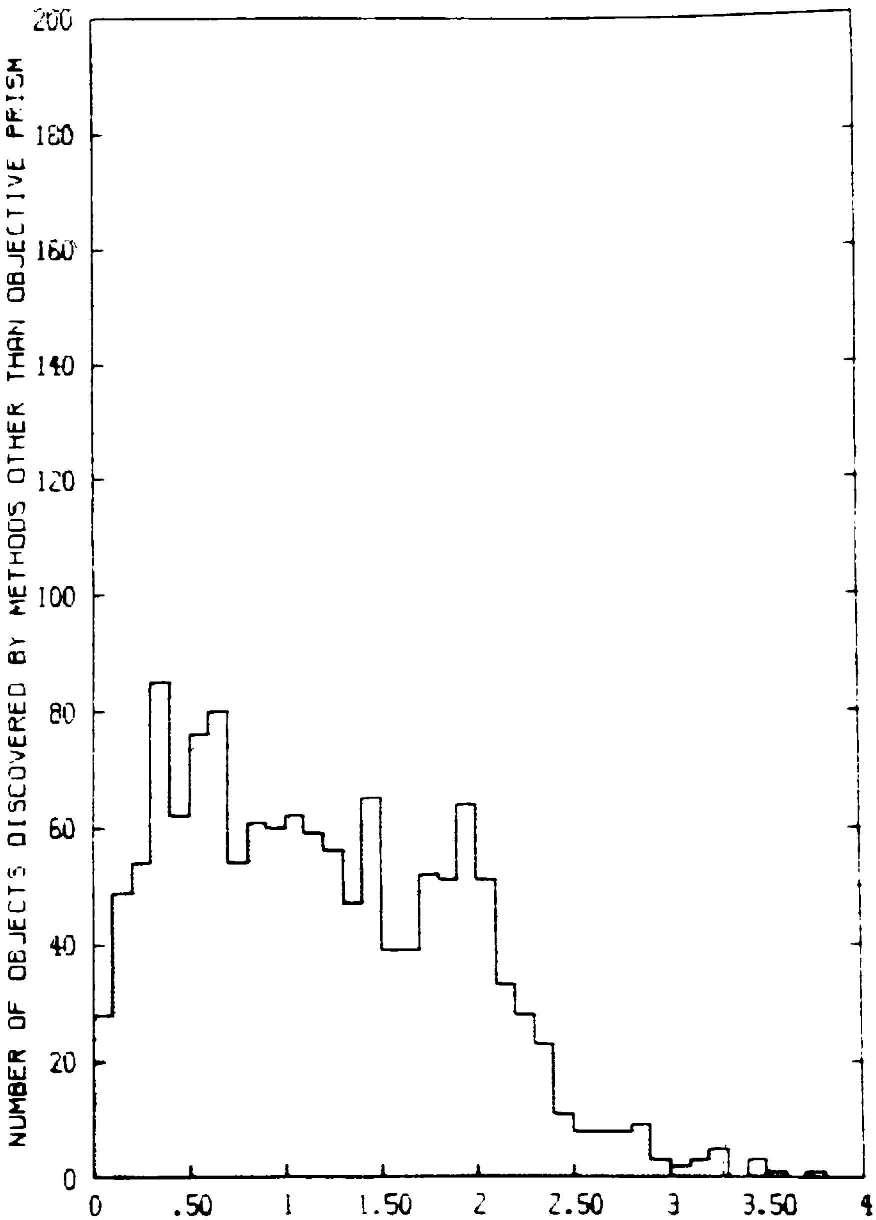

Fig. 24. The peaks in the redshift distribution of QSOs identified by colour and position. Figure taken from Hewitt and Burbidge (1986).

be explained away by selection effects (e.g., whether they were found by objective prism or grism tehcnique). Figures 23 and 24 show histograms that illustrate this point. Figure 23 shows that the QSOs detected by the objective prism method have a peak at $z > 1.8$, although there is a peak at $z = 0.3$ also. Figure 24 shows the histogram of QSOs identified by position and colour. A uniform distribution over $0.1 < z < 2$ in this histogram is seen to be superposed with another that peaks at $z = 0.3$, 0.6, 1.4, and 1.95.

This apparent non-uniformity, peaking or periodicity in the redshift distribution of QSOs and the Tifft-effect are at present two unrelated phenomena. There is no consistent theory for either of them and it is too early yet to speculate about a possible link between them. Neither of them (if real) can be understood in terms of the CH.

5.3. Galaxy-galaxy associations (C)

Although we find a tight Hubble relation for first ranked galaxies in clusters, there are a number of apparently discordant cases involving galaxies. We may broadly classify the evidence into two categories depending on whether there is a physical connection between two galaxies with different redshifts. In what follows we will consider the redshift differences between two neighbouring galaxies significant if they cannot be accounted for by a random relative motion of up to $\sim 10^3$ km s^{-1} between the two.

5.3.1. *Unconnected Neighbours*

Figures 25–27 show photographs of unusual groups of galaxies. For a detailed discussion of their features see Arp (1987b). The first photograph shows the barred galaxy NGC 3718 near the chain of smaller galaxies VV 150 from the atlas of interacting galaxies compiled by Vorontsov-Velyaminov (1959). The chain appears to have tidal interaction with NGC 3718, disrupting it in the process. However, the chain has excess redshift $cz \simeq 7 \times 10^3$ km s^{-1} with respect to the Galaxy.

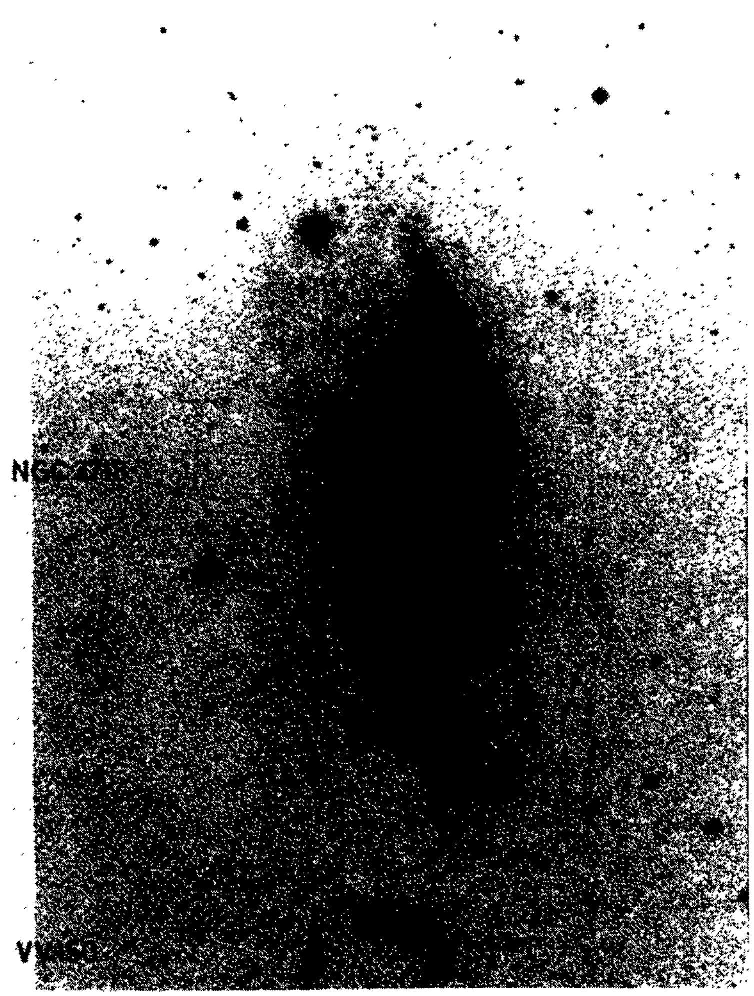

Fig. 25. The chain of galaxies VV 150 with NGC 3718 (a barred galaxy) shown nearby. The chain has excess redshift with respect to NGC 3718. (Photo by courtesy of Chip Arp.)

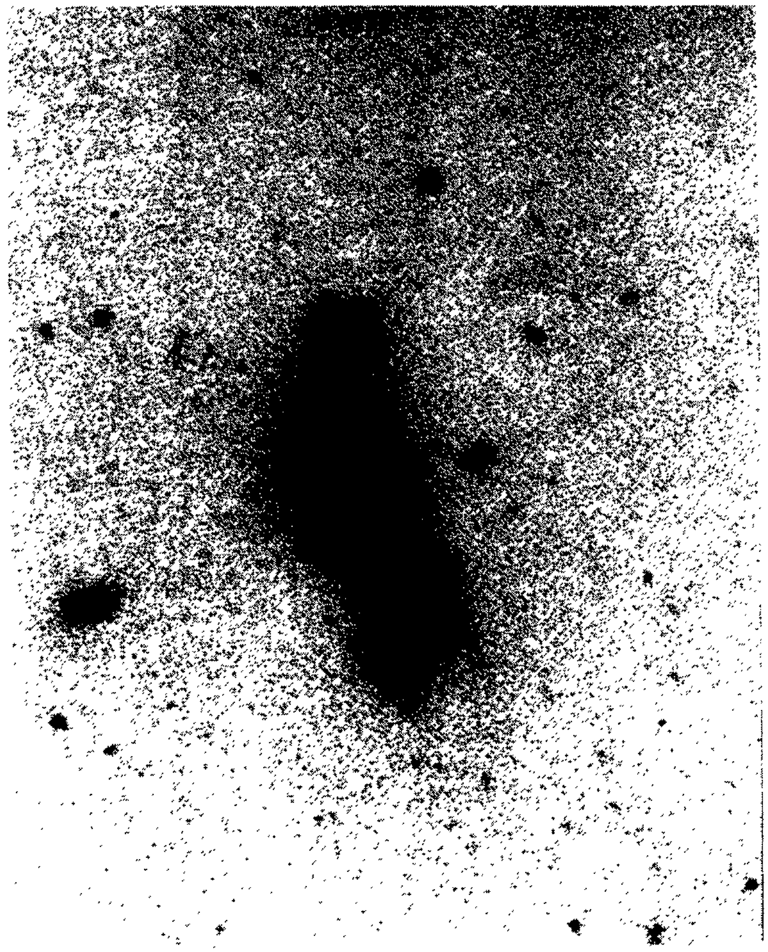

Fig. 26. The chain VV 172 with one member (second from top) having an excess redshift with respect to the rest. (Photo by courtesy of Chip Arp.)

The more famous chain VV 172 is shown in Figure 26. Of the five galaxies seen there, four have redshift $cz = 16070 \text{ km s}^{-1}$, 15820 km s^{-1}, 15690 km s^{-1}, and 15480 km s^{-1} as expected for neighbours in a group. However, the redshift of the second galaxy from top is $cz = 36880 \text{ km s}^{-1}$. According to Hubble's law, this should not be part of the chain but a more distant object and as such it should have comparatively more reddish colour. Instead, it is abnormally blue in colour! Also if it were really as far away as suggested by the CH, then it would have to be abnormally large! More spectroscopic work needs to be done on VV 172 to settle the issue one way or the other.

The most famous example of this kind, known as Stephan's quintet was first discovered by M. E. Stephan in 1877. Figure 27 shows a photograph of these five galaxies. Its peculiarity became noticeable when Burbidge and Burbidge (1961) studied it spectroscopically. They found that the largest member of the quintet (the one in bottom left-hand corner) is the galaxy NGC 7320 with a redshift cz of only 800 km s^{-1} whereas the cz-values of the other four members are 5700, 6700, 6700 and 6700 km s^{-1}. The galaxies all appear to be interacting considering their unusual shapes. As CH would have it, NGC 7320 is a foreground object with no physical proximity to the others. Arp (1987b) has argued strongly that this interpretation is incorrect.

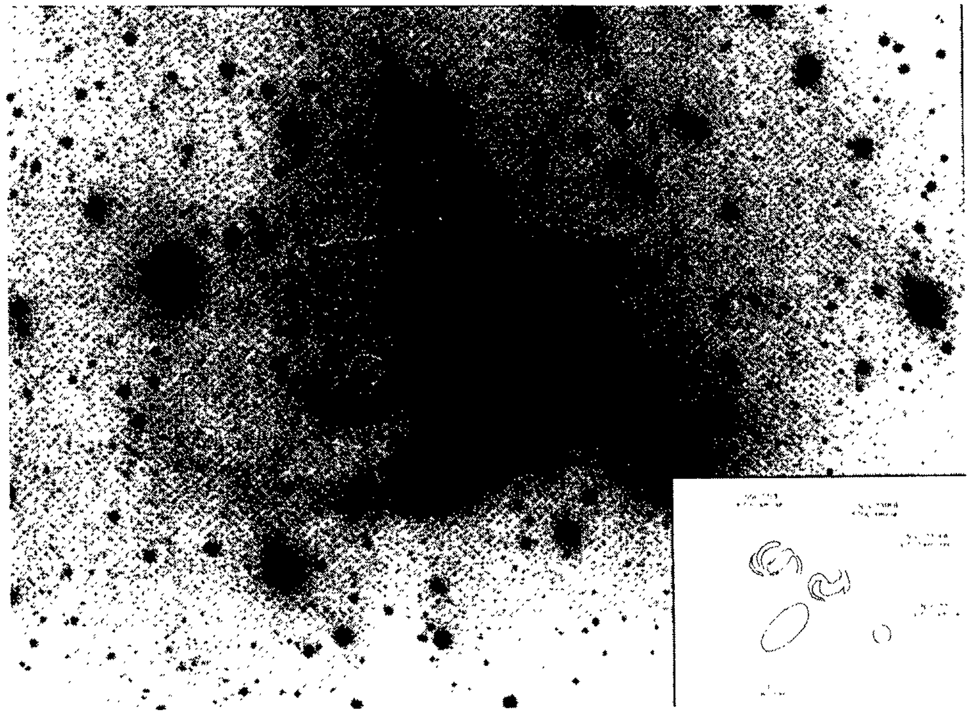

Fig. 27. Stephan's quintet with some other galaxies with discrepant reshifts. (Photo by courtesy of Chip Arp.)

According to Arp, NGC 7320 is a perfectly normal dwarf galaxy while the other four have irregular shapes indicating that their excess redshifts (with respect to NGC 7320) are noncosmological. Indeed, Arp finds that the Stephan quinted along with a few other redshift companions are all satellite members of NGC 7331, a large Sb galaxy with redshift nearly the same as that of NGC 7320. As supporting evidence he points to radio-emitting material connecting NGC 7331 to the quintet. Again, this point of view is hotly debated and no definite conclusion is yet possible.

The feature of companion (satellite) galaxies having excess redshifts with respect to the main galaxy in a group has been noticed by several observers. For example, Collin-Souffrin *et al.* (1974) have pointed out this aspect and have tried to link the companions with their excess redshift. In Figure 28 we see the distribution of redshift differences ($c\Delta z$) of companions in the M31 and M81 groups. Arp (1987b) points out that 12 out of 12 certain cases and 18 out of 20 probable group members have $c\Delta z > 0$, on an average $c\Delta z \sim 120\ \mathrm{km\ s^{-1}}$.

These velocities are not large and one could try to fit a Doppler model to them. But then why are some random relative motions not negative? To get out of this difficulty, Byrd and Valtonen (1985) tried to explain the M31 system by suggesting that these companions were ejected in an expanding shell with respect to the central galaxy (M31) and the shell has now become so large that we are inside it. This argument would explain the dominance of $cz > 0$, but would require the whole shell to have a radius at least 2–3 Mpc, whereas the size of the Local Group is estimated at ~ 1 Mpc.

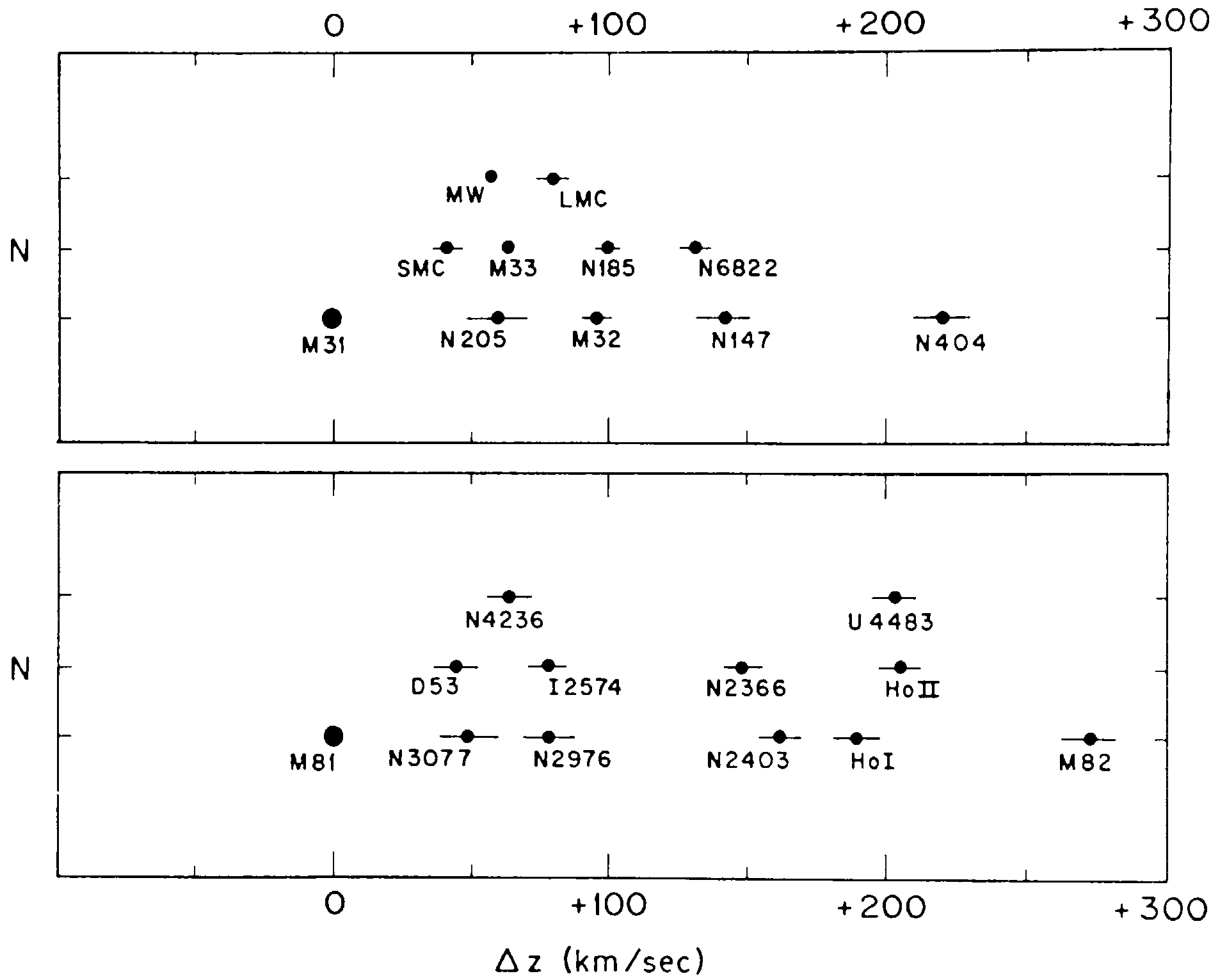

Fig. 28. Excess redshifts of companion galaxies in the M31 and M81 groups.

