STRING COSMOLOGY IN A STATIONARY CYLINDRICALLY SYMMETRIC SPACE-TIME

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Abstract :

A one parameter family of solutions are obtained for String cosmology in a stationary cylindrically symmetric space-time.

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1. Introduction

The study of stationary cylindrically symmetric space-time was originated long ago by Lewis [1] in 1932. He solved Einstein s equations for vacuum space-time. Later, Van Stockum [2] and Bonnor [3] has done an extensive analysis of the solution. Recently, Santos [4] has extended the solution to Einstein s equations with a cosmological constant. In this paper, we have made an attempt to find solutions of Einstein s equations in a string cosmological model.

The study of early universe has to consider cosmic strings as they generate density fluctuations leading to galaxy formation [5]. Further,strings can explain double quasar problem [6] in Astrophysics. Usually, strings are considered as line singularity, generated in the phase transition (as the universe passes through its critical temperture) [7] at the early stages of the universe. Though there is no direct evidence of strings in the present day universe, yet the presence of strings in the early universe do not contradict present day observations.

The world sheets of the strings are two dimensional time-like surfaces [7]. Here the gravitational effects that arise from strings are considered by the couplimg of the stress energy of the strings to the gravitational field for a cylindrically symmetric space-time . So the energy - momentum tensor for a cloud of massive strings can be written as

$$T_{\mu}^{\nu} = \rho V_{\mu} V^{\nu} - \lambda x_{\mu} x^{\nu}. \tag{1}$$

Here ρ stands for rest energy for cloud of strings with particles attached to them

and λ is string tension density. The connecting relation is [9]

$$\rho = \rho_p + \lambda, \tag{2}$$

with ρ_p as the particle energy density . The four velocity vector V^μ for the cloud of particles and the four vector x^μ ,the direction of string will satisfy

$$V_\mu V^\mu = -1 = -x_\mu x^\mu, \tag{3}$$

and $V_\mu x^\mu = 0$ in + 2 signature for the space-time metric.

2. The Field Equations

We consider string cosmology for the stationary cylindrically symmetric space-time with metric ansatz [4]

$$ds^2 = -adt^2 + 2bdtd\phi + e^\alpha(dr^2 + dz^2) + cd\phi^2, \tag{4}$$

with a= a(r), b = b(r) ,c = c(r) and $\alpha = \alpha(r)$ are functions of the radial co-ordinate r only. The co-ordinates are identified as follows (with the ranges):

$$x^0 = t \in (-\infty, +\infty),$$

$$x^1 = r \in (-\infty, +\infty), \tag{5}$$

$$x^2 = z \in (-\infty, +\infty),$$

$$x^3 = \phi \in [0, 2\pi].$$

In a co-moving co-ordinate system we set

$$V^\mu = (a^{-1/2}, 0, 0, 0).$$

Also, X^μ is taken along the direction of $\frac{\partial}{\partial x^2}$ i.e.

$$x^\mu = (0, 0, e^{-\alpha/2}, 0).$$

Now,for the above metric, the non-vanishing components of the einstein s equations with a cosmological constant

$$R^\nu_\mu - (R/2)\delta^\nu_\mu = -8\pi T^\nu_\mu + \Lambda\delta^\nu_\mu, \tag{6}$$

can be written as

$$\frac{a''}{a} + \frac{1}{2\alpha^2}(c'a' - \frac{ca'}{a} - \frac{2bb'a'}{a} + 2b'^2) = 8\pi(\lambda - \rho a^{-1}e^\alpha) - 2 \wedge e^\alpha, \tag{7}$$

$$-\alpha'' + \alpha'\frac{\mu'}{\mu} - 2\frac{\mu''}{\mu} + \frac{1}{\mu^2}(b' + a'c') = -8\pi(\lambda + \rho a^{-1}e^\alpha) + 2 \wedge e^\alpha, \tag{8}$$

$$-\alpha'' - \alpha'\frac{\mu'}{\mu} = 8\pi(\lambda - \rho a^{-1}e^\alpha) + 2 \wedge e^\alpha, \tag{9}$$

$$-\frac{c''}{c} - \frac{1}{2\mu^2}(a'c' - \frac{ac'^2}{c} - \frac{2bb'c'}{c} + 2b'^2) = -8\pi(\lambda + \rho a^{-1}e^\alpha) + 2 \wedge e^\alpha, \tag{10}$$

$$-\frac{b''}{b}+\frac{1}{2\mu^2}(ac'\frac{b'}{b}+\frac{ca'b'}{b}-2a'c')=-8\pi(\lambda+\rho a^{-1}e^{\alpha})+2\wedge e^{\alpha}. \qquad (11)$$

Here ' / ' stands for differentiation with respect to r and

$$\mu^2=b^2+ac.$$

As there are five field equations containing 6 unknowns, so we look for solutions with one unknown function as a parameter.

3. Solution to the field equations.

We first observe that from the field equations (7), (10) and (11) the expression for $R_0^0+R_3^3$ leads to a simple expression

$$\frac{\mu''}{\mu}=8\pi(\rho a^{-1}e^{\alpha}+2\lambda)-2\wedge e^{\alpha}. \qquad (12)$$

Now, in order to solve the field equations let us introduce [2,4]

$$x=a/c, y=b/c. \qquad (13)$$

The addition of (7) and (10), after a simple algebra results

$$\frac{d}{dr}\left[\frac{\mu.x'}{(x+y^2)}\right]=-(16\pi\rho)\frac{e^{\alpha}}{(x+y^2)^{1/2}}. \qquad (14)$$

Similarly, subtraction of (10) and (11) and addition of (7) and (11) result

$$\frac{d}{dr}\left[\frac{\mu y'}{(x+y^2)}\right]=0, \qquad (15)$$

and

$$\frac{d}{dr}\left[\frac{\mu w'}{1+w/y}\right] = -(16\pi\rho)\frac{e^{\alpha}}{(1+\frac{w}{y})^{1/2}}. \tag{16}$$

It is to be noted that equations (14)- (16) are not independent as one can obtain any one of them from the remaining two. Now a first integral of (15) results

$$y' = A\frac{\mu}{c^2} = A\phi'(r)(say), \tag{17}$$

where we have considered μ/c^2 as the function $\phi'(r)$. Hence we write

$$y = A\phi(r). \tag{18}$$

(A, an integration constant)

Now from (14) the expression for x is

$$x = B[\int \phi(r)dr - r\phi(r)], \tag{19}$$

with $B = 16\pi\rho e^{\alpha}c/\mu$, assumed to be constant. Thus the expression for the metric coefficients are

$$a = x(x+y^2)^{1/2}/\phi'(r),$$

$$b = y(x+\bar{y}^2)^{1/2}/\phi'(r),$$

$$c = (x+y^2)^{1/2}/\phi'(r), \tag{20}$$

and

$$\alpha = \int \{\frac{24\pi}{\mu}[\int \mu\lambda dr]\}dr - (ln\mu).$$

The expression for energy density is

$$\rho = \left(\frac{B}{16\pi}\right)e^{-\alpha}/(x+y^2)^{1/2}. \tag{21}$$

The string tension λ and cosmological constant Λ can be obtained from any one of the field equations (7)- (11) and equation (12) using the above expressions for the metric coefficients .

A simple solution set can be obtained if we take $\phi(r) = 1/r$. For future work ,it will be interesting to consider a detailed physical interpretation of the solution.

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