5.3.2. *Connected Neighbours*

Typical examples of this kind are shown in Figures 29–31. Again, for details of these systems see Arp (1987b). The standard pattern running through these cases is the following. The photographic evidence shows some filamentary structure joining two galaxies, one of them a large NGC galaxy and the other a small, compact, galaxy. The filament starts at the former and ends at the latter, suggesting that the compact member was somehow ejected from the bigger object.

Filamentary structures linking two galaxies are not uncommon and it is customary to see the pair of galaxies as interacting tidally. Figure 32 is a well-known example of this kind, known as the 'toadstool'. However, what distinguishes the cases found by Arp (and shown in Figures 29–31) from a case like the toadstool is that in the latter there is no discrepancy regarding redshifts. Not so in Figures 29–31! For example, the *cz*-values of the main and the companion galaxies in Figure 29 are 8700 km s^{-1} and 16 900 km s^{-1}.

Are we seeing in such cases a breakdown of the CH? If the differences in the *cz* motions as large as in the above examples are accepted as normal, then one should not

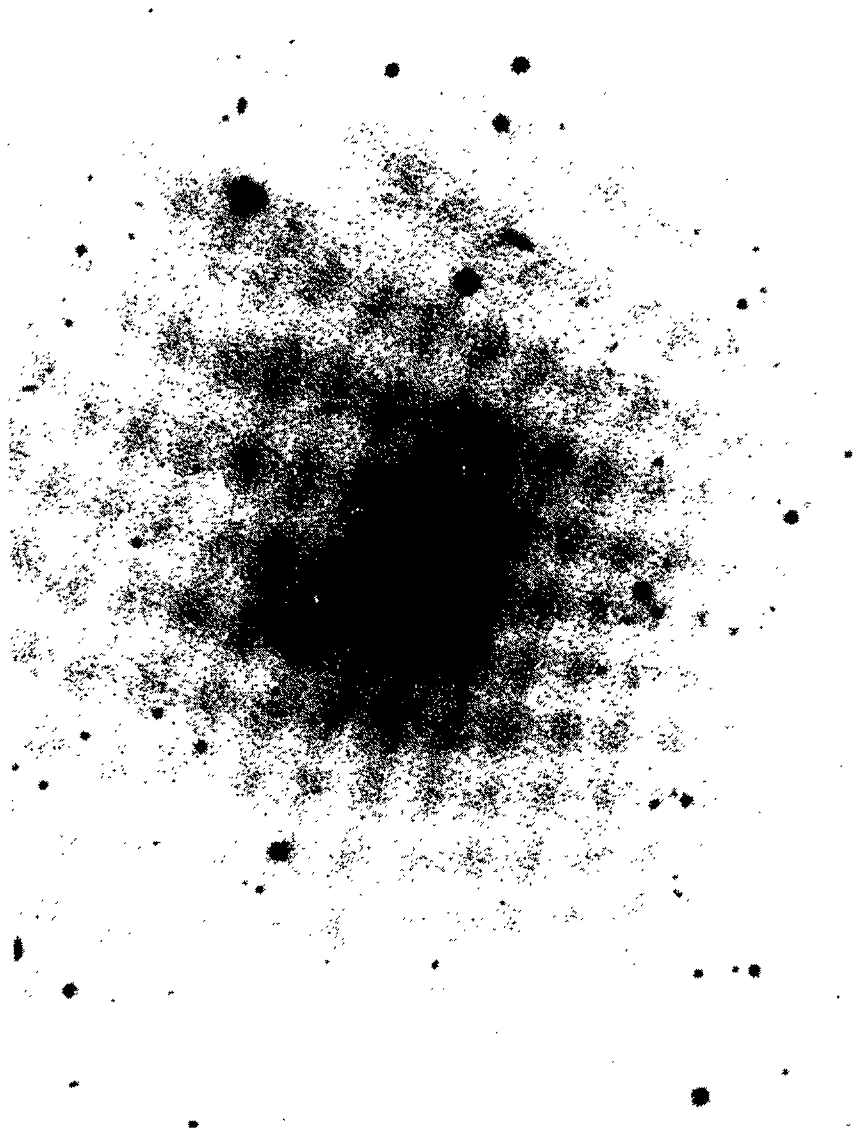

Fig. 29. The large galaxy NGC 7602 (cz = 8700 km s^{-1}) appears connected to a compact companion (cz = 16900 km s^{-1}). (Photo by courtesy of Chip Arp.)

be seeing the Hubble expansion effect for these nearby galaxies at all. So the only way to sustain the CH is to argue either that the filamentary connection is not real but an artifact or that the compact galaxy is really much farther away and happens by chance to have been projected at the end of the filament.

The former alternative was indeed advocated for the system shown in Figure 33 wherein a filament linking NGC 4319 with a QSO like object Markarian 205 was found by Arp (1971a). The redshifts of these objects are quite different and hence it was argued that the filament is a photographic artifact. However, the system has been subjected to CCD pictures by Sulentic (1984) who has demonstrated the reality of the filament (see Figure 34).

If excess (non-cosmological) redshifts are indeed present then they might systematically modify the (basically linear) Hubble law and make it look quadratic. This has been

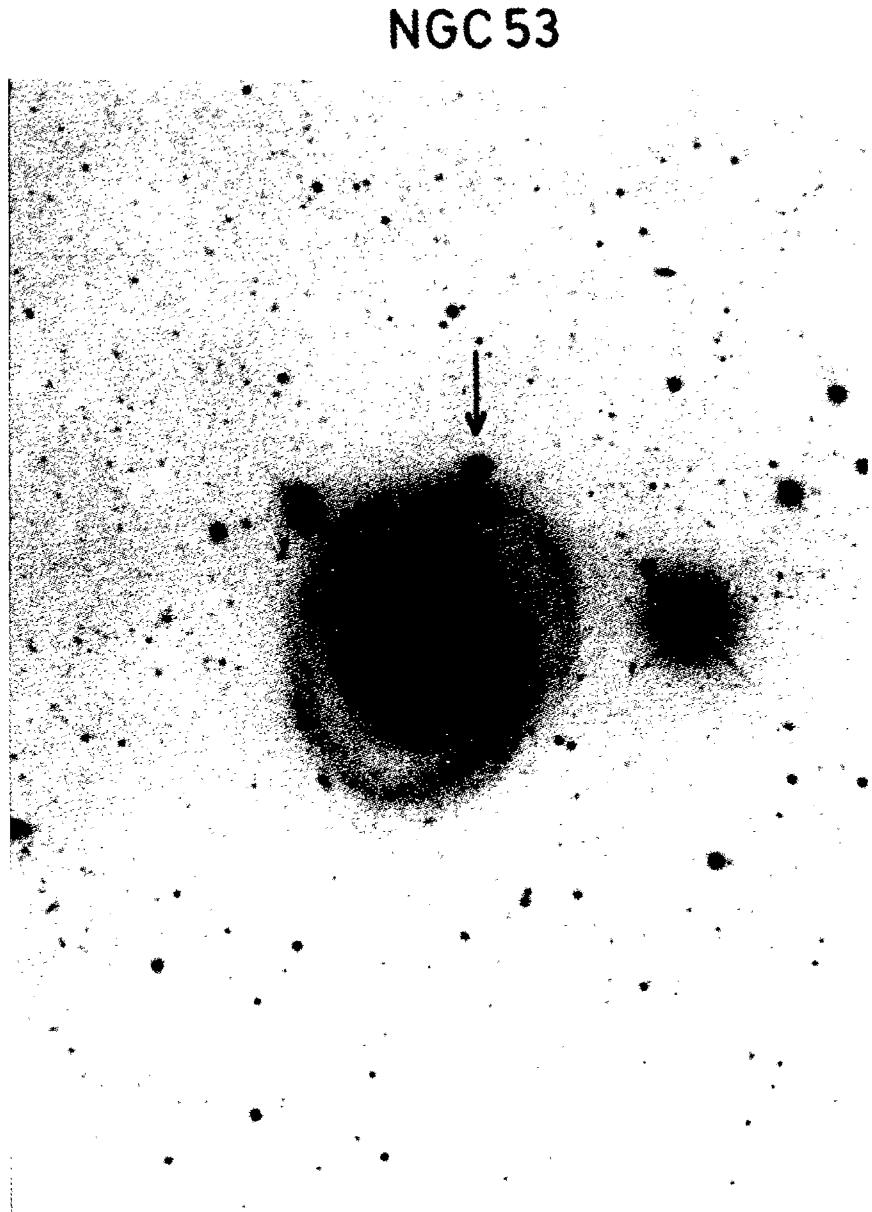

Fig. 30. The large galaxy NGC 53 (cz = 4160 km s^{-1}) has a filament extending up to a small galaxy (cz = 37000 km s^{-1}. (Photo by courtesy of Chip Arp.)

suggested as a possible reason why, as discussed in Section 5.1, many observers claim to see a quadratic Hubble law for bright galaxies.

We next present evidence suggesting physical association between bright, low redshift galaxies and QSOs of substantially higher redshifts. In that sense the case of NGC 4319 + Markarian 205 might belong to the next subsection.

5.4. Associations between QSOs and bright galaxies (C)

In Section 3.5 we presented evidence for close associations between QSOs and galaxies of nearly equal redshifts. In establishing association statistical arguments were used (e.g., Stockton, 1978) claiming that two objects must be real neighbours if their angular

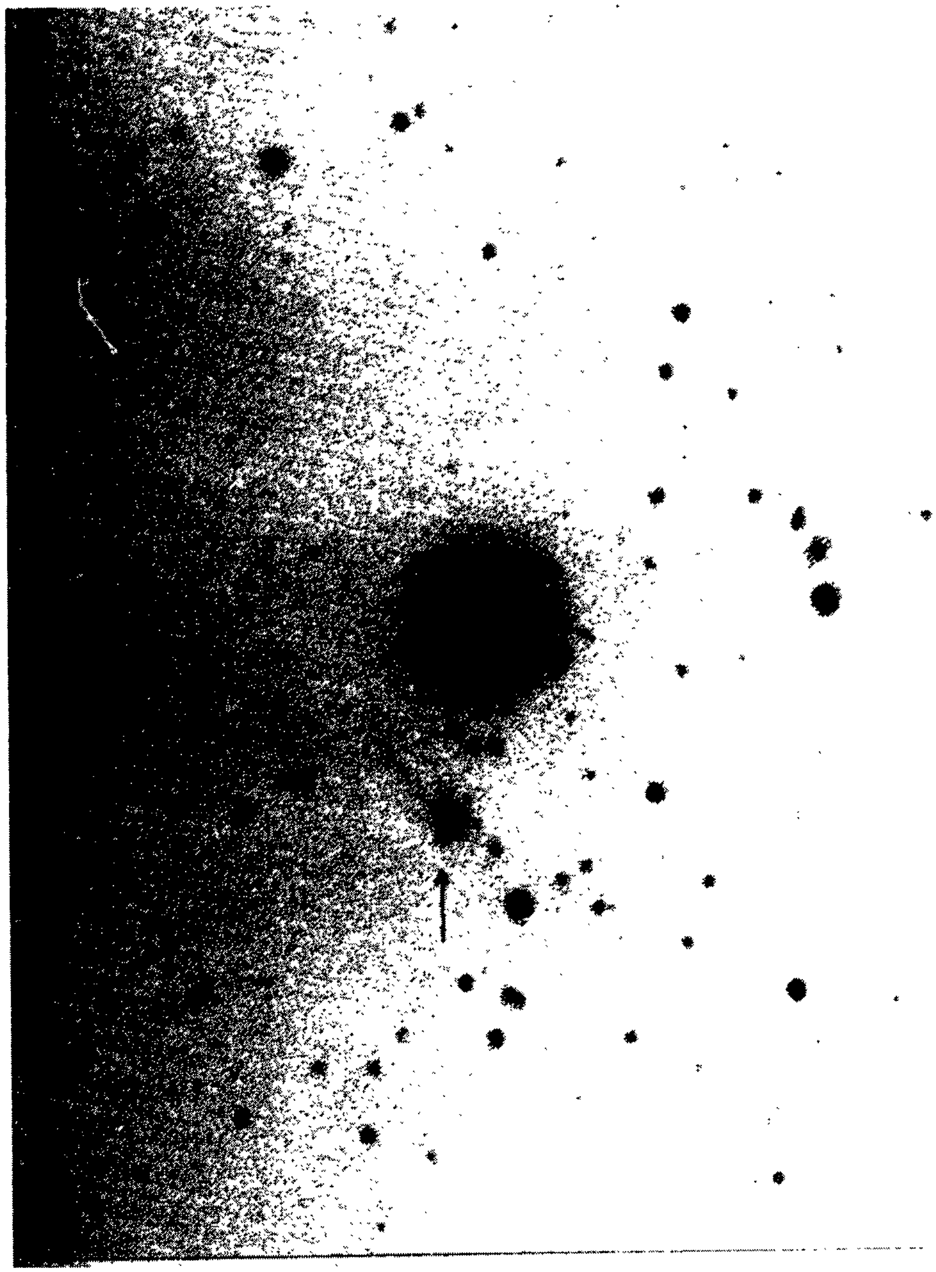

Fig. 31. The galaxy and its interacting companion (AM2054−2210) in the catalogue of Southern Peculiar Galaxies and Associations) have respective redshifts $cz = 10400$ km s^{-1} and 46900 km s^{-1}. (Photograph by courtesy of Chip Arp.)

positions are so close that the probability of this happening by chance is small. Somewhat similar probability arguments have been invoked on the other side of the debate to demonstrate that a QSO and a galaxy with very different redshifts are real neighbours. As we mentioned in Section 3.5, one should not prejudice the 'nearness' argument by bringing in the redshifts right from the beginning – if one really wants to test the validity of the CH.

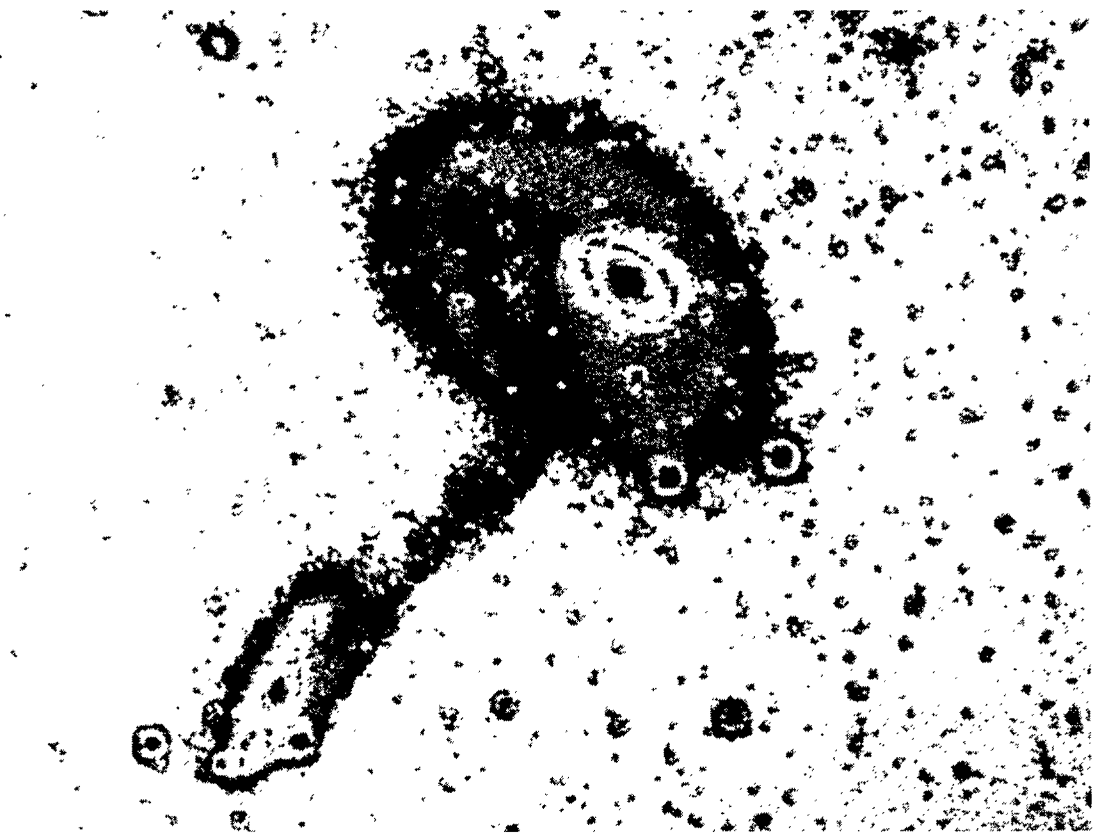

Fig. 32. The interacting galaxies known as 'toadstool'.

5.4.1. *Distributions*

The first systematic attempt to look for close pairs of QSOs and galaxies, *irrespective of redshifts*, was made by Burbidge *et al.* (1971). These authors looked for distributions rather than isolated instances of QSOs and galaxies; in this case they compared the radio loud QSOs from the 3CR catalogue with the galaxies in the Shapley–Ames catalogue. They found a significant statistical effect which was later confirmed also by Kippenhahn and de Vries (1974). However, a similar analysis using the QSOs from the Parkes radio catalogue and the galaxies in the Zwicky catalogue yielded a much less pronounced effect (Burbidge *et al.*, 1972). Hazard and Sanitt (1972) also discounted the effect after a comparison of other QSO samples with the Shapley–Ames galaxies. As Burbidge *et al.* (1972) themselves point out, comparison of samples containing further galaxies or/and fainter QSOs increases the population densities and hence the probability of a chance association.

Amongst other attempts to look for associations between populations of QSOs and galaxies may be mentioned the work of Seldner and Peebles (1979) who found a statistically significant evidence for association. Their QSO population numbered 382 with $|b| > 40°$, $\delta > -23°$ and the galaxies came from the Lick counts going down to 19^m. These authors found on the average 1.45 ± 0.39 more galaxies within $15'$ of a QSO than expected under a random arrangement. The effect does not seem related to the redshifts of the QSOs in the sample.

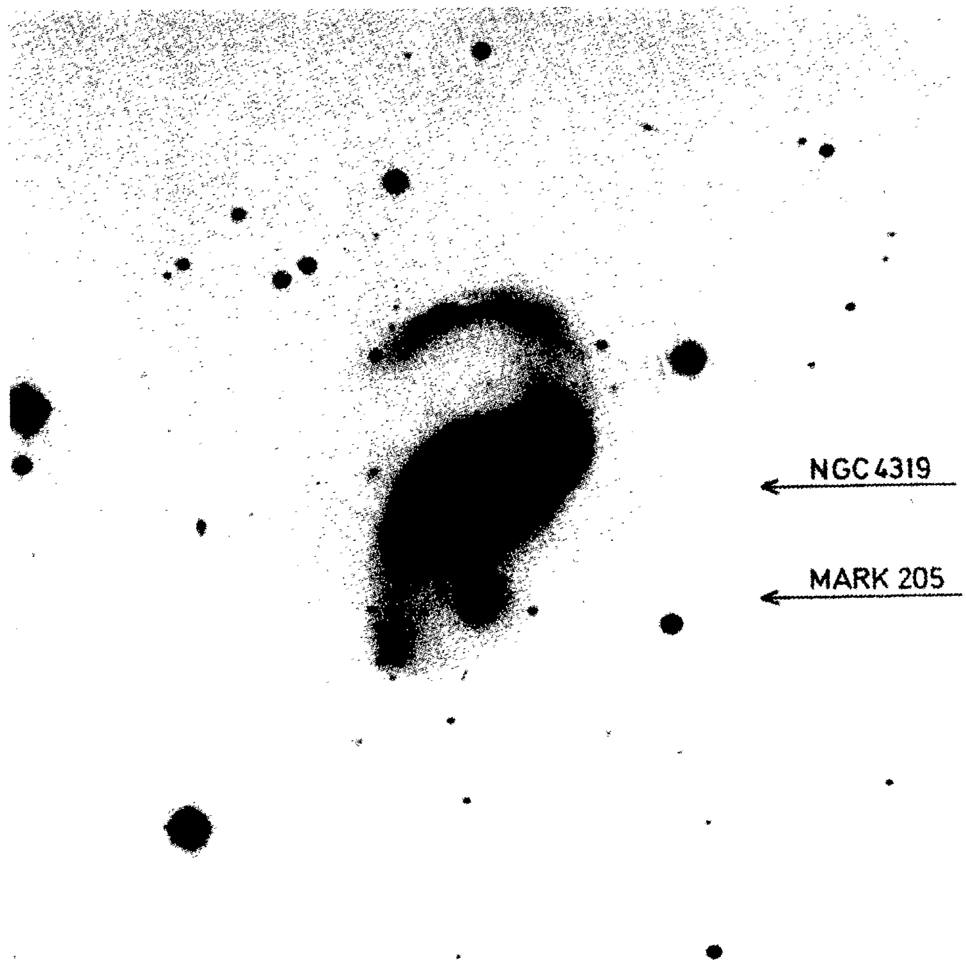

Fig. 33. The luminous connection found by Arp between NGC 4319 and Markarian 205. (Photo by courtesy of Chip Arp.)

Subsequently Chu *et al.* (1984) compared the distribution of QSOs in the Hewitt–Burbidge (1980) catalogue with the bright galaxies in the *Second Reference Catalogue* of de Vaucouleurs *et al.* (1976). The galaxies in this catalogue are brighter than 16^m while their redshifts are less than $cz = 15\,000$ km s^{-1}. Using the technique of cross-correlation function and the separation from nearest neighbour, these authors found that the QSOs and bright galaxies are associated. For example, there are 0.36 ± 0.09 more bright galaxies within 40′ of QSOs than expected by chance on the basis of the CH.

5.4.2. *Particular Cases*

We will now consider cases where particular galaxies are seen near particular QSOs, close enough to make one suspect physical association. Here one must ask, what is the average number $\langle n \rangle$ of QSOs one would expect to see within a given angular separation from the galaxy under a purely random arrangement. Should the answer turn out to be very small, we suspect the observed association to be genuine. The formula which

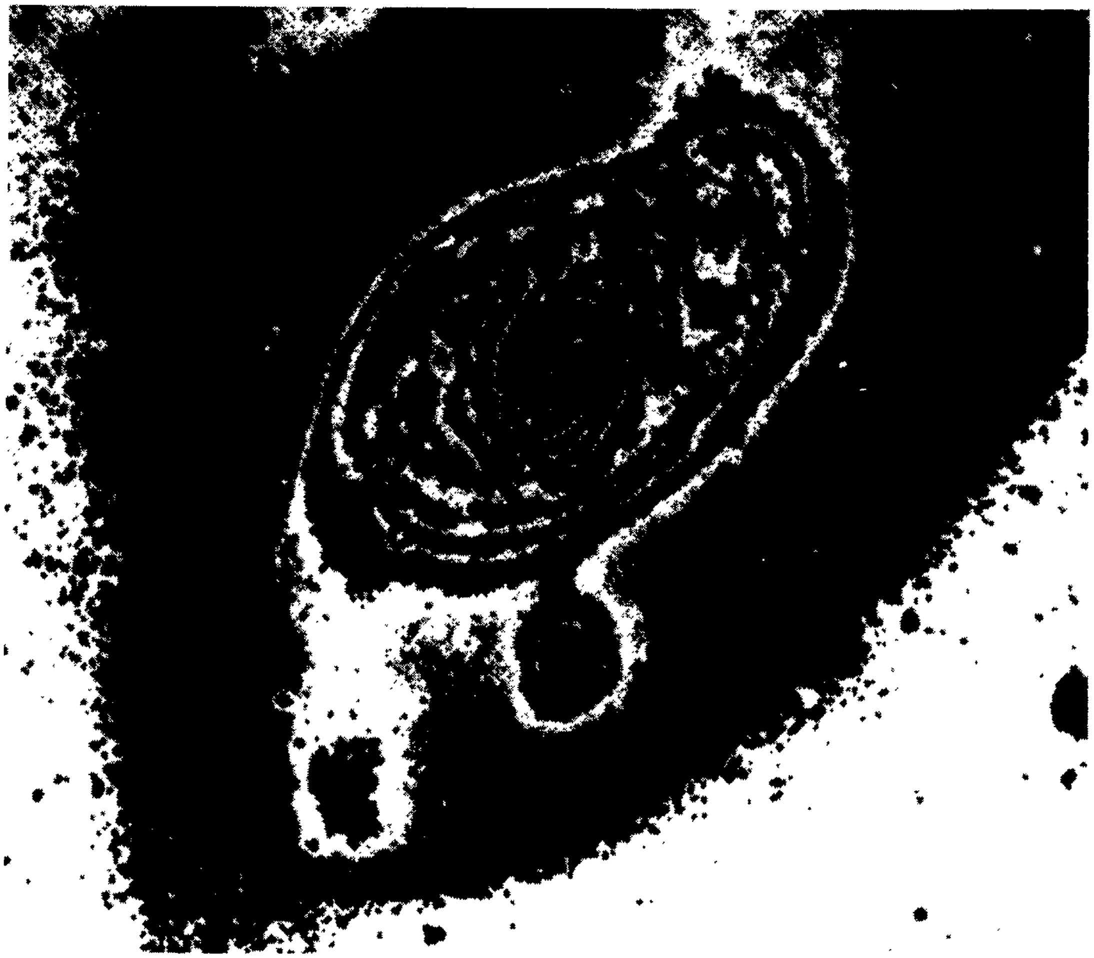

Fig. 34. The confirmation of the connection of Figure 33 by Sulentic. The isophotal contours from image processing techniques clearly link the two objects.

computes $\langle n \rangle$ is a simple one, first given by Burbidge *et al.* (1974):

$$\langle n \rangle = 2.4 \times 10^{-7} N\Gamma(<m)\theta^2 . \quad (36)$$

Here θ'' is the separation within which each of the N galaxies (or any other specified points on the sky) have been searched for QSOs with apparent magnitudes $\leq m$ and $\Gamma(<m)$ is their number per square degree. Thus Equation (36) requires the QSO surface density to be known. We illustrate the application of this formula as given by Burbidge (1981) by taking data from Tables 1 and 2 of Burbidge's paper. (The tables are too detailed to be reproduced here.)

Table I gives a list of all cases of close associations of QSOs and galaxies known up to 1980. Setting $\theta'' = 600''$ $(=10')$ we get the shorter list of Table II. Making the conservative assumption that 300 galaxies have been looked at carefully for close QSO neighbours, out of a total of 2000 bright galaxies, we get $\langle n \rangle$ as a function of θ from the formula (36). Following Table 3 of Burbidge (1981) we get Table I.

From this table we see that up to $\theta'' = 120''$ the observed numbers significantly exceed the expected numbers. For $\theta'' > 300''$, the expected values exceed the observed

TABLE I

A comparison between the number (N_0) of QSOs found near bright galaxies and the number $\langle n \rangle$ expected by chance

m	$\theta = 60$		$61 \leq \theta \leq 120$		$121 \leq \theta \leq 180$		$181 \leq \theta \leq 300$		$301 \leq \theta \leq 600$	
	N_0	$\langle n \rangle$	N_0	$\langle n \rangle$	N_0	$\langle n \rangle$	N_0	$\langle n \rangle$	N_0	$\langle n \rangle$
≤ 17	3	0.08	1	0.24	1	0.36	1	1.23	5	5.8
≤ 18	7	0.25	6	0.78	1	1.29	2	4.2	12	19.5
≤ 19	11	0.78	12	2.32	7	3.6	9	12.3	19	58.6
≤ 20	17	2.60	19	7.8	9	12.9	14	41.2	24	19.4

ones. Thus the 'near-neighbour' requirement implies that for small angular separations we do see more QSOs near galaxies than expected by chance. Why does N_0 become less than $\langle n \rangle$ at larger values of θ? This could imply that our estimate of $\langle n \rangle$ was perhaps two high, i.e., we overestimated the QSO number density $\Gamma(<m)$. If that is the case, then $\langle n \rangle$ should be lower than given in Table I and so the statistical significance of the difference $N_0 - \langle n \rangle$ becomes even more for $\theta \leq 120$.

The use of the formula (36) circumvents a criticism often made of the process of computation of probabilities. The criticism is that the computations are made *post facto* – after a certain observation has been made. It is true that calculations of *a posteriori* probabilities can be misleading after the event has already occurred. However, with the formula (36) one can lay down the prior probabilities, i.e., the expected numbers $\langle n \rangle$, which can be compared with actual data anytime.

Arp (1981) approached the problem of QSO-galaxy associations in a different way. He selected a sample of bright NGC galaxies in a given part of the sky and looked for QSOs in fields in their neighbourhoods. In some cases he found a QSO near the NGC galaxy or near a companion galaxy in the field. If, for the magnitude of the QSO in question $\Gamma(<m)$ was such that the chance of finding a QSO of that apparent magnitude (or brighter) within the observed separation θ'' turned out to be less than 0.01, then Arp regarded the association a 'success'. In this way, he registered 13 successes out of 22 QSOs found near some 34 candidate galaxies. Thus, if the CH is correct then the chance of registering upto 13 successes out of 34 trials with a success probability per trial of 0.01 turns out to be as low as 10^{-17}.

Arp's success criterion depends on the magnitude of the QSO and a conclusion based on such a criterion has been criticised by other authors. Browne (1982) for example, has concluded that the chance of obtaining Arp's results by accident is 10^{-7} if one does not bring in magnitudes into the calculation *a posteriori*. Webster (1982b) has reinterpreted Arp's search procedure to come up with an even higher probability of 0.05. Since the probability computations are critically dependent on the search procedures, extreme caution is needed in interpreting such data. The above episode illustrates this aspect but at the same time suggests that the association discovered by Arp cannot be entirely dismissed away as being due to change.

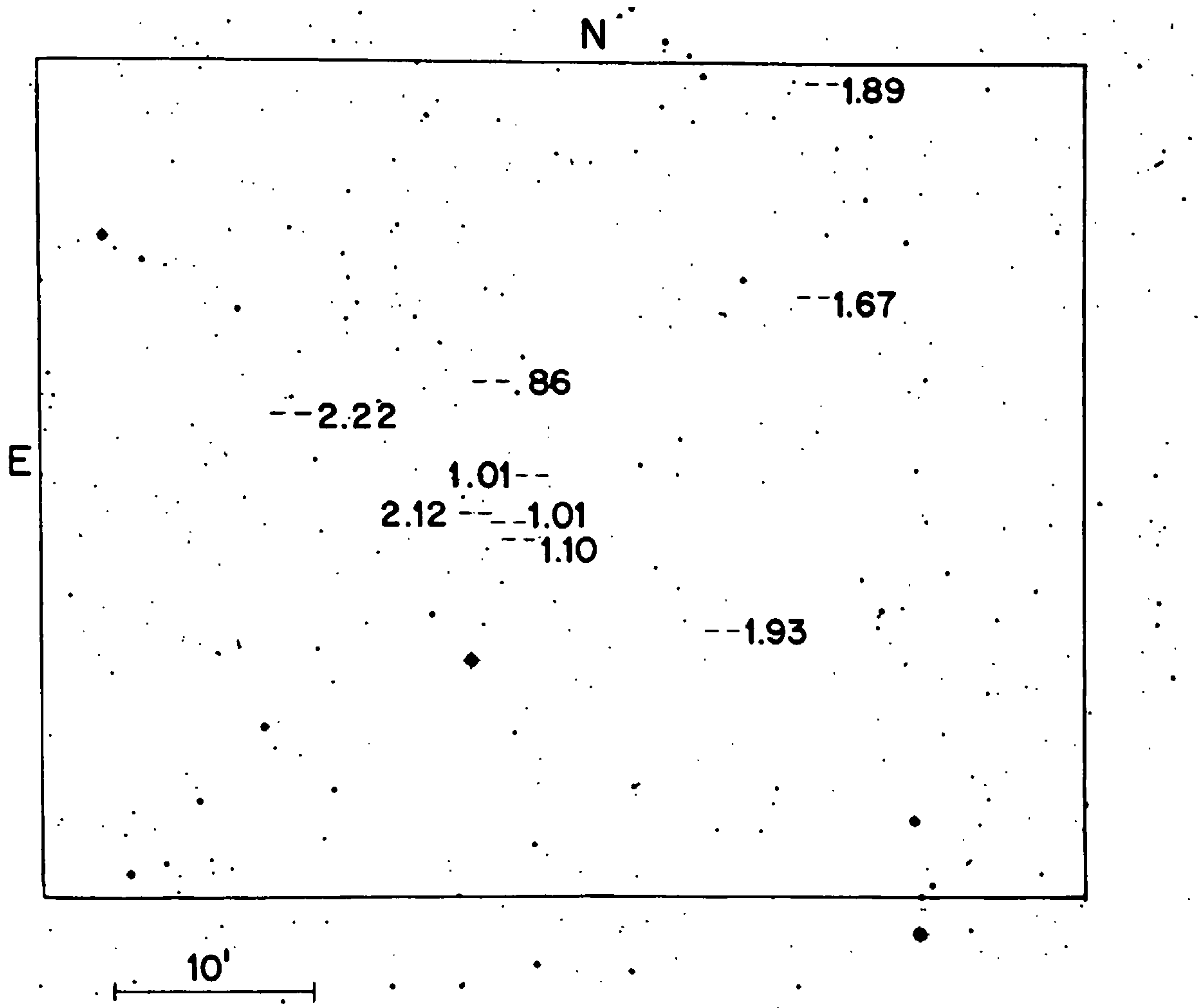

Fig. 35. The concentration of QSOs (with redshifts marked) in the vicinity of the region $\alpha = 11^h46^m14^s$, $\delta = 11°11'42''$ found by Arp and Hazard (1980).

Arp and his coworkers have produced several isolated examples of dense concentrations of QSOs around companion galaxies near the NGC galaxies. For a review of such cases see Arp (1987b). A striking example is shown in Figure 35 where we see a dense concentration of 9 QSOs discovered by Arp and Hazard (1980). There are four galaxies in the field. Is the concentration significant? We will return to this particular example in Section 6.1 when we look at a Doppler ejection model.

5.4.3. *The $\theta - z$ Relation*

In the early 1970s, Burbidge *et al.* (1972) reported a peculiar empirical relation between the angular separation θ between QSO and galaxy in a close pair and the redshift z of the galaxy. The relation obtained for five pairs with the QSOs taken from the 3CR catalogue is shown in Figure 36. The five points lie very close to the line

$$\log\theta + \log z = \text{constant}\,. \tag{37}$$

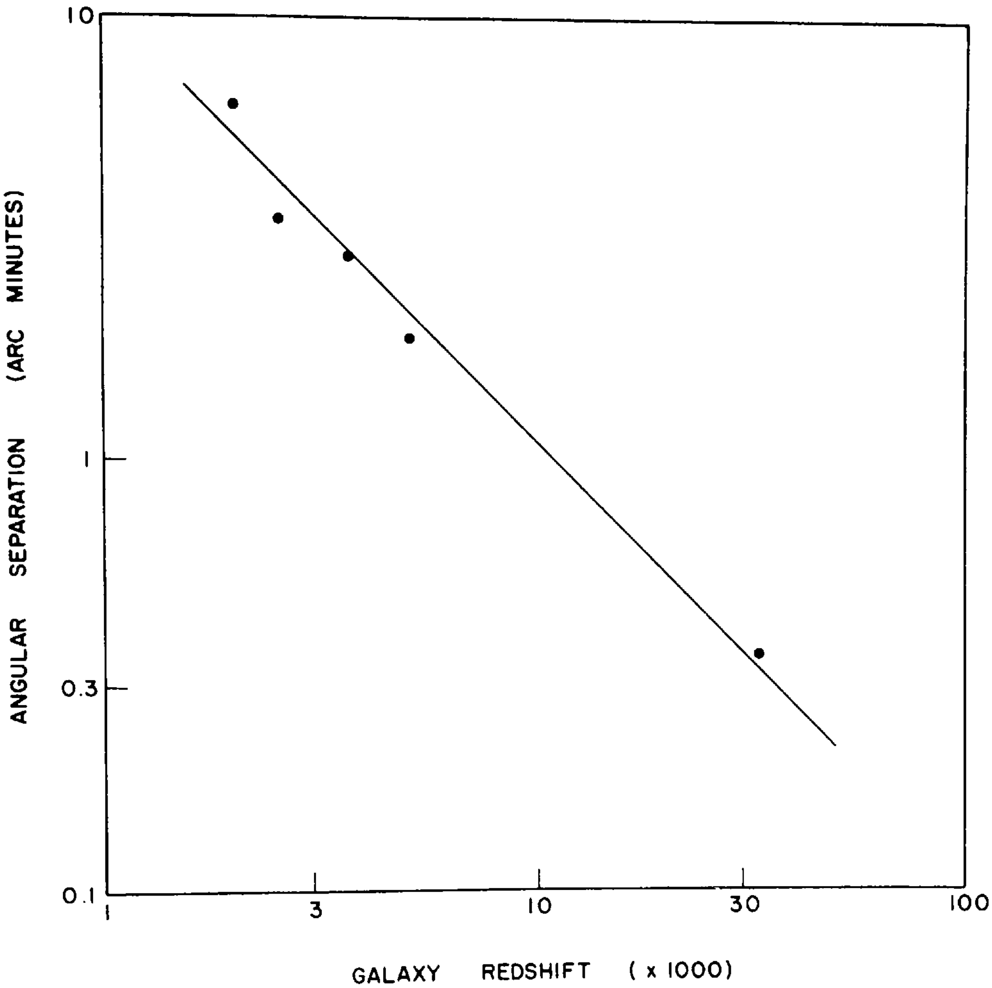

Fig. 36. The angular separation θ between QSO-galaxy pair plotted against the redshift of the galaxy on a log-log scale. (From Burbidge *et al.*, 1972.)

Why should such a relation emerge, if the QSOs (according to the CH) happen to be projected by chance near the galaxies?

If the two members of the pair are at the same distance (as indicated by the galaxy redshift) than (37) implies that the linear separation between the two, projected perpendicular to the line of sight, is constant. It may be that in a local theory, the linear separation stays within narrow limits, in which case, due to projection effects we expect to see a scatter around the line (37).

When the ($\log \theta$, $\log z$) were plotted for *all* the close QSO-galaxy pairs in Burbidge's Table 1 (Burbidge, 1981), there was indeed more scatter in the distribution. However, the best fit line has the slope $d \log \theta / d \log z = -1.2$ which is within statistical fluctuations of the slope -1. For details of this plot see Burbidge (1980) and the Figure 4 therein.

5.4.4. *The 'Lensed' QSO*

We end this subsection with the remarkable case of the close pair of QSO + galaxy found by Huchra *et al.* (1985). The QSO of redshift $z = 1.695$ was found by these authors, within 0.3″ of the nucleus of a galaxy 2237 + 0305 with $z = 0.0394$. Had the two redshifts turned out to be equal, this system would have been reported as a case of a QSO forming the nucleus of the Galaxy. We encountered such cases in Section 3.6 However, here the redshifts were discrepant.

The only alternative for the survival of the CH in this case is to argue that the QSO was projected by chance near the nucleus of the Galaxy. The probability for this to happen is, as low as 10^{-5} if the QSO must fall within 0.3″ of the nucleus and somewhat larger at if it is to fall within the slit area of observation. In either case the probability is too low to justifty chance projection.

To boost the probability the authors hit upon a new loophole apparently provided by gravitational lensing. If the QSO were not really as bright as it seemed at 17^m, but was boosted in its brightness by a suitable gravitational lens, then it really belongs to a much fainter population. Thus the factor $\Gamma(<m)$ in the formula (36) goes up and the probability of chance projection improves by a large factor. In this way the authors boosted the probability by a factor 20.

Notice that in this example there are no basic hallmarks of gravitational lensing present. There are no two images, let alone images having the same spectra*. There is no intervening lensing galaxy. The QSO is not so abnormally bright as to force one to suspect a gravitational lens. The role of the lens here is that of an epicycle to prop up the CH. Yet the paper was titled as if a lens was definitely confirmed: '2237 + 0305. A new and unusual Gravitational Lens'.

A closer scrutiny reveals the weakness of the lens alternative. Claims were made (Tyson and Gorenstein, 1985) subsequently that the lensing galaxy was found by deep CCD imaging – yet a paper in a scientific journal never appeared. What is more, the spectrum of the QSO shows a steep slope ($\sim \nu^{-3.5}$) in the optical band, thus implying that interstellar dust in the intervening galaxy would have absorbed considerable light from it (Edmunds, 1985). This would make the QSO intrinsically much brighter and hence bring down the factor $\Gamma(<m)$ again, thereby vitiating the advantage gained by lensing.

Subramanian (1985) has pointed out that the absence of a secondary image puts constraints on the lensing models. The probability that a suitable galaxy (a barred spiral is needed) comes in at the right place to do the right lensing has to be folded in with the total probability. These factors in the end bring the probability down to $\sim 10^{-5}$.

On general grounds, Schneider (1985) has argued that it is improbable that gravitational amplification can sufficiently account for the puzzling features of the QSO-galaxy associations in this way.

5.5. Close pairs of QSOs (C)

Over the last 15 years several close pairs of QSOs have been found. By close, we mean a pair in which the angular separation is less than 1′ or 2′. Table II in the paper by Burbidge *et al.* (1985) gives the list upto 1985. The conventional view with regard to the

* Recently, Yee (1988) has claimed to have resolved the QSO image into four components though spectroscopic study is still awaited.

close pairs is briefly discussed in Section 3.4. We restate it here since it is appropriate as a background to the data to be discussed now. Basically speaking the close pairs are treated in three different ways: (i) Pairs with very small separation ($\theta < 7''$) but with identical redshifts are looked upon as gravitational lens candidates. (ii) Pairs with similar but not identical redshifts and with $\theta \gg 7''$ are considered distinct members of a supercluster. (iii) Pairs with different redshifts but small separation, $\theta \lesssim 2'$ are either ignored as of no significance or considered cases of chance projection.

The first two cases were the pair Ton 155, 156 with $z = 1.703$, 0.549 found by Stockton (1972) and 1548 + 114 A, B with $z = 0.436$, 1.901 found by Wampler *et al.* (1973). The angular separation in the former case was 35″ and in the latter 5.5″. While the authors emphasized that the probability of chance projection is small, the conclusions were ignored largely on the grounds that the probabilities were computed *a posteriori*.

However, the formula (36) bypasses this criticism by offering a rule for probability computation. Burbidge *et al.* (1985) find that the probability of finding 6 close pairs with $\theta < 120''$ and one member of the pair a radio source and both members brighter than 18.5^m is as low as 1.3×10^{-4} whereas 6 such pairs have actually been found. Likewise, if one drops the more stringent radio-selection criterion and limits to $\theta < 60''$ and $m < 18.5^m$, then we have 6 pairs. The probability for this to happen by chance is only 8×10^{-8}.

These low probabilities cast doubts on the fundamental assumption of the cosmological hypothesis. However, the conclusions arrived at in the above work have been challenged by Shaver (1985a).

Shaver has argued that as indicated by formula (36) the number of close pairs of QSOs should increase in proportion to θ^2 if there is no special clustering effect. He points to several studies (Osmer, 1981; Webster, 1982a, Chu and Zhu, 1983; Savage *et al.*, 1984; and Shaver, 1985b) to show that the cumulative number of QSO pairs does follow the above rule. If one sets the coefficient A in $\langle n \rangle = A\theta^2$ from the above studies, one finds that the number $\langle n \rangle$ is considerably (by ~ a factor 10) lower than has been found by Burbidge *et al.* (1985, *op. cit.*). He, therefore, goes on to say that the sample chosen by these authors has a strong bias and selection effects. Earlier Wills (1978) had also cautioned against the unestimated bias that creeps in when the observer specially looks for close pairs; for such pairs get more attention compared to others.

The criticism of Shaver is not entirely valid since the studies quoted by him in favour of his $\langle n \rangle = A\theta^2$ law extend to larger angle separations ($\theta \sim 2'$–$1000'$) and over such separations the effect of local clustering tends to disappear. Burbidge (1981) has already drawn attention to this fact for QSO-galaxy separations (see Section 5.4, Table I). Thus it is unfair to deduce 'bias' and 'selection effect' for a sample that deals with smaller separations.

So far as the 'special attention' effect is concerned, it is avoided at least for radio QSOs, since the process of optical identification automatically requires the observer to search the field around the radio positions carefully. Hence any QSOs of small or large separations would get equal attention in the search process.

5.6. Alignments and Redshift Bunching (F)

In astronomy we often interpret alignments of objects on the sky in terms of some physical mechanism of ejection. The most striking example of this practice is found in the double lobed radio sources whose lobe centres are very often seen as aligned across a central galaxy. The usual interpretation of these objects is in terms of a central black hole ejecting plasma in a highly collimated fashion in opposite directions.

Highly suggestive alignments have been found between QSOs in relation to a central bright galaxy and in some cases between QSOs without reference to a galaxy. Figures 37–40 illustrate QSO-galaxy alignments while Figure 41 shows a remarkable double alignment of two triplets found on the same photographic plate by Arp and Hazard (1980).

In Figures 37–40 the separations between the QSOs and the galaxy are of the order of several arc minutes ranging upto a degree or so. Notice also that in many cases the QSO redshifts are bunched around in relatively small intervals. Not all of these data can be put down to selection effects. Are these alignments across the galaxy and the

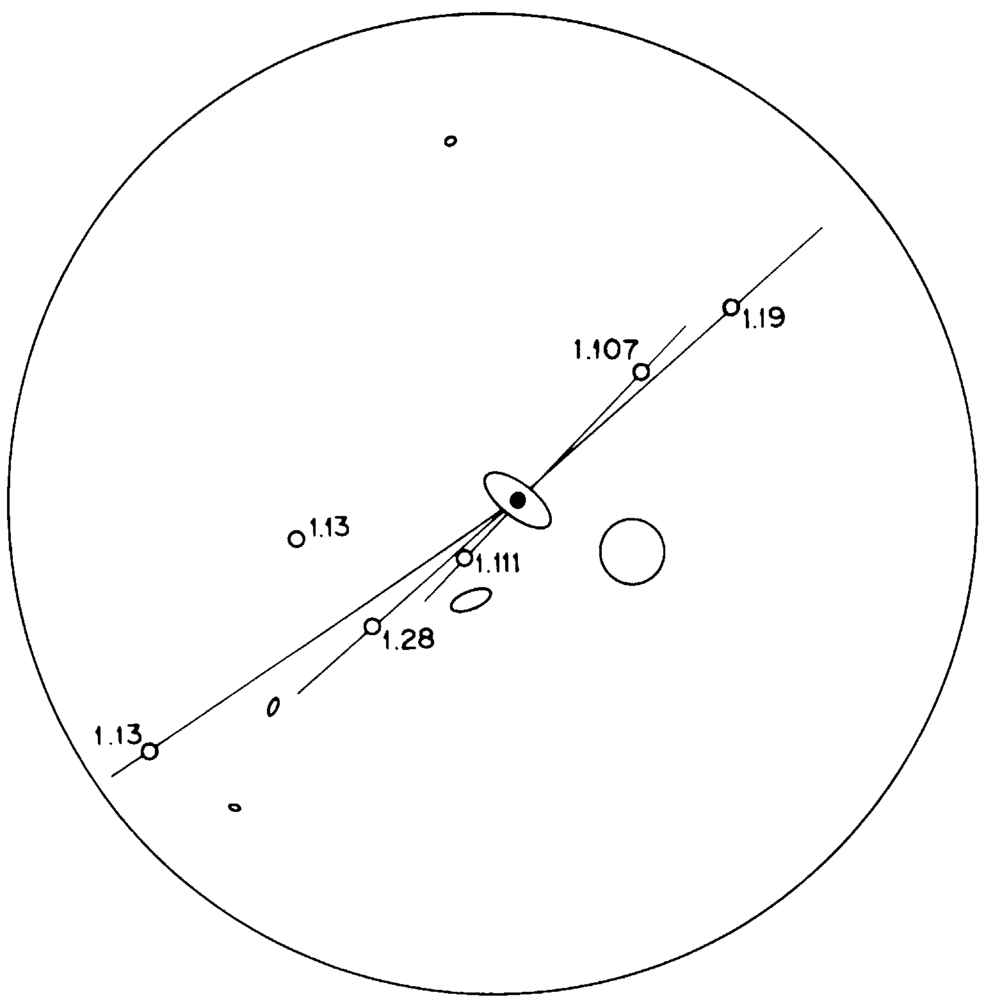

Fig. 37. Six bright QSOs with bunched redshifts are seen here aligned across NGC 3384. (From Arp *et al.*, 1979.)

Fig. 38. Three QSOs with redshifts shown are found within 2″ of the centre of the spiral galaxy NGC 1073. Another remarkable feature is that the redshifts of the QSOs are at peaks of the redshift distribution (cf. Section 5.2). (Photo by courtesy of Chip Arp.)

redshift values entirely coincidental? It is fair to say that although *a posteriori* probabilities for these configurations have been computed and shown to be low, they are also subject to considerable debate.

The configuration of two aligned triplets in Figure 41 is relatively more clearcut. Here also the three redshifts of the 'upper' triplet closely match those of the 'lower' one. Leaving aside this peculiar feature, we can ask the question as to what is the probability of finding three randomly selected points to lie on a straight line, given the observational uncertainty of alignment to be $\pm 1''$. The answer depends naturally on the number

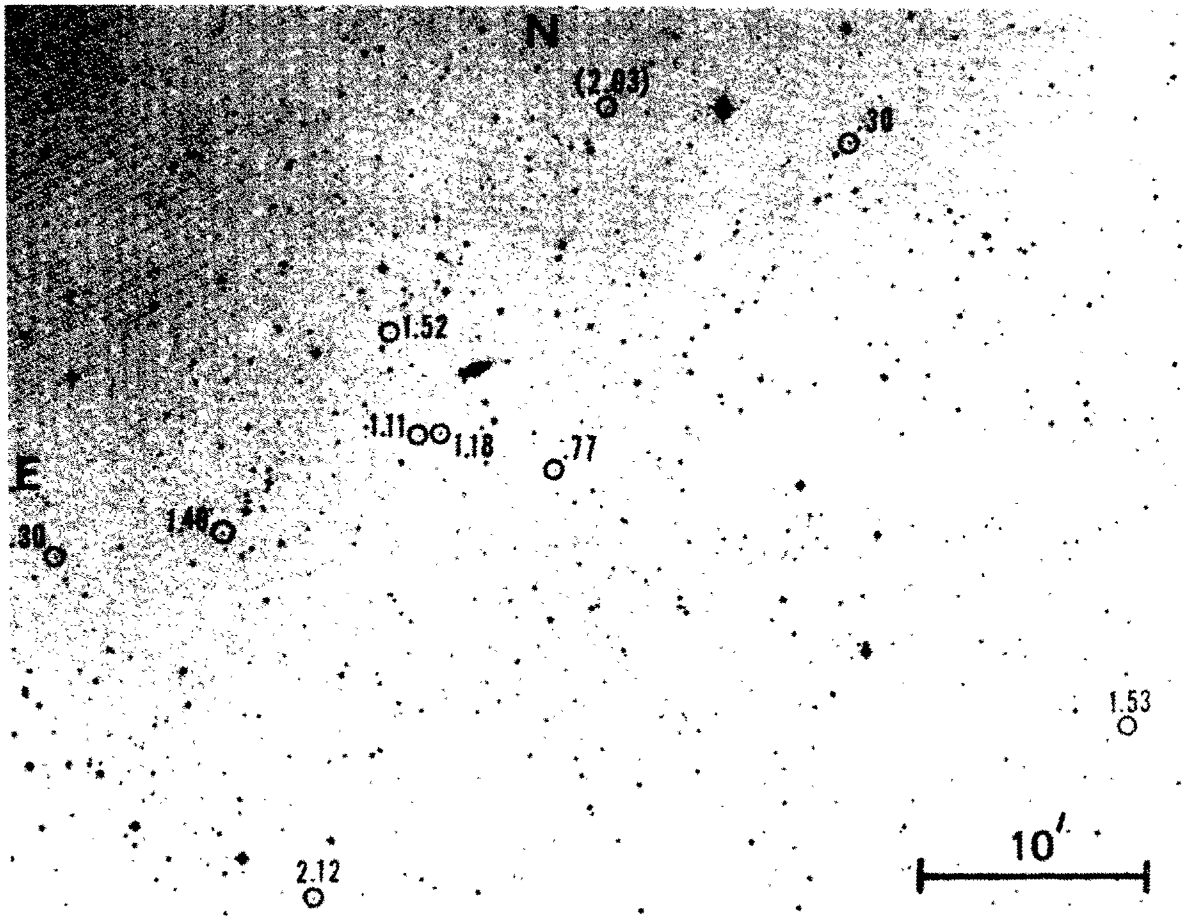

Fig. 39. Ten QSOs surrounding the campanion galaxy to the spiral galaxy NGC 2639. There is alignment of QSOs across the galaxy and similarity of redshifts.

density of the distribution of QSOs on the plate and the area of the plate searched. Hoyle (1980) computed the probability with the following result.

Let Σ square degrees be the angular area of the photographic plate with N_0 QSOs brighter than a given magnitude m randomly distributed on it. Let d' be the linear extent of the triplet expressed for convenience in arc minutes. Similarly we will denote the 'thickness' of the triplet by p'', i.e., p'' is the positional uncertainty from a strict straight line. Then the probability for finding a triplet (or the 'mean' number of triplets) expected by purely random process is

$$P = \frac{2\pi}{3} \frac{d^3 p N_0^3}{(60^5)\Sigma^2} . \tag{38}$$

In general $P \ll 1$. If N triplets are found we can use the Poisson distribution to compute their probability. Hoyle noted that Saslaw had subsequently reported two more triplets on the same plate. Thus for $N = 4$ he got the probability $< 10^{-4}$.

Narlikar and Subramanian (1982) did a Monte-Carlo simulation of random points on the 'photographic plate' as well as made different analytical estimates to arrive at approximately the same value of the probability. Edmunds and George (1981) on the

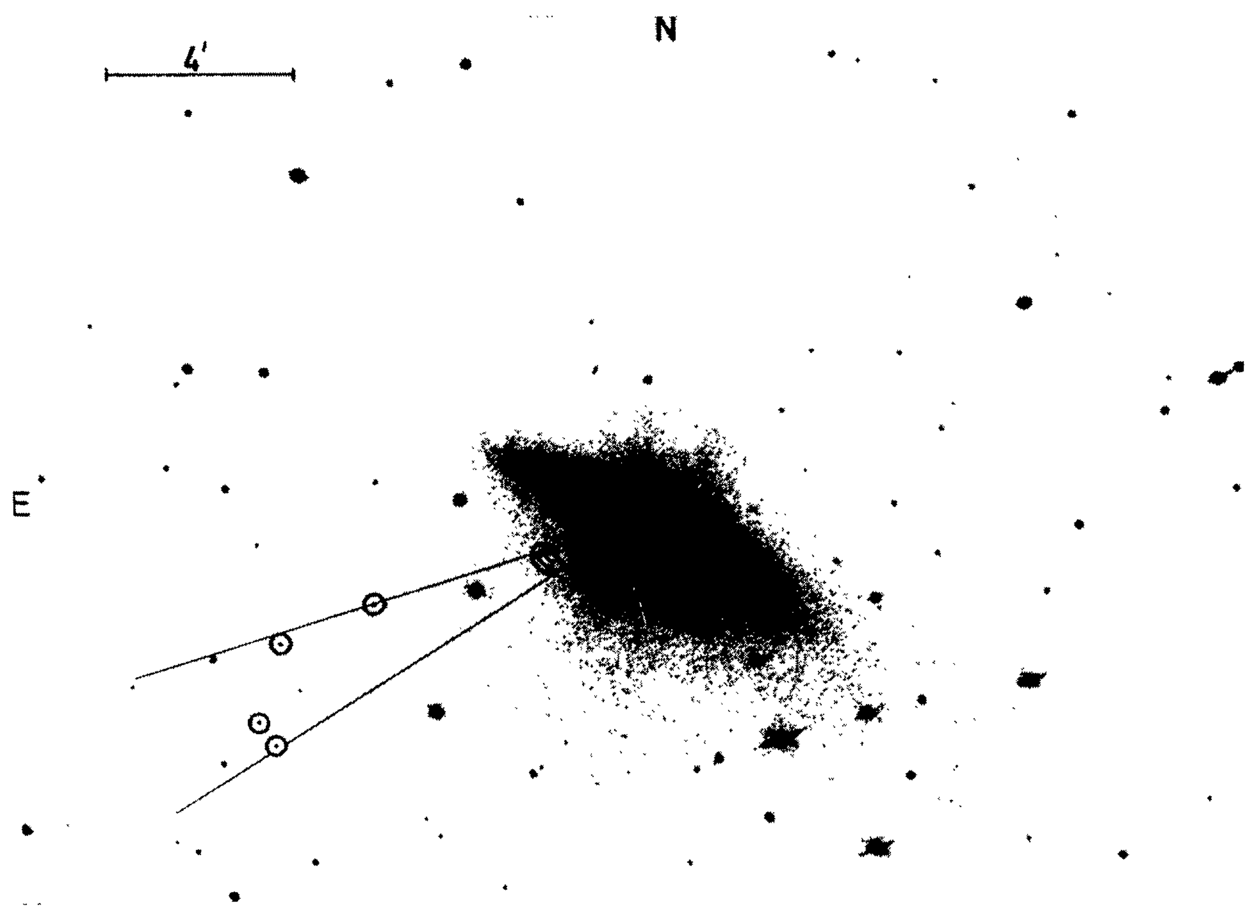

Fig. 40. The four QSOs found near the galaxy M82 have redshifts 2.05, 2.1, 2.04 and 0.85. Of these the first three were found by Burbidge *et al.* (1980) and the fourth one by Arp (1983). (Photo by courtesy of Chip Arp.)

other hand did a similar exercise and found the probability as high as 10%. The reason for the discrepancy can be traced to some extent to the analytical formula (38). Here the probability goes as cube of the QSO surface density as well as the cube of the length of the triplet. If both quantities are raised by factor 2, the probability shoots up by 64. We, therefore, leave it to the reader to form his own judgement as to the improbability or otherwise of finding four aligned triplets on the same plate.

Searches for aligned triplets have been made by Clube *et al.* (1983) who also found significantly more triplets of aligned QSOs in certain regions, compared to what is expected by chance. It seems, in view of the formula (38) that to disprove the CH decisively one needs to produce triplets no longer than 20′–30′ of QSOs not fainter than 19^m. It is also necessary, in view of its importance in probability estimation, to get as precise a value as possible for surface density of QSOs brighter than the specified magnitude m.

On a much longer scale, Arp (1984) has highlighted the apparent anisotropy of radio selected QSOs in the Southern Galactic Hemisphere (SGH). He finds that the QSOs in the redshift magnitude window $1.4 < z < 2.7$, $17.5^m < V < 19^m$ taken from the Parkes and 3CR surveys show a jet like distribution extending from R.A. $1^h 30^m$, $\delta = 30°$ to

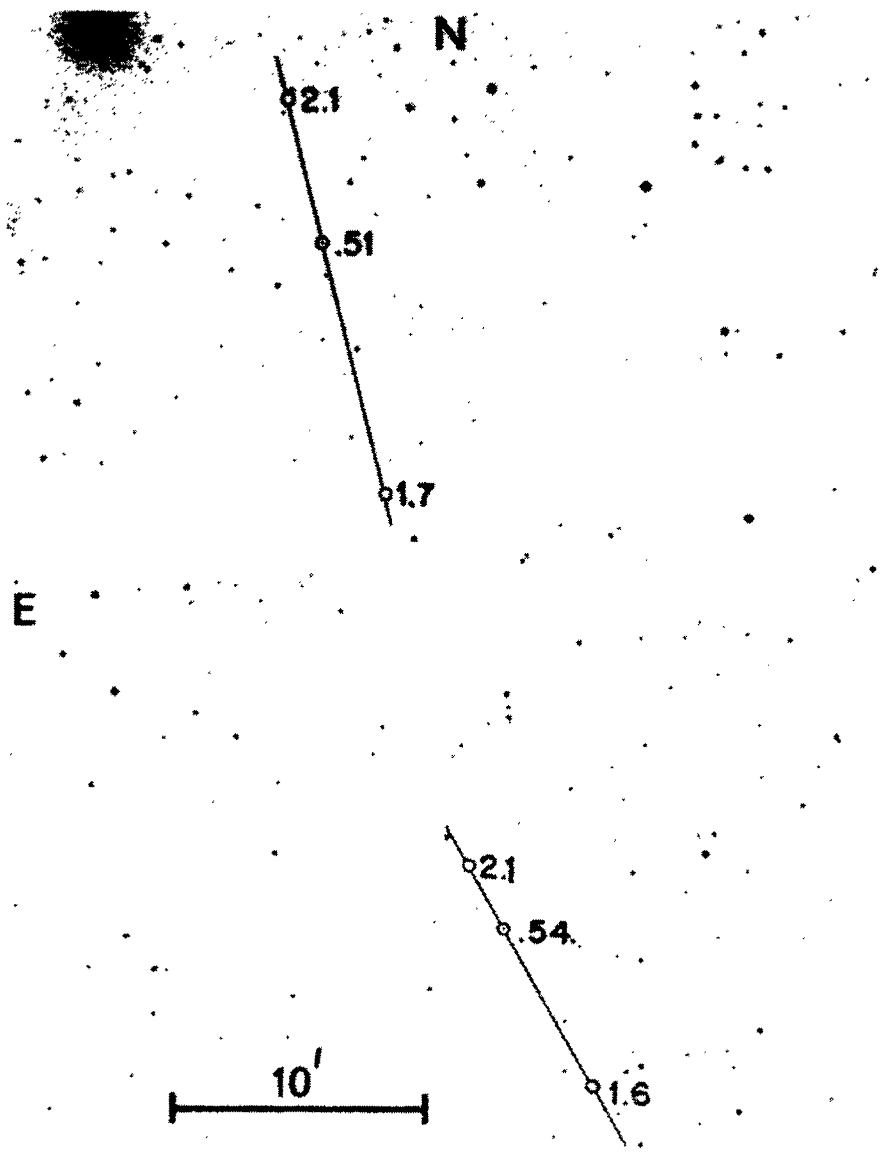

Fig. 41. The two triplets of QSOs with their redshifts marked discovered on the same plate by Arp and Hazard (1980).

R.A. 23^{h}, $\delta = 0°$. Figure 42 illustrates this distribution but it also indicates a curious feature. The galaxy M33 from the Local Group is sitting at one end of the jet. Arp poses the question as to whether all these QSOs are very close by and may have been ejected from M33.

That there is anisotropy in the radio QSO distribution at high flux densities has also been noticed by Shastri and Gopal Krishna (1983). Arp claims that the number of QSOs in the M33 region of the SGH is far larger (above $\sim 11\sigma$ difference) than the number in the diametrically opposite region. Even if we ignore M33 and simply estimate on the basis of the CH, the length of the QSO jet, we find it to be at least ~ 1000 Mpc! Thus on the basis of the CH, Arp reasons that the above inhomogeneous distribution violates the cosmological principle.

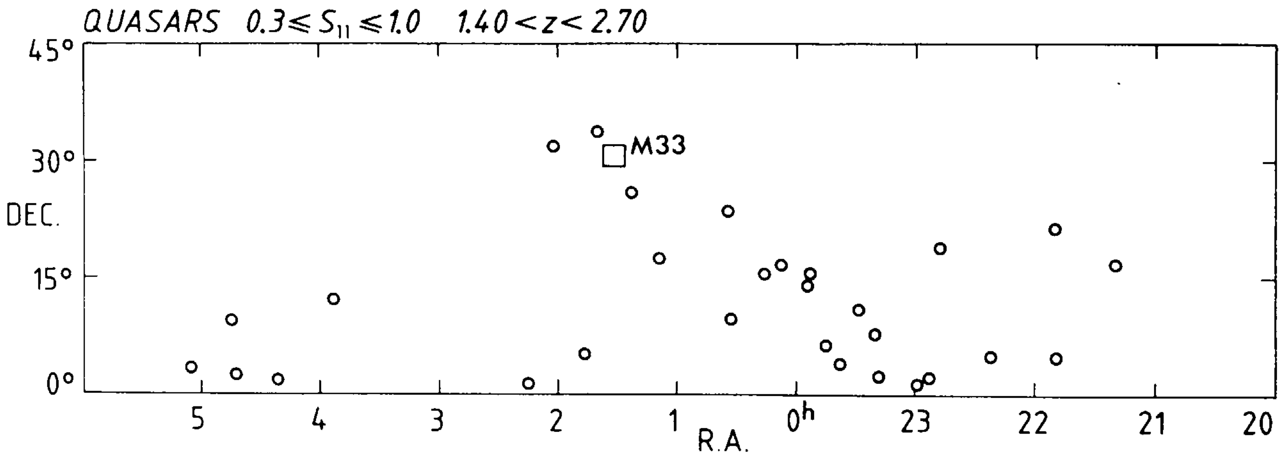

Fig. 42. The inhomogeneous distribution of a complete sample of radio loud QSOs within specified redshift and magnitude intervals. Arp claims that this could be a jet of QSOs issuing from a galaxy like M33 which happens to be suitably located.

To what extent is the claimed jet-like concentration of QSOs in Figure 42 statistically significant? The standard statistical techniques like the χ^2 or the Poisson statistic do not tell us about the reality of a large-scale jet. To arrive at a suitable criterion, Narlikar and Subramanian (1985) derived a new technique. This tells us the probability (under random processes) for arriving at a long chain-like structure or a straight extended jet. Using this technique these authors find that the jet-like structure appearing in the z window chosen by Arp is indeed significant. But at the same time, if we enlarge the window or choose a different one, the effect either gets diluted or disappears altogether.

Arp (1987c) has extended this work to the motions of high velocity hydrogen clouds in the vicinity of 3C-120 which also, he claims, show the tendency of being ejected. Further, like the M33 and the Local Group, he finds another jet (a smaller one in angular extent) apparently emanating from the galaxy NGC 300 in the Sculptor Group. Arp argues that if the characteristic jet is of a fixed linear size then the angle subtended by it at the observer will reduce in inverse proportion to the distance of the source galaxy from us. Such an effect appears to hold for the jets associated with NGC 300 and M33.

Skeptics would require more significant evidence to make one believe that ejection process involving several QSOs occurred in the group of apparently normal galaxies like the Local Group. However, the alignments and redshift groupings reported in this section can have no explanation on the basis of the CH: all of them would have to be put aside as chance coincidences.

5.7. Morphology of Galaxies (E)

If galaxies are at the distances implied by the CH, then their measured angular diameters can be converted to linear sizes and compared with those of standard galaxies like our own or the M31; for which reliable sizes are known. Arp (1988a) has drawn attention to certain discrepancies that result from such a comparison.

The galaxy NGC 262 (Markarian 348) has a large neutral hydrogen (H I) envelope around it (Heckman *et al.*, 1978; Morris and Wannier, 1980; Heckman *et al.*, 1981).

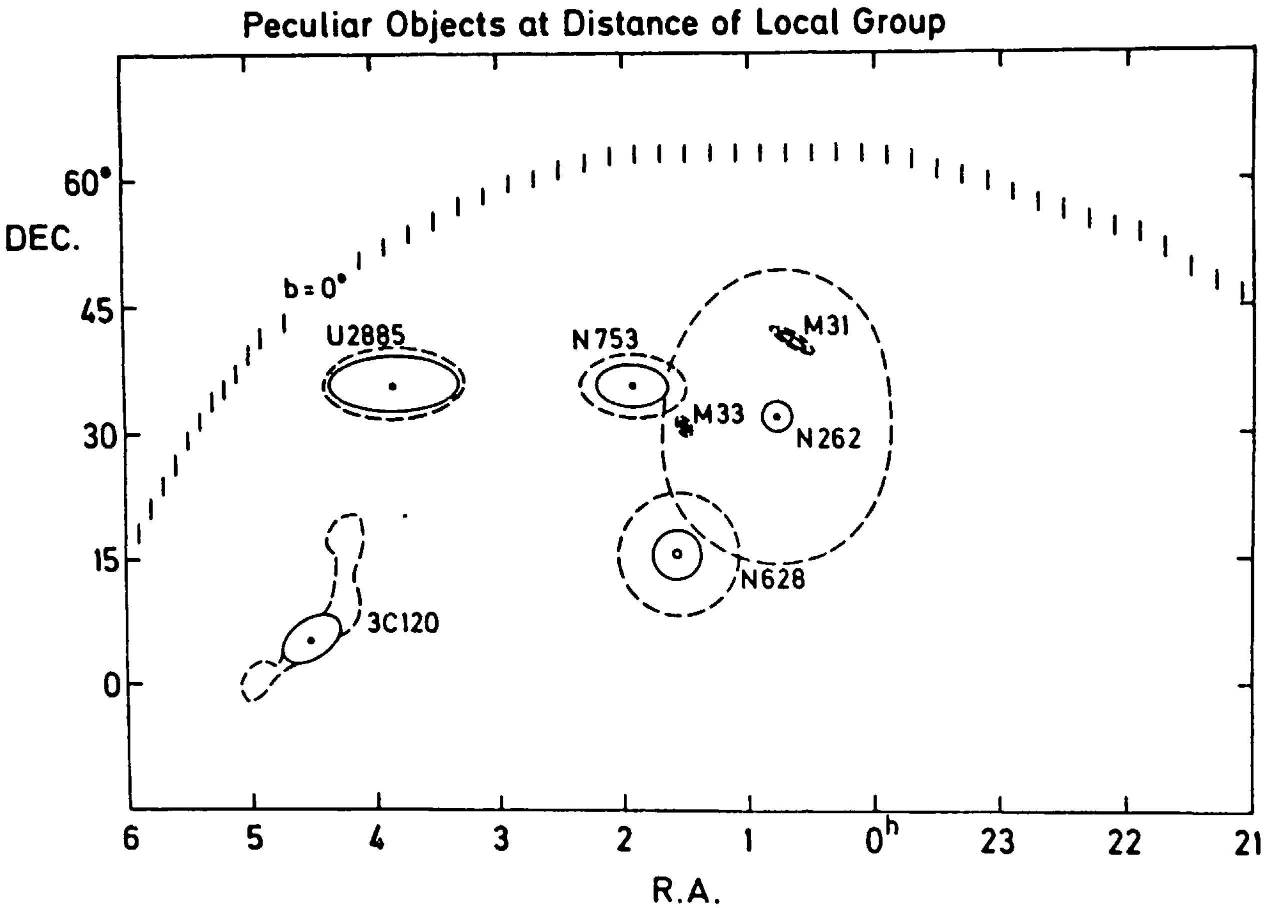

Fig. 43. This shows how large NGC 262 is compared to the Local Group galaxies if its size is estimated from the cosmological hypothesis.

Recently Simkin *et al.* (1987) reported measures both with the VLA and Schmidt photography that make it the largest known galaxy (of diameter ~400 kpc). Of course, these conclusions are based on assuming that the Galaxy is at its redshift distance ($cz = 4700$ km s^{-1}, $H_0 = 50$ km s^{-1} Mpc^{-1}). How large this galaxy is can be seen in Figure 43 where it is 'brought forward' and superposed on the Local Group. It is so large as to engulf both M31 and M33. Certain other large galaxies are also similarly placed for comparison in this diagram.

An independent measure of the distance of NGC 262 is given by the Tully–Fisher method of the rotational broadening of its H I line width. This method, however, leads to a very small distance since the measured velocity difference across the entire galaxy is only 100 km s^{-1}. It could be argued that the galaxy is rotating almost face-on with respect to us. But this assumption is not consistent with observed distorted and noncircular optical and H I features.

This example illustrates the possible checks morphological studies may provide on extragalactic distances. A further point made by Arp (*op. cit.*) is that as the optical diameter of NGC 262 is ~10 times that of M31, its volume is ~10^3 times larger and so its supernova rate must be as high as 5–50 per year! All these numbers get scaled down if we assume that NGC 262 is actually much closer than its redshift-distance.

Is there any likely correlation between the structure of a galaxy (e.g., spiral or elliptical, etc.) and the extent of non-cosmological redshift it has? As we shall see in Section 6 there is no theory that would provide an answer; but one may look at it empirically. Arp (*op. cit.*) presents evidence from the Local Group and the Virgo Cluster to argue that the SCI galaxies have the larges z_{NC}, and that normally we should expect $z_{NC} \neq 0$ in the SC and later morphological types of galaxies. Figure 44 shows the results obtained by Giraud (1983) for galaxies from three major clusters: Virgo, Perseus, and Hercules. The non-cosmological components seems to be correlated with galaxy type. Tifft and Cocke (1987) find a similar correlation for the member galaxies of the cluster Abell 262.

How is the excess redshift derived? Normally the excess could be measured with respect to the number-weighted average for the whole cluster. Arp, however, argues that the number-weighted average gives equal weightage to small and massive galaxies, whereas, dynamically the 'centre of mass' motion is given by a mass-weighted average of all velocities. Taking masses as proportional to luminosities, Arp evaluates the average redshift for the Virgo cluster to be around $cz = 863$ km s^{-1} – much lower than the number-weighted average redshift of $cz = 967$–1165 km s^{-1}. Thus Arp finds larger values of z_{NC} in relation to the mass-weighted average.

Arp's argument is dynamically sound. Its one weakness, in practical terms, lies in the fact that we do not know galaxy-masses (M). Thus unless we are sure of the possible ranges in M/L, our mass-weighted average may also be incorrectly estimated.

5.8. Faraday rotations (D)

In the late 1960s, Sofue *et al.* (1968), Kawabata *et al.* (1969), and Reinhardt and Thiel (1970) showed that if one avoided the low galactic latitudes (to prevent or reduce the contribution from the galactic magnetic field and the plasma) then the Faraday rotation of the plane of polarization of the radiation from a QSO tended to increase with redshift. Thus these early results seemed to go in favour of the CH.

Later Arp (1971b) found that the increase of Faraday rotation seemed to peak at $z \sim 1$ and that for QSOs with $z \sim 2$ the rotation was no greater; sometimes even less. A later systematic analysis by Kronberg and Perry (1982) averaged from four to eight rotation measures in the immediate area of each QSO and removed the effect of the Galaxy. These measures were compared with the predicted values depending on four different Friedman models (Welter *et al.*, 1984). The fit between theory and observation was not satisfactory. The peak at $z = 1$ still remained.

More recently Arp (1988b) has reanalyzed the same data within the framework of the local hypothesis, i.e., not assuming that the QSO redshifts are related to their distances from us. He finds empirically that the peak at $z = 1$ is there and that the strongest concentration of QSOs with $z \sim 1$ is in an area of the sky covering a solid angle of diameter 40° centred on the Virgo cluster. Arp concludes that these QSOs are located in the Local Supercluster. Further, the rotation measure for all of them has the same sign and that indicates an ordered magnetic field in the Local Supercluster direction. By contrast, Arp finds that the Faraday rotation measures for QSOs of $z \sim 0.5$ and 2

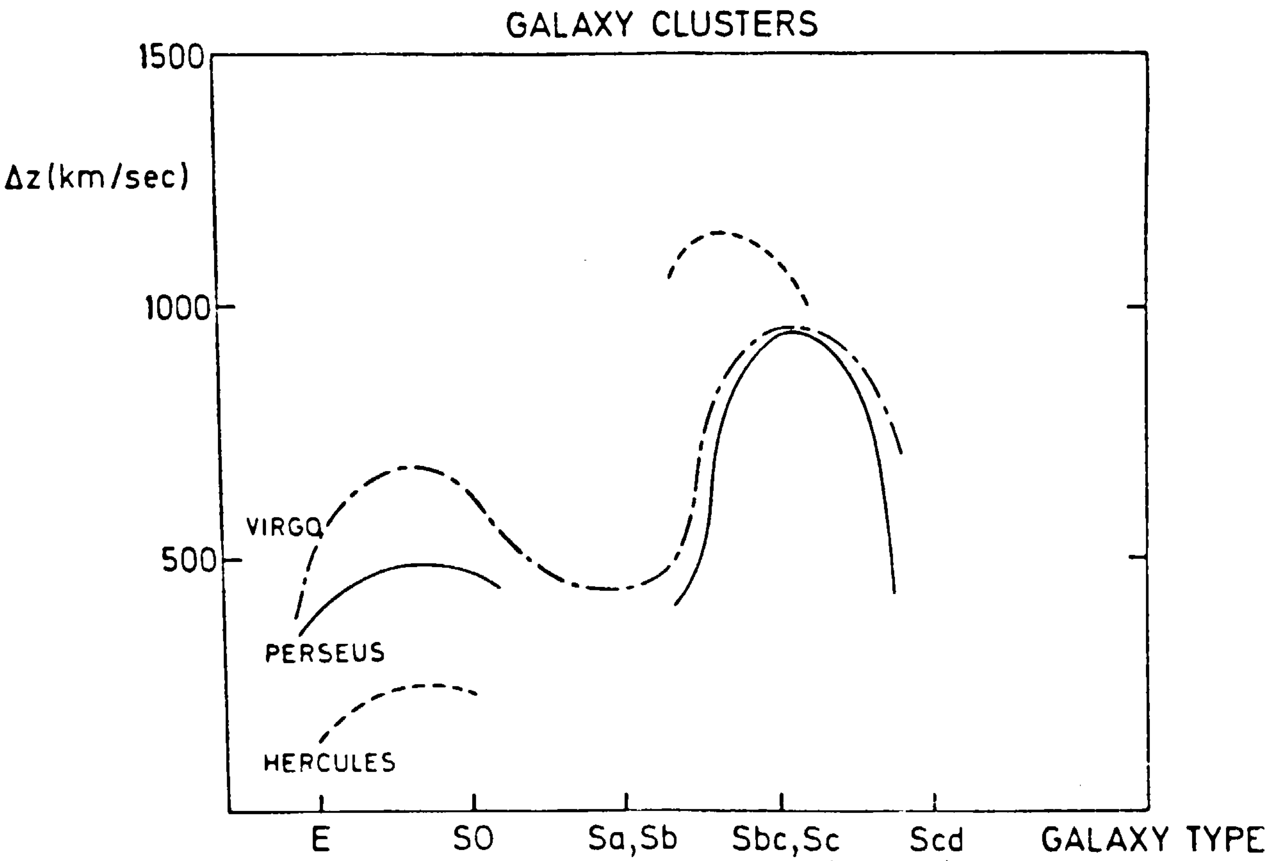

Fig. 44. This picture (from E. Giraud's Ph.D Thesis) shows how the excess redshift in different clusters varies with galaxy types. The effect is maximum for *Sbc* and *Sc* galaxies.

are small and not systematic and concludes that these QSOs are in fact much closer to us.

Arp's analysis is based on a subset of the original data (92 out of 115 in the sample were used). Critics may not agree with his proposed criteria for choosing this subset. Furthermore, the skeptic may also point to an unknown (and not easy to estimate) part of the Faraday rotation coming from the source itself. This component may vary from QSO to QSO and could be substantial because the magnetic field and the plasma density within the QSO would be considerably larger than in the intergalactic space and the intracluster space.

Nevertheless the Faraday rotation measurements of larger samples of QSOs need to be made to follow up the type of analysis Arp has made to check for consistency or otherwise with the CH.

5.9. Summary

We have discussed the discordant evidence in its various aspects. Technically, to disprove a well established hypothesis one discordant piece of evidence is sufficient: provided the discordant nature of the evidence is clearly established and generally accepted after a critical examination.

This has not been the case with the different topics discussed in this section. Rejection of any discordant evidence is of course justified provided it has been subjected to proper scrutiny and debate. This has not happened in this case. The discussion in many cases has been cursory and in some cases the results are simply ignored.

There is, however, a point of view usually shared by observers that such discordant data cannot be important because there is no theory for it. To correct that impression we now discuss alternatives to the cosmological hypothesis.

6. Noncosmological Alternatives

We have completed our presentation of observational evidence and now wish to come to theoretical issues. The Cosmological Hypothesis (CH) may be rephrased in two forms. In the *strong* form it requires that all redshifts of extragalactic objects are cosmological, i.e., follow Hubble's law, subject to small contributions $cz_{NC} \lesssim 1000 \text{ km s}^{-1}$ of Doppler or gravitational origin. In the *weak* form the CH applies to a large subset of galaxies whose redshifts are given by Hubble's law, but it leaves the question open in the case of QSOs and some galaxies like the compact companions. The weaker alternative thus allows the presence of $z_{\rm NC}$ in varying magnitudes in such objects.

We shall take the view that the present evidence does not justify confidence in the strong form of the CH. The various theoretical alternatives for z_{NC} will accordingly be discussed in this section. These alternatives fall in two classes. The ideas described in Sections 6.1–6.3 fall within the scope of established conventional physics while those discussed in Sections 6.4–6.6 require 'new physics'.

6.1. The Doppler Effect

Of all the different theoretical explanations of spectral shift, the Doppler effect is the one best tested in the laboratory. Typically, we have a source of light (S) moving with speed v in a direction making an angle θ with the radially outward direction from the observer (O) to the source. The spectrum of the source then shows a shift z_D given by the formula

$$1 + z_D = \frac{1 + \frac{v}{c} \cos\theta}{\sqrt{1 - \frac{v^2}{c^2}}} . \tag{39}$$

The astronomers are familiar with Doppler spectral shifts (redshifts as well as blueshifts) in stellar motions. Indeed, when Hubble and Humason obtained the spectral shifts in galaxies they interpreted them as due to velocities of recession. The practice of expressing redshifts in velocity units owes its origin to the Doppler effect. Even today the Hubble constant is expressed in units of velocity/distance even though the CH *per se* does not associate redshifts with velocities.

It was Terrell (1964) who first proposed that the QSOs derive their redshifts from the Doppler effect because they are stellar objects fired with $v \lesssim c$ from the nucleus of the Galaxy. Although basically simple and attractive, the idea ran into difficulties. For example, for a typical solar mass star to be ejected at relativistic speed, energy of the order of $\sim 2 \times 10^{54}$ erg is needed. If all the > 3000 QSOs are to come from our Galaxy, the energy requirement goes up to $\sim 10^{58}$ erg. An activity as energetic as this should have left some disruptive relics in the nuclear region besides disturbing stellar orbits in the Galaxy. By contrast the nucleus of the Galaxy and its entire structure do not show any abnormality suggestive of such violent ejection activity. Nor is any directional asymmetry reflecting the disc structure of the Galaxy seen in the angular distribution of the QSOs.

To make the model more realistic Hoyle and Burbidge (1966) shifted the ejection process to galaxies whose nuclei show signs of violent activity, like NGC 5128 and M87. The problems of energy and disruption then disappear. Hoyle and Burbidge assumed that QSOs normally seen are all within $\sim$30–100 Mpc. Thus in formula (10) we have $z_D \gg z_C$.

By bringing the QSOs near, we reduce their energy requirements as seen from formula (25). By reducing D by a factor 100, for example, we reduce L by a factor 10^4. In this way, the energy problems of Section 4.6 either disappear or become surmountable. Likewise the so-called superluminal motions become subluminal and hence do not call for esoteric scenarios. The redshift being uncorrelated with distance, the Hubble diagram for QSOs would be a scatter diagram as it actually is (Section 4.1).

Violent activity in AGNs can be related to the ejection of QSOs. In a momentum conserving ejection we might expect QSOs ejected in opposite directions, which may account for the alignments seen. (We may again compare this process with two oppositely directed jets in a radio source.) It also becomes easier to understand the triplets of QSOs found by Arp and Hazard (1980). Indeed, the configuration given in Figure 41 can be modelled with Doppler velocities (Narlikar and Edmunds, 1981; Narlikar and Subramanian, 1982).

Can the gravitationally lensed QSOs in fact be two distinct objects ejected from the same source with nearly equal speeds? This has been suggested by Narasimha and Narlikar (1989). To illustrate their point they consider the field of nine QSOs in the 1146 + 111 field, shown earlier in Figure 35. This field has two QSOs with nearly equal redshifts (1.012, 1.013) and an angular separation of 157″. There are four galaxies close by in the field one of which could have been responsible for the ejection of all nine QSOs. A detailed Doppler modelling shows that one of the four galaxies would meet all the dynamical and geometrical constraints of the model. The authors conclude that an explosion $\sim 3.4 \times 10^7$ yr ago ($H_0 = 50$ km s^{-1} Mpc^{-1}) ejected all the QSOs including the close pair that was once considered candidate for gravitational lensing (cf. Section 3.4). It may be worthwhile therefore looking for Doppler ejection pictures in (i) cases where gravitational lensing has been considered as established and (ii) cases where dense QSO concentration is found near a galaxy (e.g., Figures 37–40).

The Doppler effect, however, generates blueshifts as well as redshifts. In Terrell's theory blueshifts did not present much problem since they would be seen only in those

QSOs which are travelling from the galactic centre towards us and their number would be negligible. In the scenario of Hoyle and Burbidge, the lack of any observed blueshifts becomes a problem. The blueshifted QSOs would be considerably brighter and hence more easily detectable than their redshifted counterparts. A precise calculation by Strittmatter (1967) shows that if the spectral index of the QSO population is α and the largest redshift seen in a flux limited sample is $z_{\max}$, then the ratio of the numbers N_B and N_R of the blueshifted and redshifts QSOs will be

$$\frac{N_B}{N_R} = (1 + z_{\max})^{3+\alpha} . \tag{40}$$

For example, with $\alpha = 1$, $z_{\max} = 2$, the blueshifted QSOs should be 81 times more populous than the redshifted ones.

To date no blueshifted QSO has been seen. This could be due to several causes: (i) Observers do not examine spectra specifically for long wavelength lines (even going to the infrared) because they are only looking for redshifted lines. (ii) There may not be many good lines that blueshift into the visible part of the spectrum. (iii) The continuum part of the spectrum becomes very bright under a blueshift, thus making it difficult to pick out specific lines.

Even so, the absence of any blueshifts in a Doppler model calls for further explanation. A loophole from his argument was suggested by Strittmatter, that a QSO selectively radiates in the backward direction. This idea was quantified by Hoyle (1980) who showed that if the QSO is moving at speed v relative to the intergalactic medium (IGM) then no observer at rest in the IGM would see the QSO blueshifted provided it radiates within a backward cone of semi-vertical angle

$$\theta_H = \cos^{-1} \frac{c - \sqrt{c^2 - v^2}}{v} \tag{41}$$

measured in the rest frame of the QSO. For example, for $v = 0.8c$, $\theta_H = 60°$. As $v \to c$, the backward cone expands in its vertical angle to $\theta_H = 90°$.

Thus the required backward emission need not be very narrowly directed. But how can it be so directed in the first place? Narlikar and Subramanian (1983) have suggested that the twin exhaust jet model of Blandford and Rees (1974) may be modified to a one-(backward) jet model for a QSO travelling rapidly in the IGM. This happens because the IGM provides a ram pressure to stop any forward jet from developing. Thus the number ratio N_B/N_R in this model drops to ~ 0.01. Hence, for about 3000 redshifted QSOs we may expect to see $\lesssim 30$ blueshifted ones. It has been suggested (Narlikar and Edmunds, 1981) that such QSOs may be essentially lineless objects with powerful continuum emission. The BL Lac objects might in fact be blueshifted QSOs.

It is significant that QSOs as a rule show only one jet in their radio maps (or, wherever, optical jets are seen). This would be a mystery under the CH where the QSO is at rest with respect to the IGM and should have two jets as in a radio galaxy.

The single jet model can be tested by looking for absolute proper motion against the

IGM and checking that the jet is in the opposite direction. As yet the technique of measuring proper motion by the VLBI is not able to check this fact unequivocally. Proper motion of the order of 0.6 milli arc sec yr^{-1} is predicted by this model (Narlikar and Subramanian, 1983).

The model also predicts a gaseous fuzz around the central QSO that decreases in size for faster moving QSOs, i.e., for QSOs with large redshifts. This prediction is consistent with the fact that so far fuzz has been seen around low redshift QSOs (see Section 3.6).

This Doppler model also explains why the phenomenon of radio-loud QSOs is not all that common. The typical QSO in this model emits radio waves but the backward cone, in which the observer has to be in order to detect them, is relatively narrow.

The Doppler model still needs to account for the actual ejection phenomenon in the galaxies. What is it that makes a nucleus disgorge a bound object at high speed? Can the backward jet itself accelerate the QSO to near-light speeds? Saslaw (1988) has proposed an accelerating mechanism of this kind but the entire issue needs more theoretical inputs.

6.2. The gravitational option

According to Einstein's general relativity, the spacetime geometry in and round a static spherically-symmetric matter distribution is given by the line element

$$\mathrm{d}s^2 = c^2 e^{\nu}\,\mathrm{d}t^2 - e^{\lambda}\,\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\,. \tag{42}$$

Here θ, ϕ are the angular coordinates; r the radial coordinate and t the time coordinate. The functions ν and λ depend on r only. A radially outgoing light photon leaving the point (r_1, θ, ϕ) on the source and reaching an observer at (r_2, θ, ϕ) will be spectral shifted by z_G, where $\nu_1 = \nu(r_1)$, $\nu_2 = \nu(r_2)$ and

$$1 + z_G = \exp\{(\nu_2 - \nu_1)/2\}\,. \tag{43}$$

For a spherical mass M confined to $r \leq r_1$, and $r_2 \gg r_1$, the above formula approximates to

$$1 + z_G = \left(1 - \frac{2GM}{c^2 r_1}\right)^{-1/2}. \tag{44}$$

Thus the light leaving the surface of a massive source gets redshifted. This formula found applications in astronomy in the spectral shifts seen in the radiation from white dwarf stars, although the shift was very small ($z_G < 10^{-4}$). However, in principle Equations (43) and (44) allow z_G to have substantially larger values.

The gravitational redshift was considered as a possible explanation soon after the QSOs were discovered. In practical terms, however, two difficulties prevented the ready acceptance of the idea.

The first difficulty was observational and was highlighted by Greenstein and Schmidt (1964) for the first two QSOs to be discovered and studied: 3C273 and 3C48. They calculated the gravitational mass of the source by taking into account (i) the emission line widths, (ii) the emissivity and number density of the emitting plasma, (iii) the limits up to which the objects could be brought close to the Galaxy (or even within it) without disrupting the stellar dynamics in it and of course (iv) the redshift of the object. The minimum mass consistent with all data turned out to be $\sim 10^{10} M_{\odot}$! Thus, on this hypothesis the QSOs could not be considered highly collapsed stars in or in the vicinity of the Galaxy.

The second, theoretical point was made by Bondi (1964) who proved a general theorem limiting the maximum z_G to be expected from the surface of a spherical mass to 0.62 provided the matter within the massive object satisfied the reasonable condition that pressure never exceeded energy density. Clearly, from this result it would be impossible to account for the large redshifts of QSOs going up to $\sim$4.4.

However, Hoyle and Fowler (1967) proposed another version of the gravitational redshift in which the light from the source came from deep interior. Thus Equation (43) would apply [but not Equation (44)] with ν_1 being evaluated in the core region of the object. Hence, Bondi's theorem was circumvented: the redshift was not from the surface but from the central core. But this requirement meant that the object had to be transparent to its core radiation. To achieve transparency while maintaining a deep gravitational well at the core, these authors had compact stars (e.g., neutron stars) form a cluster round the core leaving sufficient gaps for the radiation to emerge. They also showed in a qualitative way, that the Greenstein–Schmidt constraints could also be met. Figure 45 illustrates the model.

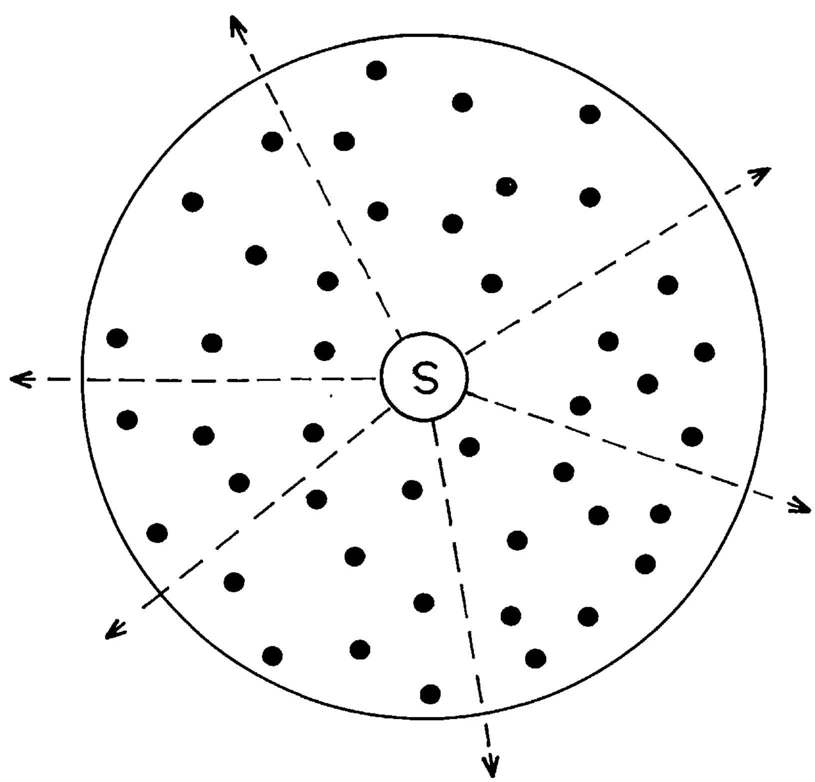

Fig. 45. Figures illustrating the Hoyle–Fowler model with a central emitting source S surrounded by gravitating matter in the form of compact subunits (dark discs) like neutron stars.

Fackerell (1968) studied such models in detail and pointed out another constraint. The star cluster surrounding the central core would have to have a non-negative velocity distribution function. It turned out that in many models that gave high values of z_G, the distribution function became negative for some velocities. Further, it was also necessary, as first pointed out by Chandrasekhar (1964), the models had to satisfy the stability criteria against small oscillations.

Das and Narlikar (1975) and Das (1975, 1976) studied the Hoyle–Fowler models extensively and were able to construct models that satisfied the constraints of stability and non-negative distribution functions, were optically thin for radiation to emerge from the core and gave central gravitational redshifts as high as 2–2.5. (It is not clear from the work of Das whether these were limiting redshifts. In the mid-1970s, most QSOs had redshifts less than 2.5, and this may have been considered sufficient to demonstrate the viability of z_G.) However, the masses of these objects turn out to be of galactic order ($10^{11}\,M_\odot$) and their typical distances 10–100 Mpc. Subsequently Hoyle (1983) obtained an even more massive model (10^{13}–$10^{14}\,M_\odot$) made of magnetic monopoles. With this model Hoyle was able to explain certain spectroscopic features of the QSOs like their multiple absorption line systems.

The gravitational model naturally produces only redshifts and hence does not suffer from the 'blueshift catastrophe' that troubles the Doppler models. They also naturally lead to a scatter diagram on the $m - z$ plot. But it is hard to see why and how the alignments and associations found by Arp and others came about.

By and large the gravitational option may work well for $z_G \ll 1$ and as such it needs to be tried for companion galaxies (Section 5.3) and the quantized redshift phenomenon (Section 5.2).

6.3. The Spectral Coherence Effect

Wolf (1986, 1987) has pointed out that if the fluctuations in a source of light are correlated within the source region, the resulting spectrum (in its normalized form) gets modified as the light waves carrying it travel through free space. This effect has been tested in the laboratory by Bocks *et al.* (1987) and by Morris and Faklis (1987). The underlying principle is as follows

Suppose two point sources P_1 and P_2 have identical spectra $S_Q(\omega)$ and that measurements of radiation are made at some point P located at distances R_1, R_2 from P_1, P_2, respectively. If $Q(P_1, \omega)$ and $Q(P_2, \omega)$ are the strengths of the sources in the angular frequency ω, then the spectrum of light at P is given by $S_V(P, \omega) = \langle V^*(P, \omega) V(P, \omega) \rangle$, where

$$V(P, \omega) = Q(P_1, \omega) \frac{e^{i\omega R_1/c}}{R_1} + Q(P_2, \omega) \frac{e^{i\omega R_2/c}}{R_2} . \qquad (45)$$

Therefore,

$$S_V(P, \omega) = \left\{ \frac{1}{R_1^2} + \frac{1}{R_2^2} \right\} S_Q(\omega) + $$
$$+ 2 \times \text{real part of} \left\{ \langle Q^*(P_1, \omega) Q(P_2, \omega) \rangle \frac{e^{i\omega(R_2 - R_1)/c}}{R_1 R_2} \right\} . \qquad (46)$$

Thus in general S_V is not proportional to S_Q, but is modified by the extra cross-product term on the right-hand side, whose contribution depends on the distances of P_1, P_2 from P.

The relevant feature of the Wolf effect here is that when a certain scaling law is violated, the spectrum in the far zone will be different from that at the source and can generate spectral shifts. Both blueshifts and redshifts are possible and Wolf has suggested that the mechanism may apply to QSO redshifts where $z_{NC} > 0$.

It is too early to judge how effective this mechanism will be in explaining non-cosmological redshifts. As yet no model of an emitting source at extragalactic distance and with the necessary correlation conditions has been constructed. The model will have to avoid blueshifts and should be able to generate $z_{NC} > 1$ to make a significant impact in this field. It might stand a better chance in explaining small values of z_{NC}, e.g., in compact companion galaxies.

6.4. The Chronometric Cosmology

An alternative to the expanding universe idea has been proposed by Segal (1976) under the above name. Segal's approach is formal and abstract, giving axiomatic statements for causality and direction of time, then developing a spacetime structure based on group theory and finally relating the various abstract concepts to physical reality. Segal has been critical of the standard development of the relativity theory on the grounds that it draws on human intuition in working with such concepts as causality, simultaneity, propagation of light signals, etc., without giving an axiomatic treatment.

This is not the appropriate place for describing and discussing the formal aspects of chronometric cosmology (the reader may refer to the above reference for these details). Broadly speaking, the word 'chronometric' is indicative of the temporal order of events in the cosmos, looked at globally. Mathematically, a time-coordinate τ taking real values from $-\infty$ to $+\infty$ does this ordering. The global spatial structure is that of the 3-surface of a 4-sphere.

At any point P in space one can talk of two observers, O and O'_P. The observer O is the global observer while O'_P is the unique local relativistic observer who uses a local Minkowski spacetime. (The analogy of a tangent plane to a sphere would help!) O and O'_P agree up to second order in distances measured from P, but O'_P uses a different time coordinate t, for all local measurements.

Thus when a source S emits a photon which is received by the relativistic observer at P, the frequency measured by him will not be the same as the frequency measured by a similar observer at S. Segal calculates a redshift

$$z = \tan^2 \frac{r}{2R} , \tag{47}$$

where r is the distance between S and P measured as per the geometry of space and R is the radius of the 4-sphere on whose surface the space resides. Thus for $r \ll R$, we get a quadratic law for redshift-distance, as discussed in Section 5.1. This can be converted to a $z - m$ relation

$$m = 2.5 \log z + \dots , \tag{48}$$

where the additional terms on the right-hand side include details of spectral shape, geometrical factors, etc.

The mathematical foundations apart, Segal has extensively discussed the statistics of data reduction in extragalactic astronomy and claimed that within the present observational uncertainties his cosmology gives a better fit to the various curves like the $(m - z)$, $N(m)$, $\theta(z)$ curves that we discussed in Section 3. However, being of global origin like the Friedman cosmology, it is hard to see how the chronometric theory can account for the specific cases of discordant evidence described in Section 5.

6.5. The tired light theory

The idea that the photon may have a small but nonzero rest mass and that it would lose energy by travelling through space has been proposed by many authors from time to time. For a survey of the various theories see Pecker (1976) and the references therein.

In this 'tired light' hypothesis the photon loses energy and is redshifted by its interaction with other ambient matter. The actual interaction has been the subject of speculations but the effect – of a redshift increasing with distance – is predicted more or less in the same fashion by most theories. Zwicky (1929) had advocated it as an alternative to the expanding universe theory.

It is worth mentioning that Finlay-Freundlich (1954) and Born (1954) had also deduced the existence of a microwave background from their version of the theory. We will briefly outline the theory favoured by Pecker (1976) in his review.

The basic idea here is based on the photon having a small rest mass m and that while travelling through the intergalactic space it is supposed to lose energy by collisional interaction with a specific from of matter. It appears that this matter cannot be made of the usually known particles like the electrons or protons. Denoting the species by ϕ, the mass m_ϕ of the ϕ-particle must satisfy the inequality

$$0 < m_\gamma \ll m_\phi \ll m_e , \tag{49}$$

where m_e is the mass of the electron.

In Pecker's (*op. cit.*) ϕ-mechanism the standard cosmological redshift and Hubble's law arise from the progressive loss in energy of a photon from the resistance offered by the ϕ-bath that fills the Universe. The Hubble constant is thus an averaged distance effect depending on the strength of the $\phi - \gamma$ interaction, and the density of the ϕ-population along the path from the source to the observer.

Thus anomalous (excess) redshifts can arise from the local inhomogeneities of the ϕ-bath near these sources: the ϕ-distribution will be denser in sources with excess redshifts. The effect of tired light can also be expected in galactic objects and indeed Pecker *et al.* (1972) were led to the hypothesis through the study of anomalous effects in solar and stellar spectra. La Violette (1986) has studied the consequences of this mechanism for cosmological tests like the $m - z$, $N(m)$, $\theta(z)$ relations and concluded that the tired light mechanism offers a better fit with the data than standard cosmology without evolution.

The tired light theory in this form has, however, run into insurmountable difficulties on two counts. First, the frequent scattering of the photon by the ϕ-bath smears out the coherence of the radiation from the source, and so all images of distant objects look blurred. Secondly, the scattering effect and the consequent loss of energy is frequency dependent, thus making a 21 cm redshift different from the redshift of Lα, say.

Vigier (1988) has given a different version of the tired light hypothesis. It requires the vacuum to behave like a stochastic covariant superfluid aether whose excitations can interfere with the propagation of particles or light waves through it. The interaction is dissipative, taking energy away from the wave. Thus over cosmological distances the effect appears as a redshift. It is claimed that the two difficulties of the original mechanism are absent in this version.

Again, it is too new an idea to judge critically. Its merit lies in that it can in principle be tested by laboratory experiments (Section 7). For explaining anomalous redshifts, the vacuum must show inhomogeneity in its interaction with light. This has not yet been demonstrated.

6.6. THE VARIABLE-MASS HYPOTHESIS

Hoyle and Narlikar (1964) developed a new theory of gravitation based on Mach's principle. According to Mach (1883) the inertia of a particle is not entirely its intrinsic property but also requires a universal (non-empty) background of other matter to define it. This qualitative concept of Mach finds a mathematical expression in the Hoyle–Narlikar (HN) theory. The theory of gravity is conformally invariant in the sense that if the spacetime metric g_{ik} and particle mass functions $m_1, m_2, \ldots$ are solutions of the HN equations then so are the metric $\Omega^2 g_{ik}$ and masses $\Omega^{-1}m_1, \Omega^{-1}m_2, \ldots$ for any well-behaved function Ω of spacetime that satisfies the condition $O < \Omega < \infty$.

For a homogeneous isotropic universe the simplest solution in the HN theory is the Minkowski metric η_{ik} and particle masses $m \sim t^2$ where t is a cosmic time. The epoch $t = 0$ is such that all particle masses vanish at that time. Such zero-mass epochs exist in general solutions of the HN equations.

The flat spacetime solution described above does lead to redshifts in the following way. In Figure 46 we see worldlines of two galaxies separated by a distance r. Notice that the world lines exist all the way from $t = -\infty$ to $t = +\infty$. The observer O on the world line of galaxy G_0 receives radiation from source point S_1 on galaxy G_1. The times of reception and emission of radiation are, respectively, t_0 and t_1. Because the radiation is travelling in flat spacetime $t_1 = t_0 - r/c$. However, because particle masses vary with cosmic time, the radiation wavelength of a particular spectral line at S_1 will be different from that at O. Let these wavelengths be λ_1 and λ_0, respectively.

Now, in a typical atom the wavelength will be inversely proportional to the mass of the electron and hence we get the result

$$1 + z_1 = \frac{\lambda_1}{\lambda_0} = \frac{m_0}{m_1} = \frac{t_0^2}{\left(t_0 - \frac{r}{c}\right)^2} . \tag{50}$$

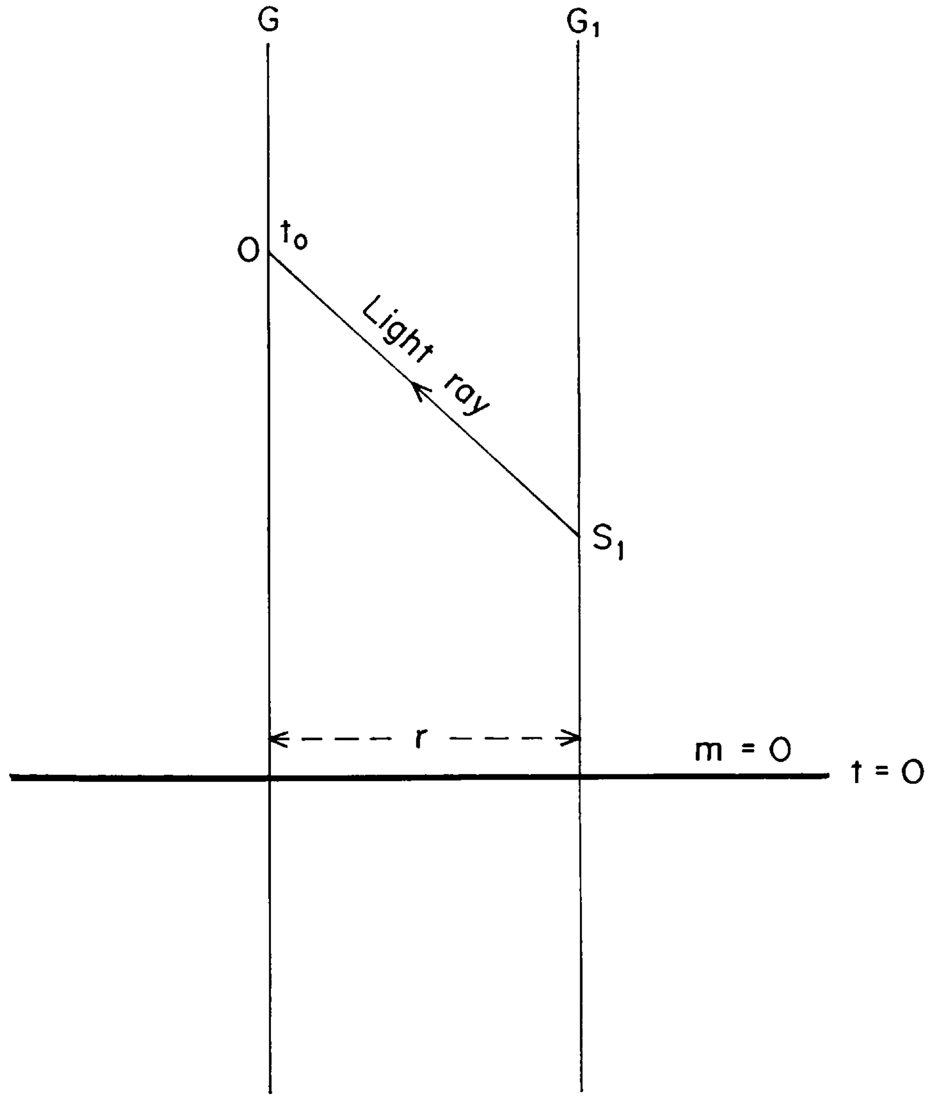

Fig. 46. In the HN cosmology, the light from source S_1 in the world line of galaxy G_1 reaches observer O on the world line of galaxy G_0 without spectral shift. However, being emitted by smaller particle masses, the wavelength of that light is longer than similar light emitted in O's vicinity.

Here the redshift, z_1 comes because the wave from S_1 has had a fixed and longer wavelength than a similar wave would have at Q. The wave has not increased its wavelength during propagation.

By a Taylor expansion near $r = 0$ we easily recover Hubble's law with $H_0 = 2/t_0$. It can be shown (cf. Hoyle and Narlikar, 1974) that for $t > 0$, this model is conformally transformable to the $k = 0$ Friedman model with $\Omega \sim t^2$. All particle masses are then constant and we get the usual cosmological redshift through light propagation in curved spacetime. However, the transformation function Ω vanishes at $t = 0$: hence, the condition $0 < \Omega < \infty$ is violated there. If we insist on pushing through the transformation, we get the singularity of spacetime there. Indeed, as shown by Kembhavi (1978), the forcing through of a conformal function to a zero-mass hypersurface leads to the singular solutions of general relativity.

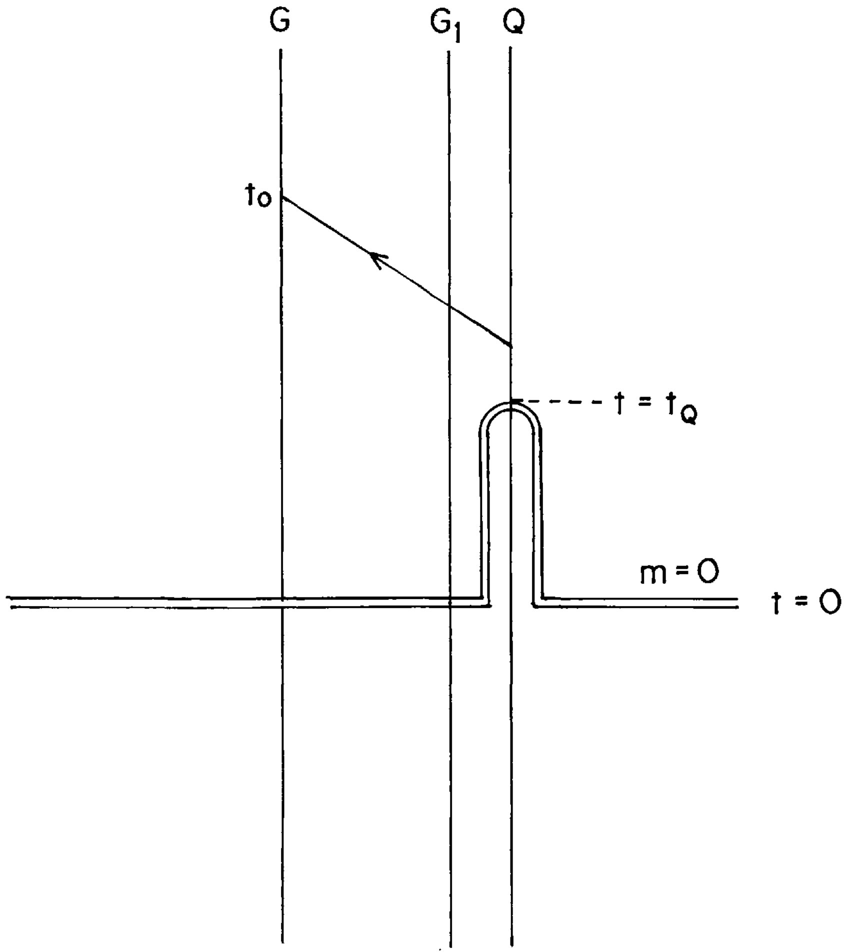

Fig. 47. A kink in the zero-mass hypersurface can lead to anomalous redshifts as discussed in the text.

A departure from the homogeneous situation considered above can lead to anomalous redshifts, however. Figure 47 illustrates what happens when a kink appears in a zero-mass hypersurface. There we have a QSO, whose world line Q passes through the kink, crossing it at a point $t_Q > 0$. For the neighbouring galaxy G_1, however, the crossing of zero-mass hypersurface occurs as before, at $t = 0$. In physical terms such a situation arises when a QSO is ejected from the nucleus of a galaxy at $t_Q > 0$.

It was shown by Narlikar (1977) that the particle masses in the QSO will increase as $(t - t_Q)^2$, *not* as t^2. Thus the redshift of Q as seen by O will be given by

$$1 + z_Q = \frac{t_0^2}{\left(t_0 - t_Q - \frac{r}{c}\right)^2} . \tag{51}$$

Hence, Q will have higher redshift than G_1. What is more, the later the creation of Q, the higher will be the anomalous (or excess) redshift.

Narlikar and Das (1980) have investigated the dynamics of ejection within the framework of the HN theory. At the moment of ejection the QSO has zero mass and, hence, the speed of light. However, it quickly acquires inertia and slows down. Whether it is finally trapped by the parent galaxy or escapes its gravitational pull will depend on an ejection parameter η analogous to the escape speed in Newtonian dynamics. For η exceeding a critical value η_c, the QSO escapes; otherwise it is trapped into an eccentric orbit around the galaxy. This orbit steadily shrinks as the QSO acquires larger inertia.

Thus while all QSOs may be products of ejection, those with $\eta > \eta_c$ are no longer seen near their parent galaxies while those with $\eta < \eta_c$ appear as anomalous companions in the vicinity of the galaxy. Since the excess redshifts of companion galaxies are small, their epoch of ejection was well into the past. Thus they have acquired galactic forms from an initial QSO structure. Also, if more QSOs are ejected at the same epoch, their redshifts would tend to be bunched round the same value. In this connection Figure 48 provides a second look on the two triplets of Figure 41. The lines joining QSOs of similar redshifts pass close to each other in the shaded region. It might be worth searching that region for possible debris of a galaxy which had undergone three separate explosions.

Since the QSOs are fired with zero mass and acquire their inertia subsequently, the energy requirements for the explosion are not very severe. Further, the radiating particle masses like electrons in a QSO being much lighter, the energetics of radiation is less constrained within it. In particular the energy problems discussed in Section 4.5 are considerably alleviated. Since the separation QG_1 decreases with time while z_Q increases, an inverse correlation is expected between QSO-galaxy separation and z_Q. Arp (1983) reportred finding such a correlation.

The application of this theory to the ejection of hydrogen clouds from the galaxies like 3C120, however, runs into one difficulty. The excess redshifts of these clouds being small, the ejection should have occurred several billion years ago. The evidence on the rotation of the galaxies, on the other hand suggests time-scales of the order 10^7 years. This would suggest that the clouds material was created much earlier but was blown out later. Arp and Narlikar (1988) suggest that other forces like buoyancy forces and/or magnetic forces may be responsible for the process.

The variable mass concept can, in principle, be subjected to quantization. Basically the problem is very similar to the quantization of the conformal degree of freedom (Narlikar and Padmanabhan, 1986). Discrete values of m would, in this theory, lead to discrete values of z. A periodicity in $\log(1 + z)$ would require that the mass function takes a discrete set of values in a geometric series $\{k^n\}$ where k is a constant. From (50) we then have a periodicity of $\log(1 + z)$:

$$\Delta \log(1 + z_n) = \log k = \text{constant} . \qquad (52)$$

From the empirical findings of Karlsson (1977) and Depaquit *et al.* (1985) one would expect $\log k = 0.089$.

However, for the Tifft effect one has $|z_n| \ll 1$ and the above formula approximates to $\Delta z_n = \ln k$, with $\ln k = (72 \text{ km s}^{-1}/c)$. It is difficult to see from the present theory why k should have either of these values; nor can we see whether the two effects are in fact linked. Thus the quantum version of the theory needs to be developed further.

It is worth remarking here that attempts to explain the Tifft effect theoretically have

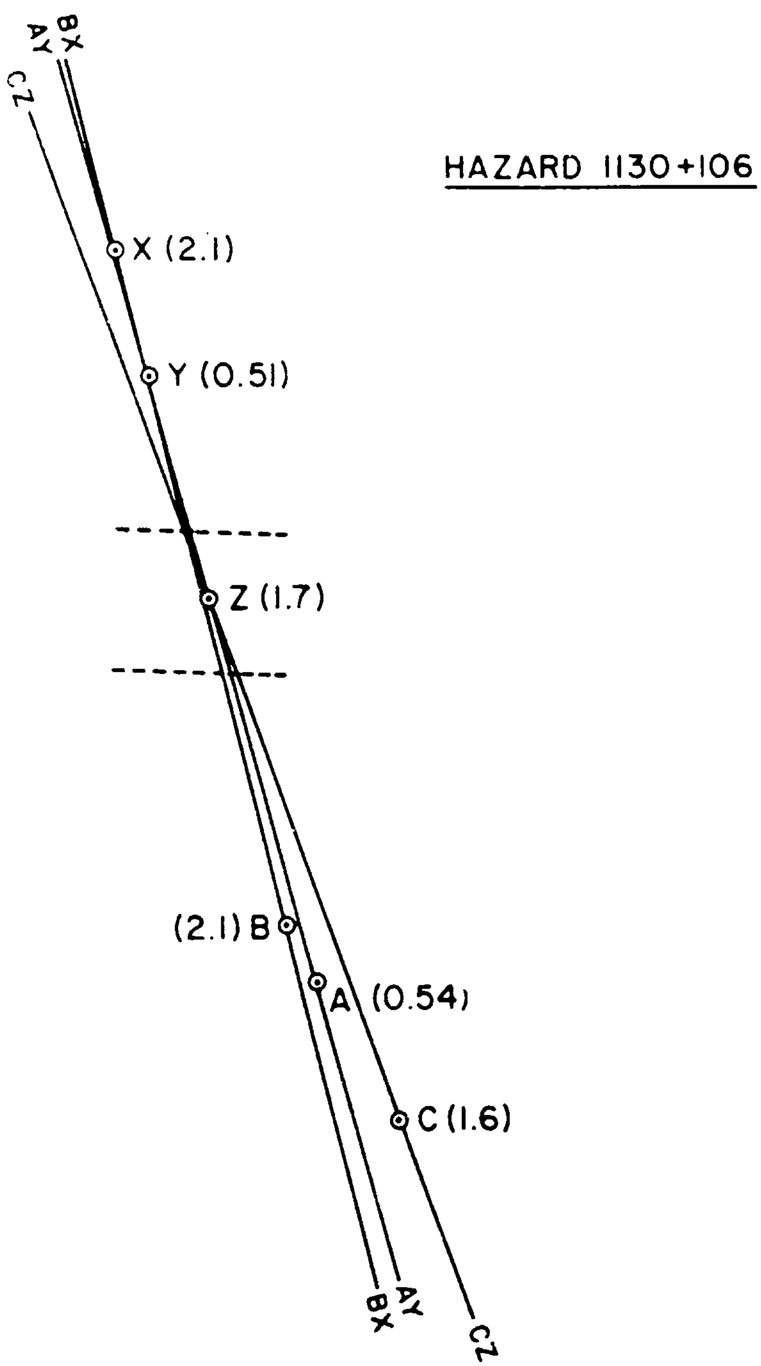

Fig. 48. A different look at the Arp–Hazard triplets of Figure 41. The two are not independent, according to this version, but were emitted in a triple explosion releasing pairs of QSOs of nearly equal redshift each time. The centre of explosion is where the three lines intersect the shaded region in the figure.

been made by Cocke (1983, 1985) and by Nieto (1986). The approach incorporates quantum operators to restrict the z-values. One prediction from this work was of a z-dependence in the quantization intervals. It is too early to say whether this approach – or any other – works well enough to explain the phenomenon of redshift periodicities.

6.7. Summary

The noncosmological alternatives described here do not exhaust the field, but they are the ones more commonly discussed in the context of observations. It is clear, that none of them alone can account for all the different types of discrepant phenomena discussed. It is tempting to bring in Occam's razor to reduce the possible alternatives. But that stage has not yet arrived. It is necessary to enlarge the scope of present day observations to focuss the discrepancies more clearly and to make the theoretical predictions more crisp in order to distinguish between them. In the following section we discuss suggestions for future work in this area.

7. Future Tests

Tests that can help resolve the complex issues of extragalactic redshifts can be broadly classified into two categories: cosmological and local.

7.1. Cosmological tests

(a) The most direct test of relevance to Hubble's law is the test of its applicability to nearby galaxies. It is increasingly becoming clear that the Universe is not as homogeneous and isotropic in the distribution of discrete objects, as assumed by theoreticians. So does the Hubble constant really become a constant on a large enough scale? To be able to apply Friedman cosmology to observations, that scale has to be small compared with c/H_0.

(b) Tests of the large scale geometry of the universe need to be sharpened and made as free from evolutionary parameters as possible. If evolution is indeed taking place, it should be explicable in terms of the astrophysics of the sources (...Baade's dictum paraphrased!). Only when that part is understood and (if necessary) taken out, can one really talk about the large scale geometry of the Universe.

(c) The value of H_0 that enters into the calculation of the age of the universe is at present inconsistent with the astrophysical age estimates using either stellar evolution or nuclear cosmochronology. Just as in (a) we emphasized the need to know the 'true' value of H_0 so we stress here the need to reduce the uncertainties in astrophysical age determinations.

7.2. Local tests

(a) The VLBI measurements of absolute proper motions of QSOs are needed to place limits on their distances, especially from the Doppler model. Likewise the structure of 'superluminal' QSOs need to be probed further to check the validity of the beaming or the gravitational bending hypothesis.

(b) Nuclear dating methods and other techniques like synchrotron lifetimes, measurements of magnetic fields etc. may be brought to bear on the prediction of the variable mass hypothesis that high redshift QSOs contain 'young' and 'light' matter.

(c) Sulentic (1988) has proposed *twelve* tests of the discordant redshift hypothesis that is framed in a model independent form: it simply assumes that compact objects are ejected from active galactic nuclei, to evolve later to QSOs and compact companion galaxies. Broadly speaking these tests fall into various categories like direct/statistical tests of controlled samples for QSO–QSO/QSO–galaxy/galaxy–galaxy associations. Sulentic suggests looking for radio and optical evidence of activity in the nuclei of spirals and special studies of companions on the ends of spiral arms.

(d) It is necessary to develop the interesting idea of Wolf (Section 6.3) for possible applications to astrophysics. The idea has been tested and confirmed in the laboratory but its relevance to the non-cosmological redshifts still needs to be demonstrated.

(e) Laboratory experiments of vacuum gravitational drag that can lead to a tired light theory are possible and are being done. For the cosmologist, it is important to know whether the effect, if it exists, can be large enough to compete with or even replace the CH.

8. Conclusions

Table II gives the general picture on theory and observations as presented here. The reader, after going through this review would arrive at one of the following alternative conclusions.

(i) There is nothing in the data available today to warrant any anxiety about the validity of Hubble's law and the expanding universe hypothesis of cosmology. The so called discrepancies or anomalies reported from time to time are either observational artifacts (that will go away when the errors are reduced with improved techniques) or represent chance effect of projection that have no theoretical significance.

(ii) While Hubble's law and the standard cosmology based on it are basically correct, *some* of the anomalies in the data are possibly real and should be taken seriously. It is hoped that these anomalies will be explained away with the physics we know and that no new (or unconventional) ideas would be required.

(iii) In spite of the derision and suspicion by the majority of astronomers and astrophysicists, the list of discrepant observations has steadily grown and can no longer be ignored. Perhaps nature is trying to reveal some new facts that may one day substantially change some of our basic physical assumptions.

Such a mixed reaction is inevitable whenever the established viewpoint is being confronted with discrepant observations. It is, however necessary to break out of the viscious circle wherein the observers suspect the data because they have no theory for them and the theoreticians ignore new ideas because they do not know of any outstanding discrepant data that might require them.

We can think of no better way of ending this review than reproducing the uncommitted view expressed by Hubble himself on his law:

"If the estimations of density were completely reliable, a radius of curvature of the necessary dimensions would be ruled out by the evidence. But so definitive a solution is probably unwarranted. The crucial data are surrounded by uncertainties. By pressing the data to the limit of their tolerance, always in one direction, we might force the velocity-shift into the framework of the surveys. The universe would then be small, and filled with matter to the very threshold of perception.

TABLE II

Summary of observational evidence and theories on extragalactic redshifts

Type of test	Evidence consistent with the CH	Neutral evidence	Discordant evidence	Alternative theory
A: $m - z$ relation	Good for first ranked cluster member galaxies	Scatter diagram for QSOs but can be reconciled with CH	Nonlinear Hubble law for nearby galaxies?	Segal's chronometric cosmology. Any local theory explains scatter diagram of QSOs
B: Non-euclidean geometry	Counts of optical and radio galaxies, QSOs; in some cases evolution postulated	$\theta - z$ relation with evolution or size-Luminosity correlation	–	–
C: Near-neighbour criterion	Work of Stockton and Yee and Green for QSOs and galaxies	–	QSOs and bright galaxies, pairs and close groups of QSOs and galaxies, compact companion galaxies of excess redshift	Doppler effect, tired light (?), variable mass hypothesis
D: Background vs foreground systems	Absorption lines in QSO spectra, gravitational lensing	Lack of continuum absorption blue-ward of Lα: IGM may be made differently	No systematic increase of Faraday rotation with increase of QSO redshift	Any local theory. Will explain the Faraday rotation and Lα observation
E: Physics and morphology	Evolutionary link between QSOs and AGNs	Is the age of the universe too short? Epicycles are needed to accommodate 'superluminal' motions and energy constraints of QSOs	Evidence that non-cosmological component of redshift depends on galaxy types	In a local theory 'superluminals' and energy constraints are easier to handle
F: Extraordinary effects	–	–	Redshift periodicites and quantization; alignments and redshift bunching	Variable mass hypothesis can explain the latter problem but periodicities and quantization problems not yet solved

On the other hand, if the interpretation as velocity-shifts is abandoned, we find in the redshifts a hitherto unrecognized principle whose implications are unknown. The expanding universe of general relativity would still persist in theory, but the rate of expansion would not be indicated by observations."

In *The Realm of the Nebulae* (Silliman Lectures, 1936) EDWIN HUBBLE

Acknowledgement

This review is the outcome of several discussions with leading workers in the field, too numerous to mention by names. The opinion in this field is strongly polarized, with the majority favouring the status-quo so far as the CH is concerned and I am grateful to the proponents from both sides for informing and educating me on details.

Note added in proof: Dr Arp has pointed out that some of the redshifts in Figures 37 and 39 have been revised (cf. his article in Proc. 24th Liège Astrophysical Colloquium on Quasars and Gravitational Lenses, 1983, p. 343). He also points out that an HI map on the VLA shows an HI filament leading directly from the galaxy NGC 3067 to the quasar 3C232.

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